Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.5-Secant/119-4.5.1.3-d-sin-^n-a+b-sec-^m

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [306]. This is test number [119].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (306)	0.00 (0)
Rubi	98.37 (301)	1.63 (5)
Maple	87.25 (267)	12.75 (39)
Fricas	79.41 (243)	20.59 (63)
Mupad	63.07 (193)	36.93 (113)
Giac	62.42 (191)	37.58 (115)
Maxima	57.19 (175)	42.81 (131)
Sympy	2.61 (8)	97.39 (298)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

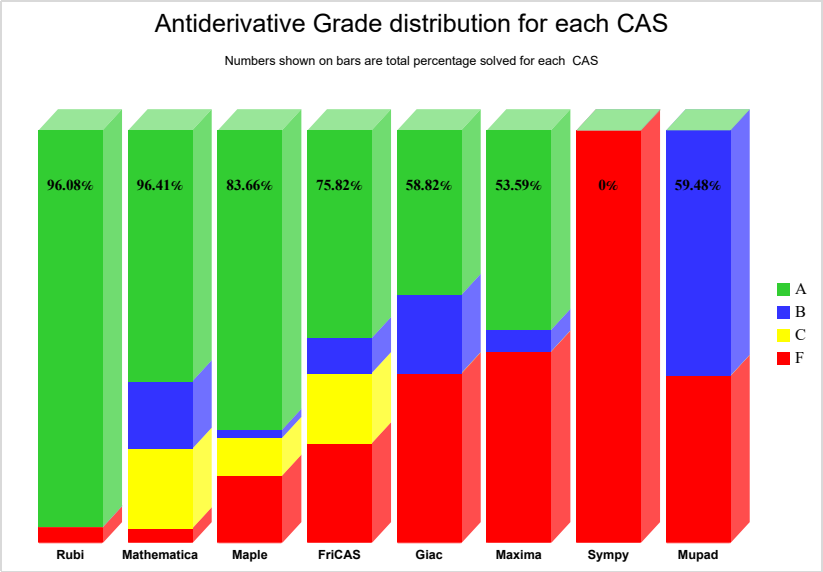
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

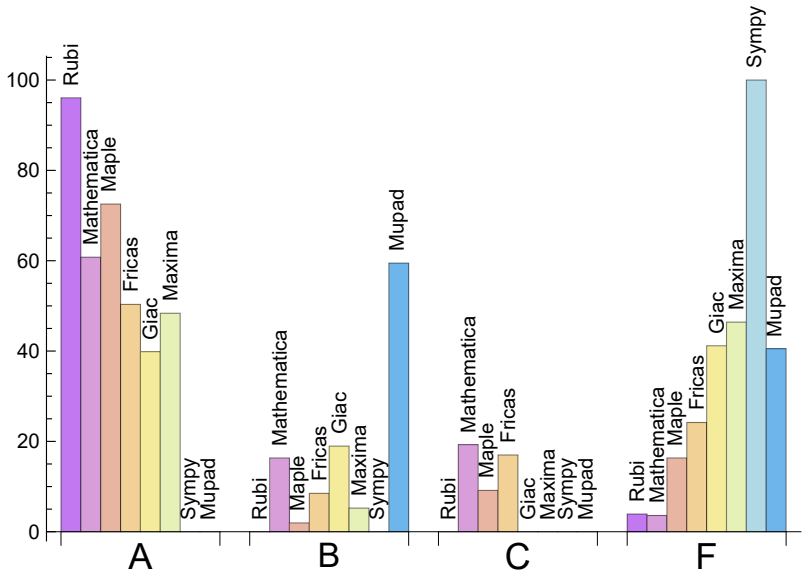
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.771	0.000	0.000	5.229
Maple	72.549	1.961	9.150	16.340
Mathematica	60.784	16.340	19.281	3.595
Fricas	50.327	8.497	16.993	24.183
Maxima	48.366	5.229	0.000	46.405
Giac	39.869	18.954	0.000	41.176
Mupad	0.000	59.477	0.000	40.523
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	5	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Fricas	63	74.60	25.40	0.00
Mupad	113	0.00	100.00	0.00
Giac	115	97.39	0.00	2.61
Maxima	131	70.99	16.79	12.21
Sympy	298	62.75	37.25	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Maxima	0.30
Rubi	0.72
Giac	1.14
Mathematica	4.09
Maple	4.09
Mupad	11.26
Sympy	46.23

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	22.50	0.96	22.00	0.96
Maxima	128.71	1.15	106.00	1.00
Rubi	170.29	0.99	125.00	1.00
Maple	240.84	1.27	128.00	1.01
Giac	249.48	1.76	149.00	1.42
Fricas	250.22	1.67	140.00	1.27
Mupad	250.50	1.49	107.00	1.05
Mathematica	324.61	1.82	136.50	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

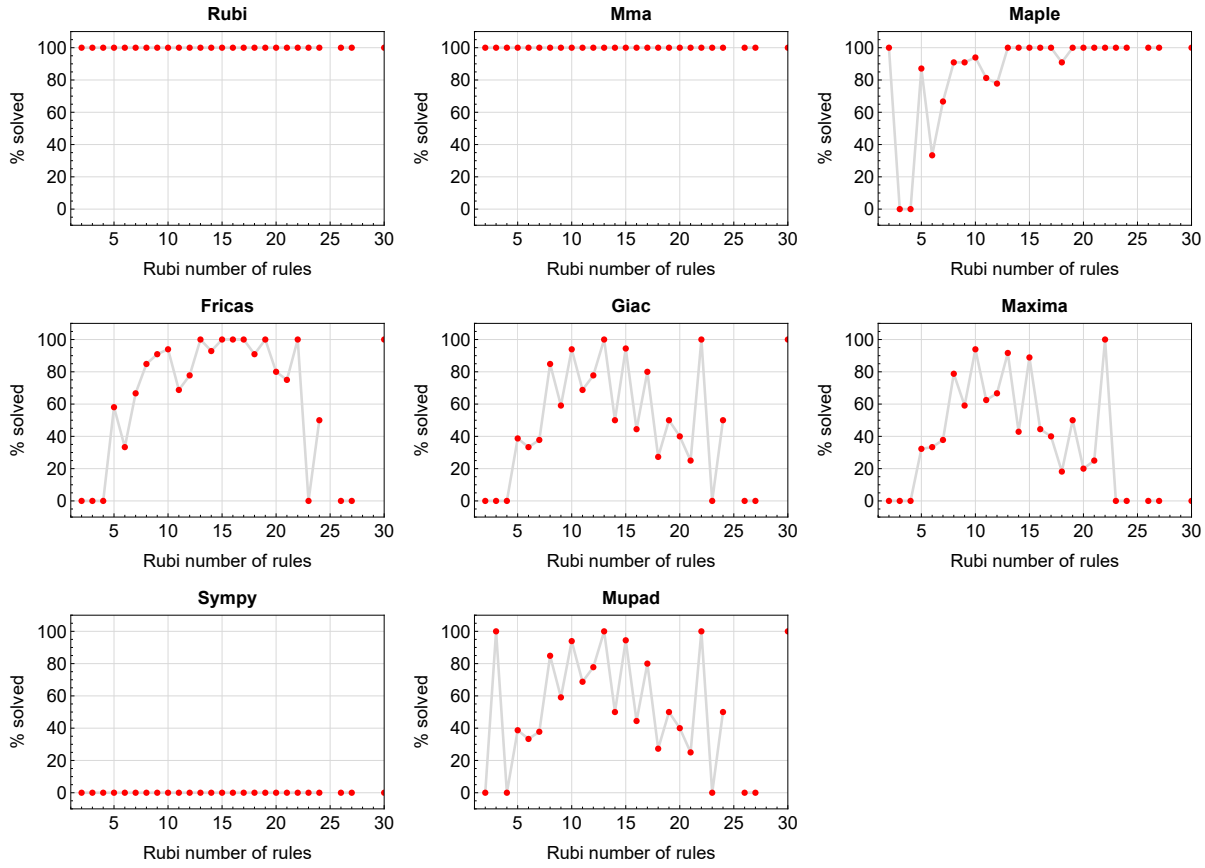


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

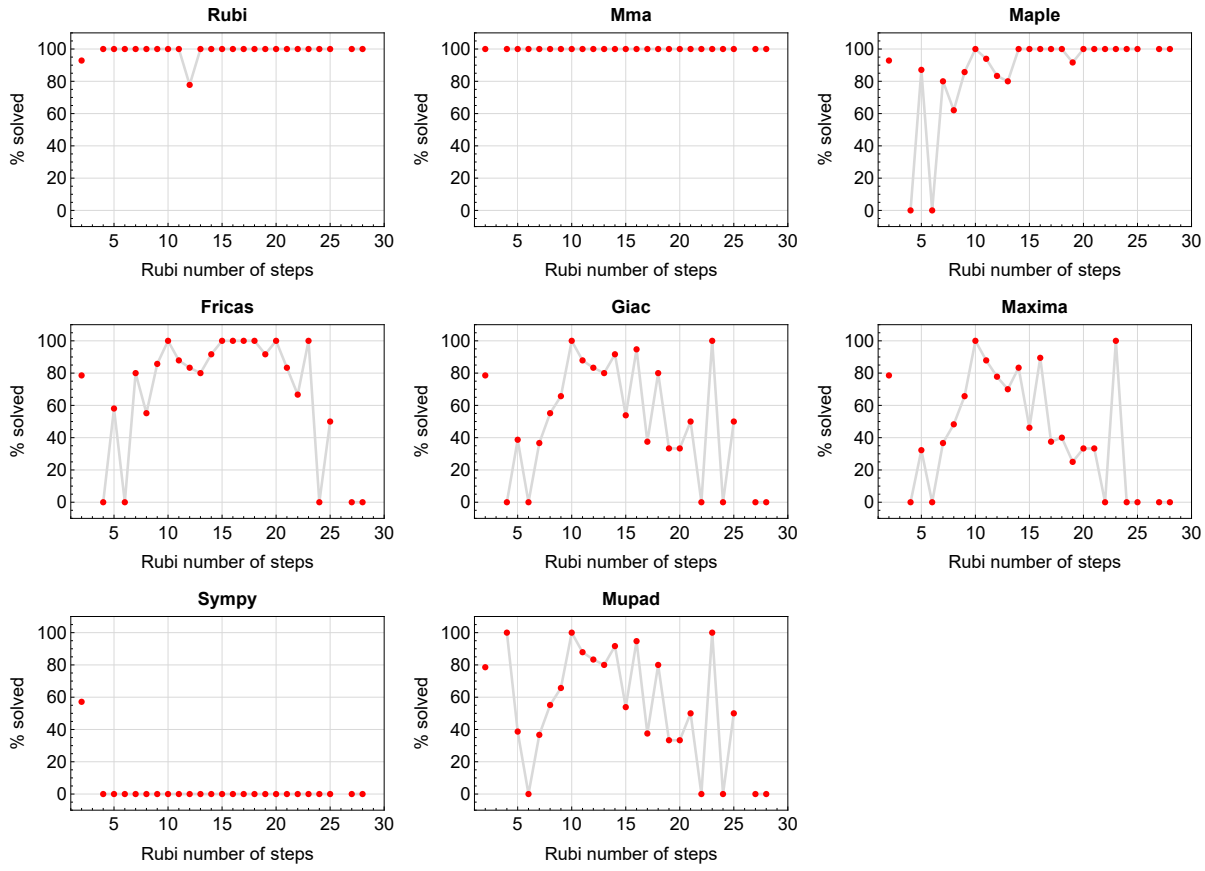


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

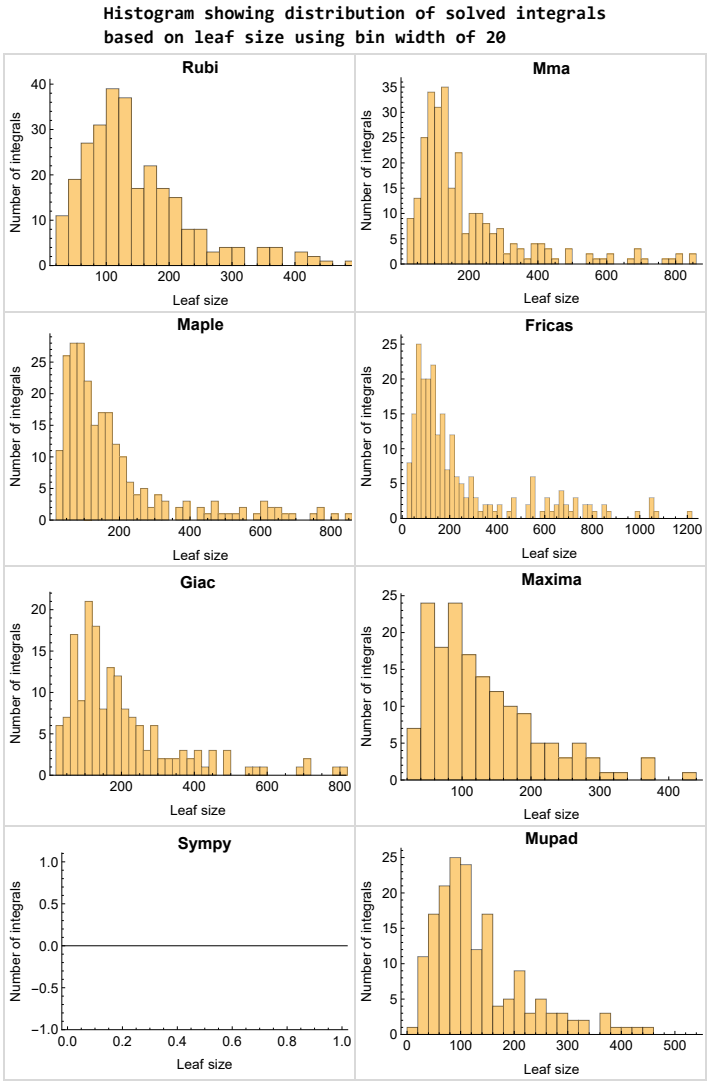


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

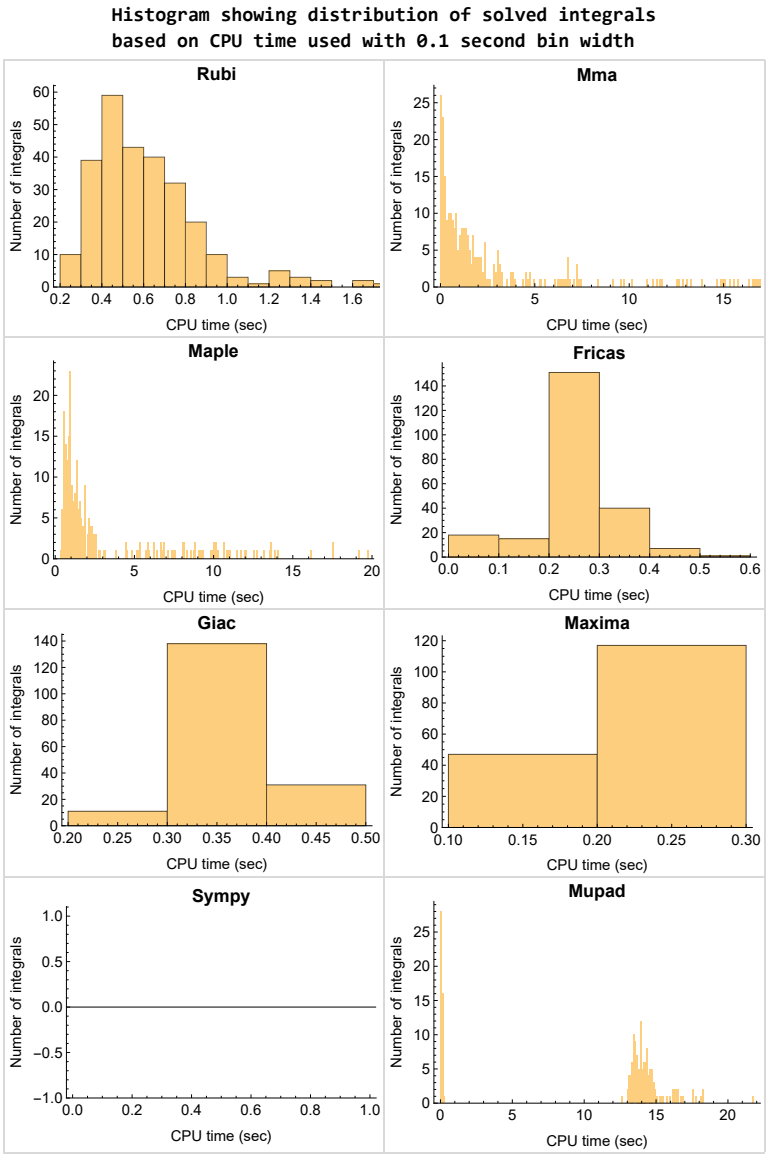


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

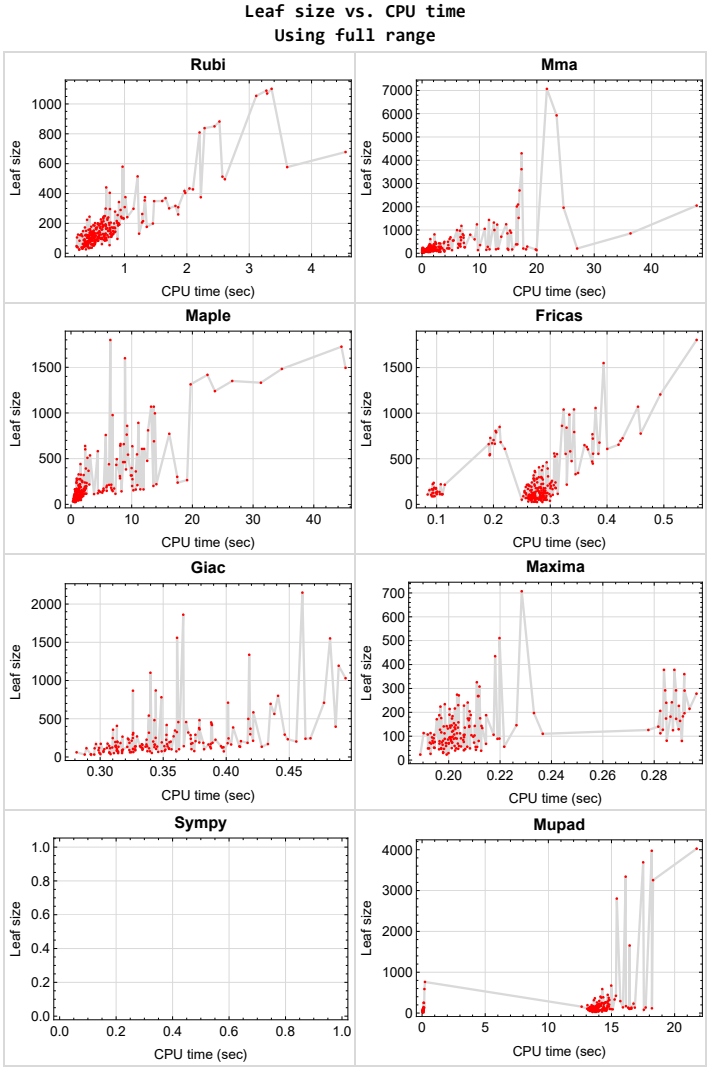


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {108, 109, 110, 111, 112, 113, 166, 167, 233, 234, 235, 236, 237, 238, 239, 240}

Mathematica {111, 114, 115, 116, 117, 118, 119, 139, 140, 141, 142, 143, 144, 152, 153, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 260, 261, 262, 272, 275, 276, 287, 288, 289, 290, 291, 292}

Maple {241, 242, 243, 244, 245, 246, 247, 248, 255, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 300, 301}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

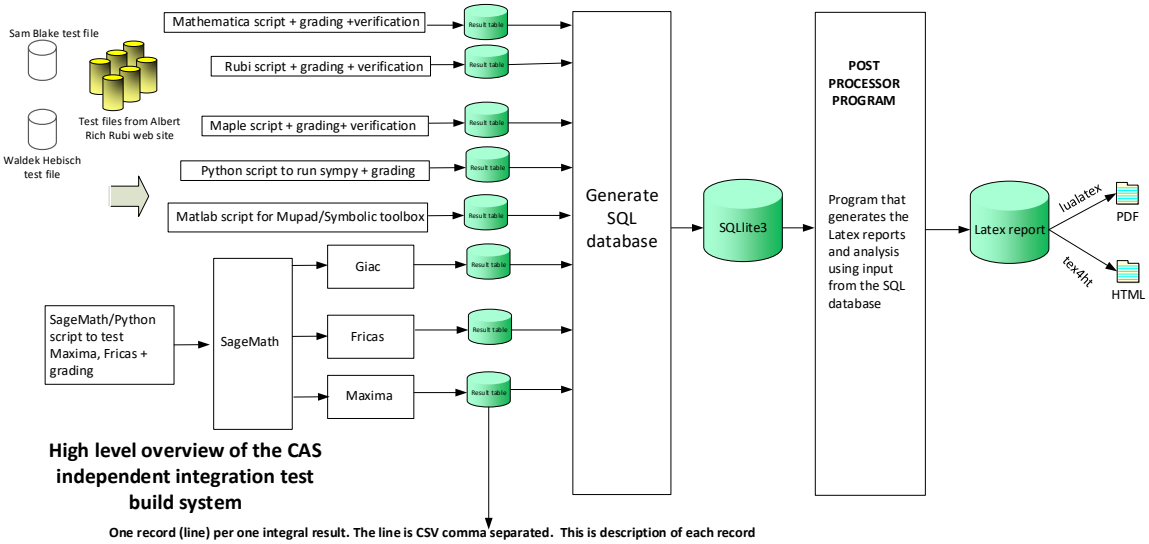
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.2 Mma 22
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2.1.4 Fricas 23
2.1.5 Maxima 23
2.1.6 Giac 24
2.1.7 Mupad 25
2.1.8 Sympy 25

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

B grade { }

C grade { }

F normal fail { 104, 105, 106, 107, 276 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 120, 122, 124, 126, 127, 129, 131, 133, 134, 135, 136, 145, 146, 147, 148, 150, 154, 155, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 179, 180, 181, 185, 186, 187, 188, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 230, 231, 232, 249, 250, 252, 253, 254, 256, 257, 258, 259, 269, 270, 271, 281, 283, 285, 293, 295, 297, 299, 300, 302, 304, 306 }

B grade { 6, 32, 33, 34, 35, 36, 37, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 140, 141, 142, 143, 144, 149, 151, 156, 157, 158, 159, 164, 178, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 229, 251, 255, 260, 261, 262, 268, 272, 275, 276 }

C grade { 14, 15, 16, 17, 18, 111, 114, 115, 116, 117, 118, 119, 121, 123, 125, 128, 130, 132, 137, 138, 139, 152, 153, 171, 172, 173, 226, 227, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 282, 284, 286, 287, 288, 289, 290, 291, 292, 294, 296, 298, 301, 303, 305 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253 }

B grade { 203, 251, 252, 254, 255, 256 }

C grade { 44, 56, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F normal fail { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232 }

B grade { 9, 16, 17, 18, 28, 37, 46, 47, 64, 71, 72, 73, 82, 97, 98, 99, 165, 166, 167, 202, 208, 214, 215, 225, 226, 227 }

C grade { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F normal fail { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

F(-1) timedout fail { 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226 }

B grade { 65, 66, 67, 68, 71, 72, 73, 83, 84, 85, 86, 100, 101, 102, 215, 227 }

C grade { }

F normal fail { 108, 109, 110, 111, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 127, 128, 129, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 242, 243, 244, 245, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 283, 284, 285, 286, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 303, 304, 305, 306 }

F(-1) timedout fail { 112, 118, 119, 125, 126, 131, 132, 133, 238, 239, 240, 241, 246, 247, 248, 282, 287, 288, 294, 300, 301, 302 }

F(-2) exception fail { 203, 204, 205, 206, 207, 208, 216, 217, 218, 219, 220, 228, 229, 230, 231, 232 }

2.1.6 Giac

A grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 162, 163, 168, 175, 176, 177, 180, 186, 187, 191, 193, 194, 195, 198, 199, 200, 201, 206, 211, 212, 216, 217, 218, 219, 220, 223, 224, 229, 231, 232 }

B grade { 1, 2, 3, 19, 20, 21, 24, 38, 40, 43, 44, 76, 77, 81, 93, 160, 161, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 202, 203, 204, 205, 207, 208, 209, 210, 213, 214, 215, 221, 222, 225, 226, 227, 228, 230 }

C grade { }

F normal fail { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F(-1) timedout fail { }

F(-2) exception fail { 143, 147, 269 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 148, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 270 }

C grade { }

F normal fail { }

F(-1) timedout fail { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 41, 42, 43, 44, 50, 51, 52, 53, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 110, 111, 112, 116, 117, 118, 122, 123, 124, 125, 130, 131, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 148, 149, 150, 153, 154, 157, 158, 159, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 186, 187, 188, 189, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 245, 246, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 275, 282, 283, 284, 285, 286, 289, 290, 291, 294, 295, 296, 297, 301, 302, 303, 304 }

F(-1) timedout fail { 1, 2, 9, 10, 17, 18, 19, 20, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 73, 74, 75, 76, 77, 78, 83, 90, 91, 92, 93, 94, 95, 100, 107, 108,

109, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 132, 133, 140, 145, 146, 147, 151, 152, 155, 156, 160, 167, 179, 184, 185, 190, 195, 196, 197, 198, 209, 210, 211, 221, 222, 223, 233, 234, 239, 240, 241, 242, 243, 244, 247, 248, 252, 263, 268, 269, 272, 273, 276, 277, 281, 287, 288, 292, 293, 298, 299, 300, 305, 306 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	150	106	107	113	115	0	293	111
N.S.	1	0.99	0.70	0.70	0.74	0.76	0.00	1.93	0.73
time (sec)	N/A	0.377	0.484	1.585	0.190	0.313	0.000	0.326	0.138

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	120	113	87	91	93	0	247	89
N.S.	1	1.01	0.95	0.73	0.76	0.78	0.00	2.08	0.75
time (sec)	N/A	0.379	0.183	1.462	0.194	0.289	0.000	0.310	0.082

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	90	83	67	69	71	0	201	67
N.S.	1	1.03	0.95	0.77	0.79	0.82	0.00	2.31	0.77
time (sec)	N/A	0.354	0.076	1.358	0.193	0.290	0.000	0.314	0.057

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	62	57	47	47	49	0	66	45
N.S.	1	1.07	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.328	0.050	1.308	0.197	0.292	0.000	0.303	0.053

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	25	37	24	23	25	0	32	20
N.S.	1	0.96	1.42	0.92	0.88	0.96	0.00	1.23	0.77
time (sec)	N/A	0.279	0.015	0.618	0.199	0.286	0.000	0.293	0.042

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	39	65	33	26	31	0	58	17
N.S.	1	1.30	2.17	1.10	0.87	1.03	0.00	1.93	0.57
time (sec)	N/A	0.302	0.043	0.423	0.193	0.261	0.000	0.307	0.122

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	79	120	59	52	93	0	102	53
N.S.	1	1.08	1.64	0.81	0.71	1.27	0.00	1.40	0.73
time (sec)	N/A	0.372	0.045	1.060	0.194	0.273	0.000	0.303	0.074

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	176	83	95	193	0	149	99
N.S.	1	1.00	1.49	0.70	0.81	1.64	0.00	1.26	0.84
time (sec)	N/A	0.385	0.036	0.821	0.194	0.267	0.000	0.313	0.103

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	157	232	103	136	307	0	196	142
N.S.	1	0.96	1.42	0.63	0.83	1.88	0.00	1.20	0.87
time (sec)	N/A	0.418	0.199	1.125	0.204	0.296	0.000	0.312	13.578

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	163	106	116	127	123	0	174	150
N.S.	1	0.99	0.64	0.70	0.77	0.75	0.00	1.05	0.91
time (sec)	N/A	0.697	1.095	1.855	0.197	0.289	0.000	0.306	13.774

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	125	86	96	106	102	0	146	120
N.S.	1	0.98	0.68	0.76	0.83	0.80	0.00	1.15	0.94
time (sec)	N/A	0.582	0.674	1.553	0.194	0.287	0.000	0.307	13.664

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	87	86	76	81	79	0	118	90
N.S.	1	0.98	0.97	0.85	0.91	0.89	0.00	1.33	1.01
time (sec)	N/A	0.487	0.293	1.471	0.198	0.281	0.000	0.305	13.455

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	54	55	59	55	0	88	80
N.S.	1	0.96	1.06	1.08	1.16	1.08	0.00	1.73	1.57
time (sec)	N/A	0.381	0.228	0.923	0.204	0.292	0.000	0.299	13.291

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	41	42	50	63	0	50	29
N.S.	1	0.95	1.11	1.14	1.35	1.70	0.00	1.35	0.78
time (sec)	N/A	0.422	0.156	0.749	0.201	0.278	0.000	0.311	13.688

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	60	69	63	76	108	0	79	65
N.S.	1	0.87	1.00	0.91	1.10	1.57	0.00	1.14	0.94
time (sec)	N/A	0.462	0.100	0.958	0.195	0.270	0.000	0.313	14.297

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	84	91	83	96	190	0	107	97
N.S.	1	0.83	0.90	0.82	0.95	1.88	0.00	1.06	0.96
time (sec)	N/A	0.477	0.081	0.889	0.201	0.283	0.000	0.326	14.525

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	104	113	103	116	281	0	136	128
N.S.	1	0.79	0.86	0.79	0.89	2.15	0.00	1.04	0.98
time (sec)	N/A	0.486	0.248	1.005	0.197	0.271	0.000	0.328	14.439

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	132	135	123	136	366	0	164	159
N.S.	1	0.80	0.82	0.75	0.82	2.22	0.00	0.99	0.96
time (sec)	N/A	0.516	0.302	1.028	0.206	0.273	0.000	0.337	14.790

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	161	127	181	146	167	0	370	146
N.S.	1	0.88	0.69	0.99	0.80	0.91	0.00	2.02	0.80
time (sec)	N/A	0.468	1.209	2.364	0.226	0.304	0.000	0.419	14.383

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	118	107	151	107	128	0	320	105
N.S.	1	0.90	0.82	1.15	0.82	0.98	0.00	2.44	0.80
time (sec)	N/A	0.441	0.389	2.054	0.192	0.283	0.000	0.370	14.311

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	103	87	121	94	115	0	270	91
N.S.	1	0.92	0.78	1.08	0.84	1.03	0.00	2.41	0.81
time (sec)	N/A	0.431	0.207	1.894	0.200	0.273	0.000	0.358	14.264

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	91	56	76	0	74	54
N.S.	1	1.00	1.05	1.47	0.90	1.23	0.00	1.19	0.87
time (sec)	N/A	0.381	0.210	2.018	0.222	0.269	0.000	0.349	0.060

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	36	31	34	41	51	0	51	41
N.S.	1	0.84	0.72	0.79	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.331	0.296	0.826	0.206	0.273	0.000	0.317	0.059

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	36	67	43	61	0	115	35
N.S.	1	1.04	0.75	1.40	0.90	1.27	0.00	2.40	0.73
time (sec)	N/A	0.367	0.131	0.448	0.211	0.262	0.000	0.308	14.252

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	68	75	95	68	112	0	135	61
N.S.	1	0.99	1.09	1.38	0.99	1.62	0.00	1.96	0.88
time (sec)	N/A	0.403	0.450	0.815	0.215	0.298	0.000	0.336	13.300

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	110	103	118	104	209	0	191	109
N.S.	1	0.96	0.90	1.03	0.90	1.82	0.00	1.66	0.95
time (sec)	N/A	0.425	1.214	0.967	0.209	0.261	0.000	0.349	0.109

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	149	136	172	143	289	0	238	147
N.S.	1	0.93	0.85	1.08	0.89	1.81	0.00	1.49	0.92
time (sec)	N/A	0.448	1.130	1.352	0.213	0.285	0.000	0.372	0.118

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	188	164	210	197	461	0	291	203
N.S.	1	0.92	0.80	1.02	0.96	2.25	0.00	1.42	0.99
time (sec)	N/A	0.517	2.620	1.267	0.233	0.294	0.000	0.376	14.143

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	203	144	164	215	185	0	225	293
N.S.	1	1.02	0.72	0.82	1.08	0.93	0.00	1.13	1.47
time (sec)	N/A	0.654	1.065	2.291	0.294	0.280	0.000	0.391	15.701

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	161	124	142	174	158	0	193	235
N.S.	1	1.03	0.79	0.90	1.11	1.01	0.00	1.23	1.50
time (sec)	N/A	0.555	0.469	2.128	0.285	0.288	0.000	0.355	14.710

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	94	116	126	133	0	161	177
N.S.	1	1.03	0.82	1.01	1.10	1.16	0.00	1.40	1.54
time (sec)	N/A	0.533	0.150	2.142	0.278	0.288	0.000	0.343	14.164

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	243	78	81	104	0	128	117
N.S.	1	1.05	3.33	1.07	1.11	1.42	0.00	1.75	1.60
time (sec)	N/A	0.378	1.224	1.273	0.285	0.304	0.000	0.343	13.581

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	401	77	74	101	0	90	70
N.S.	1	1.00	7.04	1.35	1.30	1.77	0.00	1.58	1.23
time (sec)	N/A	0.490	6.655	0.779	0.201	0.269	0.000	0.323	13.502

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	96	228	98	113	159	0	104	91
N.S.	1	1.10	2.62	1.13	1.30	1.83	0.00	1.20	1.05
time (sec)	N/A	0.881	1.971	1.335	0.193	0.267	0.000	0.334	15.097

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	317	154	144	206	0	136	124
N.S.	1	1.00	2.46	1.19	1.12	1.60	0.00	1.05	0.96
time (sec)	N/A	0.503	1.425	1.327	0.201	0.263	0.000	0.345	13.727

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	428	191	175	272	0	168	159
N.S.	1	1.00	2.63	1.17	1.07	1.67	0.00	1.03	0.98
time (sec)	N/A	0.543	2.955	1.326	0.212	0.287	0.000	0.353	13.467

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	1050	217	204	406	0	200	194
N.S.	1	1.00	5.22	1.08	1.01	2.02	0.00	1.00	0.97
time (sec)	N/A	0.566	10.916	1.484	0.208	0.280	0.000	0.356	13.097

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	178	148	217	158	182	0	396	157
N.S.	1	0.88	0.73	1.07	0.78	0.90	0.00	1.95	0.77
time (sec)	N/A	0.472	1.932	3.018	0.204	0.309	0.000	0.487	12.635

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	115	106	118	107	121	0	239	107
N.S.	1	0.88	0.81	0.90	0.82	0.92	0.00	1.82	0.82
time (sec)	N/A	0.427	0.587	2.361	0.202	0.269	0.000	0.463	13.503

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	119	108	172	106	130	0	297	107
N.S.	1	0.89	0.81	1.28	0.79	0.97	0.00	2.22	0.80
time (sec)	N/A	0.424	0.299	2.108	0.206	0.272	0.000	0.419	13.356

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	87	86	114	80	104	0	102	80
N.S.	1	0.89	0.88	1.16	0.82	1.06	0.00	1.04	0.82
time (sec)	N/A	0.357	0.156	2.169	0.212	0.263	0.000	0.382	13.276

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	51	65	46	55	65	0	64	52
N.S.	1	0.82	1.05	0.74	0.89	1.05	0.00	1.03	0.84
time (sec)	N/A	0.355	0.176	0.980	0.210	0.276	0.000	0.326	0.057

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	66	81	90	56	76	0	142	49
N.S.	1	0.99	1.21	1.34	0.84	1.13	0.00	2.12	0.73
time (sec)	N/A	0.388	0.115	0.492	0.207	0.266	0.000	0.332	13.299

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	84	88	132	84	132	0	189	75
N.S.	1	0.95	1.00	1.50	0.95	1.50	0.00	2.15	0.85
time (sec)	N/A	0.458	0.733	0.787	0.206	0.267	0.000	0.379	0.095

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	104	100	153	103	177	0	186	96
N.S.	1	0.94	0.90	1.38	0.93	1.59	0.00	1.68	0.86
time (sec)	N/A	0.437	0.723	0.996	0.210	0.279	0.000	0.389	0.111

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	146	129	164	145	297	0	243	151
N.S.	1	0.93	0.82	1.04	0.92	1.89	0.00	1.55	0.96
time (sec)	N/A	0.470	0.781	1.122	0.201	0.280	0.000	0.402	13.293

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	185	159	210	189	419	0	292	195
N.S.	1	0.92	0.79	1.04	0.94	2.07	0.00	1.45	0.97
time (sec)	N/A	0.510	0.886	1.251	0.209	0.287	0.000	0.446	13.426

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	214	156	197	291	204	0	244	320
N.S.	1	1.02	0.74	0.94	1.39	0.97	0.00	1.16	1.52
time (sec)	N/A	0.658	1.794	2.484	0.292	0.301	0.000	0.467	14.836

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	186	136	199	240	177	0	212	261
N.S.	1	1.02	0.75	1.09	1.32	0.97	0.00	1.16	1.43
time (sec)	N/A	0.587	0.570	2.516	0.285	0.300	0.000	0.421	14.533

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	142	114	153	182	152	0	180	204
N.S.	1	1.03	0.83	1.11	1.32	1.10	0.00	1.30	1.48
time (sec)	N/A	0.498	0.234	2.265	0.286	0.265	0.000	0.401	14.384

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	300	126	127	125	0	102	90
N.S.	1	1.04	3.06	1.29	1.30	1.28	0.00	1.04	0.92
time (sec)	N/A	0.436	2.197	1.548	0.283	0.295	0.000	0.372	13.396

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	77	244	103	137	122	0	106	98
N.S.	1	0.96	3.05	1.29	1.71	1.52	0.00	1.32	1.22
time (sec)	N/A	0.456	1.521	0.805	0.202	0.257	0.000	0.355	14.622

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	290	125	188	178	0	123	116
N.S.	1	1.05	2.64	1.14	1.71	1.62	0.00	1.12	1.05
time (sec)	N/A	0.501	6.706	1.487	0.215	0.288	0.000	0.397	18.203

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	177	353	137	228	225	0	141	136
N.S.	1	1.07	2.14	0.83	1.38	1.36	0.00	0.85	0.82
time (sec)	N/A	1.368	2.013	1.399	0.206	0.287	0.000	0.396	17.708

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	430	192	268	278	0	169	169
N.S.	1	1.00	2.24	1.00	1.40	1.45	0.00	0.88	0.88
time (sec)	N/A	0.968	3.788	1.556	0.211	0.276	0.000	0.433	16.142

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	1000	259	308	375	0	202	204
N.S.	1	1.00	4.31	1.12	1.33	1.62	0.00	0.87	0.88
time (sec)	N/A	0.988	12.602	1.751	0.212	0.285	0.000	0.456	14.120

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	75	62	89	89	89	0	141	110
N.S.	1	0.82	0.68	0.98	0.98	0.98	0.00	1.55	1.21
time (sec)	N/A	0.519	3.217	0.830	0.198	0.268	0.000	0.312	0.090

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	63	52	69	69	69	0	119	84
N.S.	1	0.86	0.71	0.95	0.95	0.95	0.00	1.63	1.15
time (sec)	N/A	0.490	1.116	0.659	0.204	0.257	0.000	0.299	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	51	42	49	49	49	0	97	58
N.S.	1	0.93	0.76	0.89	0.89	0.89	0.00	1.76	1.05
time (sec)	N/A	0.494	0.172	0.533	0.203	0.274	0.000	0.318	0.072

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	29	29	0	32	26
N.S.	1	1.00	0.86	0.78	0.78	0.78	0.00	0.86	0.70
time (sec)	N/A	0.438	0.161	0.506	0.200	0.273	0.000	0.288	13.499

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	32	30	28	0	34	25
N.S.	1	1.00	0.90	1.03	0.97	0.90	0.00	1.10	0.81
time (sec)	N/A	0.320	0.057	0.325	0.198	0.271	0.000	0.293	0.055

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	35	47	60	0	56	33
N.S.	1	1.00	1.16	0.60	0.81	1.03	0.00	0.97	0.57
time (sec)	N/A	0.508	0.078	0.535	0.204	0.283	0.000	0.299	13.502

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	84	92	55	86	138	0	129	75
N.S.	1	1.02	1.12	0.67	1.05	1.68	0.00	1.57	0.91
time (sec)	N/A	0.681	0.377	0.557	0.200	0.281	0.000	0.313	0.100

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	110	122	79	130	217	0	182	115
N.S.	1	1.04	1.15	0.75	1.23	2.05	0.00	1.72	1.08
time (sec)	N/A	0.793	0.358	0.578	0.203	0.261	0.000	0.316	13.383

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	131	132	99	360	91	0	139	132
N.S.	1	1.05	1.06	0.79	2.88	0.73	0.00	1.11	1.06
time (sec)	N/A	0.761	1.613	0.666	0.292	0.261	0.000	0.337	16.326

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	103	112	77	278	70	0	113	106
N.S.	1	1.04	1.13	0.78	2.81	0.71	0.00	1.14	1.07
time (sec)	N/A	0.641	0.511	0.591	0.296	0.285	0.000	0.296	16.459

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	75	83	44	196	51	0	87	55
N.S.	1	1.03	1.14	0.60	2.68	0.70	0.00	1.19	0.75
time (sec)	N/A	0.528	0.417	0.605	0.292	0.250	0.000	0.313	13.995

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	33	112	27	0	58	30
N.S.	1	1.00	1.55	0.75	2.55	0.61	0.00	1.32	0.68
time (sec)	N/A	0.407	0.190	0.608	0.282	0.267	0.000	0.307	13.794

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	66	35	49	41	0	37	32
N.S.	1	1.00	1.78	0.95	1.32	1.11	0.00	1.00	0.86
time (sec)	N/A	0.461	0.823	0.511	0.208	0.284	0.000	0.308	13.813

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	52	116	60	96	89	0	74	60
N.S.	1	0.95	2.11	1.09	1.75	1.62	0.00	1.35	1.09
time (sec)	N/A	0.500	0.684	0.530	0.200	0.253	0.000	0.317	13.991

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	64	158	86	136	131	0	103	153
N.S.	1	0.88	2.16	1.18	1.86	1.79	0.00	1.41	2.10
time (sec)	N/A	0.515	0.828	0.556	0.201	0.255	0.000	0.313	14.038

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	76	200	112	176	177	0	132	201
N.S.	1	0.84	2.20	1.23	1.93	1.95	0.00	1.45	2.21
time (sec)	N/A	0.526	2.066	0.607	0.197	0.310	0.000	0.322	13.927

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	88	242	138	216	219	0	161	139
N.S.	1	0.81	2.22	1.27	1.98	2.01	0.00	1.48	1.28
time (sec)	N/A	0.544	3.185	0.767	0.202	0.272	0.000	0.344	15.907

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	122	72	89	89	89	0	185	109
N.S.	1	0.89	0.53	0.65	0.65	0.65	0.00	1.35	0.80
time (sec)	N/A	0.461	3.896	1.160	0.202	0.301	0.000	0.382	0.090

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	102	62	79	79	79	0	141	96
N.S.	1	0.89	0.54	0.69	0.69	0.69	0.00	1.24	0.84
time (sec)	N/A	0.451	2.930	0.959	0.196	0.272	0.000	0.366	0.072

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	69	53	49	49	49	0	141	58
N.S.	1	0.95	0.73	0.67	0.67	0.67	0.00	1.93	0.79
time (sec)	N/A	0.413	1.373	0.850	0.192	0.288	0.000	0.351	0.074

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	42	39	39	39	0	119	36
N.S.	1	0.98	0.76	0.71	0.71	0.71	0.00	2.16	0.65
time (sec)	N/A	0.404	0.411	0.831	0.213	0.299	0.000	0.333	13.970

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	63	51	48	51	48	0	75	56
N.S.	1	0.95	0.77	0.73	0.77	0.73	0.00	1.14	0.85
time (sec)	N/A	0.414	0.099	0.962	0.195	0.265	0.000	0.314	0.066

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	64	44	46	58	0	52	46
N.S.	1	0.92	1.23	0.85	0.88	1.12	0.00	1.00	0.88
time (sec)	N/A	0.351	0.196	0.434	0.206	0.280	0.000	0.341	0.086

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	55	83	46	74	106	0	87	60
N.S.	1	0.92	1.38	0.77	1.23	1.77	0.00	1.45	1.00
time (sec)	N/A	0.377	0.215	0.515	0.197	0.263	0.000	0.328	0.097

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	44	38	46	59	60	0	82	58
N.S.	1	1.05	0.90	1.10	1.40	1.43	0.00	1.95	1.38
time (sec)	N/A	0.360	0.206	0.527	0.202	0.264	0.000	0.351	13.814

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	138	152	100	167	283	0	207	152
N.S.	1	0.95	1.04	0.68	1.14	1.94	0.00	1.42	1.04
time (sec)	N/A	0.484	0.621	0.611	0.208	0.277	0.000	0.376	13.638

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	171	131	99	378	90	0	139	133
N.S.	1	1.02	0.78	0.59	2.26	0.54	0.00	0.83	0.80
time (sec)	N/A	0.750	3.045	1.134	0.288	0.273	0.000	0.354	16.849

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	111	111	77	292	71	0	113	107
N.S.	1	1.07	1.07	0.74	2.81	0.68	0.00	1.09	1.03
time (sec)	N/A	0.791	0.619	0.933	0.288	0.259	0.000	0.339	16.577

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	91	91	55	206	50	0	87	81
N.S.	1	1.05	1.05	0.63	2.37	0.57	0.00	1.00	0.93
time (sec)	N/A	0.518	0.583	0.805	0.282	0.289	0.000	0.317	17.564

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	61	121	45	140	61	0	75	91
N.S.	1	0.88	1.75	0.65	2.03	0.88	0.00	1.09	1.32
time (sec)	N/A	0.628	0.456	0.800	0.282	0.266	0.000	0.337	13.160

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	105	58	90	71	0	74	71
N.S.	1	1.05	1.44	0.79	1.23	0.97	0.00	1.01	0.97
time (sec)	N/A	0.458	0.733	0.538	0.203	0.263	0.000	0.340	13.180

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	95	149	84	134	108	0	105	121
N.S.	1	1.04	1.64	0.92	1.47	1.19	0.00	1.15	1.33
time (sec)	N/A	0.637	1.110	0.638	0.213	0.258	0.000	0.344	13.187

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	191	110	174	169	0	134	106
N.S.	1	1.04	1.75	1.01	1.60	1.55	0.00	1.23	0.97
time (sec)	N/A	0.656	1.316	0.720	0.208	0.261	0.000	0.411	14.385

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	129	233	112	174	204	0	134	201
N.S.	1	1.03	1.86	0.90	1.39	1.63	0.00	1.07	1.61
time (sec)	N/A	0.684	2.321	0.623	0.207	0.273	0.000	0.387	14.015

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	124	120	89	89	89	0	207	110
N.S.	1	0.89	0.86	0.64	0.64	0.64	0.00	1.49	0.79
time (sec)	N/A	0.460	3.902	1.035	0.219	0.284	0.000	0.412	13.475

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	99	100	69	69	69	0	185	84
N.S.	1	0.91	0.92	0.63	0.63	0.63	0.00	1.70	0.77
time (sec)	N/A	0.428	1.856	0.991	0.200	0.265	0.000	0.417	13.185

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	69	80	49	49	49	0	163	58
N.S.	1	0.95	1.10	0.67	0.67	0.67	0.00	2.23	0.79
time (sec)	N/A	0.424	0.847	0.969	0.192	0.268	0.000	0.386	13.419

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	94	73	68	73	70	0	172	82
N.S.	1	0.92	0.72	0.67	0.72	0.69	0.00	1.69	0.80
time (sec)	N/A	0.437	0.361	1.047	0.210	0.272	0.000	0.372	13.356

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	83	99	60	72	82	0	94	75
N.S.	1	0.93	1.11	0.67	0.81	0.92	0.00	1.06	0.84
time (sec)	N/A	0.432	0.209	0.965	0.194	0.269	0.000	0.334	0.073

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	67	103	59	71	96	0	63	59
N.S.	1	0.89	1.37	0.79	0.95	1.28	0.00	0.84	0.79
time (sec)	N/A	0.368	0.150	0.446	0.200	0.268	0.000	0.325	0.089

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	75	97	61	98	151	0	113	83
N.S.	1	0.91	1.18	0.74	1.20	1.84	0.00	1.38	1.01
time (sec)	N/A	0.423	0.243	0.607	0.210	0.278	0.000	0.355	0.128

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	111	138	87	146	240	0	182	130
N.S.	1	0.88	1.10	0.69	1.16	1.90	0.00	1.44	1.03
time (sec)	N/A	0.412	0.743	0.680	0.203	0.268	0.000	0.373	0.170

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	116	137	91	188	317	0	232	173
N.S.	1	0.91	1.07	0.71	1.47	2.48	0.00	1.81	1.35
time (sec)	N/A	0.468	3.777	0.989	0.196	0.271	0.000	0.449	13.757

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	161	131	99	378	91	0	139	132
N.S.	1	1.03	0.83	0.63	2.41	0.58	0.00	0.89	0.84
time (sec)	N/A	0.755	5.223	0.970	0.284	0.265	0.000	0.410	16.578

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	133	111	77	292	70	0	113	106
N.S.	1	1.03	0.86	0.60	2.26	0.54	0.00	0.88	0.82
time (sec)	N/A	0.574	1.728	0.955	0.284	0.261	0.000	0.388	16.396

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	102	173	65	227	83	0	101	98
N.S.	1	0.94	1.60	0.60	2.10	0.77	0.00	0.94	0.91
time (sec)	N/A	0.602	0.614	0.909	0.290	0.283	0.000	0.363	14.959

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	101	177	66	164	99	0	96	115
N.S.	1	1.04	1.82	0.68	1.69	1.02	0.00	0.99	1.19
time (sec)	N/A	0.593	0.491	1.064	0.290	0.271	0.000	0.352	13.317

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	0	137	58	90	95	0	73	84
N.S.	1	0.00	1.54	0.65	1.01	1.07	0.00	0.82	0.94
time (sec)	N/A	0.000	1.003	0.575	0.220	0.268	0.000	0.360	13.481

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	0	175	60	92	146	0	73	105
N.S.	1	0.00	1.70	0.58	0.89	1.42	0.00	0.71	1.02
time (sec)	N/A	0.000	1.354	0.806	0.203	0.255	0.000	0.398	13.077

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	0	223	110	174	191	0	134	201
N.S.	1	0.00	1.76	0.87	1.37	1.50	0.00	1.06	1.58
time (sec)	N/A	0.000	1.405	0.899	0.201	0.257	0.000	0.428	13.935

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	265	136	214	214	0	163	249
N.S.	1	0.00	1.83	0.94	1.48	1.48	0.00	1.12	1.72
time (sec)	N/A	0.000	3.100	1.111	0.200	0.259	0.000	0.404	14.481

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	151	106	210	0	649	0	0	0
N.S.	1	0.96	0.68	1.34	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.746	0.518	10.939	0.000	0.360	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	149	170	154	0	603	0	0	0
N.S.	1	0.97	1.10	1.00	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.739	0.622	10.279	0.000	0.366	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	97	69	142	0	555	0	0	0
N.S.	1	0.93	0.66	1.37	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.582	0.279	8.324	0.000	0.375	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	103	97	226	115	0	550	0	0	0
N.S.	1	0.94	2.19	1.12	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.607	0.951	7.324	0.000	0.194	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	150	143	201	0	670	0	0	0
N.S.	1	0.97	0.92	1.30	0.00	4.32	0.00	0.00	0.00
time (sec)	N/A	0.786	0.567	10.071	0.000	0.198	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	156	120	164	0	705	0	0	0
N.S.	1	0.98	0.75	1.02	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.772	0.569	11.421	0.000	0.200	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	205	265	0	769	0	0	0
N.S.	1	1.00	1.06	1.37	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.703	17.639	19.127	0.000	0.374	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	204	201	0	721	0	0	0
N.S.	1	1.00	1.06	1.05	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.659	27.023	13.577	0.000	0.375	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	168	220	0	677	0	0	0
N.S.	1	1.00	1.22	1.59	0.00	4.91	0.00	0.00	0.00
time (sec)	N/A	0.559	11.755	14.067	0.000	0.387	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	164	163	0	660	0	0	0
N.S.	1	1.00	1.18	1.17	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.587	19.808	11.951	0.000	0.192	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	135	238	0	790	0	0	0
N.S.	1	1.00	0.60	1.06	0.00	3.53	0.00	0.00	0.00
time (sec)	N/A	0.719	19.948	17.572	0.000	0.205	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	169	301	0	849	0	0	0
N.S.	1	1.00	0.72	1.29	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.701	18.127	17.500	0.000	0.211	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	142	122	128	0	126	0	0	0
N.S.	1	1.02	0.88	0.92	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.848	1.735	5.149	0.000	0.094	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	106	193	173	0	111	0	0	0
N.S.	1	1.02	1.86	1.66	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.674	3.189	5.362	0.000	0.100	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	104	69	112	0	88	0	0	0
N.S.	1	1.02	0.68	1.10	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.682	1.261	3.802	0.000	0.092	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	103	210	149	0	123	0	0	0
N.S.	1	1.08	2.21	1.57	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.693	4.750	4.590	0.000	0.093	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	109	77	121	0	108	0	0	0
N.S.	1	1.08	0.76	1.20	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.744	1.132	4.429	0.000	0.084	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	141	124	187	0	163	0	0	0
N.S.	1	1.04	0.92	1.39	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.851	1.518	5.841	0.000	0.093	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	145	91	136	0	215	0	0	0
N.S.	1	1.07	0.67	1.01	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.860	1.738	4.814	0.000	0.114	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	168	94	145	0	125	0	0	0
N.S.	1	1.04	0.58	0.90	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.849	2.539	6.867	0.000	0.094	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	198	446	173	0	171	0	0	0
N.S.	1	1.06	2.39	0.93	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.890	7.280	7.114	0.000	0.099	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	200	119	153	0	140	0	0	0
N.S.	1	1.06	0.63	0.81	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.864	2.836	5.378	0.000	0.101	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	202	234	205	0	179	0	0	0
N.S.	1	1.07	1.24	1.09	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.876	6.265	5.924	0.000	0.099	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	204	82	148	0	165	0	0	0
N.S.	1	1.07	0.43	0.78	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.866	1.967	5.886	0.000	0.091	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	235	163	213	0	222	0	0	0
N.S.	1	1.05	0.73	0.95	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.983	1.991	6.276	0.000	0.093	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	235	113	160	0	236	0	0	0
N.S.	1	1.05	0.50	0.71	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.948	1.648	6.726	0.000	0.095	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	160	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	0.687	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	127	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	97	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	383	0	0	0	0	0	0
N.S.	1	1.00	3.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	16.385	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	216	612	0	0	0	0	0	0
N.S.	1	1.04	2.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.875	4.799	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	236	245	995	0	0	0	0	0	0
N.S.	1	1.04	4.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.924	6.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	168	1243	0	0	0	0	0	0
N.S.	1	1.58	11.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.701	9.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	107	169	433	0	0	0	0	0	0
N.S.	1	1.58	4.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	2.373	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	178	277	0	0	0	0	0	0
N.S.	1	1.55	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.612	2.849	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	120	180	484	0	0	0	0	0	0
N.S.	1	1.50	4.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	2.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	130	197	276	0	0	0	0	0	0
N.S.	1	1.52	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	2.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	173	116	0	0	0	0	0	0
N.S.	1	0.96	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	127	84	0	0	0	0	0	0
N.S.	1	1.03	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	84	67	0	0	0	0	0	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.229	0.039	0.000	0.000	0.000	0.000	0.000	14.428

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	92	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.506	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	123	179	0	0	0	0	0	0
N.S.	1	1.10	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	1.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	245	492	0	0	0	0	0	0
N.S.	1	1.02	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	3.891	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	230	317	7069	0	0	0	0	0	0
N.S.	1	1.38	30.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.809	21.752	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	95	121	4297	0	0	0	0	0	0
N.S.	1	1.27	45.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	17.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	142	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	1.509	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	441	350	0	0	0	0	0	0
N.S.	1	1.26	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.692	4.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	382	0	0	0	0	0	0
N.S.	1	1.00	3.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	16.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	214	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	3.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.507	3.532	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	133	212	0	0	0	0	0	0
N.S.	1	1.27	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	4.385	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	124	113	87	91	93	0	317	89
N.S.	1	1.04	0.95	0.73	0.76	0.78	0.00	2.66	0.75
time (sec)	N/A	0.376	0.146	1.855	0.205	0.293	0.000	0.339	14.350

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	93	83	67	69	71	0	248	67
N.S.	1	1.07	0.95	0.77	0.79	0.82	0.00	2.85	0.77
time (sec)	N/A	0.352	0.067	1.576	0.196	0.276	0.000	0.326	13.937

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	64	57	47	47	49	0	66	45
N.S.	1	1.10	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.333	0.037	1.639	0.198	0.273	0.000	0.314	0.054

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	25	37	26	23	25	0	32	23
N.S.	1	0.96	1.42	1.00	0.88	0.96	0.00	1.23	0.88
time (sec)	N/A	0.267	0.025	0.782	0.189	0.277	0.000	0.295	0.039

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	33	45	51	0	61	63
N.S.	1	1.00	2.50	1.27	1.73	1.96	0.00	2.35	2.42
time (sec)	N/A	0.377	0.019	0.567	0.196	0.280	0.000	0.281	0.112

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	120	61	71	123	0	169	76
N.S.	1	0.97	1.88	0.95	1.11	1.92	0.00	2.64	1.19
time (sec)	N/A	0.514	0.015	1.155	0.199	0.290	0.000	0.297	0.101

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	176	83	110	201	0	266	117
N.S.	1	1.01	1.76	0.83	1.10	2.01	0.00	2.66	1.17
time (sec)	N/A	0.635	0.020	1.296	0.237	0.264	0.000	0.318	13.909

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	232	103	143	284	0	357	148
N.S.	1	0.99	1.66	0.74	1.02	2.03	0.00	2.55	1.06
time (sec)	N/A	0.772	0.053	1.245	0.210	0.271	0.000	0.310	13.473

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	125	118	96	106	102	0	228	332
N.S.	1	0.98	0.93	0.76	0.83	0.80	0.00	1.80	2.61
time (sec)	N/A	0.612	0.465	1.887	0.217	0.283	0.000	0.323	15.246

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	87	86	76	81	79	0	172	267
N.S.	1	0.98	0.97	0.85	0.91	0.89	0.00	1.93	3.00
time (sec)	N/A	0.498	0.174	1.726	0.201	0.283	0.000	0.300	14.541

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	54	55	59	55	0	114	83
N.S.	1	0.96	1.06	1.08	1.16	1.08	0.00	2.24	1.63
time (sec)	N/A	0.391	0.122	1.217	0.199	0.284	0.000	0.289	13.933

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	41	42	50	63	0	77	60
N.S.	1	0.95	1.11	1.14	1.35	1.70	0.00	2.08	1.62
time (sec)	N/A	0.426	0.079	1.191	0.197	0.281	0.000	0.303	14.566

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	60	69	63	76	125	0	133	101
N.S.	1	0.87	1.00	0.91	1.10	1.81	0.00	1.93	1.46
time (sec)	N/A	0.475	0.038	0.967	0.206	0.285	0.000	0.315	14.849

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	84	91	83	96	174	0	194	142
N.S.	1	0.83	0.90	0.82	0.95	1.72	0.00	1.92	1.41
time (sec)	N/A	0.485	0.045	1.056	0.206	0.277	0.000	0.314	14.363

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	128	112	120	105	125	0	418	104
N.S.	1	1.03	0.90	0.97	0.85	1.01	0.00	3.37	0.84
time (sec)	N/A	0.462	2.221	1.802	0.206	0.292	0.000	0.389	13.948

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	82	72	90	71	92	0	100	69
N.S.	1	1.02	0.90	1.12	0.89	1.15	0.00	1.25	0.86
time (sec)	N/A	0.409	0.678	1.799	0.199	0.285	0.000	0.357	14.113

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	41	37	40	40	50	0	50	40
N.S.	1	0.98	0.88	0.95	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.332	0.107	0.538	0.203	0.294	0.000	0.314	0.055

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	89	91	66	73	97	0	124	62
N.S.	1	1.20	1.23	0.89	0.99	1.31	0.00	1.68	0.84
time (sec)	N/A	0.435	0.833	0.511	0.193	0.265	0.000	0.310	14.601

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	148	329	116	119	205	0	314	120
N.S.	1	1.30	2.89	1.02	1.04	1.80	0.00	2.75	1.05
time (sec)	N/A	0.578	1.385	0.942	0.201	0.295	0.000	0.326	0.126

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	188	193	163	173	176	0	379	231
N.S.	1	1.07	1.10	0.93	0.99	1.01	0.00	2.17	1.32
time (sec)	N/A	0.901	2.140	2.136	0.289	0.291	0.000	0.378	16.744

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	199	157	133	125	142	0	285	207
N.S.	1	1.12	0.88	0.75	0.70	0.80	0.00	1.60	1.16
time (sec)	N/A	1.429	1.457	1.801	0.287	0.289	0.000	0.372	14.822

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	121	77	80	108	0	159	143
N.S.	1	1.00	1.57	1.00	1.04	1.40	0.00	2.06	1.86
time (sec)	N/A	0.384	0.975	1.385	0.291	0.282	0.000	0.334	14.663

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	138	76	73	104	0	167	108
N.S.	1	0.92	2.34	1.29	1.24	1.76	0.00	2.83	1.83
time (sec)	N/A	0.761	1.143	0.995	0.201	0.267	0.000	0.333	14.577

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	87	259	116	112	178	0	226	182
N.S.	1	0.87	2.59	1.16	1.12	1.78	0.00	2.26	1.82
time (sec)	N/A	0.694	1.283	1.667	0.203	0.274	0.000	0.337	14.683

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	122	368	154	143	241	0	326	248
N.S.	1	0.85	2.57	1.08	1.00	1.69	0.00	2.28	1.73
time (sec)	N/A	0.781	1.439	1.763	0.208	0.277	0.000	0.361	14.213

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	169	154	174	142	175	0	695	143
N.S.	1	0.99	0.91	1.02	0.84	1.03	0.00	4.09	0.84
time (sec)	N/A	0.543	1.390	2.446	0.199	0.284	0.000	0.435	13.583

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	103	102	116	98	123	0	128	99
N.S.	1	0.89	0.88	1.00	0.84	1.06	0.00	1.10	0.85
time (sec)	N/A	0.404	0.860	2.534	0.202	0.292	0.000	0.398	0.075

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	61	56	57	57	67	0	66	57
N.S.	1	0.95	0.88	0.89	0.89	1.05	0.00	1.03	0.89
time (sec)	N/A	0.350	0.154	0.823	0.205	0.269	0.000	0.346	13.584

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	118	89	92	112	139	0	250	85
N.S.	1	1.16	0.87	0.90	1.10	1.36	0.00	2.45	0.83
time (sec)	N/A	0.527	0.957	0.651	0.198	0.274	0.000	0.344	0.131

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	188	669	162	171	290	0	482	159
N.S.	1	1.16	4.13	1.00	1.06	1.79	0.00	2.98	0.98
time (sec)	N/A	0.719	7.220	1.311	0.195	0.285	0.000	0.379	13.956

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	818	234	242	241	0	563	373
N.S.	1	1.00	2.74	0.78	0.81	0.81	0.00	1.88	1.25
time (sec)	N/A	0.698	7.091	2.570	0.287	0.302	0.000	0.438	14.780

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	260	696	194	183	196	0	431	281
N.S.	1	1.10	2.95	0.82	0.78	0.83	0.00	1.83	1.19
time (sec)	N/A	1.836	6.749	2.285	0.291	0.301	0.000	0.388	14.697

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	131	327	128	129	151	0	346	202
N.S.	1	0.95	2.37	0.93	0.93	1.09	0.00	2.51	1.46
time (sec)	N/A	1.226	1.203	1.856	0.290	0.284	0.000	0.359	13.879

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	121	406	129	139	151	0	225	181
N.S.	1	0.91	3.05	0.97	1.05	1.14	0.00	1.69	1.36
time (sec)	N/A	0.717	2.442	1.049	0.208	0.279	0.000	0.352	14.088

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	610	187	190	260	0	361	260
N.S.	1	1.00	2.98	0.91	0.93	1.27	0.00	1.76	1.27
time (sec)	N/A	0.762	1.438	1.600	0.200	0.272	0.000	0.379	13.855

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	812	243	230	354	0	498	363
N.S.	1	1.00	2.91	0.87	0.82	1.27	0.00	1.78	1.30
time (sec)	N/A	0.872	2.156	1.872	0.204	0.273	0.000	0.418	13.684

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	202	282	275	224	222	0	1559	249
N.S.	1	0.91	1.26	1.23	1.00	1.00	0.00	6.99	1.12
time (sec)	N/A	0.514	1.809	1.355	0.197	0.284	0.000	0.361	0.152

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	137	172	160	141	140	0	867	151
N.S.	1	0.90	1.13	1.05	0.93	0.92	0.00	5.70	0.99
time (sec)	N/A	0.452	0.311	0.834	0.198	0.280	0.000	0.326	13.419

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	80	89	82	80	78	0	102	79
N.S.	1	0.90	1.00	0.92	0.90	0.88	0.00	1.15	0.89
time (sec)	N/A	0.399	0.098	0.775	0.194	0.288	0.000	0.326	13.510

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	31	30	48	33	31	0	38	30
N.S.	1	0.91	0.88	1.41	0.97	0.91	0.00	1.12	0.88
time (sec)	N/A	0.322	0.018	0.471	0.198	0.274	0.000	0.300	0.057

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	63	57	64	64	0	100	68
N.S.	1	1.03	0.85	0.77	0.86	0.86	0.00	1.35	0.92
time (sec)	N/A	0.375	0.225	0.722	0.198	0.282	0.000	0.328	13.674

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	152	123	110	132	216	0	202	133
N.S.	1	1.31	1.06	0.95	1.14	1.86	0.00	1.74	1.15
time (sec)	N/A	0.502	0.889	0.720	0.200	0.329	0.000	0.342	13.695

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	227	207	172	268	469	0	419	297
N.S.	1	1.27	1.16	0.96	1.50	2.62	0.00	2.34	1.66
time (sec)	N/A	0.605	4.624	0.943	0.212	0.374	0.000	0.353	14.030

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	263	268	441	0	553	0	781	3341
N.S.	1	1.14	1.17	1.92	0.00	2.40	0.00	3.40	14.53
time (sec)	N/A	1.295	2.318	1.563	0.000	0.327	0.000	0.349	16.123

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	186	172	264	0	393	0	407	317
N.S.	1	1.16	1.07	1.64	0.00	2.44	0.00	2.53	1.97
time (sec)	N/A	0.898	0.752	0.970	0.000	0.295	0.000	0.313	14.780

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	115	96	142	0	258	0	185	147
N.S.	1	1.15	0.96	1.42	0.00	2.58	0.00	1.85	1.47
time (sec)	N/A	0.584	0.195	0.704	0.000	0.303	0.000	0.323	13.945

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	95	118	96	0	300	0	129	109
N.S.	1	1.13	1.40	1.14	0.00	3.57	0.00	1.54	1.30
time (sec)	N/A	0.460	0.477	0.545	0.000	0.284	0.000	0.332	13.813

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	165	162	165	0	558	0	269	219
N.S.	1	1.18	1.16	1.18	0.00	3.99	0.00	1.92	1.56
time (sec)	N/A	0.725	1.071	0.663	0.000	0.307	0.000	0.329	13.998

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	242	277	282	0	861	0	541	387
N.S.	1	1.20	1.38	1.40	0.00	4.28	0.00	2.69	1.93
time (sec)	N/A	1.097	1.397	0.938	0.000	0.321	0.000	0.339	14.349

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	240	417	321	271	344	0	1861	588
N.S.	1	0.90	1.56	1.20	1.01	1.29	0.00	6.97	2.20
time (sec)	N/A	0.628	3.185	2.094	0.204	0.349	0.000	0.366	0.187

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	173	280	198	184	240	0	1102	253
N.S.	1	0.89	1.44	1.02	0.95	1.24	0.00	5.68	1.30
time (sec)	N/A	0.540	1.123	1.387	0.201	0.309	0.000	0.340	0.121

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	103	167	112	112	150	0	139	113
N.S.	1	0.87	1.40	0.94	0.94	1.26	0.00	1.17	0.95
time (sec)	N/A	0.482	0.323	1.386	0.204	0.291	0.000	0.328	0.086

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	48	76	67	55	75	0	61	60
N.S.	1	0.84	1.33	1.18	0.96	1.32	0.00	1.07	1.05
time (sec)	N/A	0.363	0.072	0.515	0.211	0.292	0.000	0.324	0.076

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	134	165	98	123	210	0	213	103
N.S.	1	1.23	1.51	0.90	1.13	1.93	0.00	1.95	0.94
time (sec)	N/A	0.498	0.554	0.731	0.202	0.306	0.000	0.325	13.777

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	190	224	151	274	630	0	456	228
N.S.	1	1.13	1.33	0.90	1.63	3.75	0.00	2.71	1.36
time (sec)	N/A	0.748	1.803	0.973	0.203	0.364	0.000	0.362	14.042

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	298	320	221	511	1205	0	710	447
N.S.	1	1.15	1.24	0.85	1.97	4.65	0.00	2.74	1.73
time (sec)	N/A	1.164	1.621	1.470	0.220	0.494	0.000	0.401	14.702

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	577	402	509	0	793	0	870	3692
N.S.	1	1.22	0.85	1.08	0.00	1.68	0.00	1.84	7.81
time (sec)	N/A	3.636	6.316	2.711	0.000	0.342	0.000	0.344	17.510

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	301	282	325	0	581	0	482	2804
N.S.	1	1.15	1.08	1.25	0.00	2.23	0.00	1.85	10.74
time (sec)	N/A	1.733	3.056	1.635	0.000	0.335	0.000	0.343	15.420

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	215	178	199	0	551	0	240	1655
N.S.	1	1.41	1.17	1.31	0.00	3.62	0.00	1.58	10.89
time (sec)	N/A	1.308	0.732	1.295	0.000	0.312	0.000	0.322	16.439

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
N.S.	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.764	0.917	0.661	0.000	0.308	0.000	0.353	14.317

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	281	242	0	1040	0	457	403
N.S.	1	1.00	0.82	0.71	0.00	3.03	0.00	1.33	1.17
time (sec)	N/A	0.871	1.380	0.896	0.000	0.324	0.000	0.368	14.125

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	296	550	367	326	447	0	2150	762
N.S.	1	0.90	1.67	1.12	0.99	1.36	0.00	6.53	2.32
time (sec)	N/A	0.697	5.501	2.889	0.211	0.375	0.000	0.460	0.229

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	212	388	239	234	331	0	1337	315
N.S.	1	0.89	1.62	1.00	0.98	1.38	0.00	5.59	1.32
time (sec)	N/A	0.578	2.048	2.280	0.198	0.345	0.000	0.418	13.600

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	138	208	144	154	226	0	170	167
N.S.	1	0.87	1.32	0.91	0.97	1.43	0.00	1.08	1.06
time (sec)	N/A	0.504	0.880	1.658	0.202	0.299	0.000	0.388	13.549

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	67	111	85	87	126	0	77	93
N.S.	1	0.81	1.34	1.02	1.05	1.52	0.00	0.93	1.12
time (sec)	N/A	0.368	0.155	0.747	0.201	0.281	0.000	0.375	13.479

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	185	203	148	241	474	0	452	182
N.S.	1	1.13	1.25	0.91	1.48	2.91	0.00	2.77	1.12
time (sec)	N/A	0.643	0.862	1.070	0.207	0.338	0.000	0.388	14.161

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	256	332	196	435	1071	0	800	378
N.S.	1	1.12	1.45	0.86	1.90	4.68	0.00	3.49	1.65
time (sec)	N/A	0.891	6.493	1.465	0.218	0.455	0.000	0.441	14.201

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	349	496	255	707	1803	0	1551	673
N.S.	1	1.12	1.58	0.81	2.26	5.76	0.00	4.96	2.15
time (sec)	N/A	1.464	4.514	1.847	0.228	0.558	0.000	0.482	14.980

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	678	599	582	0	1057	0	1030	3975
N.S.	1	1.26	1.11	1.08	0.00	1.96	0.00	1.91	7.37
time (sec)	N/A	4.096	9.152	4.422	0.000	0.380	0.000	0.495	18.191

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	375	1178	392	0	1041	0	584	3255
N.S.	1	1.13	3.54	1.18	0.00	3.13	0.00	1.75	9.77
time (sec)	N/A	2.200	6.857	2.448	0.000	0.342	0.000	0.422	18.295

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	308	282	276	0	984	0	1193	4026
N.S.	1	1.15	1.06	1.03	0.00	3.69	0.00	4.47	15.08
time (sec)	N/A	1.827	3.291	1.525	0.000	0.334	0.000	0.489	21.743

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	231	234	0	841	0	386	423
N.S.	1	1.00	0.61	0.62	0.00	2.24	0.00	1.03	1.12
time (sec)	N/A	1.007	1.205	0.911	0.000	0.329	0.000	0.405	15.354

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	388	328	0	1550	0	709	588
N.S.	1	1.00	0.75	0.64	0.00	3.01	0.00	1.38	1.14
time (sec)	N/A	1.186	1.304	1.259	0.000	0.394	0.000	0.478	14.270

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	516	513	2049	772	0	0	0	0	0
N.S.	1	0.99	3.97	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.491	47.845	16.193	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	417	853	690	0	0	0	0	0
N.S.	1	0.97	1.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.945	36.321	13.672	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	444	434	1959	636	0	0	0	0	0
N.S.	1	0.98	4.41	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.026	24.660	10.104	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	356	350	351	439	0	0	0	0	0
N.S.	1	0.98	0.99	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.623	10.107	6.260	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	370	369	546	494	0	0	0	0	0
N.S.	1	1.00	1.48	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.665	5.373	7.544	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	404	711	661	0	0	0	0	0
N.S.	1	0.94	1.65	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.969	13.803	8.162	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	452	428	1233	607	0	0	0	0	0
N.S.	1	0.95	2.73	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.019	13.086	12.038	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	511	496	797	892	0	0	0	0	0
N.S.	1	0.97	1.56	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.554	8.397	11.093	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1070	1070	974	1725	0	0	0	0	0
N.S.	1	1.00	0.91	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.219	15.325	44.553	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1101	1101	2095	1494	0	0	0	0	0
N.S.	1	1.00	1.90	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.305	16.726	45.234	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	850	850	886	1482	0	0	0	0	0
N.S.	1	1.00	1.04	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.365	15.507	34.740	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	882	882	2012	1350	0	0	0	0	0
N.S.	1	1.00	2.28	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.488	16.510	26.564	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	809	809	854	1313	0	0	0	0	0
N.S.	1	1.00	1.06	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.162	14.872	19.706	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	838	838	1246	1239	0	0	0	0	0
N.S.	1	1.00	1.49	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.236	14.655	23.712	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1054	1054	772	1417	0	0	0	0	0
N.S.	1	1.00	0.73	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.995	7.229	22.484	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1089	1089	1320	1331	0	0	0	0	0
N.S.	1	1.00	1.21	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.155	12.517	31.287	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.559	6.610	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	120	215	0	0	0	0	0
N.S.	1	1.00	0.99	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	1.744	6.640	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	684	1599	0	0	0	0	0
N.S.	1	1.00	2.21	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.977	6.733	8.893	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	240	276	763	0	0	0	0	0
N.S.	1	1.05	1.21	3.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.717	11.446	9.183	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	138	147	0	0	0	0	0
N.S.	1	1.00	1.30	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.392	5.247	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	255	290	259	759	0	0	0	0	0
N.S.	1	1.14	1.02	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.958	7.392	5.731	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	376	972	1799	0	0	0	0	0
N.S.	1	1.08	2.80	5.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.336	6.598	6.481	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	354	259	977	0	0	0	0	0
N.S.	1	1.11	0.81	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.328	7.468	6.858	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	182	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	206	134	0	0	0	0	0	0
N.S.	1	1.08	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.261	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	687	0	0	0	0	0	0
N.S.	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	4.957	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	405	405	1433	0	0	0	0	0	0
N.S.	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	11.663	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	580	580	2700	0	0	0	0	0	0
N.S.	1	1.00	4.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.964	16.986	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	0	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	1.00	1.08
time (sec)	N/A	0.247	39.216	0.908	1.077	0.404	0.000	1.071	18.518

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.236	0.749	0.973	1.011	0.358	5.958	0.660	15.792

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.239	21.347	0.842	1.101	0.423	1.359	78.037	18.982

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	1.08
time (sec)	N/A	0.241	20.853	0.847	1.128	0.521	30.712	38.514	25.881

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.219	7.656	0.909	1.517	0.424	161.887	0.638	15.168

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	148	562	0	0	0	0	0	0
N.S.	1	0.99	3.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	6.795	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	155	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	1.763	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	72	0	0	0	0	0	73
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.224	0.672	0.000	0.000	0.000	0.000	0.000	14.483

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	114	132	0	0	0	0	0	0
N.S.	1	0.99	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	1.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	231	224	1520	0	0	0	0	0	0
N.S.	1	0.97	6.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	16.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	35	0	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.67	0.00	1.10	1.19
time (sec)	N/A	0.212	7.952	2.449	3.220	0.343	0.000	0.488	18.334

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	26	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.24	0.95	1.10	1.19
time (sec)	N/A	0.215	6.252	1.209	1.953	0.283	37.698	0.485	15.724

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	3614	0	0	0	0	0	0
N.S.	1	1.00	26.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	17.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	424	0	5928	0	0	0	0	0	0
N.S.	1	0.00	13.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	23.443	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	0	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.00	1.00	1.09
time (sec)	N/A	0.217	8.893	0.822	1.595	0.357	0.000	22.551	16.293

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.215	11.612	0.859	1.606	0.350	16.753	0.509	15.532

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.213	10.182	0.819	1.598	0.365	3.621	0.540	16.219

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	36	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.57	0.96	1.00	1.09
time (sec)	N/A	0.213	10.084	0.790	1.656	0.465	111.854	10.804	16.773

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	190	132	135	536	0	682	0	0	0
N.S.	1	0.69	0.71	2.82	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	0.684	2.337	3.137	0.000	0.213	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	124	146	639	0	609	0	0	0
N.S.	1	0.73	0.86	3.78	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.664	11.201	2.329	0.000	0.220	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	84	111	162	0	538	0	0	0
N.S.	1	0.69	0.92	1.34	0.00	4.45	0.00	0.00	0.00
time (sec)	N/A	0.556	1.186	10.672	0.000	0.193	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	122	83	130	544	0	554	0	0	0
N.S.	1	0.68	1.07	4.46	0.00	4.54	0.00	0.00	0.00
time (sec)	N/A	0.545	0.756	9.390	0.000	0.385	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	182	127	135	547	0	608	0	0	0
N.S.	1	0.70	0.74	3.01	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.654	9.737	10.881	0.000	0.400	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	129	165	602	0	654	0	0	0
N.S.	1	0.65	0.84	3.06	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.647	1.266	2.351	0.000	0.420	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	270	189	195	607	0	806	0	0	0
N.S.	1	0.70	0.72	2.25	0.00	2.99	0.00	0.00	0.00
time (sec)	N/A	0.655	13.270	11.702	0.000	0.204	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	177	195	1069	0	729	0	0	0
N.S.	1	0.74	0.81	4.45	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.634	14.905	13.196	0.000	0.194	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	154	109	168	476	0	664	0	0	0
N.S.	1	0.71	1.09	3.09	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.556	12.886	12.555	0.000	0.202	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	153	108	287	996	0	698	0	0	0
N.S.	1	0.71	1.88	6.51	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.561	18.326	13.827	0.000	0.424	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	222	158	164	810	0	724	0	0	0
N.S.	1	0.71	0.74	3.65	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.617	15.706	12.700	0.000	0.427	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	236	160	152	1069	0	776	0	0	0
N.S.	1	0.68	0.64	4.53	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.619	15.032	13.642	0.000	0.459	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	127	131	360	0	163	0	0	0
N.S.	1	0.82	0.85	2.32	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.793	1.786	10.678	0.000	0.094	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	121	230	642	0	127	0	0	0
N.S.	1	0.83	1.59	4.43	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.783	1.812	8.114	0.000	0.100	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	96	60	285	0	105	0	0	0
N.S.	1	0.91	0.57	2.71	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.673	0.956	8.081	0.000	0.090	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	90	95	432	0	87	0	0	0
N.S.	1	0.91	0.96	4.36	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.676	1.148	7.465	0.000	0.094	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	97	70	275	0	89	0	0	0
N.S.	1	0.92	0.66	2.59	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.687	1.045	10.124	0.000	0.096	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	99	100	462	0	111	0	0	0
N.S.	1	0.82	0.83	3.85	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.678	1.537	8.586	0.000	0.106	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	129	91	316	0	111	0	0	0
N.S.	1	0.87	0.61	2.12	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.793	1.488	10.010	0.000	0.110	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	268	202	115	382	0	220	0	0	0
N.S.	1	0.75	0.43	1.43	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.862	2.106	9.063	0.000	0.107	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	200	247	859	0	182	0	0	0
N.S.	1	0.80	0.99	3.44	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.846	2.363	9.250	0.000	0.096	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	173	82	309	0	155	0	0	0
N.S.	1	0.86	0.41	1.54	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.796	1.437	8.061	0.000	0.087	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	171	252	659	0	129	0	0	0
N.S.	1	0.86	1.27	3.31	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.818	2.269	8.700	0.000	0.099	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	172	101	397	0	135	0	0	0
N.S.	1	0.81	0.47	1.86	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.827	1.585	9.988	0.000	0.105	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	170	125	462	0	111	0	0	0
N.S.	1	0.79	0.58	2.15	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.835	2.821	8.864	0.000	0.110	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	148	94	316	0	111	0	0	0
N.S.	1	0.86	0.55	1.84	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.813	3.013	9.865	0.000	0.109	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [228] had the largest ratio of [1.42856999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	10	0.99	19	0.526
2	A	11	10	1.01	19	0.526
3	A	11	10	1.03	19	0.526
4	A	11	10	1.07	19	0.526
5	A	10	9	0.96	17	0.529
6	A	13	12	1.30	17	0.706
7	A	12	11	1.08	19	0.579
8	A	12	11	1.00	19	0.579
9	A	12	11	0.96	19	0.579
10	A	19	18	0.99	19	0.947
11	A	17	16	0.98	19	0.842
12	A	15	14	0.98	19	0.737
13	A	13	12	0.96	19	0.632
14	A	16	15	0.95	19	0.789
15	A	16	15	0.87	19	0.789
16	A	16	15	0.83	19	0.789
17	A	16	15	0.79	19	0.789
18	A	16	15	0.80	19	0.789
19	A	9	8	0.88	21	0.381
20	A	9	8	0.90	21	0.381
21	A	9	8	0.92	21	0.381
22	A	9	8	1.00	21	0.381

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	8	0.84	19	0.421
24	A	8	7	1.04	19	0.368
25	A	8	7	0.99	21	0.333
26	A	8	7	0.96	21	0.333
27	A	8	7	0.93	21	0.333
28	A	8	7	0.92	21	0.333
29	A	5	5	1.02	21	0.238
30	A	5	5	1.03	21	0.238
31	A	5	5	1.03	21	0.238
32	A	5	5	1.05	21	0.238
33	A	5	5	1.00	21	0.238
34	A	16	15	1.10	21	0.714
35	A	5	5	1.00	21	0.238
36	A	5	5	1.00	21	0.238
37	A	5	5	1.00	21	0.238
38	A	11	10	0.88	21	0.476
39	A	11	10	0.88	21	0.476
40	A	11	10	0.89	21	0.476
41	A	10	9	0.89	21	0.429
42	A	11	10	0.82	19	0.526
43	A	12	11	0.99	19	0.579
44	A	12	11	0.95	21	0.524
45	A	12	11	0.94	21	0.524
46	A	12	11	0.93	21	0.524
47	A	12	11	0.92	21	0.524
48	A	7	7	1.02	21	0.333
49	A	7	7	1.02	21	0.333
50	A	7	7	1.03	21	0.333
51	A	7	7	1.04	21	0.333
52	A	8	8	0.96	21	0.381
53	A	8	8	1.05	21	0.381
54	A	23	22	1.07	21	1.048
55	A	8	8	1.00	21	0.381
56	A	8	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	15	14	0.82	21	0.667
58	A	15	14	0.86	21	0.667
59	A	15	14	0.93	21	0.667
60	A	14	13	1.00	21	0.619
61	A	11	10	1.00	19	0.526
62	A	16	15	1.00	19	0.789
63	A	18	17	1.02	21	0.810
64	A	20	19	1.04	21	0.905
65	A	18	17	1.05	21	0.810
66	A	16	15	1.04	21	0.714
67	A	14	13	1.03	21	0.619
68	A	10	10	1.00	21	0.476
69	A	15	14	1.00	21	0.667
70	A	16	15	0.95	21	0.714
71	A	16	15	0.88	21	0.714
72	A	16	15	0.84	21	0.714
73	A	16	15	0.81	21	0.714
74	A	9	8	0.89	21	0.381
75	A	9	8	0.89	21	0.381
76	A	9	8	0.95	21	0.381
77	A	9	8	0.98	21	0.381
78	A	9	8	0.95	21	0.381
79	A	9	8	0.92	19	0.421
80	A	8	7	0.92	19	0.368
81	A	7	6	1.05	21	0.286
82	A	8	7	0.95	21	0.333
83	A	7	7	1.02	21	0.333
84	A	13	13	1.07	21	0.619
85	A	7	7	1.05	21	0.333
86	A	9	9	0.88	21	0.429
87	A	5	5	1.05	21	0.238
88	A	7	7	1.04	21	0.333
89	A	7	7	1.04	21	0.333
90	A	7	7	1.03	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	11	10	0.89	21	0.476
92	A	11	10	0.91	21	0.476
93	A	11	10	0.95	21	0.476
94	A	11	10	0.92	21	0.476
95	A	11	10	0.93	21	0.476
96	A	11	10	0.89	19	0.526
97	A	12	11	0.91	19	0.579
98	A	10	9	0.88	21	0.429
99	A	12	11	0.91	21	0.524
100	A	9	9	1.03	21	0.429
101	A	9	9	1.03	21	0.429
102	A	10	10	0.94	21	0.476
103	A	9	9	1.04	21	0.429
104	F	0	0	N/A	0.000	N/A
105	F	0	0	N/A	0.000	N/A
106	F	0	0	N/A	0.000	N/A
107	F	0	0	N/A	0.000	N/A
108	A	20	19	0.96	23	0.826
109	A	20	19	0.97	23	0.826
110	A	17	16	0.93	23	0.696
111	A	19	18	0.94	23	0.783
112	A	22	21	0.97	23	0.913
113	A	22	21	0.98	23	0.913
114	A	5	5	1.00	25	0.200
115	A	5	5	1.00	25	0.200
116	A	5	5	1.00	25	0.200
117	A	5	5	1.00	25	0.200
118	A	5	5	1.00	25	0.200
119	A	5	5	1.00	25	0.200
120	A	17	16	1.02	25	0.640
121	A	15	14	1.02	25	0.560
122	A	15	14	1.02	25	0.560
123	A	15	14	1.08	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	17	16	1.08	25	0.640
125	A	19	18	1.04	25	0.720
126	A	19	18	1.07	25	0.720
127	A	7	7	1.04	25	0.280
128	A	7	7	1.06	25	0.280
129	A	7	7	1.06	25	0.280
130	A	7	7	1.07	25	0.280
131	A	7	7	1.07	25	0.280
132	A	7	7	1.05	25	0.280
133	A	7	7	1.05	25	0.280
134	A	7	7	1.00	23	0.304
135	A	5	5	1.00	23	0.217
136	A	12	11	1.00	21	0.524
137	A	11	10	1.00	23	0.435
138	A	7	7	1.04	23	0.304
139	A	9	9	1.04	23	0.391
140	A	8	7	1.58	25	0.280
141	A	8	7	1.58	25	0.280
142	A	8	7	1.55	25	0.280
143	A	8	7	1.50	25	0.280
144	A	8	7	1.52	23	0.304
145	A	9	8	0.96	21	0.381
146	A	8	7	1.03	21	0.333
147	A	7	6	1.01	21	0.286
148	A	4	3	1.00	19	0.158
149	A	6	5	1.00	19	0.263
150	A	9	8	1.10	21	0.381
151	A	12	11	1.02	21	0.524
152	A	19	18	1.38	21	0.857
153	A	11	10	1.27	21	0.476
154	A	9	8	1.00	21	0.381
155	A	13	12	1.26	21	0.571
156	A	8	7	1.00	23	0.304
157	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	8	7	1.00	23	0.304
159	A	8	7	1.27	23	0.304
160	A	11	10	1.04	19	0.526
161	A	11	10	1.07	19	0.526
162	A	11	10	1.10	19	0.526
163	A	10	9	0.96	17	0.529
164	A	13	12	1.00	17	0.706
165	A	16	15	0.97	19	0.789
166	A	19	18	1.01	19	0.947
167	A	21	20	0.99	19	1.053
168	A	17	16	0.98	19	0.842
169	A	15	14	0.98	19	0.737
170	A	13	12	0.96	19	0.632
171	A	16	15	0.95	19	0.789
172	A	16	15	0.87	19	0.789
173	A	16	15	0.83	19	0.789
174	A	9	8	1.03	21	0.381
175	A	9	8	1.02	21	0.381
176	A	9	8	0.98	19	0.421
177	A	10	9	1.20	19	0.474
178	A	10	9	1.30	21	0.429
179	A	17	16	1.07	21	0.762
180	A	16	16	1.12	21	0.762
181	A	5	5	1.00	21	0.238
182	A	14	13	0.92	21	0.619
183	A	14	13	0.87	21	0.619
184	A	14	13	0.85	21	0.619
185	A	11	10	0.99	21	0.476
186	A	10	9	0.89	21	0.429
187	A	11	10	0.95	19	0.526
188	A	13	12	1.16	19	0.632
189	A	14	13	1.16	21	0.619
190	A	7	7	1.00	21	0.333
191	A	21	21	1.10	21	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	19	19	0.95	21	0.905
193	A	8	8	0.91	21	0.381
194	A	8	8	1.00	21	0.381
195	A	8	8	1.00	21	0.381
196	A	11	10	0.91	21	0.476
197	A	11	10	0.90	21	0.476
198	A	11	10	0.90	21	0.476
199	A	11	10	0.91	19	0.526
200	A	13	12	1.03	19	0.632
201	A	14	13	1.31	21	0.619
202	A	14	13	1.27	21	0.619
203	A	18	17	1.14	21	0.810
204	A	15	14	1.16	21	0.667
205	A	13	12	1.15	21	0.571
206	A	12	11	1.13	21	0.524
207	A	14	13	1.18	21	0.619
208	A	16	15	1.20	21	0.714
209	A	9	8	0.90	21	0.381
210	A	9	8	0.89	21	0.381
211	A	9	8	0.87	21	0.381
212	A	9	8	0.84	19	0.421
213	A	10	9	1.23	19	0.474
214	A	9	8	1.13	21	0.381
215	A	10	9	1.15	21	0.429
216	A	25	24	1.22	21	1.143
217	A	19	18	1.15	21	0.857
218	A	18	17	1.41	21	0.810
219	A	5	5	1.00	21	0.238
220	A	5	5	1.00	21	0.238
221	A	11	10	0.90	21	0.476
222	A	11	10	0.89	21	0.476
223	A	11	10	0.87	21	0.476
224	A	11	10	0.81	19	0.526
225	A	14	13	1.13	19	0.684

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	11	10	1.12	21	0.476
227	A	14	13	1.12	21	0.619
228	A	31	30	1.26	21	1.429
229	A	25	24	1.13	21	1.143
230	A	21	20	1.15	21	0.952
231	A	8	8	1.00	21	0.381
232	A	8	8	1.00	21	0.381
233	A	27	26	0.99	25	1.040
234	A	24	23	0.97	25	0.920
235	A	24	23	0.98	25	0.920
236	A	21	20	0.98	25	0.800
237	A	22	21	1.00	25	0.840
238	A	25	24	0.94	25	0.960
239	A	25	24	0.95	25	0.960
240	A	28	27	0.97	25	1.080
241	A	5	5	1.00	25	0.200
242	A	5	5	1.00	25	0.200
243	A	5	5	1.00	25	0.200
244	A	5	5	1.00	25	0.200
245	A	5	5	1.00	25	0.200
246	A	5	5	1.00	25	0.200
247	A	5	5	1.00	25	0.200
248	A	5	5	1.00	25	0.200
249	A	2	2	1.00	14	0.143
250	A	5	5	1.00	23	0.217
251	A	8	8	1.00	14	0.571
252	A	8	8	1.05	23	0.348
253	A	2	2	1.00	14	0.143
254	A	11	11	1.14	23	0.478
255	A	11	11	1.08	14	0.786
256	A	14	14	1.11	23	0.609
257	A	7	7	1.00	23	0.304
258	A	13	12	1.08	23	0.522
259	A	12	11	1.00	21	0.524

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	9	9	1.00	23	0.391
261	A	5	5	1.00	23	0.217
262	A	7	7	1.00	23	0.304
263	N/A	2	0	1.00	25	0.000
264	N/A	2	0	1.00	25	0.000
265	N/A	2	0	1.00	25	0.000
266	N/A	2	0	1.00	25	0.000
267	N/A	2	0	1.00	23	0.000
268	A	5	4	0.99	21	0.190
269	A	6	5	1.00	21	0.238
270	A	4	3	1.00	19	0.158
271	A	7	6	0.99	19	0.316
272	A	5	4	0.97	21	0.190
273	N/A	2	0	1.00	21	0.000
274	N/A	2	0	1.00	21	0.000
275	A	8	7	1.00	21	0.333
276	F	0	0	N/A	0.000	N/A
277	N/A	2	0	1.00	23	0.000
278	N/A	2	0	1.00	23	0.000
279	N/A	2	0	1.00	23	0.000
280	N/A	2	0	1.00	23	0.000
281	A	21	20	0.69	23	0.870
282	A	21	20	0.73	23	0.870
283	A	18	17	0.69	23	0.739
284	A	16	15	0.68	23	0.652
285	A	19	18	0.70	23	0.783
286	A	19	18	0.65	23	0.783
287	A	7	7	0.70	25	0.280
288	A	7	7	0.74	25	0.280
289	A	7	7	0.71	25	0.280
290	A	7	7	0.71	25	0.280
291	A	7	7	0.71	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
292	A	7	7	0.68	25	0.280
293	A	19	18	0.82	25	0.720
294	A	19	18	0.83	25	0.720
295	A	17	16	0.91	25	0.640
296	A	15	14	0.91	25	0.560
297	A	15	14	0.92	25	0.560
298	A	15	14	0.82	25	0.560
299	A	17	16	0.87	25	0.640
300	A	9	9	0.75	25	0.360
301	A	9	9	0.80	25	0.360
302	A	9	9	0.86	25	0.360
303	A	9	9	0.86	25	0.360
304	A	9	9	0.81	25	0.360
305	A	9	9	0.79	25	0.360
306	A	9	9	0.86	25	0.360

CHAPTER 3

Listing of integrals

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3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$ . . . 232
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3.19 $\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx$ . . . 248
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3.21 $\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx$ . . . 262
3.22 $\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx$ . . . 269
3.23 $\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$ . . . 275
3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 281
3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 287
3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 293
3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 299
3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 305

3.29 $\int (a+a\sec(c+dx))^2 \sin^8(c+dx)\,dx$ . . . 312
3.30 $\int (a+a\sec(c+dx))^2 \sin^6(c+dx)\,dx$ . . . 319
3.31 $\int (a+a\sec(c+dx))^2 \sin^4(c+dx)\,dx$ . . . 325
3.32 $\int (a+a\sec(c+dx))^2 \sin^2(c+dx)\,dx$ . . . 331
3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 337
3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 344
3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 352
3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 358
3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$ . . . 365
3.38 $\int (a+a\sec(c+dx))^3 \sin^9(c+dx)\,dx$ . . . 372
3.39 $\int (a+a\sec(c+dx))^3 \sin^7(c+dx)\,dx$ . . . 380
3.40 $\int (a+a\sec(c+dx))^3 \sin^5(c+dx)\,dx$ . . . 386
3.41 $\int (a+a\sec(c+dx))^3 \sin^3(c+dx)\,dx$ . . . 393
3.42 $\int (a+a\sec(c+dx))^3 \sin(c+dx)\,dx$ . . . 399
3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 405
3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 412
3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 419
3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 426
3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 433
3.48 $\int (a+a\sec(c+dx))^3 \sin^8(c+dx)\,dx$ . . . 441
3.49 $\int (a+a\sec(c+dx))^3 \sin^6(c+dx)\,dx$ . . . 448
3.50 $\int (a+a\sec(c+dx))^3 \sin^4(c+dx)\,dx$ . . . 455
3.51 $\int (a+a\sec(c+dx))^3 \sin^2(c+dx)\,dx$ . . . 461
3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 468
3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 474
3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 480
3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 490
3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$ . . . 497
3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 505
3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 512
3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 519
3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 526
3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 532
3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 538
3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 545
3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 552
3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 560
3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 568

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 575
3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 582
3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 588
3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 594
3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 601
3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 608
3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$ . . . 615
3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 623
3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 629
3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 635
3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 641
3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 647
3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 653
3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 659
3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 665
3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 671
3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 677
3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 684
3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 691
3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 697
3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 703
3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 708
3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 714
3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ . . . 720
3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 726
3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 732
3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 738
3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 744
3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 750
3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 756
3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 762
3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 768
3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 775

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 782
3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 789
3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 795
3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 802
3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 809
3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 815
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3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ . . . 828
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3.110 $\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$ . . . 853
3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$ . . . 861
3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$ . . . 869
3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$ . . . 878
3.114 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$ . . . 888
3.115 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$ . . . 894
3.116 $\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$ . . . 900
3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$ . . . 906
3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$ . . . 912
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3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$ . . . 1000
3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$ . . . 1006
3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$ . . . 1012

3.134 $\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$ . . . 1018
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3.141 $\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$ . . . 1059
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3.145 $\int (a+a\sec(c+dx))^n\sin^7(c+dx)\,dx$ . . . 1083
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3.147 $\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx$ . . . 1094
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3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$ . . . 1141
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3.176 $\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$ . . . 1285
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3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1477
3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1484
3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1492
3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1502
3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1510
3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1517
3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1524
3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 1532
3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1542

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1551
3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1559
3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1565
3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1571
3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1578
3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1586
3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1595
3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1611
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3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1632
3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 1639
3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1647
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3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1671
3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1677
3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1685
3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1694
3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1705
3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1725
3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1740
3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1752
3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ . . . 1760
3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$ . . . 1769
3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$ . . . 1788
3.235 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$ . . . 1802
3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$ . . . 1816
3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$ . . . 1827
3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$ . . . 1838
3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$ . . . 1850
3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$ . . . 1863
3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$ . . . 1877
3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$ . . . 1886

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$ . . . 1895
3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$ . . . 1904
3.245 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$ . . . 1913
3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$ . . . 1922
3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$ . . . 1930
3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$ . . . 1939
3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$ . . . 1948
3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$ . . . 1953
3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$ . . . 1958
3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$ . . . 1966
3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$ . . . 1973
3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$ . . . 1978
3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$ . . . 1986
3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$ . . . 1995
3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$ . . . 2004
3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$ . . . 2010
3.259 $\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx$ . . . 2017
3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$ . . . 2023
3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$ . . . 2030
3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$ . . . 2036
3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$ . . . 2043
3.264 $\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$ . . . 2047
3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2051
3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2055
3.267 $\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$ . . . 2059
3.268 $\int (a+b\sec(c+dx))^n\sin^5(c+dx)\,dx$ . . . 2063
3.269 $\int (a+b\sec(c+dx))^n\sin^3(c+dx)\,dx$ . . . 2068
3.270 $\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx$ . . . 2073
3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2077
3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2082
3.273 $\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx$ . . . 2087
3.274 $\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$ . . . 2091
3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2095
3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2101
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3.278 $\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$ . . . 2109
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3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2117
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3.283 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$ . . . 2141
3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$ . . . 2150
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3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$ . . . 2204
3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$ . . . 2211
3.293 $\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$ . . . 2218
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3.295 $\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx$ . . . 2234
3.296 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx$ . . . 2241
3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$ . . . 2248
3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$ . . . 2255
3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$ . . . 2262
3.300 $\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$ . . . 2269
3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$ . . . 2276
3.302 $\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$ . . . 2283
3.303 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$ . . . 2289
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3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$ . . . 2303
3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$ . . . 2310

3.1 $\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx$

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3.1.1 Optimal result

Integrand size = 19, antiderivative size = 152

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^2(c+dx)}{d} + \frac{4a\cos^3(c+dx)}{3d} - \frac{3a\cos^4(c+dx)}{2d} - \frac{6a\cos^5(c+dx)}{5d} + \frac{2a\cos^6(c+dx)}{3d} + \frac{4a\cos^7(c+dx)}{7d} - \frac{a\cos^8(c+dx)}{8d} - \frac{a\cos^9(c+dx)}{9d} - \frac{a\log(\cos(c+dx))}{d}$$

output

```
-a*cos(d*x+c)/d+2*a*cos(d*x+c)^2/d+4/3*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)^4
/d-6/5*a*cos(d*x+c)^5/d+2/3*a*cos(d*x+c)^6/d+4/7*a*cos(d*x+c)^7/d-1/8*a*co
s(d*x+c)^8/d-1/9*a*cos(d*x+c)^9/d-a*ln(cos(d*x+c))/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.70

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx = -\frac{a(39690\cos(c+dx) - 161280\cos^2(c+dx) + 120960\cos^4(c+dx) - 53760\cos^6(c+dx) + 10080\cos^8(c$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

output
```
-1/80640*(a*(39690*Cos[c + d*x] - 161280*Cos[c + d*x]^2 + 120960*Cos[c + d
*x]^4 - 53760*Cos[c + d*x]^6 + 10080*Cos[c + d*x]^8 - 8820*Cos[3*(c + d*x)
] + 2268*Cos[5*(c + d*x)] - 405*Cos[7*(c + d*x)] + 35*Cos[9*(c + d*x)] + 8
0640*Log[Cos[c + d*x]]))/d
```

3.1.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^9(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^9\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^8(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sin^8(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^8(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^9\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^4(\cos(c+dx)a+a)^5\sec(c+dx)d(a\cos(c+dx))}{a^9d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^5\sec(c+dx)}{a}d(a\cos(c+dx))}{a^8d}$$

$$\downarrow 99$$

$$-\frac{\int (\cos^8(c+dx)a^8+\cos^7(c+dx)a^8-4\cos^6(c+dx)a^8-4\cos^5(c+dx)a^8+6\cos^4(c+dx)a^8+6\cos^3(c+dx)a^8}{a^8d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{9}a^9\cos^9(c+dx)+\frac{1}{8}a^9\cos^8(c+dx)-\frac{4}{7}a^9\cos^7(c+dx)-\frac{2}{3}a^9\cos^6(c+dx)+\frac{6}{5}a^9\cos^5(c+dx)+\frac{3}{2}a^9\cos^4(c+d}{a^8d}$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

output
```
-((a^9*Cos[c + d*x] - 2*a^9*Cos[c + d*x]^2 - (4*a^9*Cos[c + d*x]^3)/3 + (3
*a^9*Cos[c + d*x]^4)/2 + (6*a^9*Cos[c + d*x]^5)/5 - (2*a^9*Cos[c + d*x]^6)
/3 - (4*a^9*Cos[c + d*x]^7)/7 + (a^9*Cos[c + d*x]^8)/8 + (a^9*Cos[c + d*x]
^9)/9 + a^9*Log[a*Cos[c + d*x]])/(a^8*d))
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.1.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}\right.}{9}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}\right.}{9}}{d}$
parts	$-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx}{d}\right.}$
parallelrisch	$-\frac{a\left(-322560\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+322560\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+322560\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+269777+28980\cos(4dx+\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{65a\,e^{2i(dx+c)}}{256d}+\frac{65a\,e^{-2i(dx+c)}}{256d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{63a\cos(dx+c)}{128d}-\frac{a\cos(9dx+9c)}{2304d}-\frac{a\,c}{}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/8*sin(d*x+c)^8-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)
^2-ln(cos(d*x+c)))-1/9*a*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d
*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx =$$

$$-\frac{280\,a\cos(dx+c)^9 + 315\,a\cos(dx+c)^8 - 1440\,a\cos(dx+c)^7 - 1680\,a\cos(dx+c)^6 + 3024\,a\cos(dx+$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)
^7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4
 - 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2
520*a*log(-cos(d*x + c)))/d
```

3.1.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**9,x)
```

output
```
Timed out
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx =$$
$$-\frac{280\,a\cos(dx+c)^9 + 315\,a\cos(dx+c)^8 - 1440\,a\cos(dx+c)^7 - 1680\,a\cos(dx+c)^6 + 3024\,a\cos(dx+$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)
^7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4
 - 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2
520*a*log(cos(d*x + c)))/d
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(138) = 276.

Time = 0.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.93

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx$$
$$= \frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9177\,a - \frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{375732\,a(\cos(dx+c)-1)}{(\cos(dx+c)+1)^2}}{}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="giac")
```

output
```
1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520
*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a - 87633*
a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 - 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
1594782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1336734*a*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x +
 c) + 1)^6 - 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*
(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 7129*a*(cos(d*x + c) - 1)^9/(c
os(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx =$$

$$-\frac{a\cos(c+dx) - 2a\cos(c+dx)^2 - \frac{4a\cos(c+dx)^3}{3} + \frac{3a\cos(c+dx)^4}{2} + \frac{6a\cos(c+dx)^5}{5} - \frac{2a\cos(c+dx)^6}{3} - \frac{4a\cos(c+d}{7}}{d}$$

input
```
int(sin(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - 2*a*cos(c + d*x)^2 - (4*a*cos(c + d*x)^3)/3 + (3*a*cos(
c + d*x)^4)/2 + (6*a*cos(c + d*x)^5)/5 - (2*a*cos(c + d*x)^6)/3 - (4*a*cos
(c + d*x)^7)/7 + (a*cos(c + d*x)^8)/8 + (a*cos(c + d*x)^9)/9 + a*log(cos(c
 + d*x)))/d
```

3.2 $\int (a + a\sec(c+dx))\sin^7(c+dx)\,dx$

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3.2.1 Optimal result

Integrand size = 19, antiderivative size = 119

$$\int (a + a\sec(c+dx))\sin^7(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} + \frac{3a\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{d} - \frac{3a\cos^4(c+dx)}{4d} - \frac{3a\cos^5(c+dx)}{5d} + \frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^7(c+dx)}{7d} - \frac{a\log(\cos(c+dx))}{d}$$

output

```
-a*cos(d*x+c)/d+3/2*a*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*a*cos(d*x+c)^4/d
-3/5*a*cos(d*x+c)^5/d+1/6*a*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-a*ln(cos(d
*x+c))/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c+dx))\sin^7(c+dx)\,dx = -\frac{35a\cos(c+dx)}{64d} + \frac{7a\cos(3(c+dx))}{64d} - \frac{7a\cos(5(c+dx))}{320d} + \frac{a\cos(7(c+dx))}{448d} - \frac{a\left(-\frac{3}{2}\cos^2(c+dx) + \frac{3}{4}\cos^4(c+dx) - \frac{1}{6}\cos^6(c+dx) + \log(\cos(c+dx))\right)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

output
```
(-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(
c + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (a*((-3*Cos[c + d*x]^2
)/2 + (3*Cos[c + d*x]^4)/4 - Cos[c + d*x]^6/6 + Log[Cos[c + d*x]]))/d
```

3.2.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^7\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -\left(\sin^6(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\sin^6(c+dx)\tan(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \sin^6(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^7\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^3(\cos(c+dx)a+a)^4\sec(c+dx)d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^4\sec(c+dx)}{a}d(a\cos(c+dx))}{a^6d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6-\cos^5(c+dx)a^6+3\cos^4(c+dx)a^6+3\cos^3(c+dx)a^6-3\cos^2(c+dx)a^6-3\cos(c+dx)a\right.}{a^6d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)-\frac{1}{6}a^7\cos^6(c+dx)+\frac{3}{5}a^7\cos^5(c+dx)+\frac{3}{4}a^7\cos^4(c+dx)-a^7\cos^3(c+dx)-\frac{3}{2}a^7\cos^2(c+d}{a^6d}$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

output
```
-((a^7*Cos[c + d*x] - (3*a^7*Cos[c + d*x]^2)/2 - a^7*Cos[c + d*x]^3 + (3*a
^7*Cos[c + d*x]^4)/4 + (3*a^7*Cos[c + d*x]^5)/5 - (a^7*Cos[c + d*x]^6)/6 -
 (a^7*Cos[c + d*x]^7)/7 + a^7*Log[a*Cos[c + d*x]])/(a^6*d))
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.2.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}}{d}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+$
parallelrisch	$\frac{\left(192\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{5732}{35}-12\cos(4dx+4c)+21\cos(3dx+3c)-\right.}{192d}$
risch	$iax+\frac{2iac}{d}+\frac{29a\,e^{2i(dx+c)}}{128d}+\frac{29a\,e^{-2i(dx+c)}}{128d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{a\,c}{}$
norman	$\frac{-\frac{32a}{35d}-\frac{128a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{166a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{224a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{42a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5d}-\frac{14a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))
)-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx$$
$$= \frac{60\,a\cos(dx+c)^7 + 70\,a\cos(dx+c)^6 - 252\,a\cos(dx+c)^5 - 315\,a\cos(dx+c)^4 + 420\,a\cos(dx+c)^3 + 6}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 -
315*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a
*cos(d*x + c) - 420*a*log(-cos(d*x + c)))/d
```

3.2.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**7,x)
```

output
```
Timed out
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx$$
$$= \frac{60\,a\cos(dx+c)^7 + 70\,a\cos(dx+c)^6 - 252\,a\cos(dx+c)^5 - 315\,a\cos(dx+c)^4 + 420\,a\cos(dx+c)^3 + 6}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 -
315*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a
*cos(d*x + c) - 420*a*log(cos(d*x + c)))/d
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(109) = 218$.

Time $= 0.31$ (sec) , antiderivative size $= 247$, normalized size of antiderivative $= 2.08$

$$\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx$$

$$= \frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1473\,a - \frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - }{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

output
```
1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a - 11151*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035
*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*a*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
- 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1)^7)/d
```

3.2.9 Mupad [B] (verification not implemented)

Time $= 0.08$ (sec) , antiderivative size $= 89$, normalized size of antiderivative $= 0.75$

$$\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx =$$

$$-\frac{a\cos(c+dx) - \frac{3\,a\cos(c+dx)^2}{2} - a\cos(c+dx)^3 + \frac{3\,a\cos(c+dx)^4}{4} + \frac{3\,a\cos(c+dx)^5}{5} - \frac{a\cos(c+dx)^6}{6} - \frac{a\cos(c+dx)^7}{7}}{d}$$

input
```
int(sin(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - (3*a*cos(c + d*x)^2)/2 - a*cos(c + d*x)^3 + (3*a*cos(c
+ d*x)^4)/4 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d*x)^6)/6 - (a*cos(c + d
*x)^7)/7 + a*log(cos(c + d*x)))/d
```

3.3 $\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx$

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3.3.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} + \frac{a\cos^2(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^4(c+dx)}{4d} - \frac{a\cos^5(c+dx)}{5d} - \frac{a\log(\cos(c+dx))}{d}$$

output

```
-a*cos(d*x+c)/d+a*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d
-1/5*a*cos(d*x+c)^5/d-a*ln(cos(d*x+c))/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{5a\cos(c+dx)}{8d} + \frac{5a\cos(3(c+dx))}{48d} - \frac{a\cos(5(c+dx))}{80d} - \frac{a\left(-\cos^2(c+dx) + \frac{1}{4}\cos^4(c+dx) + \log(\cos(c+dx))\right)}{d}$$

input

```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

output
```
(-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c +
d*x)])/(80*d) - (a*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]
))/d
```

3.3.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^4(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sin^4(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^4(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^2(\cos(c+dx)a+a)^3\sec(c+dx)d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^3\sec(c+dx)}{a}d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\cos^4(c+dx)a^4+\cos^3(c+dx)a^4-2\cos^2(c+dx)a^4-2\cos(c+dx)a^4+\sec(c+dx)a^4+a^4\right)d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)+\frac{1}{4}a^5\cos^4(c+dx)-\frac{2}{3}a^5\cos^3(c+dx)-a^5\cos^2(c+dx)+a^5\cos(c+dx)+a^5\log(a\cos(c+dx))}{a^4d}$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

output
```
-((a^5*Cos[c + d*x] - a^5*Cos[c + d*x]^2 - (2*a^5*Cos[c + d*x]^3)/3 + (a^5
*Cos[c + d*x]^4)/4 + (a^5*Cos[c + d*x]^5)/5 + a^5*Log[a*Cos[c + d*x]])/(a^
4*d))
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.3.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}}{d}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$-\frac{a\left(-480\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+480\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+480\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+421+6\cos(5dx+5c)+15\cos(4dx+4\right.}{480d}$
risch	$iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{3a\,e^{-2i(dx+c)}}{16d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{a\cos(4}{3}$
norman	$\frac{-\frac{16a}{15d}-\frac{22a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{62a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{10a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{a\ln\left(ta\right.}{}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a*(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx =$$
$$-\frac{12\,a\cos(dx+c)^5 + 15\,a\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 60\,a\cos(dx+c)^2 + 60\,a\cos(dx+c) + 60}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 6
0*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(-cos(d*x + c)))/d
```

3.3.6 Sympy [F]

$$\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx = a\left(\int \sin^5(c+dx)\sec(c+dx)\,dx + \int \sin^5(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**5,x)
```

output
```
a*(Integral(sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**5, x
))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx =$$
$$-\frac{12\,a\cos(dx+c)^5 + 15\,a\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 60\,a\cos(dx+c)^2 + 60\,a\cos(dx+c) + 60}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 6
0*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(cos(d*x + c)))/d
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(81) = 162$.

Time $= 0.31$ (sec) , antiderivative size $= 201$, normalized size of antiderivative $= 2.31$

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx$$

$$= \frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{201\,a - \frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{1970\,a(}{(\cos($$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a - 1125*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d
*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*(cos(d*x + c) - 1)^5/(cos(d*x
+ c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

3.3.9 Mupad [B] (verification not implemented)

Time $= 0.06$ (sec) , antiderivative size $= 67$, normalized size of antiderivative $= 0.77$

$$\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx =$$

$$-\frac{a\cos(c+dx) - a\cos(c+dx)^2 - \frac{2\,a\cos(c+dx)^3}{3} + \frac{a\cos(c+dx)^4}{4} + \frac{a\cos(c+dx)^5}{5} + a\ln(\cos(c+dx))}{d}$$

input
```
int(sin(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - a*cos(c + d*x)^2 - (2*a*cos(c + d*x)^3)/3 + (a*cos(c +
d*x)^4)/4 + (a*cos(c + d*x)^5)/5 + a*log(cos(c + d*x)))/d
```

3.4 $\int (a + a\sec(c+dx))\sin^3(c+dx)\,dx$

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3.4.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a + a\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} + \frac{a\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{3d} - \frac{a\log(\cos(c+dx))}{d}$$

output
```
-a*cos(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-a*ln(cos(d*x+c))
/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (a + a\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{3a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} - \frac{a\left(-\frac{1}{2}\cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

output
```
(-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*(-1/2*Cos[c +
 d*x]^2 + Log[Cos[c + d*x]]))/d
```

3.4.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^2(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sin^2(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^2(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))(\cos(c+dx)a+a)^2\sec(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))(\cos(c+dx)a+a)^2\sec(c+dx)}{a}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 84$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2-\cos(c+dx)a^2+\sec(c+dx)a^2+a^2\right)d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)-\frac{1}{2}a^3\cos^2(c+dx)+a^3\cos(c+dx)+a^3\log(a\cos(c+dx))}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

output
```
-((a^3*Cos[c + d*x] - (a^3*Cos[c + d*x]^2)/2 - (a^3*Cos[c + d*x]^3)/3 + a^
3*Log[a*Cos[c + d*x]])/(a^2*d))
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.4.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
parts	$-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$\frac{a\left(12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-11+\cos(3dx+3c)+3\cos(2dx+2c)-9\cos(dx$
risch	$iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$
norman	$\frac{-\frac{4a}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{6a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c
))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx$$
$$=\frac{2\,a\cos(dx+c)^3+3\,a\cos(dx+c)^2-6\,a\cos(dx+c)-6\,a\log(-\cos(dx+c))}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(
-cos(d*x + c)))/d
```

3.4.6 Sympy [F]

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = a\left(\int \sin^3(c+dx)\sec(c+dx)\,dx + \int \sin^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**3,x)
```

output
```
a*(Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**3, x
))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))\sin^3(c+dx)\,dx$$
$$= \frac{2\,a\cos(dx+c)^3 + 3\,a\cos(dx+c)^2 - 6\,a\cos(dx+c) - 6\,a\log(\cos(dx+c))}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(
cos(d*x + c)))/d
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (a + a\sec(c+dx))\sin^3(c+dx)\,dx$$
$$= -\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{2\,ad^2\cos(dx+c)^3 + 3\,ad^2\cos(dx+c)^2 - 6\,ad^2\cos(dx+c)}{6\,d^3}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-a*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*a*d^2
*cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c + dx))\sin^3(c + dx)\,dx$$
$$= -\frac{a\cos(c + dx) - \frac{a\cos(c+dx)^2}{2} - \frac{a\cos(c+dx)^3}{3} + a\ln(\cos(c + dx))}{d}$$

input
```
int(sin(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - (a*cos(c + d*x)^2)/2 - (a*cos(c + d*x)^3)/3 + a*log(cos
(c + d*x)))/d
```

3.5 $\int (a + a\sec(c + dx))\sin(c + dx)\,dx$

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3.5.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int (a + a\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\cos(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

output
```
-a*cos(d*x+c)/d-a*ln(cos(d*x+c))/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c)\cos(dx)}{d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\sin(c)\sin(dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

output
```
-((a*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3186, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4359$$

$$\int -(\tan(c+dx)(a(-\cos(c+dx))-a))dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\tan(c+dx))dx$$

$$\downarrow 25$$

$$\int \tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\tan\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3186$$

$$-\frac{\int \frac{(\cos(c+dx)a+a)\sec(c+dx)}{a}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{\int(\sec(c+dx)+1)d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a\cos(c+dx)+a\log(a\cos(c+dx))}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

output
```
-((a*Cos[c + d*x] + a*Log[a*Cos[c + d*x]])/d)
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.5.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
default	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
parts	$-\frac{a\cos(dx+c)}{d}+\frac{a\ln(\sec(dx+c))}{d}$	26
risch	$iax+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{a\cos(dx+c)}{d}$	45
parallelrisch	$-\frac{a\left(-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+1+\cos(dx+c)\right)}{d}$	53
norman	$-\frac{2a}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	78

input
```
int((a+a*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(-1/sec(d*x+c)+ln(sec(d*x+c)))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)+a\log(-\cos(dx+c))}{d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c) + a*log(-cos(d*x + c)))/d
```

3.5.6 Sympy [F]

$$\int (a + a \sec(c + dx)) \sin(c + dx)\, dx = a\left(\int \sin(c + dx) \sec(c + dx)\, dx + \int \sin(c + dx)\, dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c),x)
```

output
```
a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x), x))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a + a \sec(c + dx)) \sin(c + dx)\, dx = -\frac{a \cos(dx + c) + a \log(\cos(dx + c))}{d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c) + a*log(cos(d*x + c)))/d
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (a + a \sec(c + dx)) \sin(c + dx)\, dx = -\frac{a \cos(dx + c)}{d} - \frac{a \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

output
```
-a*cos(d*x + c)/d - a*log(abs(cos(d*x + c))/abs(d))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\left(\cos\left(c + d\,x\right) + \ln\left(\cos\left(c + d\,x\right)\right)\right)}{d}$$

input
```
int(sin(c + d*x)*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(cos(c + d*x) + log(cos(c + d*x))))/d
```

3.6 $\int \csc(c+dx)(a+a\sec(c+dx))\,dx$

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3.6.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

output
```
a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\sin(c+dx))}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Log[Cos[c/2 + (d*x)/2]])/d) - (a*Log[Cos[c + d*x]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d + (a*Log[Sin[c + d*x]])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -(\csc(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a))dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\csc(c+dx)\sec(c+dx))dx$$

$$\downarrow 25$$

$$\int \csc(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$\frac{a\int -\frac{\sec(c+dx)}{a-a\cos(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a\int \frac{\sec(c+dx)}{a-a\cos(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^2 \int \frac{\sec(c+dx)}{a(a-a\cos(c+dx))} d(a\cos(c+dx))}{d}$$

$$\downarrow 47$$

$$-\frac{a^2\left(\frac{\int \frac{1}{a-a\cos(c+dx)} d(a\cos(c+dx))}{a} + \frac{\int \frac{\sec(c+dx)}{a} d(a\cos(c+dx))}{a}\right)}{d}$$

$$\downarrow 14$$

$$-\frac{a^2\left(\frac{\int \frac{1}{a-a\cos(c+dx)} d(a\cos(c+dx))}{a} + \frac{\log(a\cos(c+dx))}{a}\right)}{d}$$

$$\downarrow 16$$

$$-\frac{a^2\left(\frac{\log(a\cos(c+dx))}{a} - \frac{\log(a-a\cos(c+dx))}{a}\right)}{d}$$

input
```
Int[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
-((a^2*(Log[a*Cos[c + d*x]]/a - Log[a - a*Cos[c + d*x]]/a))/d)
```

3.6.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.6.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{a\ln(\tan(dx+c))+a\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
default	$\frac{a\ln(\tan(dx+c))+a\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
risch	$\frac{2a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	38
parallelrisch	$\frac{a\left(2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$	47
norman	$\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	54

input
```
int(csc(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(tan(d*x+c))+a*ln(-cot(d*x+c)+csc(d*x+c)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log\left(-\cos\left(dx+c\right)\right) - a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*log(-cos(d*x + c)) - a*log(-1/2*cos(d*x + c) + 1/2))/d
```

3.6.6 Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc\left(c+dx\right)\sec\left(c+dx\right)\,dx + \int \csc\left(c+dx\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x), x))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\cos\left(dx+c\right)-1\right) - a\log\left(\cos\left(dx+c\right)\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(cos(d*x + c) - 1) - a*log(cos(d*x + c)))/d
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}(1-2\,\cos(c+d\,x))}{d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x),x)
```

output
```
(2*a*atanh(1 - 2*cos(c + d*x)))/d
```

3.7 $\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$

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3.7.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^2}{2d(a-a\cos(c+dx))} + \frac{3a\log(1-\cos(c+dx))}{4d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log(1+\cos(c+dx))}{4d}$$

output
```
-1/2*a^2/d/(a-a*cos(d*x+c))+3/4*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+1/
4*a*ln(1+cos(d*x+c))/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log(\sin(c+dx))}{d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
-1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Cos[(c +
 d*x)/2]])/(2*d) - (a*Log[Cos[c + d*x]])/d + (a*Log[Sin[(c + d*x)/2]])/(2*
d) + (a*Log[Sin[c + d*x]])/d + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^3(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\csc^3(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^3(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$\frac{a^3 \int -\frac{\sec(c+dx)}{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)} d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^3 \int \frac{\sec(c+dx)}{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)} d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^4 \int \frac{\sec(c+dx)}{a(a-a\cos(c+dx))^2(\cos(c+dx)a+a)} d(a\cos(c+dx))}{d}$$

$$\downarrow 93$$

$$-\frac{a^4 \int \left(\frac{\sec(c+dx)}{a^4} + \frac{3}{4a^3(a-a\cos(c+dx))} - \frac{1}{4a^3(\cos(c+dx)a+a)} + \frac{1}{2a^2(a-a\cos(c+dx))^2}\right) d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a^4\left(\frac{\log(a\cos(c+dx))}{a^3} - \frac{3\log(a-a\cos(c+dx))}{4a^3} - \frac{\log(a\cos(c+dx)+a)}{4a^3} + \frac{1}{2a^2(a-a\cos(c+dx))}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
-((a^4*(1/(2*a^2*(a - a*Cos[c + d*x])) + Log[a*Cos[c + d*x]]/a^3 - (3*Log[
a - a*Cos[c + d*x]])/(4*a^3) - Log[a + a*Cos[c + d*x]]/(4*a^3)))/d)
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.7.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{4d}$	59
derivativedivides	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$	61
default	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$	61
norman	$-\frac{a}{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	71
risch	$\frac{a\,e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}+\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	83

input
```
int(csc(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*(cot(1/2*d*x+1/2*c)^2-6*ln(tan(1/2*d*x+1/2*c))+4*ln(tan(1/2*d*x+1/2
*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))/d
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{4\,(a\cos(dx+c)-a)\log(-\cos(dx+c))-(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\,(a\cos(dx$$
$$4\,(d\cos(dx+c)-d)$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(4*(a*cos(d*x + c) - a)*log(-cos(d*x + c)) - (a*cos(d*x + c) - a)*log
(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) +
1/2) - 2*a)/(d*cos(d*x + c) - d)
```

3.7.6 Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^3(c+dx)\sec(c+dx)\,dx + \int \csc^3(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**3, x
))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\log(\cos(dx+c)+1)+3\,a\log(\cos(dx+c)-1)-4\,a\log(\cos(dx+c))+\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 4*a*log(cos(d*x
 + c)) + 2*a/(cos(d*x + c) - 1))/d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a - 3*a*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{\frac{a}{2\,(\cos(c+d\,x)-1)} - a\,\ln\left(\cos\left(c+d\,x\right)\right) + \frac{3\,a\,\ln(\cos(c+d\,x)-1)}{4} + \frac{a\,\ln(\cos(c+d\,x)+1)}{4}}{d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^3,x)
```

output
```
(a/(2*(cos(c + d*x) - 1)) - a*log(cos(c + d*x)) + (3*a*log(cos(c + d*x) -
1))/4 + (a*log(cos(c + d*x) + 1))/4)/d
```

3.8 $\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$

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3.8.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^3}{8d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} - \frac{a^2}{8d(a+a\cos(c+dx))} + \frac{11a\log(1-\cos(c+dx))}{16d} - \frac{a\log(\cos(c+dx))}{d} + \frac{5a\log(1+\cos(c+dx))}{16d}$$

output

```
-1/8*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a*cos(d*x+c))-1/8*a^2/d/(a+a*co
s(d*x+c))+11/16*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+5/16*a*ln(1+cos(d*
x+c))/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.49

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^4(c+dx)}{4d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{a\log(\cos(c+dx))}{d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\sin(c+dx))}{d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

output
```
(-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[
c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^4)/(4*d) - (3*a*Log[Cos[(c + d*x)/2]])
/(8*d) - (a*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (a*
Log[Sin[c + d*x]])/d + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/
2]^4)/(64*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^5(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\csc^5(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3315$$

$$\frac{a^5\int -\frac{\sec(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^5\int \frac{\sec(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^6\int \frac{\sec(c+dx)}{a(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^6\int \left(\frac{\sec(c+dx)}{a^6}+\frac{11}{16a^5(a-a\cos(c+dx))}-\frac{5}{16a^5(\cos(c+dx)a+a)}+\frac{1}{2a^4(a-a\cos(c+dx))^2}-\frac{1}{8a^4(\cos(c+dx)a+a)^2}+\frac{1}{4a^3(a-a\cos(c+d}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^6\left(\frac{\log(a\cos(c+dx))}{a^5}-\frac{11\log(a-a\cos(c+dx))}{16a^5}-\frac{5\log(a\cos(c+dx)+a)}{16a^5}+\frac{1}{2a^4(a-a\cos(c+dx))}+\frac{1}{8a^4(a\cos(c+dx)+a)}+\frac{1}{8a^3(a-a\cos(c-}\right.}{d}$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

output
```
-((a^6*(1/(8*a^3*(a - a*Cos[c + d*x])^2) + 1/(2*a^4*(a - a*Cos[c + d*x]))
+ 1/(8*a^4*(a + a*Cos[c + d*x])) + Log[a*Cos[c + d*x]]/a^5 - (11*Log[a - a
*Cos[c + d*x]])/(16*a^5) - (5*Log[a + a*Cos[c + d*x]])/(16*a^5)))/d)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.8.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c}{8}\right.}{d}$
default	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c}{8}\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-44\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\ln\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{32d}$
norman	$\frac{-\frac{a}{32d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{11a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$\frac{a\left(3e^{5i(dx+c)}+2e^{4i(dx+c)}-18e^{3i(dx+c)}+2e^{2i(dx+c)}+3e^{i(dx+c)}\right)}{4d\left(e^{i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+1\right)^2}+\frac{11a\ln\left(e^{i(dx+c)}-1\right)}{8d}+\frac{5a\ln\left(e^{i(dx+c)}+1\right)}{8d}-\frac{a}{}$

input
```
int(csc(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*((-1/4*csc(d*
x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.64

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{6\,a\cos(dx+c)^2+2\,a\cos(dx+c)-16\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c)+a\right)\log(-\cos$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(6*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - 16*(a*cos(d*x + c)^3 - a*cos
(d*x + c)^2 - a*cos(d*x + c) + a)*log(-cos(d*x + c)) + 5*(a*cos(d*x + c)^3
 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) + 11
*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d
*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x +
c) + d)
```

3.8.6 Sympy [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^5(c+dx)\sec(c+dx)\,dx + \int \csc^5(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**5, x
))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-16\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(3\,a\cos(dx+c)^2+a\cos(dx+c)-\right.}{\cos(dx+c)^3-\cos(dx+c)^2-\cos(dx}}{16\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 16*a*log(co
s(d*x + c)) + 2*(3*a*cos(d*x + c)^2 + a*cos(d*x + c) - 6*a)/(cos(d*x + c)^
3 - cos(d*x + c)^2 - cos(d*x + c) + 1))/d
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a-\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}+}{32\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a - 10*a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 2*a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{11\,a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d} - \frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d} + \frac{5\,a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d} + \frac{\frac{3\,a\cos(c+d\,x)^2}{8}+\frac{a\,\cos(c+d\,x)}{8}-\frac{3\,a}{4}}{d\,\left(\cos\left(c+d\,x\right)^3-\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)^2\right)}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^5,x)
```

output
```
(11*a*log(cos(c + d*x) - 1))/(16*d) - (a*log(cos(c + d*x)))/d + (5*a*log(c
os(c + d*x) + 1))/(16*d) + ((a*cos(c + d*x))/8 - (3*a)/4 + (3*a*cos(c + d*
x)^2)/8)/(d*(cos(c + d*x)^3 - cos(c + d*x) + sin(c + d*x)^2))
```

3.9 $\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$

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3.9.1 Optimal result

Integrand size = 19, antiderivative size = 163

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^4}{24d(a-a\cos(c+dx))^3} - \frac{5a^3}{32d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} - \frac{a^3}{32d(a+a\cos(c+dx))^2} - \frac{3a^2}{16d(a+a\cos(c+dx))} + \frac{21a\log(1-\cos(c+dx))}{32d} - \frac{a\log(\cos(c+dx))}{d} + \frac{11a\log(1+\cos(c+dx))}{32d}$$

output

```
-1/24*a^4/d/(a-a*cos(d*x+c))^3-5/32*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-
a*cos(d*x+c))-1/32*a^3/d/(a+a*cos(d*x+c))^2-3/16*a^2/d/(a+a*cos(d*x+c))+21
/32*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+11/32*a*ln(1+cos(d*x+c))/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = &-\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}\\ &- \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\csc^2(c+dx)}{2d}\\ &- \frac{a\csc^4(c+dx)}{4d} - \frac{a\csc^6(c+dx)}{6d}\\ &- \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}\\ &- \frac{a\log(\cos(c+dx))}{d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}\\ &+ \frac{a\log(\sin(c+dx))}{d} + \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}\\ &+ \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

output
```
(-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[
(c + d*x)/2]^6)/(384*d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^4)/(4
*d) - (a*Csc[c + d*x]^6)/(6*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (a*L
og[Cos[c + d*x]])/d + (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Log[Sin[c +
d*x]])/d + (5*a*Sec[(c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
 + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{a - a\csc\left(c + dx - \frac{\pi}{2}\right)}{\cos\left(c + dx - \frac{\pi}{2}\right)^7} dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^7(c + dx)\sec(c + dx)(a(-\cos(c + dx)) - a)\right) dx$$

$$\downarrow 25$$

$$-\int -\left(\left(\cos(c + dx)a + a\right)\csc^7(c + dx)\sec(c + dx)\right) dx$$

$$\downarrow 25$$

$$\int \csc^7(c + dx)\sec(c + dx)(a\cos(c + dx) + a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a - a\sin\left(c + dx - \frac{\pi}{2}\right)}{\sin\left(c + dx - \frac{\pi}{2}\right)\cos\left(c + dx - \frac{\pi}{2}\right)^7} dx$$

$$\downarrow 25$$

$$-\int \frac{a - a\sin\left(\frac{1}{2}(2c - \pi) + dx\right)}{\cos\left(\frac{1}{2}(2c - \pi) + dx\right)^7 \sin\left(\frac{1}{2}(2c - \pi) + dx\right)} dx$$

$$\downarrow 3315$$

$$\frac{a^7 \int -\frac{\sec(c+dx)}{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^3} d(a\cos(c + dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^7 \int \frac{\sec(c+dx)}{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^3} d(a\cos(c + dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^8 \int \frac{\sec(c+dx)}{a(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^3} d(a\cos(c + dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^8 \int \left(\frac{\sec(c+dx)}{a^8} + \frac{21}{32a^7(a-a\cos(c+dx))} - \frac{11}{32a^7(\cos(c+dx)a+a)} + \frac{1}{2a^6(a-a\cos(c+dx))^2} - \frac{3}{16a^6(\cos(c+dx)a+a)^2} + \frac{5}{16a^5(a-a\cos(c+\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^8\left(\frac{\log(a\cos(c+dx))}{a^7}-\frac{21\log(a-a\cos(c+dx))}{32a^7}-\frac{11\log(a\cos(c+dx)+a)}{32a^7}+\frac{1}{2a^6(a-a\cos(c+dx))}+\frac{3}{16a^6(a\cos(c+dx)+a)}+\frac{5}{32a^5(a-a\co}{d}$$

input
```
Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

output
```
-((a^8*(1/(24*a^4*(a - a*Cos[c + d*x])^3) + 5/(32*a^5*(a - a*Cos[c + d*x])
^2) + 1/(2*a^6*(a - a*Cos[c + d*x])) + 1/(32*a^5*(a + a*Cos[c + d*x])^2) +
 3/(16*a^6*(a + a*Cos[c + d*x])) + Log[a*Cos[c + d*x]]/a^7 - (21*Log[a - a
*Cos[c + d*x]])/(32*a^7) - (11*Log[a + a*Cos[c + d*x]])/(32*a^7)))/d)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.9.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{a\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c\right.}{d}$
default	$\frac{a\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{21\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+66\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-252\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+192\right.}{192d}$
norman	$\frac{-\frac{a}{192d}-\frac{7a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{128d}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32d}-\frac{7a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{128d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}+\frac{21a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{a\ln\left(\tan\left(\right.\right.}$
risch	$\frac{a\left(15\,e^{9i(dx+c)}+18\,e^{8i(dx+c)}-136\,e^{7i(dx+c)}-34\,e^{6i(dx+c)}+402\,e^{5i(dx+c)}-34\,e^{4i(dx+c)}-136\,e^{3i(dx+c)}+18\,e^{2i(dx+c)}+15\right.}{24d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^4}$

input
```
int(csc(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c))
)+a*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16
*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(152) = 304$.

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.88

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{30\,a\cos(dx+c)^4+18\,a\cos(dx+c)^3-98\,a\cos(dx+c)^2-22\,a\cos(dx+c)-96\left(a\cos(dx+c)^5-a\cos\right.}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(30*a*cos(d*x + c)^4 + 18*a*cos(d*x + c)^3 - 98*a*cos(d*x + c)^2 - 22
*a*cos(d*x + c) - 96*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x +
c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-cos(d*x + c)) + 33*(a
*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)
^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 63*(a*cos(d*x + c)^
5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x
 + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^5 - d*cos(
d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-96\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(15\,a\cos(dx+c)^4+9\,a\cos(dx+\right.}{\cos(dx+c)^5-\cos(dx+c)^4-2}}{96\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 96*a*log(c
os(d*x + c)) + 2*(15*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^3 - 49*a*cos(d*x
+ c)^2 - 11*a*cos(d*x + c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*co
s(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c) - 1))/d
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{\left(2\,a - \frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{462\,a(\cos(dx+c)}{(\cos(dx+c)+}\right.}{(\cos(dx+c)-1)^3}}{384\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a - 21*a*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^
3/(cos(d*x + c) - 1)^3 + 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$

$$= \frac{\frac{5\,a\cos(c+d\,x)^4}{16} + \frac{3\,a\cos(c+d\,x)^3}{16} - \frac{49\,a\cos(c+d\,x)^2}{48} - \frac{11\,a\,\cos(c+d\,x)}{48} + \frac{11\,a}{12}}{d\,\left(\cos(c+d\,x)^5 - \cos(c+d\,x)^4 - 2\cos(c+d\,x)^3 + 2\cos(c+d\,x)^2 + \cos(c+d\,x) - 1\right)} - \frac{a\,\ln(\cos(c+d\,x))}{d} + \frac{21\,a\,\ln(\cos(c+d\,x)-1)}{32\,d} + \frac{11\,a\,\ln(\cos(c+d\,x)+1)}{32\,d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^7,x)
```

output
```
((11*a)/12 - (11*a*cos(c + d*x))/48 - (49*a*cos(c + d*x)^2)/48 + (3*a*cos(
c + d*x)^3)/16 + (5*a*cos(c + d*x)^4)/16)/(d*(cos(c + d*x) + 2*cos(c + d*x
)^2 - 2*cos(c + d*x)^3 - cos(c + d*x)^4 + cos(c + d*x)^5 - 1)) - (a*log(co
s(c + d*x)))/d + (21*a*log(cos(c + d*x) - 1))/(32*d) + (11*a*log(cos(c + d
*x) + 1))/(32*d)
```

3.10 $\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx$

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3.10.1 Optimal result

Integrand size = 19, antiderivative size = 165

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx = \frac{35ax}{128} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} - \frac{a\sin^5(c+dx)}{5d} - \frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\sin^7(c+dx)}{7d} - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d}$$

output

```
35/128*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-35/128*a*cos(d*x+c)*sin(
d*x+c)/d-1/3*a*sin(d*x+c)^3/d-35/192*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin
(d*x+c)^5/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d-1/8*a*co
s(d*x+c)*sin(d*x+c)^7/d
```

3.10.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx$$
$$= \frac{a(107520\operatorname{arctanh}(\sin(c+dx)) - 107520\sin(c+dx) - 35840\sin^3(c+dx) - 21504\sin^5(c+dx) - 15360\sin$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

output
```
(a*(107520*ArcTanh[Sin[c + d*x]] - 107520*Sin[c + d*x] - 35840*Sin[c + d*x
]^3 - 21504*Sin[c + d*x]^5 - 15360*Sin[c + d*x]^7 + 35*(840*c + 840*d*x -
672*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] - 32*Sin[6*(c + d*x)] + 3*Sin[
8*(c + d*x)])))/(107520*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 254, 2009, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$
$$\downarrow 4360$$
$$\int -(\sin^7(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))\,dx$$
$$\downarrow 25$$
$$-\int -((\cos(c+dx)a+a)\sin^7(c+dx)\tan(c+dx))\,dx$$
$$\downarrow 25$$

$$\int \sin^7(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^8\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^8(c+dx)dx + a\int \sin^7(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^8dx + a\int \sin(c+dx)^7\tan(c+dx)dx$$

$$\downarrow 3072$$

$$\frac{a\int \frac{\sin^8(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d} + a\int \sin(c+dx)^8dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\sin^6(c+dx)-\sin^4(c+dx)-\sin^2(c+dx)+\frac{1}{1-\sin^2(c+dx)}-1\right)d\sin(c+dx)}{d} + a\int \sin(c+dx)^8dx$$

$$\downarrow 2009$$

$$a\int \sin(c+dx)^8dx + \frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{7}\sin^7(c+dx)-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{7}{8}\int \sin^6(c+dx)dx - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{7}\sin^7(c+dx)-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{7}{8}\int \sin(c+dx)^6dx - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{7}\sin^7(c+dx)-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$\frac{a\left(\frac{7}{8}\left(\frac{5}{6}\left(\int \sin^4(c+dx)dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}}{d}$$

$$\downarrow 3042$$

$$\frac{a\left(\frac{7}{8}\left(\frac{5}{6}\left(\int \sin(c+dx)^4dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(c+dx)^2dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\sin^7(c+dx)\cos(c+dx)}{8d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \sin^7(\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 24$$

$$\frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{7}\sin^7(c+dx) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d} + a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \sin^7(c+\right)$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

output
```
(a*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3 - Sin[c + d*x]
^5/5 - Sin[c + d*x]^7/7))/d + a*(-1/8*(Cos[c + d*x]*Sin[c + d*x]^7)/d + (7
*(-1/6*(Cos[c + d*x]*Sin[c + d*x]^5)/d + (5*(-1/4*(Cos[c + d*x]*Sin[c + d*
x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6))/8)
```

3.10.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.10.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35}{6}\right.}{d}\right.}$
default	$\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35}{6}\right.}{d}\right.}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\right.}$
parallelrisch	$-\frac{a\left(-\frac{105dx}{4}+\sin(6dx+6c)-\frac{3\sin(7dx+7c)}{14}-\frac{3\sin(8dx+8c)}{32}+96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{279\sin(dx+}{2}\right.}{96d}$
risch	$\frac{35ax}{128}-\frac{93ia\,e^{-i(dx+c)}}{128d}+\frac{93ia\,e^{i(dx+c)}}{128d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a\sin(8dx+8c)}{1024d}+\frac{a\sin(7dx+}{448d}$
norman	$\frac{35ax}{128}-\frac{163a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{1335a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64d}-\frac{24223a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{320d}-\frac{359453a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2240d}-\frac{724649a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6720d}-\frac{45859a}{}$

input

```
int((a+a*sec(d*x+c))*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c)))+a*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d
*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx$$
$$= \frac{3675\,adx + 6720\,a\log(\sin(dx+c)+1) - 6720\,a\log(-\sin(dx+c)+1) + (1680\,a\cos(dx+c)^7 + 1920\,a$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/13440*(3675*a*d*x + 6720*a*log(sin(d*x + c) + 1) - 6720*a*log(-sin(d*x +
 c) + 1) + (1680*a*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 - 7000*a*cos(d*x
 + c)^5 - 8448*a*cos(d*x + c)^4 + 11410*a*cos(d*x + c)^3 + 15616*a*cos(d*x
 + c)^2 - 9765*a*cos(d*x + c) - 22528*a)*sin(d*x + c))/d
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**8,x)
```

output
```
Timed out
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx =$$
$$-\frac{512\,(30\sin(dx+c)^7 + 42\sin(dx+c)^5 + 70\sin(dx+c)^3 - 105\log(\sin(dx+c)+1) + 105\log(\sin($$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="maxima")
```

output
```
-1/107520*(512*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c)
)*a - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) +
168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a)/d
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

$$\int (a + a\sec(c + dx))\sin^8(c + dx)\,dx$$

$$= \frac{3675\,(dx + c)a + 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(9765\,a\tan(\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="giac")
```

output
```
1/13440*(3675*(d*x + c)*a + 13440*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 1
3440*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a*tan(1/2*d*x + 1/2*c)
^15 + 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 +
 724649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 + 5086
83*a*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 + 17115*a*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91

$$\int (a + a\sec(c + dx))\sin^8(c + dx)\,dx = \frac{35\,a\,x}{128} + \frac{2\,a\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{7\,a\,\sin(2\,c + 2\,d\,x)}{32\,d} + \frac{37\,a\,\sin(3\,c + 3\,d\,x)}{192\,d} + \frac{7\,a\,\sin(4\,c + 4\,d\,x)}{128\,d} - \frac{9\,a\,\sin(5\,c + 5\,d\,x)}{320\,d} - \frac{a\,\sin(6\,c + 6\,d\,x)}{96\,d} + \frac{a\,\sin(7\,c + 7\,d\,x)}{448\,d} + \frac{a\,\sin(8\,c + 8\,d\,x)}{1024\,d} - \frac{93\,a\,\sin(c + d\,x)}{64\,d}$$

input
```
int(sin(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

output
```
(35*a*x)/128 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a
*sin(2*c + 2*d*x))/(32*d) + (37*a*sin(3*c + 3*d*x))/(192*d) + (7*a*sin(4*c
 + 4*d*x))/(128*d) - (9*a*sin(5*c + 5*d*x))/(320*d) - (a*sin(6*c + 6*d*x))
/(96*d) + (a*sin(7*c + 7*d*x))/(448*d) + (a*sin(8*c + 8*d*x))/(1024*d) - (
93*a*sin(c + d*x))/(64*d)
```

3.11 $\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx$

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3.11.1 Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5ax}{16} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin^3(c+dx)}{3d} - \frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\sin^5(c+dx)}{5d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d}$$

output
```
5/16*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+
c)/d-1/3*a*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d*x+c
)^5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{a(960\operatorname{arctanh}(\sin(c+dx)) - 960\sin(c+dx) - 320\sin^3(c+dx) - 192\sin^5(c+dx) + 5(60c+60dx-45$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

output
```
(a*(960*ArcTanh[Sin[c + d*x]] - 960*Sin[c + d*x] - 320*Sin[c + d*x]^3 - 19
2*Sin[c + d*x]^5 + 5*(60*c + 60*d*x - 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d
*x)] - Sin[6*(c + d*x)])))/(960*d)
```

3.11.3 Rubi [A] (verified)

Time $= 0.58$ (sec) , antiderivative size $= 125$, normalized size of antiderivative $= 0.98$, number of steps used $= 17$, number of rules used $= 16$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used $= \{3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 254, 2009, 3115, 3042, 3115, 3042, 3115, 24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -\left(\sin^5(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\sin^5(c+dx)\tan(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \sin^5(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^6(c+dx)dx + a\int \sin^5(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^6dx + a\int \sin(c+dx)^5\tan(c+dx)dx$$

$$\downarrow 3072$$

$$\frac{a\int \frac{\sin^6(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d} + a\int \sin(c+dx)^6dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\sin^4(c+dx)-\sin^2(c+dx)+\frac{1}{1-\sin^2(c+dx)}-1\right)d\sin(c+dx)}{d} + a\int \sin(c+dx)^6dx$$

$$\downarrow 2009$$

$$a\int \sin(c+dx)^6dx + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\int \sin^4(c+dx)dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\int \sin(c+dx)^4dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(c+dx)^2dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{a\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)+\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 24$$

$$\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

output
```
(a*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3 - Sin[c + d*x]
^5/5))/d + a*(-1/6*(Cos[c + d*x]*Sin[c + d*x]^5)/d + (5*(-1/4*(Cos[c + d*x
]*Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.11.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a  Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d  Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.11.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos$
default	$\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec$
parallelrisch	$-\frac{a\left(-60dx+\sin(6dx+6c)+192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+264\sin(dx+c)+45\sin(2dx+2c)-28\sin\right.}{192d}$
risch	$\frac{5ax}{16}+\frac{11ia\,e^{i(dx+c)}}{16d}-\frac{11ia\,e^{-i(dx+c)}}{16d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\sin(5dx+5}{80d}$
norman	$\frac{5ax}{16}-\frac{21a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{389a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24d}-\frac{853a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20d}-\frac{523a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20d}-\frac{73a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{8d}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c)))+a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx$$
$$= \frac{75\,adx + 120\,a\log(\sin(dx+c)+1) - 120\,a\log(-\sin(dx+c)+1) - (40\,a\cos(dx+c)^5 + 48\,a\cos(dx+$$
$$240\,d$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(75*a*d*x + 120*a*log(sin(d*x + c) + 1) - 120*a*log(-sin(d*x + c) +
1) - (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 1
76*a*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*a)*sin(d*x + c))/d
```

3.11.6 Sympy [F]

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = a\left(\int \sin^6(c+dx)\sec(c+dx)\,dx + \int \sin^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**6,x)
```

output
```
a*(Integral(sin(c + d*x)**6*sec(c + d*x), x) + Integral(sin(c + d*x)**6, x
))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx =$$

$$-\frac{32\left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 30\sin(dx+c)\right)a - 5\left(4\sin(2dx+2c)^3 + 60dx + 60c + 9\sin(4dx+4c) - 48\sin(2dx+2c)\right)a}{960\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
-1/960*(32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1
) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx$$

$$= \frac{75\,(dx+c)a + 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 1095\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 3138\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} + 5118\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 1945\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(165*a*tan(1/2*d*x + 1/2*c)^11 + 109
5*a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 + 5118*a*tan(1/
2*d*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5\,a\,x}{16} + \frac{2\,a\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{15\,a\,\sin(2\,c+2\,d\,x)}{64\,d} + \frac{7\,a\,\sin(3\,c+3\,d\,x)}{48\,d} + \frac{3\,a\,\sin(4\,c+4\,d\,x)}{64\,d} - \frac{a\,\sin(5\,c+5\,d\,x)}{80\,d} - \frac{a\,\sin(6\,c+6\,d\,x)}{192\,d} - \frac{11\,a\,\sin(c+d\,x)}{8\,d}$$

input
```
int(sin(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

output
```
(5*a*x)/16 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (15*a*
sin(2*c + 2*d*x))/(64*d) + (7*a*sin(3*c + 3*d*x))/(48*d) + (3*a*sin(4*c +
4*d*x))/(64*d) - (a*sin(5*c + 5*d*x))/(80*d) - (a*sin(6*c + 6*d*x))/(192*d
) - (11*a*sin(c + d*x))/(8*d)
```

3.12 $\int (a + a\sec(c+dx))\sin^4(c+dx)\,dx$

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3.12.1 Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (a + a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3ax}{8} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
3/8*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)
/d-1/3*a*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3a(c+dx)}{8d} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

output
```
(3*a*(c + d*x))/(8*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d -
 (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x
)])/(32*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 254, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^3(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sin^3(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^3(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^4(c+dx)dx + a\int \sin^3(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^4dx + a\int \sin(c+dx)^3\tan(c+dx)dx$$

$$\downarrow 3072$$

$$\frac{a\int \frac{\sin^4(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d} + a\int \sin(c+dx)^4dx$$

$$\downarrow 254$$

$$\frac{a\int\left(-\sin^2(c+dx)+\frac{1}{1-\sin^2(c+dx)}-1\right)d\sin(c+dx)}{d}+a\int\sin(c+dx)^4dx$$

$$\downarrow 2009$$

$$a\int\sin(c+dx)^4dx+\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{3}{4}\int\sin^2(c+dx)dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{3}{4}\int\sin(c+dx)^2dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 24$$

$$\frac{a\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+a\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

output
```
(a*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3))/d + a*(-1/4*
(Cos[c + d*x]*Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2
*d)))/4)
```

3.12.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.12.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$-\frac{a\left(-36dx+120\sin(dx+c)-96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24\sin(2dx+2c)-3\sin(4dx+4c)-8\sin(3\right.}{96d}$
risch	$\frac{3ax}{8}+\frac{5ia\,e^{i(dx+c)}}{8d}-\frac{5ia\,e^{-i(dx+c)}}{8d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}$
norman	$\frac{\frac{3ax}{8}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{137a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}-\frac{71a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3a}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/4*(s
in(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int (a + a\sec(c+dx))\sin^4(c+dx)\,dx$$
$$= \frac{9\,adx + 12\,a\log(\sin(dx+c)+1) - 12\,a\log(-\sin(dx+c)+1) + \left(6\,a\cos(dx+c)^3 + 8\,a\cos(dx+c)^2 -\right.}{24\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + 12*a*log(sin(d*x + c) + 1) - 12*a*log(-sin(d*x + c) + 1) +
 (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*a)*sin(
d*x + c))/d
```

3.12.6 Sympy [F]

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = a\left(\int \sin^4(c+dx)\sec(c+dx)\,dx + \int \sin^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**4,x)
```

output
```
a*(Integral(sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**4, x
))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx =$$
$$-\frac{16\left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) + 6\sin(dx+c)\right)a - 3\,(12\,dx+}{96\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/96*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1) + 6*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a)/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx$$
$$= \frac{9\,(dx+c)a + 24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+71\,a\right.}{24\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 + 71*a*tan(
1/2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 + 33*a*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int (a + a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3\,a\,x}{8} + \frac{2\,a\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{a\,\sin(3\,c+3\,d\,x)}{12\,d} + \frac{a\,\sin(4\,c+4\,d\,x)}{32\,d} - \frac{5\,a\,\sin(c+d\,x)}{4\,d}$$

input
```
int(sin(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(
2*c + 2*d*x))/(4*d) + (a*sin(3*c + 3*d*x))/(12*d) + (a*sin(4*c + 4*d*x))/(
32*d) - (5*a*sin(c + d*x))/(4*d)
```

3.13 $\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx$

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3.13.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{ax}{2} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)
/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{a(c+dx)}{2d} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

output
```
(a*(c + d*x))/(2*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (
a*Sin[2*(c + d*x)])/(4*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 262, 219, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin(c+dx)\tan(c+dx)(a(-\cos(c+dx))-a))dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sin(c+dx)\tan(c+dx))dx$$

$$\downarrow 25$$

$$\int \sin(c+dx)\tan(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^2(c+dx)dx + a\int \sin(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^2dx + a\int \sin(c+dx)\tan(c+dx)dx$$

$$\downarrow 3072$$

$$\frac{a\int \frac{\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d} + a\int \sin(c+dx)^2dx$$

$$\downarrow 262$$

$$\frac{a\left(\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)-\sin(c+dx)\right)}{d}+a\int \sin(c+dx)^2dx$$

$$\downarrow 219$$

$$a\int \sin(c+dx)^2dx+\frac{a(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}$$

$$\downarrow 24$$

$$\frac{a(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}+a\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

output
```
(a*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]))/d + a*(x/2 - (Cos[c + d*x]*Sin[
c + d*x])/(2*d))
```

3.13.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.13.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parallelrisch	$-\frac{a\left(-2dx+4\sin(dx+c)+\sin(2dx+2c)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{4d}$
parts	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$\frac{ax}{2}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{ax}{2}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$

input
```
int((a+a*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx$$
$$= \frac{adx + a\log(\sin(dx+c)+1) - a\log(-\sin(dx+c)+1) - (a\cos(dx+c)+2a)\sin(dx+c)}{2d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1) - (a*cos(d
*x + c) + 2*a)*sin(d*x + c))/d
```

3.13.6 Sympy [F]

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = a\left(\int \sin^2(c+dx)\sec(c+dx)\,dx + \int \sin^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)**2,x)
```

output
```
a*(Integral(sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**2, x
))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx$$
$$= \frac{(2\,dx + 2\,c - \sin(2\,dx+2\,c))a + 2\,a(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2\,\sin(dx+c))}{4\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*a*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx$$
$$= \frac{(dx+c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int (a + a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{a\,x}{2} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(sin(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

output
```
(a*x)/2 - (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2
+ (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (2*a*atanh(tan(c/2 + (d*x)/2))
)/d
```

3.14 $\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$

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3.14.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*csc(d*x+c)/d
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\begin{aligned}&\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx \\ &= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},\sin^2(c+dx)\right)}{d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Cot[c + d*x])/d) - (a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Si
n[c + d*x]^2])/d
```

3.14.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 262, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^2(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\csc^2(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^2(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^2(c+dx)dx - a\int -\csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^2(c+dx)dx + a\int \csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^2dx + a\int \csc(c+dx)^2\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^2dx - \frac{a\int -\frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{a\int \frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d} + a\int \csc(c+dx)^2dx$$

$$\downarrow 262$$

$$a\int \csc(c+dx)^2dx - \frac{a\left(\csc(c+dx) - \int \frac{1}{1-\csc^2(c+dx)}d\csc(c+dx)\right)}{d}$$

$$\downarrow 219$$

$$a\int \csc(c+dx)^2dx - \frac{a(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int 1d\cot(c+dx)}{d} - \frac{a(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 24$$

$$-\frac{a(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d} - \frac{a\cot(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Cot[c + d*x])/d) - (a*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x]))/d
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.14.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
default	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
norman	$-\frac{a}{d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	54
risch	$-\frac{2ia}{d\left(e^{i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}$	59
parallelrisch	$\frac{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-1\right)}{d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	64

input
```
int(csc(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a*cot(d*x+c))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\log(\sin(dx+c)+1)\sin(dx+c)-a\log(-\sin(dx+c)+1)\sin(dx+c)-2a\cos(dx+c)-2a}{2d\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*log(sin(d*x + c) + 1)*sin(d*x + c) - a*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 2*a*cos(d*x + c) - 2*a)/(d*sin(d*x + c))
```

3.14.6 Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**2, x
))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2a}{\tan(dx+c)}}{2d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(a*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
 2*a/tan(d*x + c))/d
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - a/tan(1/2*d*x + 1/2*c))/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^2,x)
```

output
```
(a*(2*atanh(tan(c/2 + (d*x)/2)) - cot(c/2 + (d*x)/2)))/d
```

3.15 $\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$

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3.15.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d}$$

output
```
a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-a*csc(d*x+c)/d
-1/3*a*csc(d*x+c)^3/d
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{a\csc^3(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Csc
[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

3.15.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^4(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\csc^4(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^4(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^4(c+dx)dx - a\int -\csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^4(c+dx)dx + a\int \csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^4dx + a\int \csc(c+dx)^4\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^4dx - \frac{a\int -\frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{a\int \frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d} + a\int \csc(c+dx)^4dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\csc^2(c+dx) + \frac{1}{1-\csc^2(c+dx)} - 1\right)d\csc(c+dx)}{d} + a\int \csc(c+dx)^4dx$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^4dx - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int \left(\cot^2(c+dx)+1\right)d\cot(c+dx)}{d} - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d} - \frac{a\left(\frac{1}{3}\cot^3(c+dx) + \cot(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + Cot[c + d*x]^3/3))/d) - (a*(-ArcTanh[Csc[c + d*x]] +
Csc[c + d*x] + Csc[c + d*x]^3/3))/d
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.15.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)}{d}$	63
default	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)}{d}$	63
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+12\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}$	69
norman	$\frac{-\frac{a}{12d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	90
risch	$-\frac{2ia\left(3\,e^{3i(dx+c)}-6\,e^{2i(dx+c)}-e^{i(dx+c)}+2\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	107

input
```
int(csc(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-2/3-
1/3*csc(d*x+c)^2)*cot(d*x+c))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.57

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{4\,a\cos(dx+c)^2-3\,(a\cos(dx+c)-a)\log(\sin(dx+c)+1)\sin(dx+c)+3\,(a\cos(dx+c)-a)\log(}{6\,(d\cos(dx+c)-d)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(4*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*si
n(d*x + c) + 3*(a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) +
2*a*cos(d*x + c) - 8*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

3.15.6 Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^4(c+dx)\sec(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**4, x
))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + \frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(a*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 12\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - 3\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - \frac{12\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{12\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 3*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x + 1/2*c)^2 +
a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{12}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^4,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a/12 + a*tan(c/2 + (d*x)/2)^2)/(d*tan
(c/2 + (d*x)/2)^3) - (a*tan(c/2 + (d*x)/2))/(4*d)
```

3.16 $\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$

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3.16.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d}$$

output
```
a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+
c)^5/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^5/d
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{a\csc^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sin^2(c+dx)\right)}{5d}$$

input
```
Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

output
```
(-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a
*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (a*Csc[c + d*x]^5*Hypergeometric2F1[
-5/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^6(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\csc^6(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^6(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^6(c+dx)dx - a\int -\csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^6(c+dx)dx + a\int \csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^6dx + a\int \csc(c+dx)^6\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^6dx - \frac{a\int -\frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{a\int \frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d} + a\int \csc(c+dx)^6dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\csc^4(c+dx) - \csc^2(c+dx) + \frac{1}{1-\csc^2(c+dx)} - 1\right)d\csc(c+dx)}{d} + a\int \csc(c+dx)^6dx$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^6dx - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int \left(\cot^4(c+dx) + 2\cot^2(c+dx) + 1\right)d\cot(c+dx)}{d} - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d} - \frac{a\left(\frac{1}{5}\cot^5(c+dx) + \frac{2}{3}\cot^3(c+dx) + \cot(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + (2*Cot[c + d*x]^3)/3 + Cot[c + d*x]^5/5))/d) - (a*(-A
rcTanh[Csc[c + d*x]] + Csc[c + d*x] + Csc[c + d*x]^3/3 + Csc[c + d*x]^5/5)
)/d
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.16.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c$
default	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c$
parallelrisch	$-\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+80\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-80\ln($
norman	$-\frac{a}{80d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{48d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+$ $\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5$
risch	$-\frac{2ia\left(15\,e^{7i(dx+c)}-30\,e^{6i(dx+c)}-35\,e^{5i(dx+c)}+100\,e^{4i(dx+c)}+13\,e^{3i(dx+c)}-46\,e^{2i(dx+c)}-e^{i(dx+c)}+8\right)}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{i(dx+c)}+1\right)^3}-\frac{a\ln\left(e^{i(dx+}\right.}{d}$

input
```
int(csc(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(
d*x+c)))+a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.88

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{16\,a\cos(dx+c)^4+14\,a\cos(dx+c)^3-54\,a\cos(dx+c)^2-15\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\,co\right.}{3}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/30*(16*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 54*a*cos(d*x + c)^2 - 1
5*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(sin(d*x +
 c) + 1)*sin(d*x + c) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*
x + c) + a)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 16*a*cos(d*x + c) + 46*a
)/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c)
)
```

3.16.6 Sympy [F]

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^6(c+dx)\sec(c+dx)\,dx + \int \csc^6(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(csc(c + d*x)**6*sec(c + d*x), x) + Integral(csc(c + d*x)**6, x
))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{a\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + \frac{2\left(15\tan(dx+c)^4}{\tan(}}{30\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/30*(a*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15
*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4
+ 10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+90\,a\tan$$

$$240\,d$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 - 240*a*log(abs(tan(1/2*d*x + 1/2*c) +
1)) + 240*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 90*a*tan(1/2*d*x + 1/2*c)
 + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1
/2*d*x + 1/2*c)^5)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{48\,d}$$
$$-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(16\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}\right)}{16\,d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^6,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2))/(8*d) - (a*ta
n(c/2 + (d*x)/2)^3)/(48*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + 2*a*tan(c/2 + (d
*x)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4))/(16*d)
```

3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$

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3.17.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

output

```
a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*cot(d*x+c)^3/d-3/5*a*cot(d*x+c)^5
/d-1/7*a*cot(d*x+c)^7/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+
c)^5/d-1/7*a*csc(d*x+c)^7/d
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\begin{aligned}&\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx \\ &= -\frac{16a\cot(c+dx)}{35d} - \frac{8a\cot(c+dx)\csc^2(c+dx)}{35d} \\ &\quad - \frac{6a\cot(c+dx)\csc^4(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} \\ &\quad - \frac{a\csc^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\sin^2(c+dx)\right)}{7d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

output
```
(-16*a*Cot[c + d*x])/(35*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) - (
6*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/
(7*d) - (a*Csc[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, Sin[c + d*x]^2]
)/(7*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^8(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 4360$$

$$\int -(\csc^8(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a))\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\csc^8(c+dx)\sec(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \csc^8(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^8\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^8(c+dx)dx - a\int -\csc^8(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^8(c+dx)dx + a\int \csc^8(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^8dx + a\int \csc(c+dx)^8\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^8dx - \frac{a\int -\frac{\csc^8(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{a\int \frac{\csc^8(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d} + a\int \csc(c+dx)^8dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\csc^6(c+dx)-\csc^4(c+dx)-\csc^2(c+dx)+\frac{1}{1-\csc^2(c+dx)}-1\right)d\csc(c+dx)}{d} + a\int \csc(c+dx)^8dx$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^8dx - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{7}\csc^7(c+dx)+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int\left(\cot^6(c+dx)+3\cot^4(c+dx)+3\cot^2(c+dx)+1\right)d\cot(c+dx)}{d}-\frac{a\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{7}\csc^7(c+dx)+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{7}\csc^7(c+dx)+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}-\frac{a\left(\frac{1}{7}\cot^7(c+dx)+\frac{3}{5}\cot^5(c+dx)+\cot^3(c+dx)+\cot(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + Cot[c + d*x]^3 + (3*Cot[c + d*x]^5)/5 + Cot[c + d*x]^
7/7))/d) - (a*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x] + Csc[c + d*x]^3/3 +
Csc[c + d*x]^5/5 + Csc[c + d*x]^7/7))/d
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.17.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\csc(dx+c)^6}{7}-\frac{6\csc(dx+c}{35}\right.}{d}$
default	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\csc(dx+c)^6}{7}-\frac{6\csc(dx+c}{35}\right.}{d}$
parallelrisch	$-\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\frac{56\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{203\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+448\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+203\tan\left(\frac{dx}{2}\right.\right.}{448d}$
norman	$\frac{-\frac{a}{448d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{40d}-\frac{29a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{192d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{29a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{320d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}+\frac{a\,\mathrm{l}}{}$
risch	$-\frac{2ia(105\,\mathrm{e}^{11i(dx+c)}-210\,\mathrm{e}^{10i(dx+c)}-455\,\mathrm{e}^{9i(dx+c)}+1120\,\mathrm{e}^{8i(dx+c)}+686\,\mathrm{e}^{7i(dx+c)}-2492\,\mathrm{e}^{6i(dx+c)}-274\,\mathrm{e}^{5i(dx+c)}+13}{105d\left(\mathrm{e}^{i(dx+c)}-1\right)^7\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$

input
```
int(csc(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+l
n(sec(d*x+c)+tan(d*x+c)))+a*(-16/35-1/7*csc(d*x+c)^6-6/35*csc(d*x+c)^4-8/3
5*csc(d*x+c)^2)*cot(d*x+c))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(121) = 242$.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.15

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{96\,a\cos(dx+c)^6 + 114\,a\cos(dx+c)^5 - 450\,a\cos(dx+c)^4 - 250\,a\cos(dx+c)^3 + 670\,a\cos(dx+c)^2}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/210*(96*a*cos(d*x + c)^6 + 114*a*cos(d*x + c)^5 - 450*a*cos(d*x + c)^4
- 250*a*cos(d*x + c)^3 + 670*a*cos(d*x + c)^2 - 105*(a*cos(d*x + c)^5 - a*
cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
- a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a*cos(d*x + c)^5 - a*cos(d*
x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*l
og(-sin(d*x + c) + 1)*sin(d*x + c) + 142*a*cos(d*x + c) - 352*a)/((d*cos(d
*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d
*cos(d*x + c) - d)*sin(d*x + c))
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log\left(\sin(dx+c)+1\right)+105\log\left(\sin(dx+c)-1\right)\right)}{210\,d}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/210*(a*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 +
 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1
)) + 6*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a/t
an(d*x + c)^7)/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6720\,a\log\left(\left|\tan\right.\right.}{67}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 + 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720
*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6720*a*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) + 3045*a*tan(1/2*d*x + 1/2*c) + (6720*a*tan(1/2*d*x + 1/2*c)^6 + 1
015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 + 15*a)/tan(1/
2*d*x + 1/2*c)^7)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{29\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{320\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(64\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \frac{29\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3} + \frac{8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{5} + \frac{a}{7}\right)}{64\,d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^8,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (29*a*tan(c/2 + (d*x)/2))/(64*d) - (a*
tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^5)/(320*d) - (cot(c/2
 + (d*x)/2)^7*(a/7 + (8*a*tan(c/2 + (d*x)/2)^2)/5 + (29*a*tan(c/2 + (d*x)/
2)^4)/3 + 64*a*tan(c/2 + (d*x)/2)^6))/(64*d)
```

3.18 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$

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3.18.1 Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d}$$

output

```
a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-4/3*a*cot(d*x+c)^3/d-6/5*a*cot(d*x+
c)^5/d-4/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d-a*csc(d*x+c)/d-1/3*a*csc(
d*x+c)^3/d-1/5*a*csc(d*x+c)^5/d-1/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.82

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{128a\cot(c+dx)}{315d} - \frac{64a\cot(c+dx)\csc^2(c+dx)}{315d} - \frac{16a\cot(c+dx)\csc^4(c+dx)}{105d}$$
$$- \frac{8a\cot(c+dx)\csc^6(c+dx)}{63d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d}$$
$$- \frac{a\csc^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\sin^2(c+dx)\right)}{9d}$$

input
```
Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

output
```
(-128*a*Cot[c + d*x])/(315*d) - (64*a*Cot[c + d*x]*Csc[c + d*x]^2)/(315*d)
 - (16*a*Cot[c + d*x]*Csc[c + d*x]^4)/(105*d) - (8*a*Cot[c + d*x]*Csc[c +
d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) - (a*Csc[c + d*x]^9
*Hypergeometric2F1[-9/2, 1, -7/2, Sin[c + d*x]^2])/(9*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{10}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\downarrow 3042$$
$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}\,dx$$
$$\downarrow 4360$$
$$\int -\left(\csc^{10}(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)\right)dx$$

$$\downarrow 25$$

$$-\int -\left(\left(\cos(c+dx)a+a\right)\csc^{10}(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^{10}(c+dx)\sec(c+dx)(a\cos(c+dx)+a)dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{10}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^{10}(c+dx)dx - a\int -\csc^{10}(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^{10}(c+dx)dx + a\int \csc^{10}(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^{10}dx + a\int \csc(c+dx)^{10}\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^{10}dx - \frac{a\int -\frac{\csc^{10}(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{a\int \frac{\csc^{10}(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d} + a\int \csc(c+dx)^{10}dx$$

$$\downarrow 254$$

$$\frac{a\int \left(-\csc^8(c+dx)-\csc^6(c+dx)-\csc^4(c+dx)-\csc^2(c+dx)+\frac{1}{1-\csc^2(c+dx)}-1\right)d\csc(c+dx)}{d} + a\int \csc(c+dx)^{10}dx$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^{10}dx - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{9}\csc^9(c+dx) + \frac{1}{7}\csc^7(c+dx) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int \left(\cot^8(c+dx) + 4\cot^6(c+dx) + 6\cot^4(c+dx) + 4\cot^2(c+dx) + 1\right) d\cot(c+dx)}{d} - \frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{9}\csc^9(c+dx) + \frac{1}{7}\csc^7(c+dx) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{9}\csc^9(c+dx) + \frac{1}{7}\csc^7(c+dx) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d} - \frac{a\left(\frac{1}{9}\cot^9(c+dx) + \frac{4}{7}\cot^7(c+dx) + \frac{6}{5}\cot^5(c+dx) + \frac{4}{3}\cot^3(c+dx) + \cot(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + (4*Cot[c + d*x]^3)/3 + (6*Cot[c + d*x]^5)/5 + (4*Cot[
c + d*x]^7)/7 + Cot[c + d*x]^9/9))/d) - (a*(-ArcTanh[Csc[c + d*x]] + Csc[c
 + d*x] + Csc[c + d*x]^3/3 + Csc[c + d*x]^5/5 + Csc[c + d*x]^7/7 + Csc[c +
 d*x]^9/9))/d
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.18.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\csc(dx+}{9}\right.}{d}$
default	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\csc(dx+}{9}\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^9+\frac{90\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{414\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+390\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+138\tan\left(\right.\right.}{2304d}$
risch	$-\frac{2ia\left(315\,e^{15i(dx+c)}-630\,e^{14i(dx+c)}-1995\,e^{13i(dx+c)}+4620\,e^{12i(dx+c)}+5103\,e^{11i(dx+c)}-14826\,e^{10i(dx+c)}-6303\,e^{9i(d}\right.}{315d(e}$

input
```
int(csc(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)
^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-128/315-1/9*csc(d*x+c)^8-8/
63*csc(d*x+c)^6-16/105*csc(d*x+c)^4-64/315*csc(d*x+c)^2)*cot(d*x+c))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(149) = 298$.

Time $= 0.27$ (sec) , antiderivative size $= 366$, normalized size of antiderivative $= 2.22$

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{256\,a\cos(dx+c)^8+374\,a\cos(dx+c)^7-1526\,a\cos(dx+c)^6-1204\,a\cos(dx+c)^5+3220\,a\cos(dx+$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/630*(256*a*cos(d*x + c)^8 + 374*a*cos(d*x + c)^7 - 1526*a*cos(d*x + c)^
6 - 1204*a*cos(d*x + c)^5 + 3220*a*cos(d*x + c)^4 + 1316*a*cos(d*x + c)^3
- 2996*a*cos(d*x + c)^2 - 315*(a*cos(d*x + c)^7 - a*cos(d*x + c)^6 - 3*a*c
os(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)
^2 - a*cos(d*x + c) + a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a*cos(d
*x + c)^7 - a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3
*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-sin(d*x
+ c) + 1)*sin(d*x + c) - 496*a*cos(d*x + c) + 1126*a)/((d*cos(d*x + c)^7 -
 d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x
+ c)^3 - 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\left(315\sin(dx+c)^8+105\sin(dx+c)^6+63\sin(dx+c)^4+45\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(d\right.\right.\right.}{630\,d}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/630*(a*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4
+ 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315
*log(sin(d*x + c) - 1)) + 2*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378
*tan(d*x + c)^4 + 180*tan(d*x + c)^2 + 35)*a/tan(d*x + c)^9)/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 + 630*a*tan(1/2*d*x + 1/2*c)^5 + 483
0*a*tan(1/2*d*x + 1/2*c)^3 - 80640*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
80640*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 40950*a*tan(1/2*d*x + 1/2*c)
+ (80640*a*tan(1/2*d*x + 1/2*c)^8 + 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898*
a*tan(1/2*d*x + 1/2*c)^4 + 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*
x + 1/2*c)^9)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{65\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d} - \frac{23\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{1792\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(256\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + \frac{130\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3} + \frac{46\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5} + \frac{10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{7} + \frac{a}{9}\right)}{256\,d}$$

input
```
int((a + a/cos(c + d*x))/sin(c + d*x)^10,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (65*a*tan(c/2 + (d*x)/2))/(128*d) - (2
3*a*tan(c/2 + (d*x)/2)^3)/(384*d) - (a*tan(c/2 + (d*x)/2)^5)/(128*d) - (a*
tan(c/2 + (d*x)/2)^7)/(1792*d) - (cot(c/2 + (d*x)/2)^9*(a/9 + (10*a*tan(c/
2 + (d*x)/2)^2)/7 + (46*a*tan(c/2 + (d*x)/2)^4)/5 + (130*a*tan(c/2 + (d*x)
/2)^6)/3 + 256*a*tan(c/2 + (d*x)/2)^8))/(256*d)
```

3.19 $\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx$

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3.19.1 Optimal result

Integrand size = 21, antiderivative size = 183

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx = {} & \frac{3a^2\cos(c+dx)}{d} + \frac{4a^2\cos^2(c+dx)}{d} \\ & - \frac{2a^2\cos^3(c+dx)}{3d} \\ & - \frac{3a^2\cos^4(c+dx)}{d} - \frac{2a^2\cos^5(c+dx)}{5d} \\ & + \frac{4a^2\cos^6(c+dx)}{3d} + \frac{3a^2\cos^7(c+dx)}{7d} \\ & - \frac{a^2\cos^8(c+dx)}{4d} - \frac{a^2\cos^9(c+dx)}{9d} \\ & - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}\end{aligned}$$

output

```
3*a^2*cos(d*x+c)/d+4*a^2*cos(d*x+c)^2/d-2/3*a^2*cos(d*x+c)^3/d-3*a^2*cos(d
*x+c)^4/d-2/5*a^2*cos(d*x+c)^5/d+4/3*a^2*cos(d*x+c)^6/d+3/7*a^2*cos(d*x+c)
^7/d-1/4*a^2*cos(d*x+c)^8/d-1/9*a^2*cos(d*x+c)^9/d-2*a^2*ln(cos(d*x+c))/d+
a^2*sec(d*x+c)/d
```

3.19.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx =$$
$$-\frac{a^2(-714420 - 361620\cos(2(c+dx)) - 134820\cos(3(c+dx)) + 29232\cos(4(c+dx)) + 24780\cos(5(c$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

output
```
-1/322560*(a^2*(-714420 - 361620*Cos[2*(c + d*x)] - 134820*Cos[3*(c + d*x)
] + 29232*Cos[4*(c + d*x)] + 24780*Cos[5*(c + d*x)] - 1458*Cos[6*(c + d*x)
] - 3885*Cos[7*(c + d*x)] - 380*Cos[8*(c + d*x)] + 315*Cos[9*(c + d*x)] +
70*Cos[10*(c + d*x)] + 210*Cos[c + d*x]*(205 + 3072*Log[Cos[c + d*x]]))*Se
c[c + d*x])/d
```

3.19.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^9(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^9 \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$
$$\downarrow 4360$$
$$\int \sin^7(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$
$$\downarrow 3042$$
$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^9 \left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2} dx$$
$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^4(\cos(c+dx)a+a)^6\sec^2(c+dx)d(a\cos(c+dx))}{a^9 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^6\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\cos^8(c+dx)a^8+2\cos^7(c+dx)a^8-3\cos^6(c+dx)a^8-8\cos^5(c+dx)a^8+2\cos^4(c+dx)a^8+12\cos^3(c+dx\right.}{a^7 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{9}a^9\cos^9(c+dx)+\frac{1}{4}a^9\cos^8(c+dx)-\frac{3}{7}a^9\cos^7(c+dx)-\frac{4}{3}a^9\cos^6(c+dx)+\frac{2}{5}a^9\cos^5(c+dx)+3a^9\cos^4(c+d}{a^7 d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

output
```
-((-3*a^9*Cos[c + d*x] - 4*a^9*Cos[c + d*x]^2 + (2*a^9*Cos[c + d*x]^3)/3 +
 3*a^9*Cos[c + d*x]^4 + (2*a^9*Cos[c + d*x]^5)/5 - (4*a^9*Cos[c + d*x]^6)/
3 - (3*a^9*Cos[c + d*x]^7)/7 + (a^9*Cos[c + d*x]^8)/4 + (a^9*Cos[c + d*x]^
9)/9 + 2*a^9*Log[a*Cos[c + d*x]] - a^9*Sec[c + d*x])/(a^7*d))
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.19.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin($
default	$\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin($
parallelrisch	$\frac{a^2\left(645120\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-645120\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-645120\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+$
parts	$-\frac{a^2\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)\right.\right.}$
risch	$\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{65a^2e^{2i(dx+c)}}{128d}+2ia^2x+\frac{4ia^2c}{d}+\frac{65a^2e^{-2i(dx+c)}}{128d}+\frac{311a^2e^{-i(dx+c)}}{256d}+\frac{311a^2e^{i(dx+c)}}{256d}-$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(sin(d*x+c)^10/cos(d*x+c)+(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+4
8/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^8
-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/9*a^
2*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c
)^2)*cos(d*x+c))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx =$$

$$-\frac{17920\,a^2\cos(dx+c)^{10} + 40320\,a^2\cos(dx+c)^9 - 69120\,a^2\cos(dx+c)^8 - 215040\,a^2\cos(dx+c)^7 + 64}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/161280*(17920*a^2*cos(d*x + c)^10 + 40320*a^2*cos(d*x + c)^9 - 69120*a^
2*cos(d*x + c)^8 - 215040*a^2*cos(d*x + c)^7 + 64512*a^2*cos(d*x + c)^6 +
483840*a^2*cos(d*x + c)^5 + 107520*a^2*cos(d*x + c)^4 - 645120*a^2*cos(d*x
 + c)^3 - 483840*a^2*cos(d*x + c)^2 + 322560*a^2*cos(d*x + c)*log(-cos(d*x
 + c)) + 197295*a^2*cos(d*x + c) - 161280*a^2)/(d*cos(d*x + c))
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**9,x)
```

output
```
Timed out
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx =$$

$$-\frac{140\,a^2\cos(dx+c)^9 + 315\,a^2\cos(dx+c)^8 - 540\,a^2\cos(dx+c)^7 - 1680\,a^2\cos(dx+c)^6 + 504\,a^2\cos(d}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/1260*(140*a^2*cos(d*x + c)^9 + 315*a^2*cos(d*x + c)^8 - 540*a^2*cos(d*x
 + c)^7 - 1680*a^2*cos(d*x + c)^6 + 504*a^2*cos(d*x + c)^5 + 3780*a^2*cos(
d*x + c)^4 + 840*a^2*cos(d*x + c)^3 - 5040*a^2*cos(d*x + c)^2 - 3780*a^2*c
os(d*x + c) + 2520*a^2*log(cos(d*x + c)) - 1260*a^2/cos(d*x + c))/d
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(171) = 342.

Time = 0.42 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.02

$$\int (a + a\sec(c+dx))^2 \sin^9(c+dx)\,dx$$

$$= \frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2520\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{1457\,a^2-\frac{2067}{}}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="giac")
```

output
```
1/1260*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2520*(2*a^2
+ a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 1) + (1457*a^2 - 20673*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 123012*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 421428*a^2*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 949662*a^2*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 1009134*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^
5 + 666036*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 276804*a^2*(cos
(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 66681*a^2*(cos(d*x + c) - 1)^8/(co
s(d*x + c) + 1)^8 - 7129*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^2 \sin^9(c + dx)\, dx =$$

$$-\frac{\frac{2a^2\cos(c+dx)^3}{3} - \frac{a^2}{\cos(c+dx)} - 4a^2\cos(c+dx)^2 - 3a^2\cos(c+dx) + 3a^2\cos(c+dx)^4 + \frac{2a^2\cos(c+dx)^5}{5} - }{d}$$

input
```
int(sin(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

output
```
-((2*a^2*cos(c + d*x)^3)/3 - a^2/cos(c + d*x) - 4*a^2*cos(c + d*x)^2 - 3*a
^2*cos(c + d*x) + 3*a^2*cos(c + d*x)^4 + (2*a^2*cos(c + d*x)^5)/5 - (4*a^2
*cos(c + d*x)^6)/3 - (3*a^2*cos(c + d*x)^7)/7 + (a^2*cos(c + d*x)^8)/4 + (
a^2*cos(c + d*x)^9)/9 + 2*a^2*log(cos(c + d*x)))/d
```

3.20 $\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$

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3.20.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx = {} & \frac{2a^2\cos(c+dx)}{d} + \frac{3a^2\cos^2(c+dx)}{d} \\ & - \frac{3a^2\cos^4(c+dx)}{2d} - \frac{2a^2\cos^5(c+dx)}{5d} \\ & + \frac{a^2\cos^6(c+dx)}{3d} + \frac{a^2\cos^7(c+dx)}{7d} \\ & - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}\end{aligned}$$

output
```
2*a^2*cos(d*x+c)/d+3*a^2*cos(d*x+c)^2/d-3/2*a^2*cos(d*x+c)^4/d-2/5*a^2*cos
(d*x+c)^5/d+1/3*a^2*cos(d*x+c)^6/d+1/7*a^2*cos(d*x+c)^7/d-2*a^2*ln(cos(d*x
+c))/d+a^2*sec(d*x+c)/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$$
$$= \frac{a^2(25725 + 11760\cos(2(c+dx)) + 5250\cos(3(c+dx)) - 588\cos(4(c+dx)) - 770\cos(5(c+dx)) - 48\,c}{1}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

output
```
(a^2*(25725 + 11760*Cos[2*(c + d*x)] + 5250*Cos[3*(c + d*x)] - 588*Cos[4*(
c + d*x)] - 770*Cos[5*(c + d*x)] - 48*Cos[6*(c + d*x)] + 70*Cos[7*(c + d*x
)] + 15*Cos[8*(c + d*x)] - 70*Cos[c + d*x]*(5 + 384*Log[Cos[c + d*x]]))*Se
c[c + d*x])/(13440*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^7\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^5(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^7\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^3(\cos(c+dx)a+a)^5\sec^2(c+dx)d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^5\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6-2\cos^5(c+dx)a^6+2\cos^4(c+dx)a^6+6\cos^3(c+dx)a^6+\sec^2(c+dx)a^6-6\cos(c+dx)a\right.}{a^5 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)-\frac{1}{3}a^7\cos^6(c+dx)+\frac{2}{5}a^7\cos^5(c+dx)+\frac{3}{2}a^7\cos^4(c+dx)-3a^7\cos^2(c+dx)-2a^7\cos(c+d}{a^5 d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

output
```
-((-2*a^7*Cos[c + d*x] - 3*a^7*Cos[c + d*x]^2 + (3*a^7*Cos[c + d*x]^4)/2 +
 (2*a^7*Cos[c + d*x]^5)/5 - (a^7*Cos[c + d*x]^6)/3 - (a^7*Cos[c + d*x]^7)/
7 + 2*a^7*Log[a*Cos[c + d*x]] - a^7*Sec[c + d*x])/(a^5*d))
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.20.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)}{2}\right.}{d}$
parts	$-\frac{a^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8}{5}\right.\right.}{d}$
parallelrisch	$\frac{a^2\left(26880\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-26880\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-26880\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)\right.}{}$
risch	$2ia^2x+\frac{29a^2e^{2i(dx+c)}}{64d}+\frac{117a^2e^{i(dx+c)}}{128d}+\frac{117a^2e^{-i(dx+c)}}{128d}+\frac{29a^2e^{-2i(dx+c)}}{64d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-$
norman	$\frac{-\frac{192a^2}{35d}-\frac{64a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{d}-\frac{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{172a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{3d}-\frac{264a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{292a^2t}{}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$

input

```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^2*(sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*
sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*si
n(d*x+c)^2-ln(cos(d*x+c)))-1/7*a^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5
*sin(d*x+c)^2)*cos(d*x+c))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$$
$$= \frac{120\,a^2\cos(dx+c)^8 + 280\,a^2\cos(dx+c)^7 - 336\,a^2\cos(dx+c)^6 - 1260\,a^2\cos(dx+c)^5 + 2520\,a^2\cos(dx}{840\,d\cos(}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/840*(120*a^2*cos(d*x + c)^8 + 280*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x +
 c)^6 - 1260*a^2*cos(d*x + c)^5 + 2520*a^2*cos(d*x + c)^3 + 1680*a^2*cos(d
*x + c)^2 - 1680*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 875*a^2*cos(d*x + c
) + 840*a^2)/(d*cos(d*x + c))
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**7,x)
```

output
```
Timed out
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$$
$$= \frac{30\,a^2\cos(dx+c)^7 + 70\,a^2\cos(dx+c)^6 - 84\,a^2\cos(dx+c)^5 - 315\,a^2\cos(dx+c)^4 + 630\,a^2\cos(dx+c)^2}{210\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/210*(30*a^2*cos(d*x + c)^7 + 70*a^2*cos(d*x + c)^6 - 84*a^2*cos(d*x + c)
^5 - 315*a^2*cos(d*x + c)^4 + 630*a^2*cos(d*x + c)^2 + 420*a^2*cos(d*x + c
) - 420*a^2*log(cos(d*x + c)) + 210*a^2/cos(d*x + c))/d
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.

Time $= 0.37$ (sec) , antiderivative size $= 320$, normalized size of antiderivative $= 2.44$

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$$

$$= \frac{420\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{420\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{357\,a^2-\frac{3759\,a^2(co}{\cos(dx}}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="giac")
```

output
```
1/210*(420*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 420*(2*a^2 + a^
2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1) + (357*a^2 - 3759*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1
6737*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 42595*a^2*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 + 43855*a^2*(cos(d*x + c) - 1)^4/(cos(d*x +
 c) + 1)^4 - 25389*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8043*a^
2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*a^2*(cos(d*x + c) - 1)^
7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time $= 14.31$ (sec) , antiderivative size $= 105$, normalized size of antiderivative $= 0.80$

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx$$

$$= \frac{2\,a^2\,\cos(c+d\,x) + \frac{a^2}{\cos(c+d\,x)} + 3\,a^2\cos(c+d\,x)^2 - \frac{3\,a^2\cos(c+d\,x)^4}{2} - \frac{2\,a^2\cos(c+d\,x)^5}{5} + \frac{a^2\cos(c+d\,x)^6}{3} + \frac{a^2\cos(c+d\,x}{7}}{d}$$

input
```
int(sin(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

output

```
(2*a^2*cos(c + d*x) + a^2/cos(c + d*x) + 3*a^2*cos(c + d*x)^2 - (3*a^2*cos
(c + d*x)^4)/2 - (2*a^2*cos(c + d*x)^5)/5 + (a^2*cos(c + d*x)^6)/3 + (a^2*
cos(c + d*x)^7)/7 - 2*a^2*log(cos(c + d*x)))/d
```

3.21 $\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx$

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3.21.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx = \frac{a^2\cos(c+dx)}{d} + \frac{2a^2\cos^2(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{a^2\cos^4(c+dx)}{2d} - \frac{a^2\cos^5(c+dx)}{5d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d
*x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx = -\frac{a^2(-750 - 275\cos(2(c+dx)) - 165\cos(3(c+dx)) - 2\cos(4(c+dx)) + 15\cos(5(c+dx)) + 3\cos(6(c$$
$$480d$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

output
```
-1/480*(a^2*(-750 - 275*Cos[2*(c + d*x)] - 165*Cos[3*(c + d*x)] - 2*Cos[4*
(c + d*x)] + 15*Cos[5*(c + d*x)] + 3*Cos[6*(c + d*x)] + 30*Cos[c + d*x]*(-
3 + 32*Log[Cos[c + d*x]]))*Sec[c + d*x])/d
```

3.21.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^3(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^2(\cos(c+dx)a+a)^4\sec^2(c+dx)d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^4\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{a^3 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\cos^4(c+dx)a^4+2\cos^3(c+dx)a^4-\cos^2(c+dx)a^4+\sec^2(c+dx)a^4-4\cos(c+dx)a^4+2\sec(c+dx)a^4-c\right.}{a^3 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)+\frac{1}{2}a^5\cos^4(c+dx)-\frac{1}{3}a^5\cos^3(c+dx)-2a^5\cos^2(c+dx)-a^5\cos(c+dx)-a^5\sec(c+dx)+2}{a^3d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

output
```
-((-(a^5*Cos[c + d*x]) - 2*a^5*Cos[c + d*x]^2 - (a^5*Cos[c + d*x]^3)/3 + (
a^5*Cos[c + d*x]^4)/2 + (a^5*Cos[c + d*x]^5)/5 + 2*a^5*Log[a*Cos[c + d*x]]
 - a^5*Sec[c + d*x])/(a^3*d))
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.21.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(\frac{8}{3}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(\frac{8}{3}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+\frac{2}{}$
parallelrisch	$-\frac{\left(-320\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos($
risch	$2ia^2x+\frac{3a^2e^{2i(dx+c)}}{8d}+\frac{9a^2e^{i(dx+c)}}{16d}+\frac{9a^2e^{-i(dx+c)}}{16d}+\frac{3a^2e^{-2i(dx+c)}}{8d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln$
norman	$\frac{-\frac{64a^2}{15d}-\frac{64a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{196a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{2a^2\ln\left(t$

input

```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(
d*x+c))+2*a^2*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a^2*
(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx =$$

$$-\frac{48\,a^2\cos(dx+c)^6 + 120\,a^2\cos(dx+c)^5 - 80\,a^2\cos(dx+c)^4 - 480\,a^2\cos(dx+c)^3 - 240\,a^2\cos(dx+}{240\,d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/240*(48*a^2*cos(d*x + c)^6 + 120*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x +
c)^4 - 480*a^2*cos(d*x + c)^3 - 240*a^2*cos(d*x + c)^2 + 480*a^2*cos(d*x +
 c)*log(-cos(d*x + c)) + 195*a^2*cos(d*x + c) - 240*a^2)/(d*cos(d*x + c))
```

3.21.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx = a^2\left(\int 2\sin^5(c+dx)\sec(c+dx)\,dx + \int \sin^5(c+dx)\sec^2(c+dx)\,dx + \int \sin^5(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)*
*5*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**5, x))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx =$$
$$-\frac{6\,a^2\cos(dx+c)^5 + 15\,a^2\cos(dx+c)^4 - 10\,a^2\cos(dx+c)^3 - 60\,a^2\cos(dx+c)^2 - 30\,a^2\cos(dx+c) +}{30\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/30*(6*a^2*cos(d*x + c)^5 + 15*a^2*cos(d*x + c)^4 - 10*a^2*cos(d*x + c)^
3 - 60*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) + 60*a^2*log(cos(d*x + c))
 - 30*a^2/cos(d*x + c))/d
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(106) = 212.

Time = 0.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.41

$$\int (a + a\sec(c+dx))^2 \sin^5(c+dx)\,dx$$
$$= \frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{60\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{69\,a^2-\frac{525\,a^2(\cos(dx+c}{\cos(dx+c)+1}}{30\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/30*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(2*a^2 + a^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1) + (69*a^2 - 525*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1650*a^
2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1610*a^2*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 + 745*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 137*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 1)^5)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c + dx))^2 \sin^5(c + dx)\, dx$$

$$= \frac{a^2 \cos(c + dx) + \frac{a^2}{\cos(c+dx)} + 2a^2 \cos(c + dx)^2 + \frac{a^2 \cos(c+dx)^3}{3} - \frac{a^2 \cos(c+dx)^4}{2} - \frac{a^2 \cos(c+dx)^5}{5} - 2a^2 \ln(\cos(c}{d}$$

input
```
int(sin(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*cos(c + d*x) + a^2/cos(c + d*x) + 2*a^2*cos(c + d*x)^2 + (a^2*cos(c +
 d*x)^3)/3 - (a^2*cos(c + d*x)^4)/2 - (a^2*cos(c + d*x)^5)/5 - 2*a^2*log(c
os(c + d*x)))/d
```

3.22 $\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx$

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3.22.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = \frac{a^2\cos^2(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d
*x+c)/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = \frac{a^2(27 + 4\cos(2(c+dx)) + 6\cos(3(c+dx)) + \cos(4(c+dx)) - 6\cos(c+dx)(1 + 8\log(\cos(c+dx))))\sec}{24d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

output
```
(a^2*(27 + 4*Cos[2*(c + d*x)] + 6*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 6*
Cos[c + d*x]*(1 + 8*Log[Cos[c + d*x]]))*Sec[c + d*x])/(24*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))(\cos(c+dx)a+a)^3\sec^2(c+dx)d(a\cos(c+dx))}{a^3 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))(\cos(c+dx)a+a)^3\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{ad}$$

$$\downarrow 84$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+\sec^2(c+dx)a^2-2\cos(c+dx)a^2+2\sec(c+dx)a^2\right)d(a\cos(c+dx))}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)-a^3\cos^2(c+dx)-a^3\sec(c+dx)+2a^3\log(a\cos(c+dx))}{ad}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

output
```
-((-(a^3*Cos[c + d*x]^2) - (a^3*Cos[c + d*x]^3)/3 + 2*a^3*Log[a*Cos[c + d*
x]] - a^3*Sec[c + d*x])/(a*d))
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.22.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
parts	$-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}-\frac{a^2\sin(dx+c)^2}{d}-\frac{2a^2\ln(\cos(d$
parallelrisch	$\frac{a^2\left(-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+48\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+26\cos(d$
risch	$2ia^2x+\frac{a^2e^{2i(dx+c)}}{4d}+\frac{a^2e^{i(dx+c)}}{8d}+\frac{a^2e^{-i(dx+c)}}{8d}+\frac{a^2e^{-2i(dx+c)}}{4d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{2i($
norman	$\frac{-\frac{8a^2}{3d}-\frac{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{d}$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/2
*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$=\frac{2\,a^2\cos(dx+c)^4+6\,a^2\cos(dx+c)^3-12\,a^2\cos(dx+c)\log(-\cos(dx+c))-3\,a^2\cos(dx+c)+6\,a^2}{6\,d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a^2*cos(d*x + c)^4 + 6*a^2*cos(d*x + c)^3 - 12*a^2*cos(d*x + c)*log
(-cos(d*x + c)) - 3*a^2*cos(d*x + c) + 6*a^2)/(d*cos(d*x + c))
```

3.22.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = a^2\left(\int 2\sin^3(c+dx)\sec(c+dx)\,dx + \int \sin^3(c+dx)\sec^2(c+dx)\,dx + \int \sin^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)*
*3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3, x))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = \frac{a^2\cos(dx+c)^3 + 3\,a^2\cos(dx+c)^2 - 6\,a^2\log(\cos(dx+c)) + \frac{3\,a^2}{\cos(dx+c)}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/3*(a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^2 - 6*a^2*log(cos(d*x + c)) +
 3*a^2/cos(d*x + c))/d
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = -\frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d\cos(dx+c)} + \frac{a^2d^5\cos(dx+c)^3 + 3\,a^2d^5\cos(dx+c)^2}{3\,d^6}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*x + c)) + 1/3*(a^2*d
^5*cos(d*x + c)^3 + 3*a^2*d^5*cos(d*x + c)^2)/d^6
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a + a\sec(c + dx))^2 \sin^3(c + dx)\, dx$$
$$= \frac{\frac{a^2}{\cos(c+d\,x)} + a^2\cos(c + d\,x)^2 + \frac{a^2\cos(c+d\,x)^3}{3} - 2\,a^2\,\ln(\cos(c + d\,x))}{d}$$

input
```
int(sin(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2/cos(c + d*x) + a^2*cos(c + d*x)^2 + (a^2*cos(c + d*x)^3)/3 - 2*a^2*lo
g(cos(c + d*x)))/d
```

3.23 $\int (a + a\sec(c + dx))^2 \sin(c + dx)\, dx$

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3.23.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (a+a\sec(c+dx))^2 \sin(c+dx)\, dx = -\frac{a^2\cos(c+dx)}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
-a^2*cos(d*x+c)/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int (a + a\sec(c + dx))^2 \sin(c + dx)\, dx = \frac{a^2(1 - 2\log(\cos(c + dx)) + \sin(c + dx)\tan(c + dx))}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

output
```
(a^2*(1 - 2*Log[Cos[c + d*x]] + Sin[c + d*x]*Tan[c + d*x]))/d
```

3.23.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4360, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \tan(c+dx)\sec(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3312$$

$$-\frac{\int(\cos(c+dx)a+a)^2\sec^2(c+dx)d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{a\int\frac{(\cos(c+dx)a+a)^2\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{a\int\left(\sec^2(c+dx)+2\sec(c+dx)+1\right)d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a(a\cos(c+dx)-a\sec(c+dx)+2a\log(a\cos(c+dx)))}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

output
```
-((a*(a*Cos[c + d*x] + 2*a*Log[a*Cos[c + d*x]] - a*Sec[c + d*x]))/d)
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.23.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
parts	$-\frac{a^2\cos(dx+c)}{d}+\frac{a^2\sec(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c))}{d}$
parallelrisch	$-\frac{a^2\left(4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+\cos(2dx+2$
risch	$2ia^2x-\frac{a^2e^{i(dx+c)}}{2d}-\frac{a^2e^{-i(dx+c)}}{2d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2a^2\ln\left(1+$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(sec(d*x+c)-1/sec(d*x+c)+2*ln(sec(d*x+c)))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$$
$$=-\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)\log(-\cos(dx+c))-a^2}{d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - a^2)/(d*cos
(d*x + c))
```

3.23.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sin(c+dx)\,dx = a^2\left(\int 2\sin(c+dx)\sec(c+dx)\,dx + \int \sin(c+dx)\sec^2(c+dx)\,dx + \int \sin(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c),x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)*sec
(c + d*x)**2, x) + Integral(sin(c + d*x), x))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c+dx))^2 \sin(c+dx)\,dx = -\frac{a^2\cos(dx+c) + 2\,a^2\log(\cos(dx+c)) - \frac{a^2}{\cos(dx+c)}}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*cos(d*x + c) + 2*a^2*log(cos(d*x + c)) - a^2/cos(d*x + c))/d
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (a + a\sec(c+dx))^2 \sin(c+dx)\,dx = -\frac{a^2\cos(dx+c)}{d} - \frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-a^2*cos(d*x + c)/d - 2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d
*x + c))
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c + dx))^2 \sin(c + dx)\, dx$$
$$= -\frac{a^2 \left(2\cos(c + dx)\ln(\cos(c + dx)) + \cos(c + dx)^2 - 1\right)}{d\cos(c + dx)}$$

input
```
int(sin(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

output
```
-(a^2*(2*cos(c + d*x)*log(cos(c + d*x)) + cos(c + d*x)^2 - 1))/(d*cos(c +
d*x))
```

3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.24.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(-2\log(\cos(c+dx)) + 4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \sec(c+dx)\right)}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(-2*Log[Cos[c + d*x]] + 4*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

3.24.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4360, 3042, 3315, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int \csc(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 3315$$

$$\frac{a\int \frac{(\cos(c+dx)a+a)\sec^2(c+dx)}{a-a\cos(c+dx)}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^3\int \frac{(\cos(c+dx)a+a)\sec^2(c+dx)}{a^2(a-a\cos(c+dx))}d(-a\cos(c+dx))}{d}$$

$$\downarrow 86$$

$$\frac{a^3\int \left(\frac{\sec^2(c+dx)}{a^2}+\frac{2\sec(c+dx)}{a^2}+\frac{2}{a(a-a\cos(c+dx))}\right)d(-a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$\frac{a^3\left(\frac{\sec(c+dx)}{a}-\frac{2\log(-a\cos(c+dx))}{a}+\frac{2\log(a-a\cos(c+dx))}{a}\right)}{d}$$

input
```
Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^3*((-2*Log[-(a*Cos[c + d*x])])/a + (2*Log[a - a*Cos[c + d*x]])/a + Sec[
c + d*x]/a))/d
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.24.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+2a^2\ln(\tan(dx+c))+a^2\ln(-\cot(dx+c)+\csc(dx+c))}{d}$
default	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+2a^2\ln(\tan(dx+c))+a^2\ln(-\cot(dx+c)+\csc(dx+c))}{d}$
risch	$\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
parallelrisch	$\frac{a^2\left(4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos(dx+c)+1\right)}{d\cos(dx+c)}$
norman	$-\frac{2a^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$

input
```
int(csc(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/cos(d*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+2*a^2*ln(tan(d*x+c))+a^
2*ln(-cot(d*x+c)+csc(d*x+c)))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\,a^2\cos(dx+c)\log(-\cos(dx+c)) - 2\,a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - a^2}{d\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 2*a^2*cos(d*x + c)*log(-1/2*cos(
d*x + c) + 1/2) - a^2)/(d*cos(d*x + c))
```

3.24.6 Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc(c+dx)\sec(c+dx)\,dx + \int \csc(c+dx)\sec^2(c+dx)\,dx + \int \csc(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x)*sec
(c + d*x)**2, x) + Integral(csc(c + d*x), x))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log(\cos(dx+c)-1) - 2\,a^2\log(\cos(dx+c)) + \frac{a^2}{\cos(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + a^2/cos(d*x + c))
/d
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(48) = 96.

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^2 + a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2}{d\cos(c+dx)} - \frac{4a^2\,\mathrm{atanh}(2\cos(c+dx)-1)}{d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x),x)
```

output
```
a^2/(d*cos(c + d*x)) - (4*a^2*atanh(2*cos(c + d*x) - 1))/d
```

3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.25.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^3}{d(a-a\cos(c+dx))} + \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
-a^3/d/(a-a*cos(d*x+c))+2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^
2*sec(d*x+c)/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log(\cos(c+dx))-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{8d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/8*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(Csc[(c + d*x)/2]^2 + 4*
Log[Cos[c + d*x]] - 8*Log[Sin[(c + d*x)/2]] - 2*Sec[c + d*x]))/d
```

3.25.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3315, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int \csc^3(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^3\int \frac{\sec^2(c+dx)}{(a-a\cos(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^5\int \frac{\sec^2(c+dx)}{a^2(a-a\cos(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 54$$

$$\frac{a^5\int \left(\frac{\sec^2(c+dx)}{a^4}+\frac{2\sec(c+dx)}{a^4}+\frac{2}{a^3(a-a\cos(c+dx))}+\frac{1}{a^2(a-a\cos(c+dx))^2}\right)d(-a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$\frac{a^5\left(\frac{\sec(c+dx)}{a^3}-\frac{2\log(-a\cos(c+dx))}{a^3}+\frac{2\log(a-a\cos(c+dx))}{a^3}-\frac{1}{a^2(a-a\cos(c+dx))}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^5*(-(1/(a^2*(a - a*Cos[c + d*x]))) - (2*Log[-(a*Cos[c + d*x])])/a^3 + (
2*Log[a - a*Cos[c + d*x]])/a^3 + Sec[c + d*x]/a^3))/d
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.25.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

method	result
parallelrisch	$-\frac{2a^2\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)+\frac{3\left(\cos(dx+c)-\right.}{d\cos(dx+c)}$
risch	$\frac{4a^2\left(e^{3i(dx+c)}-e^{2i(dx+c)}+e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{a^2}{2d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{d}$
derivativedivides	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2a^2\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\cot}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2a^2\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\cot}{}\right.}{d}$

input
```
int(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2*(ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+ln(tan(1/2*d*x+1/2*c)+1)*cos(d
*x+c)-2*ln(tan(1/2*d*x+1/2*c))*cos(d*x+c)+3/4*(cos(d*x+c)-2/3)*cot(1/2*d*x
+1/2*c)^2)/d/cos(d*x+c)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{2\,a^2\cos(dx+c)-a^2-2\left(a^2\cos(dx+c)^2-a^2\cos(dx+c)\right)\log(-\cos(dx+c))+2\left(a^2\cos(dx+c)^2-a\right.}{d\cos(dx+c)^2-d\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(2*a^2*cos(d*x + c) - a^2 - 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(
-cos(d*x + c)) + 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2))/(d*cos(d*x + c)^2 - d*cos(d*x + c))
```

3.25.6 Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^3(c+dx)\sec(c+dx)\,dx + \int \csc^3(c+dx)\sec^2(c+dx)\,dx + \int \csc^3(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)*
*3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3, x))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(\cos(dx+c)-1) - 2a^2\log(\cos(dx+c)) + \frac{2a^2\cos(dx+c)-a^2}{\cos(dx+c)^2-\cos(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + (2*a^2*cos(d*x +
c) - a^2)/(cos(d*x + c)^2 - cos(d*x + c)))/d
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.96

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 4a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{a^2+\frac{5a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{2d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a^2*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a^2 + 5*a^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\,a^2\,\cos(c+d\,x)-a^2}{d\,\left(\cos(c+d\,x)-\cos(c+d\,x)^2\right)} - \frac{4\,a^2\,\operatorname{atanh}(2\,\cos(c+d\,x)-1)}{d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

output
```
- (2*a^2*cos(c + d*x) - a^2)/(d*(cos(c + d*x) - cos(c + d*x)^2)) - (4*a^2*
atanh(2*cos(c + d*x) - 1))/d
```

3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.26.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^4}{4d(a-a\cos(c+dx))^2} - \frac{5a^3}{4d(a-a\cos(c+dx))} + \frac{17a^2\log(1-\cos(c+dx))}{8d} - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{a^2\log(1+\cos(c+dx))}{8d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
-1/4*a^4/d/(a-a*cos(d*x+c))^2-5/4*a^3/d/(a-a*cos(d*x+c))+17/8*a^2*ln(1-cos
(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d-1/8*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c
)/d
```

3.26.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{64d}$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/64*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(10*Csc[(c + d*x)/2]^2
+ Csc[(c + d*x)/2]^4 + 4*(Log[Cos[(c + d*x)/2]] + 8*Log[Cos[c + d*x]] - 17
*Log[Sin[(c + d*x)/2]] - 4*Sec[c + d*x])))/d
```

3.26.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 4360$$

$$\int \csc^5(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 3315$$

$$\frac{a^5\int \frac{\sec^2(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^7\int \frac{\sec^2(c+dx)}{a^2(a-a\cos(c+dx))^3(\cos(c+dx)a+a)}d(-a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$\frac{a^7\int \left(\frac{\sec^2(c+dx)}{a^6}+\frac{2\sec(c+dx)}{a^6}+\frac{17}{8a^5(a-a\cos(c+dx))}+\frac{1}{8a^5(\cos(c+dx)a+a)}+\frac{5}{4a^4(a-a\cos(c+dx))^2}+\frac{1}{2a^3(a-a\cos(c+dx))^3}\right)d(-a}{d}$$

$$\downarrow 2009$$

$$\frac{a^7\left(\frac{\sec(c+dx)}{a^5}-\frac{2\log(-a\cos(c+dx))}{a^5}+\frac{17\log(a-a\cos(c+dx))}{8a^5}-\frac{\log(a\cos(c+dx)+a)}{8a^5}-\frac{5}{4a^4(a-a\cos(c+dx))}-\frac{1}{4a^3(a-a\cos(c+dx))^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^7*(-1/4*1/(a^3*(a - a*Cos[c + d*x])^2) - 5/(4*a^4*(a - a*Cos[c + d*x]))
 - (2*Log[-(a*Cos[c + d*x])])/a^5 + (17*Log[a - a*Cos[c + d*x]])/(8*a^5) -
 Log[a + a*Cos[c + d*x]]/(8*a^5) + Sec[c + d*x]/a^5))/d
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.26.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{17\left(-\frac{8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)}{17}-\frac{8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)}{17}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)-\frac{45\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\cos($}{4d\cos(dx+c)}$
norman	$\frac{\frac{a^2}{16d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}-\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{17a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}$
risch	$\frac{a^2\left(9\,e^{5i(dx+c)}-28\,e^{4i(dx+c)}+34\,e^{3i(dx+c)}-28\,e^{2i(dx+c)}+9\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{4d}+\frac{17a^2\ln\left(e^{i(dx+c)}-1\right)}{4d}-$
derivativedivides	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{5}{8\sin(dx+c)^2\cos(dx+c)}+\frac{15}{8\cos(dx+c)}+\frac{15\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+2a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2s}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{5}{8\sin(dx+c)^2\cos(dx+c)}+\frac{15}{8\cos(dx+c)}+\frac{15\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+2a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2s}\right.}{d}$

input
```
int(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
17/4*(-8/17*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)-8/17*ln(tan(1/2*d*x+1/2*c)
+1)*cos(d*x+c)+ln(tan(1/2*d*x+1/2*c))*cos(d*x+c)-45/136*csc(1/2*d*x+1/2*c)
^2*(cos(d*x+c)-3/10*cos(2*d*x+2*c)-59/90)*cot(1/2*d*x+1/2*c)^2)*a^2/d/cos(
d*x+c)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.82

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{18\,a^2\cos(dx+c)^2-28\,a^2\cos(dx+c)+8\,a^2-16\left(a^2\cos(dx+c)^3-2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)\right.}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(18*a^2*cos(d*x + c)^2 - 28*a^2*cos(d*x + c) + 8*a^2 - 16*(a^2*cos(d*x
 + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - (a
^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + 17*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d
*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 2*d*cos(d*x + c
)^2 + d*cos(d*x + c))
```

3.26.6 Sympy [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^5(c+dx)\sec(c+dx)\,dx + \int \csc^5(c+dx)\sec^2(c+dx)\,dx + \int \csc^5(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)*
*5*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**5, x))
```

3.26.7 Maxima [A] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 104$, normalized size of antiderivative $= 0.90$

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\log(\cos(dx+c)+1) - 17\,a^2\log(\cos(dx+c)-1) + 16\,a^2\log(\cos(dx+c)) - \frac{2\left(9\,a^2\cos(dx+c)^2 - 14\,a^2\cos(dx+c) + 4\,a^2\right)}{\cos(dx+c)^3 - 2\cos(dx+c)^2 + \cos(dx+c)}}{8\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/8*(a^2*log(cos(d*x + c) + 1) - 17*a^2*log(cos(d*x + c) - 1) + 16*a^2*lo
g(cos(d*x + c)) - 2*(9*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) + 4*a^2)/(
cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c)))/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.66

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{34\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 32\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1}{(\cos(dx+c)-1)^2}}{16\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(34*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 - 12*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 51*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 32*(2*a^2 + a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1))/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{17\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{8\,d} - \frac{a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{8\,d} + \frac{\frac{9\,a^2\cos(c+d\,x)^2}{4} - \frac{7\,a^2\,\cos(c+d\,x)}{2} + a^2}{d\,\left(\cos\left(c+d\,x\right)^3 - 2\cos\left(c+d\,x\right)^2 + \cos\left(c+d\,x\right)\right)} - \frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^5,x)
```

output
```
(17*a^2*log(cos(c + d*x) - 1))/(8*d) - (a^2*log(cos(c + d*x) + 1))/(8*d) +
 (a^2 - (7*a^2*cos(c + d*x))/2 + (9*a^2*cos(c + d*x)^2)/4)/(d*(cos(c + d*x
) - 2*cos(c + d*x)^2 + cos(c + d*x)^3)) - (2*a^2*log(cos(c + d*x)))/d
```

3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.27.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^5}{12d(a-a\cos(c+dx))^3} - \frac{3a^4}{8d(a-a\cos(c+dx))^2} - \frac{23a^3}{16d(a-a\cos(c+dx))} + \frac{a^3}{16d(a+a\cos(c+dx))} + \frac{9a^2\log(1-\cos(c+dx))}{4d} - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{a^2\log(1+\cos(c+dx))}{4d} + \frac{a^2\sec(c+dx)}{d}$$

output
```
-1/12*a^5/d/(a-a*cos(d*x+c))^3-3/8*a^4/d/(a-a*cos(d*x+c))^2-23/16*a^3/d/(a
-a*cos(d*x+c))+1/16*a^3/d/(a+a*cos(d*x+c))+9/4*a^2*ln(1-cos(d*x+c))/d-2*a^
2*ln(cos(d*x+c))/d-1/4*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

3.27.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(120\csc^2\left(\frac{1}{2}(c+dx)\right)+36\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.$$

input
```
Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(120*Csc[(c + d*x)/2]^
2 + 36*Csc[(c + d*x)/2]^4 + 48*(Log[Cos[(c + d*x)/2]] + 4*Log[Cos[c + d*x]
] - 9*Log[Sin[(c + d*x)/2]]) + Csc[(c + d*x)/2]^6*(16 - 3*Sec[(c + d*x)/2]
^2*(3 + 2*Sec[c + d*x]))))/d
```

3.27.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 4360$$

$$\int \csc^7(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 3315$$

$$\frac{a^7\int \frac{\sec^2(c+dx)}{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^9\int \frac{\sec^2(c+dx)}{a^2(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$\frac{a^9 \int \left(\frac{\sec^2(c+dx)}{a^8} + \frac{2\sec(c+dx)}{a^8} + \frac{9}{4a^7(a-a\cos(c+dx))} + \frac{1}{4a^7(\cos(c+dx)a+a)} + \frac{23}{16a^6(a-a\cos(c+dx))^2} + \frac{1}{16a^6(\cos(c+dx)a+a)^2} + \frac{4a}{d}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{a^9\left(\frac{\sec(c+dx)}{a^7} - \frac{2\log(-a\cos(c+dx))}{a^7} + \frac{9\log(a-a\cos(c+dx))}{4a^7} - \frac{\log(a\cos(c+dx)+a)}{4a^7} - \frac{23}{16a^6(a-a\cos(c+dx))} + \frac{1}{16a^6(a\cos(c+dx)+a)} - \right.}{d}$$

input
```
Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^9*(-1/12*1/(a^4*(a - a*Cos[c + d*x])^3) - 3/(8*a^5*(a - a*Cos[c + d*x])
^2) - 23/(16*a^6*(a - a*Cos[c + d*x])) + 1/(16*a^6*(a + a*Cos[c + d*x])) -
 (2*Log[-(a*Cos[c + d*x])])/a^7 + (9*Log[a - a*Cos[c + d*x]])/(4*a^7) - Lo
g[a + a*Cos[c + d*x]]/(4*a^7) + Sec[c + d*x]/a^7))/d
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.27.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\frac{a^2}{96d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}+\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{32d}-\frac{95a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{9a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2a^2\ln($
parallelrisch	$\frac{a^2\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+11\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+432\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan$
derivativedivides	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)}-\frac{7}{24\sin(dx+c)^4\cos(dx+c)}-\frac{35}{48\sin(dx+c)^2\cos(dx+c)}+\frac{35}{16\cos(dx+c)}+\frac{35\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)}-\frac{7}{24\sin(dx+c)^4\cos(dx+c)}-\frac{35}{48\sin(dx+c)^2\cos(dx+c)}+\frac{35}{16\cos(dx+c)}+\frac{35\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right.}{d}$
risch	$\frac{a^2\left(15\,e^{9i(dx+c)}-48\,e^{8i(dx+c)}+32\,e^{7i(dx+c)}+40\,e^{6i(dx+c)}-62\,e^{5i(dx+c)}+40\,e^{4i(dx+c)}+32\,e^{3i(dx+c)}-48\,e^{2i(dx+c)}+15\,e^{i}\right.}{3d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}+1\right)}$

input
```
int(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(1/96/d*a^2+11/96*a^2/d*tan(1/2*d*x+1/2*c)^2+13/16*a^2/d*tan(1/2*d*x+1/2*c
)^4+1/32*a^2/d*tan(1/2*d*x+1/2*c)^10-95/32*a^2/d*tan(1/2*d*x+1/2*c)^6)/tan
(1/2*d*x+1/2*c)^6/(-1+tan(1/2*d*x+1/2*c)^2)+9/2/d*a^2*ln(tan(1/2*d*x+1/2*c
))-2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.81

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{30\,a^2\cos(dx+c)^4-48\,a^2\cos(dx+c)^3-14\,a^2\cos(dx+c)^2+46\,a^2\cos(dx+c)-12\,a^2-24\left(a^2\cos(dx\right.}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(30*a^2*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^3 - 14*a^2*cos(d*x + c)^
2 + 46*a^2*cos(d*x + c) - 12*a^2 - 24*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x
 + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-cos(d*x + c)) - 3*(
a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 27*(a^2*cos(d*x + c)^5 - 2*a^2*co
s(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 -
d*cos(d*x + c))
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{3\,a^2\log\left(\cos\left(dx+c\right)+1\right)-27\,a^2\log\left(\cos\left(dx+c\right)-1\right)+24\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(15\,a^2\cos(dx+c)^4-24\,a\right.}{\cos(dx+c)^5-}}{12\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/12*(3*a^2*log(cos(d*x + c) + 1) - 27*a^2*log(cos(d*x + c) - 1) + 24*a^2
*log(cos(d*x + c)) - 2*(15*a^2*cos(d*x + c)^4 - 24*a^2*cos(d*x + c)^3 - 7*
a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) - 6*a^2)/(cos(d*x + c)^5 - 2*cos(
d*x + c)^4 + 2*cos(d*x + c)^2 - cos(d*x + c)))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.49

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{216\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 192\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos}{(\cos(d}}{96\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/96*(216*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 192*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 3*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + (a^2 - 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + 90*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 396*a^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)
^3 + 192*(2*a^2 + a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{9\,a^2\,\ln(\cos(c+dx)-1)}{4\,d} - \frac{a^2\,\ln(\cos(c+dx)+1)}{4\,d} - \frac{2\,a^2\,\ln(\cos(c+dx))}{d} + \frac{-\frac{5\,a^2\cos(c+dx)^4}{2} + 4\,a^2\cos(c+dx)^3 + \frac{7\,a^2\cos(c+dx)^2}{6} - \frac{23\,a^2\,\cos(c+dx)}{6} + a^2}{d\left(-\cos(c+dx)^5 + 2\cos(c+dx)^4 - 2\cos(c+dx)^2 + \cos(c+dx)\right)}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^7,x)
```

output
```
(9*a^2*log(cos(c + d*x) - 1))/(4*d) - (a^2*log(cos(c + d*x) + 1))/(4*d) -
(2*a^2*log(cos(c + d*x)))/d + (a^2 - (23*a^2*cos(c + d*x))/6 + (7*a^2*cos(
c + d*x)^2)/6 + 4*a^2*cos(c + d*x)^3 - (5*a^2*cos(c + d*x)^4)/2)/(d*(cos(c
 + d*x) - 2*cos(c + d*x)^2 + 2*cos(c + d*x)^4 - cos(c + d*x)^5))
```

3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.28.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = &-\frac{a^6}{32d(a-a\cos(c+dx))^4}\\ &-\frac{7a^5}{48d(a-a\cos(c+dx))^3}\\ &-\frac{15a^4}{32d(a-a\cos(c+dx))^2}-\frac{51a^3}{32d(a-a\cos(c+dx))}\\ &+\frac{a^4}{64d(a+a\cos(c+dx))^2}+\frac{9a^3}{64d(a+a\cos(c+dx))}\\ &+\frac{303a^2\log(1-\cos(c+dx))}{128d}\\ &-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{47a^2\log(1+\cos(c+dx))}{128d}\\ &+\frac{a^2\sec(c+dx)}{d}\end{aligned}$$

output

```
-1/32*a^6/d/(a-a*cos(d*x+c))^4-7/48*a^5/d/(a-a*cos(d*x+c))^3-15/32*a^4/d/(
a-a*cos(d*x+c))^2-51/32*a^3/d/(a-a*cos(d*x+c))+1/64*a^4/d/(a+a*cos(d*x+c))
^2+9/64*a^3/d/(a+a*cos(d*x+c))+303/128*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos
(d*x+c))/d-47/128*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

3.28.2 Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(1224\csc^2\left(\frac{1}{2}(c+dx)\right)+180\csc^4\left(\frac{1}{2}(c+dx)\right)+28\csc^6\left(\frac{1}{2}(c+dx)\right)+3\csc^8\left(\frac{1}{2}(c+dx)\right)-6\left(18\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+4\left(-47\log\left[\cos\left(\frac{1}{2}(c+dx)\right)\right]-128\log[\cos(c+dx)]+303\log\left[\sin\left(\frac{1}{2}(c+dx)\right)\right]+64\sec(c+dx)\right)\right)\right)}{6144d}$$

input
```
Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1224*Csc[(c + d*x)/2
]^2 + 180*Csc[(c + d*x)/2]^4 + 28*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^
8 - 6*(18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 4*(-47*Log[Cos[(c + d*
x)/2]] - 128*Log[Cos[c + d*x]] + 303*Log[Sin[(c + d*x)/2]] + 64*Sec[c + d*
x]))))/d
```

3.28.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^9(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^9}dx$$

$$\downarrow 4360$$

$$\int \csc^9(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^9}dx$$

$$\downarrow 3315$$

$$\frac{a^9 \int \frac{\sec^2(c+dx)}{(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^3} d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^{11} \int \frac{\sec^2(c+dx)}{a^2(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^3} d(-a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$\frac{a^{11} \int \left(\frac{\sec^2(c+dx)}{a^{10}} + \frac{2\sec(c+dx)}{a^{10}} + \frac{303}{128a^9(a-a\cos(c+dx))} + \frac{47}{128a^9(\cos(c+dx)a+a)} + \frac{51}{32a^8(a-a\cos(c+dx))^2} + \frac{9}{64a^8(\cos(c+dx)a+a)^2} \right.}{d}$$

$$\downarrow 2009$$

$$\frac{a^{11} \left(\frac{\sec(c+dx)}{a^9} - \frac{2\log(-a\cos(c+dx))}{a^9} + \frac{303\log(a-a\cos(c+dx))}{128a^9} - \frac{47\log(a\cos(c+dx)+a)}{128a^9} - \frac{51}{32a^8(a-a\cos(c+dx))} + \frac{9}{64a^8(a\cos(c+dx)+} \right.}{d}$$

input
```
Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^11*(-1/32*1/(a^5*(a - a*Cos[c + d*x])^4) - 7/(48*a^6*(a - a*Cos[c + d*x
])^3) - 15/(32*a^7*(a - a*Cos[c + d*x])^2) - 51/(32*a^8*(a - a*Cos[c + d*x
])) + 1/(64*a^7*(a + a*Cos[c + d*x])^2) + 9/(64*a^8*(a + a*Cos[c + d*x]))
 - (2*Log[-(a*Cos[c + d*x])])/a^9 + (303*Log[a - a*Cos[c + d*x]])/(128*a^9)
  - (47*Log[a + a*Cos[c + d*x]])/(128*a^9) + Sec[c + d*x]/a^9))/d
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.28.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\frac{a^2}{512d}+\frac{37a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{1536d}+\frac{121a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{768d}+\frac{233a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{256d}+\frac{19a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{256d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{256d}-\frac{203a^2\tan\left(\frac{d}{2}\right.}{64d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
parallelrisch	$\frac{a^2\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^8+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+37\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+114\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+242\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+7272\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)}-\frac{3}{16\sin(dx+c)^6\cos(dx+c)}-\frac{21}{64\sin(dx+c)^4\cos(dx+c)}-\frac{105}{128\sin(dx+c)^2\cos(dx+c)}+\frac{315}{128\cos(dx+c)}+\frac{3}{}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)}-\frac{3}{16\sin(dx+c)^6\cos(dx+c)}-\frac{21}{64\sin(dx+c)^4\cos(dx+c)}-\frac{105}{128\sin(dx+c)^2\cos(dx+c)}+\frac{315}{128\cos(dx+c)}+\frac{3}{}\right.}{}$
risch	$\frac{a^2\left(525\,e^{13i(dx+c)}-1716\,e^{12i(dx+c)}+214\,e^{11i(dx+c)}+4652\,e^{10i(dx+c)}-4173\,e^{9i(dx+c)}-2552\,e^{8i(dx+c)}+4564\,e^{7i(dx+c)}-2\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^4\left(e^{2i}\right.}$

input
```
int(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(1/512/d*a^2+37/1536*a^2/d*tan(1/2*d*x+1/2*c)^2+121/768*a^2/d*tan(1/2*d*x+
1/2*c)^4+233/256*a^2/d*tan(1/2*d*x+1/2*c)^6+19/256*a^2/d*tan(1/2*d*x+1/2*c
)^12+1/256*a^2/d*tan(1/2*d*x+1/2*c)^14-203/64/d*a^2*tan(1/2*d*x+1/2*c)^8)/
tan(1/2*d*x+1/2*c)^8/(-1+tan(1/2*d*x+1/2*c)^2)+303/64/d*a^2*ln(tan(1/2*d*x
+1/2*c))-2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(193) = 386$.

Time $= 0.29$ (sec) , antiderivative size $= 461$, normalized size of antiderivative $= 2.25$

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{1050\,a^2\cos(dx+c)^6 - 1716\,a^2\cos(dx+c)^5 - 1468\,a^2\cos(dx+c)^4 + 3308\,a^2\cos(dx+c)^3 - 38\,a^2\cos(d$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/384*(1050*a^2*cos(d*x + c)^6 - 1716*a^2*cos(d*x + c)^5 - 1468*a^2*cos(d*
x + c)^4 + 3308*a^2*cos(d*x + c)^3 - 38*a^2*cos(d*x + c)^2 - 1568*a^2*cos(
d*x + c) + 384*a^2 - 768*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*
cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x
 + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - 141*(a^2*cos(d*x + c)^7 -
 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*co
s(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x +
c) + 1/2) + 909*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x +
 c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 +
 a^2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^7 - 2*d*c
os(d*x + c)^6 - d*cos(d*x + c)^5 + 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 -
 2*d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{141\,a^2\log\left(\cos\left(dx+c\right)+1\right)-909\,a^2\log\left(\cos\left(dx+c\right)-1\right)+768\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(525\,a^2\cos(dx+c)\right.}{\cos}}{384\,d}$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/384*(141*a^2*log(cos(d*x + c) + 1) - 909*a^2*log(cos(d*x + c) - 1) + 76
8*a^2*log(cos(d*x + c)) - 2*(525*a^2*cos(d*x + c)^6 - 858*a^2*cos(d*x + c)
^5 - 734*a^2*cos(d*x + c)^4 + 1654*a^2*cos(d*x + c)^3 - 19*a^2*cos(d*x + c
)^2 - 784*a^2*cos(d*x + c) + 192*a^2)/(cos(d*x + c)^7 - 2*cos(d*x + c)^6 -
 cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 2*cos(d*x + c)^2 + c
os(d*x + c)))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{3636\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-3072\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{120\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a\right.}{153}$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1536*(3636*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 3072*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 120*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - (3*a^2 - 40*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 282*a^2*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1680*a^2*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 7575*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*
(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 3072*(2*a^2 + a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{-\frac{175\,a^2\cos(c+dx)^6}{64}+\frac{143\,a^2\cos(c+dx)^5}{32}+\frac{367\,a^2\cos(c+dx)^4}{96}-\frac{827\,a^2\cos(c+dx)^3}{96}+\frac{19\,a^2\cos(c+dx)^2}{192}+\frac{49\,a^2\cos(c+d}{12}}{d\,\left(-\cos(c+dx)^7+2\cos(c+dx)^6+\cos(c+dx)^5-4\cos(c+dx)^4+\cos(c+dx)^3+2\cos(c+dx)^2-\right.}$$
$$+\frac{303\,a^2\,\ln(\cos(c+dx)-1)}{128\,d}-\frac{47\,a^2\,\ln(\cos(c+dx)+1)}{128\,d}-\frac{2\,a^2\,\ln(\cos(c+dx))}{d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^9,x)
```

output
```
((49*a^2*cos(c + d*x))/12 - a^2 + (19*a^2*cos(c + d*x)^2)/192 - (827*a^2*c
os(c + d*x)^3)/96 + (367*a^2*cos(c + d*x)^4)/96 + (143*a^2*cos(c + d*x)^5)
/32 - (175*a^2*cos(c + d*x)^6)/64)/(d*(2*cos(c + d*x)^2 - cos(c + d*x) + c
os(c + d*x)^3 - 4*cos(c + d*x)^4 + cos(c + d*x)^5 + 2*cos(c + d*x)^6 - cos
(c + d*x)^7)) + (303*a^2*log(cos(c + d*x) - 1))/(128*d) - (47*a^2*log(cos(
c + d*x) + 1))/(128*d) - (2*a^2*log(cos(c + d*x)))/d
```

3.29 $\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx$

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3.29.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx = &-\frac{245a^2x}{128} + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} \\ &- \frac{2a^2\sin(c+dx)}{d} + \frac{139a^2\cos(c+dx)\sin(c+dx)}{128d} \\ &+ \frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d} \\ &- \frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} \\ &+ \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} \\ &- \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} \\ &- \frac{2a^2\sin^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d}\end{aligned}$$

output

```
-245/128*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+139/128*a^2*
cos(d*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-17/48*a^2*cos
(d*x+c)^5*sin(d*x+c)/d+1/8*a^2*cos(d*x+c)^7*sin(d*x+c)/d-2/3*a^2*sin(d*x+c
)^3/d-2/5*a^2*sin(d*x+c)^5/d-2/7*a^2*sin(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

3.29.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx =$$

$$-\frac{a^2(1+\cos(c+dx))^2 \sec^4\left(\frac{1}{2}(c+dx)\right)\left(168000c + 168000dx + 37800\arctan(\tan(c+dx)) - 215040\text{arct}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

output
```
-1/430080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(168000*c + 168000*
d*x + 37800*ArcTan[Tan[c + d*x]] - 215040*ArcTanh[Sin[c + d*x]] + 215040*S
in[c + d*x] + 71680*Sin[c + d*x]^3 + 43008*Sin[c + d*x]^5 + 30720*Sin[c +
d*x]^7 - 55440*Sin[2*(c + d*x)] + 2520*Sin[4*(c + d*x)] + 560*Sin[6*(c + d
*x)] - 105*Sin[8*(c + d*x)] - 107520*Tan[c + d*x]))/d
```

3.29.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^8 \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^6(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^8 \left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2} dx$$

$$\downarrow 3351$$

$$\frac{\int \left(\cos^8(c+dx)a^{10}+2\cos^7(c+dx)a^{10}-3\cos^6(c+dx)a^{10}-8\cos^5(c+dx)a^{10}+2\cos^4(c+dx)a^{10}+12\cos^3(c+\right.}{a^8}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^{10}\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2a^{10}\sin^7(c+dx)}{7d}-\frac{2a^{10}\sin^5(c+dx)}{5d}-\frac{2a^{10}\sin^3(c+dx)}{3d}-\frac{2a^{10}\sin(c+dx)}{d}+\frac{a^{10}\tan(c+dx)}{d}+\frac{a^{10}\sin(c+dx)}{8d}}{a^8}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

output
```
((-245*a^10*x)/128 + (2*a^10*ArcTanh[Sin[c + d*x]])/d - (2*a^10*Sin[c + d*
x])/d + (139*a^10*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^10*Cos[c + d*
x]^3*Sin[c + d*x])/(192*d) - (17*a^10*Cos[c + d*x]^5*Sin[c + d*x])/(48*d)
 + (a^10*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^10*Sin[c + d*x]^3)/(3*d)
 - (2*a^10*Sin[c + d*x]^5)/(5*d) - (2*a^10*Sin[c + d*x]^7)/(7*d) + (a^10*T
an[c + d*x])/d)/a^8
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.29.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{a^2\left(-411600dx\cos(dx+c)-430080\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+430080\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+270480\sin$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\sin(dx+c)^7}{7}-\right.}{}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\sin(dx+c)^7}{7}-\right.}{}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7\right.\right.}{}$
risch	$-\frac{245a^2x}{128}+\frac{33ia^2e^{-2i(dx+c)}}{128d}-\frac{93ia^2e^{-i(dx+c)}}{64d}+\frac{93ia^2e^{i(dx+c)}}{64d}-\frac{33ia^2e^{2i(dx+c)}}{128d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2}{}$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/215040*a^2*(-411600*d*x*cos(d*x+c)-430080*ln(tan(1/2*d*x+1/2*c)-1)*cos(d
*x+c)+430080*ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+270480*sin(d*x+c)+35392*s
in(4*d*x+4*c)+105*sin(9*d*x+9*c)-271040*sin(2*d*x+2*c)+480*sin(8*d*x+8*c)-
5568*sin(6*d*x+6*c)-455*sin(7*d*x+7*c)-3080*sin(5*d*x+5*c)+52920*sin(3*d*x
+3*c))/d/cos(d*x+c)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx =$$

$$-\frac{25725\,a^2dx\cos(dx+c)-13440\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+13440\,a^2\cos(dx+c)\log(-\sin$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/13440*(25725*a^2*d*x*cos(d*x + c) - 13440*a^2*cos(d*x + c)*log(sin(d*x
+ c) + 1) + 13440*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (1680*a^2*cos(
d*x + c)^8 + 3840*a^2*cos(d*x + c)^7 - 4760*a^2*cos(d*x + c)^6 - 16896*a^2
*cos(d*x + c)^5 + 770*a^2*cos(d*x + c)^4 + 31232*a^2*cos(d*x + c)^3 + 1459
5*a^2*cos(d*x + c)^2 - 45056*a^2*cos(d*x + c) + 13440*a^2)*sin(d*x + c))/(
d*cos(d*x + c))
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 \sin^8(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**8,x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int (a + a\sec(c + dx))^2 \sin^8(c + dx)\, dx =$$

$$-\frac{1024\left(30\sin(dx + c)^7 + 42\sin(dx + c)^5 + 70\sin(dx + c)^3 - 105\log(\sin(dx + c) + 1) + 105\log(\sin\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="maxima")
```

output
```
-1/107520*(1024*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c
))*a^2 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c)
 + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^2 + 2240*(105*d*x + 105*
c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x +
c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c))*a^2)/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.13

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx =$$

$$-\frac{25725\,(dx+c)a^2 - 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{26880}{\tan}}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="giac")
```

output
```
-1/13440*(25725*(d*x + c)*a^2 - 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 26880*a^2*tan(1/2*d*x
+ 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(39165*a^2*tan(1/2*d*x + 1/2*c)^
15 + 300265*a^2*tan(1/2*d*x + 1/2*c)^13 + 989261*a^2*tan(1/2*d*x + 1/2*c)^
11 + 1791073*a^2*tan(1/2*d*x + 1/2*c)^9 + 1814943*a^2*tan(1/2*d*x + 1/2*c)
^7 + 670131*a^2*tan(1/2*d*x + 1/2*c)^5 + 147735*a^2*tan(1/2*d*x + 1/2*c)^3
 + 14595*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.47

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx = \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{245\,a^2\,x}{128}$$
$$+ \frac{\frac{501\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{64} + \frac{2633\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{48} + \frac{38047\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{240} + \frac{388613\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{1680} + \frac{13781\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{96}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} - 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \right.}$$

input
```
int(sin(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

output

```
(4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (245*a^2*x)/128 + ((13781*a^2*tan(c/
2 + (d*x)/2)^9)/96 - (1739*a^2*tan(c/2 + (d*x)/2)^5)/80 - (32681*a^2*tan(c
/2 + (d*x)/2)^7)/560 - (61*a^2*tan(c/2 + (d*x)/2)^3)/16 + (388613*a^2*tan(
c/2 + (d*x)/2)^11)/1680 + (38047*a^2*tan(c/2 + (d*x)/2)^13)/240 + (2633*a^
2*tan(c/2 + (d*x)/2)^15)/48 + (501*a^2*tan(c/2 + (d*x)/2)^17)/64 - (11*a^2
*tan(c/2 + (d*x)/2))/64)/(d*(7*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2
)^4 + 28*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^8 - 14*tan(c/2 + (d*
x)/2)^10 - 28*tan(c/2 + (d*x)/2)^12 - 20*tan(c/2 + (d*x)/2)^14 - 7*tan(c/2
 + (d*x)/2)^16 - tan(c/2 + (d*x)/2)^18 + 1))
```

3.30 $\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx$

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3.30.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx = &-\frac{25a^2x}{16} + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} \\ &- \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{16d} \\ &+ \frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d} \\ &- \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a^2\sin^3(c+dx)}{3d} \\ &- \frac{2a^2\sin^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d}\end{aligned}$$

output

```
-25/16*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+7/16*a^2*cos(d
*x+c)*sin(d*x+c)/d+7/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^2*cos(d*x+c)^5
*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d-2/5*a^2*sin(d*x+c)^5/d+a^2*tan(d*x+c)
/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx =$$

$$-\frac{a^2(1+\cos(c+dx))^2 \sec^4\left(\frac{1}{2}(c+dx)\right)\left(1080c + 1080dx + 420\arctan(\tan(c+dx)) - 1920\text{arctanh}(\sin(c\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

output
```
-1/3840*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1080*c + 1080*d*x +
420*ArcTan[Tan[c + d*x]] - 1920*ArcTanh[Sin[c + d*x]] + 1920*Sin[c + d*x]
+ 640*Sin[c + d*x]^3 + 384*Sin[c + d*x]^5 - 255*Sin[2*(c + d*x)] - 15*Sin[
4*(c + d*x)] + 5*Sin[6*(c + d*x)] - 960*Tan[c + d*x]))/d
```

3.30.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6 \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^4(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6 \left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2} dx$$

$$\downarrow 3351$$

$$\frac{\int \left(-\cos^6(c+dx)a^8 - 2\cos^5(c+dx)a^8 + 2\cos^4(c+dx)a^8 + 6\cos^3(c+dx)a^8 + \sec^2(c+dx)a^8 - 6\cos(c+dx)a^8\right.}{a^6}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^8\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^8\sin^5(c+dx)}{5d} - \frac{2a^8\sin^3(c+dx)}{3d} - \frac{2a^8\sin(c+dx)}{d} + \frac{a^8\tan(c+dx)}{d} - \frac{a^8\sin(c+dx)\cos^5(c+dx)}{6d} + \frac{7a^8\sin(c+d}{}}{a^6}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

output
```
((-25*a^8*x)/16 + (2*a^8*ArcTanh[Sin[c + d*x]])/d - (2*a^8*Sin[c + d*x])/d
 + (7*a^8*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^8*Cos[c + d*x]^3*Sin[c
+ d*x])/(24*d) - (a^8*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^8*Sin[c +
d*x]^3)/(3*d) - (2*a^8*Sin[c + d*x]^5)/(5*d) + (a^8*Tan[c + d*x])/d)/a^6
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.30.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{a^2\left(-3000dx\cos(dx+c)-3840\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3840\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2175\sin(dx+c)-\right.}{1920d\cos(dx+c)}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\right.}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\right.\right.}{d}$
risch	$-\frac{25a^2x}{16}+\frac{17ia^2e^{-2i(dx+c)}}{128d}-\frac{17ia^2e^{2i(dx+c)}}{128d}-\frac{11ia^2e^{-i(dx+c)}}{8d}+\frac{11ia^2e^{i(dx+c)}}{8d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln}{}$
norman	$\frac{25a^2x}{16}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{27a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{797a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{40d}-\frac{91a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}-\frac{8041a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{120d}-\frac{431a^2\tan\left(\frac{d}{2}\right.}{12d}$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/1920*a^2*(-3000*d*x*cos(d*x+c)-3840*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+
3840*ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+2175*sin(d*x+c)-2360*sin(2*d*x+2*
c)-24*sin(6*d*x+6*c)+256*sin(4*d*x+4*c)-5*sin(7*d*x+7*c)+10*sin(5*d*x+5*c)
+270*sin(3*d*x+3*c))/d/cos(d*x+c)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx =$$

$$-\frac{375\,a^2dx\cos(dx+c)-240\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+240\,a^2\cos(dx+c)\log(-\sin(dx+c}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/240*(375*a^2*d*x*cos(d*x + c) - 240*a^2*cos(d*x + c)*log(sin(d*x + c) +
 1) + 240*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (40*a^2*cos(d*x + c)^6
 + 96*a^2*cos(d*x + c)^5 - 70*a^2*cos(d*x + c)^4 - 352*a^2*cos(d*x + c)^3
- 105*a^2*cos(d*x + c)^2 + 736*a^2*cos(d*x + c) - 240*a^2)*sin(d*x + c))/(
d*cos(d*x + c))
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 \sin^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

output
```
Timed out
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c + dx))^2 \sin^6(c + dx)\, dx =$$

$$-\frac{64\left(6\sin(dx + c)^5 + 10\sin(dx + c)^3 - 15\log(\sin(dx + c) + 1) + 15\log(\sin(dx + c) - 1) + 30\sin(dx + c)\right)a^2 - 5\left(4\sin(2dx + 2c)^3 + 60dx + 60c + 9\sin(4dx + 4c) - 48\sin(2dx + 2c)\right)a^2 + 120\left(15dx + 15c - \frac{9\tan(dx + c)^3 + 7\tan(dx + c)}{\tan(dx + c)^4 + 2\tan(dx + c)^2 + 1} - 8\tan(dx + c)\right)a^2}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
-1/960*(64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1
) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c
)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 120*
(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*t
an(d*x + c)^2 + 1) - 8*tan(d*x + c))*a^2)/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.23

$$\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx =$$

$$-\frac{375\,(dx+c)a^2 - 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{480\,a^2\tan\left(\frac{1}{2}\,dx}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4
80*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 480*a^2*tan(1/2*d*x + 1/2*c)/(
tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(615*a^2*tan(1/2*d*x + 1/2*c)^11 + 3485*a^
2*tan(1/2*d*x + 1/2*c)^9 + 7926*a^2*tan(1/2*d*x + 1/2*c)^7 + 8586*a^2*tan(
1/2*d*x + 1/2*c)^5 + 2595*a^2*tan(1/2*d*x + 1/2*c)^3 + 345*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.50

$$\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx$$

$$= \frac{\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{8} + \frac{431\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{12} + \frac{8041\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{120} + \frac{91\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{2} - \frac{797\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{40} - \frac{27\,a^2\,t}{}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 9\tan\left(\right.\right.}$$

$$- \frac{25\,a^2\,x}{16} + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(sin(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

output
```
((91*a^2*tan(c/2 + (d*x)/2)^7)/2 - (797*a^2*tan(c/2 + (d*x)/2)^5)/40 - (27
*a^2*tan(c/2 + (d*x)/2)^3)/4 + (8041*a^2*tan(c/2 + (d*x)/2)^9)/120 + (431*
a^2*tan(c/2 + (d*x)/2)^11)/12 + (57*a^2*tan(c/2 + (d*x)/2)^13)/8 - (7*a^2*
tan(c/2 + (d*x)/2))/8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4
 + 5*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^
10 - 5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (25*a^2*x)/16
 + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.31 $\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx$

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3.31.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = -\frac{9a^2x}{8} + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\tan(c+dx)}{d}$$

output
```
-9/8*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/8*a^2*cos(d*x+
c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+a
^2*tan(d*x+c)/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(48c + 48dx + 60\arctan(\tan(c+dx)) - 192\operatorname{arctanh}(\sin(c+dx))\right.}{384d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

output
```
-1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(48*c + 48*d*x + 60*Ar
cTan[Tan[c + d*x]] - 192*ArcTanh[Sin[c + d*x]] + 192*Sin[c + d*x] + 64*Sin
[c + d*x]^3 - 3*Sin[4*(c + d*x)] - 96*Tan[c + d*x]))/d
```

3.31.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^2(c+dx)\tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3351$$

$$\frac{\int \left(\cos^4(c+dx)a^6+2\cos^3(c+dx)a^6-\cos^2(c+dx)a^6+\sec^2(c+dx)a^6-4\cos(c+dx)a^6+2\sec(c+dx)a^6-a^6\right.}{a^4}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^6\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^6\sin^3(c+dx)}{3d}-\frac{2a^6\sin(c+dx)}{d}+\frac{a^6\tan(c+dx)}{d}+\frac{a^6\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^6\sin(c+dx)\cos(c+dx)}{8d}-\frac{9a^6}{8}}{a^4}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

output
```
((-9*a^6*x)/8 + (2*a^6*ArcTanh[Sin[c + d*x]])/d - (2*a^6*Sin[c + d*x])/d -
 (a^6*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^6*Cos[c + d*x]^3*Sin[c + d*x])
/(4*d) - (2*a^6*Sin[c + d*x]^3)/(3*d) + (a^6*Tan[c + d*x])/d)/a^4
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.31.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{a^2\left(-72dx\cos(dx+c)-128\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+128\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(5dx+5c)+64\sin(d}{64d\cos(dx+c)}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-}{d}$
risch	$-\frac{9a^2x}{8}+\frac{5ia^2e^{i(dx+c)}}{4d}-\frac{5ia^2e^{-i(dx+c)}}{4d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+a^2$
norman	$\frac{\frac{9a^2x}{8}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{22a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{31a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}-\frac{58a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{27a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{8}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/64*a^2*(-72*d*x*cos(d*x+c)-128*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+128*l
n(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+sin(5*d*x+5*c)+64*sin(d*x+c)-224/3*sin(
2*d*x+2*c)+sin(3*d*x+3*c)+16/3*sin(4*d*x+4*c))/d/cos(d*x+c)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx =$$

$$-\frac{27\,a^2 dx\cos(dx+c)-24\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+24\,a^2\cos(dx+c)\log(-\sin(dx+c)+}{24\,d\,co}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(27*a^2*d*x*cos(d*x + c) - 24*a^2*cos(d*x + c)*log(sin(d*x + c) + 1)
 + 24*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (6*a^2*cos(d*x + c)^4 + 16
*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c)^2 - 64*a^2*cos(d*x + c) + 24*a^2)
*sin(d*x + c))/(d*cos(d*x + c))
```

3.31.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = a^2\left(\int 2\sin^4(c+dx)\sec(c+dx)\,dx + \int \sin^4(c+dx)\sec^2(c+dx)\,dx + \int \sin^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4, x))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = -\frac{32\left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) + 6\sin(dx+c)\right)a^2 - 3(12\,dx +}{96\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/96*(32*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1) + 6*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(
2*d*x + 2*c))*a^2 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) -
2*tan(d*x + c))*a^2)/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx =$$

$$-\frac{27\,(dx+c)a^2 - 48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{24\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48*a
^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 48*a^2*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1) + 2*(51*a^2*tan(1/2*d*x + 1/2*c)^7 + 187*a^2*tan(1/
2*d*x + 1/2*c)^5 + 229*a^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{9\,a^2\,x}{8}$$
$$+\frac{\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4} + \frac{58\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3} + \frac{31\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2} - \frac{22\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} - \frac{7\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

input
```
int(sin(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

output
```
(4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^2*x)/8 + ((31*a^2*tan(c/2 + (d*
x)/2)^5)/2 - (22*a^2*tan(c/2 + (d*x)/2)^3)/3 + (58*a^2*tan(c/2 + (d*x)/2)^
7)/3 + (25*a^2*tan(c/2 + (d*x)/2)^9)/4 - (7*a^2*tan(c/2 + (d*x)/2))/4)/(d*
(3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6
- 3*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1))
```

3.32 $\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx$

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3.32.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx = -\frac{a^2 x}{2} + \frac{2a^2 \operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2 \sin(c+dx)}{d} - \frac{a^2 \cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2 \tan(c+dx)}{d}$$

output
```
-1/2*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*cos(d*x+
c)*sin(d*x+c)/d+a^2*tan(d*x+c)/d
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(73) = 146.

Time = 1.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.33

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx$$
$$= \frac{1}{16}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-2x\right.$$
$$-\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$-\frac{8\cos(dx)\sin(c)}{d}-\frac{\cos(2dx)\sin(2c)}{d}-\frac{8\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\sin(2dx)}{d}$$
$$+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$\left.+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-2*x - (8*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (8*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 - (8*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (8*Cos[c]*Sin[d*x])/d
 - (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[
c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/16
```

3.32.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4359, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4359$$

$$\int \tan^2(c+dx)(a(-\cos(c+dx))-a)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\tan\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3188$$

$$\frac{\int\left(-\cos^2(c+dx)a^4+\sec^2(c+dx)a^4-2\cos(c+dx)a^4+2\sec(c+dx)a^4\right)dx}{a^2}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4x}{2}}{a^2}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

output
```
(-1/2*(a^4*x) + (2*a^4*ArcTanh[Sin[c + d*x]])/d - (2*a^4*Sin[c + d*x])/d -
 (a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d)/a^2
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.32.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2(\tan(dx+c)-dx-c)}{d}+\frac{2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$-\frac{a^2\left(4dx\cos(dx+c)+16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-7\sin(dx+c)+\sin(3dx+3c\right.}{8d\cos(dx+c)}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right.}{d}$
norman	$\frac{\frac{a^2x}{2}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}-\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}-}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+
a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx =$$
$$-\frac{a^2dx\cos(dx+c)-2\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+2\,a^2\cos(dx+c)\log(-\sin(dx+c)+1)+}{2\,d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*d*x*cos(d*x + c) - 2*a^2*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*
a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 4*a^2*cos(
d*x + c) - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.32.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx = a^2\left(\int 2\sin^2(c+dx)\sec(c+dx)\,dx + \int \sin^2(c+dx)\sec^2(c+dx)\,dx + \int \sin^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2, x))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx = \frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a^2 - 4\,(dx + c - \tan(dx + c))a^2 + 4\,a^2(\log(\sin(dx + c) + 1) - \log(\sin(dx$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*a^2
 + 4*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))
/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx =$$

$$-\frac{(dx+c)a^2 - 4\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 4\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(}{}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4*a^2*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c+dx))^2 \sin^2(c+dx)\,dx$$
$$= \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a^2\,x}{2}$$
$$+ \frac{5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

input
```
int(sin(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

output
```
(4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*x)/2 + (6*a^2*tan(c/2 + (d*x)/2
)^3 + 5*a^2*tan(c/2 + (d*x)/2)^5 - 3*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.33.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d}$$

output
```
2*a^2*arctanh(sin(d*x+c))/d-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d+a^2*tan(
d*x+c)/d
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(57) = 114$.

Time $= 6.65$ (sec) , antiderivative size $= 401$, normalized size of antiderivative $= 7.04$

$$\begin{aligned}
&\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx \\
&= -\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d} \\
&\quad+\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d} \\
&\quad+\frac{\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{2d} \\
&\quad+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/2*(Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/d + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(
2*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(2*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos
[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2
]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c
/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

3.33.3 Rubi [A] (verified)

Time $= 0.49$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4360, 3042, 3352, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4360$$

$$\int \csc^2(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$\int \left(a^2\csc^2(c+dx)+a^2\csc^2(c+dx)\sec^2(c+dx)+2a^2\csc^2(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c +
d*x])/d + (a^2*Tan[c + d*x])/d
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.33.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^2\cot(dx+c)}{d}$
default	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^2\cot(dx+c)}{d}$
parallelrisch	$-\frac{a^2\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\cos(dx+c)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\cos(dx+c)}$
norman	$\frac{\frac{2a^2}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$-\frac{2ia^2\left(2e^{2i(dx+c)}-e^{i(dx+c)}+3\right)}{d\left(e^{i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}$

input
```
int(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a^2*(-1/sin(d*x+c)+ln(se
c(d*x+c)+tan(d*x+c)))-a^2*cot(d*x+c))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.77

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c) - a^2\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c) - 3}{d\cos(dx+c)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a^2*cos(d*x + c)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) - 3*a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x +
 c) + a^2)/(d*cos(d*x + c)*sin(d*x + c))
```

3.33.6 Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^2(c+dx)\sec(c+dx)\,dx + \int \csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2, x))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + a^2\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + \frac{a^2}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a^2*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + a
^2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/(tan(1/2*d*x + 1/2*c)^3 -
tan(1/2*d*x + 1/2*c)))/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 2\,a^2}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)} + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

output
```
(4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d
*x)/2)^3)) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.34.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{10a^2\tan(c+dx)}{3d} - \frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} - \frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}$$

output
```
2*a^2*arctanh(sin(d*x+c))/d+10/3*a^2*tan(d*x+c)/d-2*a^2*tan(d*x+c)/d/(1-co
s(d*x+c))-1/3*a^4*tan(d*x+c)/d/(a-a*cos(d*x+c))^2
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(87) = 174.

Time = 1.97 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.62

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right) - (-8+7\cos(c+dx))\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-(Cot[c/2]*Csc[(c + d*x)/2]^
2) - (-8 + 7*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*Sin[(d*x)/2] + 6*(-
2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))
)/(24*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4360, 3042, 3348, 3042, 3245, 27, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4360$$

$$\int \csc^4(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 3348$$

$$a^4\int \frac{\sec^2(c+dx)}{(a-a\cos(c+dx))^2}dx$$

$$\downarrow 3042$$

$$a^4\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 3245$$

$$a^4\left(\frac{\int \frac{2(\cos(c+dx)a+2a)\sec^2(c+dx)}{a-a\cos(c+dx)}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 27$$

$$a^4\left(\frac{2\int \frac{(\cos(c+dx)a+2a)\sec^2(c+dx)}{a-a\cos(c+dx)}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 3042$$

$$a^4\left(\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a+2a}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 3457$$

$$a^4\left(\frac{2\left(\frac{\int\left(3\cos(c+dx)a^2+5a^2\right)\sec^2(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 3042$$

$$a^4\left(\frac{2\left(\frac{\int \frac{3\sin\left(c+dx+\frac{\pi}{2}\right)a^2+5a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 3227$$

$$a^4\left(\frac{2\left(\frac{5a^2\int\sec^2(c+dx)dx+3a^2\int\sec(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 3042$$

$$a^4\left(\frac{2\left(\frac{3a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 4254$$

$$a^4\left(\frac{2\left(\frac{3a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{5a^2\int 1d(-\tan(c+dx))}{d}}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 24$$

$$a^4\left(\frac{2\left(\frac{3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

$$\downarrow 4257$$

$$a^4\left(\frac{2\left(\frac{\frac{3a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(1-\cos(c+dx))}\right)}{3a^2}-\frac{\tan(c+dx)}{3d(a-a\cos(c+dx))^2}\right)$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
a^4*(-1/3*Tan[c + d*x]/(d*(a - a*Cos[c + d*x])^2) + (2*((-3*Tan[c + d*x])/
(d*(1 - Cos[c + d*x])) + ((3*a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c +
 d*x])/d)/a^2))/(3*a^2))
```

3.34.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3348
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a^(2*m)   Int[(d*
Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.34.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

method	result
parallelrisch	$-\frac{2\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\frac{7\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\cos(dx+c)-\frac{5\cos(2dx+2c)}{14}-\frac{4}{7}\right)\cot}{6}\right.}{d\cos(dx+c)}$
norman	$\frac{\frac{a^2}{6d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
derivativedivides	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx\right.}{d}$
risch	$-\frac{4ia^2\left(3e^{4i(dx+c)}-9e^{3i(dx+c)}+11e^{2i(dx+c)}-12e^{i(dx+c)}+5\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$

input
```
int(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)-ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c
)+7/6*csc(1/2*d*x+1/2*c)^2*(cos(d*x+c)-5/14*cos(2*d*x+2*c)-4/7)*cot(1/2*d*
x+1/2*c))*a^2/d/cos(d*x+c)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.83

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{10\,a^2\cos(dx+c)^3-4\,a^2\cos(dx+c)^2-11\,a^2\cos(dx+c)-3\left(a^2\cos(dx+c)^2-a^2\cos(dx+c)\right)\log(}{3\left(d\cos(dx+c)^2-}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c) -
 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x +
 c) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin
(d*x + c) + 3*a^2)/((d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

3.34.6 Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^4(c+dx)\sec(c+dx)\,dx + \int \csc^4(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4, x))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + a^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3} - 3\tan(d\right.}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(a^2*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) + a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 12*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1
) - (15*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{-9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{14\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{a^2}{3}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

output
```
(4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((14*a^2*tan(c/2 + (d*x)/2)^2)/3 - 9
*a^2*tan(c/2 + (d*x)/2)^4 + a^2/3)/(d*(2*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2
+ (d*x)/2)^5))
```

3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.35.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d}$$

output

```
2*a^2*arctanh(sin(d*x+c))/d-4*a^2*cot(d*x+c)/d-5/3*a^2*cot(d*x+c)^3/d-2/5*
a^2*cot(d*x+c)^5/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d
*x+c)^5/d+a^2*tan(d*x+c)/d
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 317 vs. 2(129) = 258.

Time = 1.42 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.46

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-3840\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-3840*Cos[c + d
*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3840*Cos[c + d*x]*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c + d*x]
*(320*Sin[2*c] - 596*Sin[d*x] + 864*Sin[2*d*x] + 216*Sin[c - d*x] - 416*Si
n[c + d*x] + 624*Sin[2*(c + d*x)] - 416*Sin[3*(c + d*x)] + 104*Sin[4*(c +
d*x)] - 596*Sin[2*c + d*x] - 680*Sin[3*c + d*x] + 894*Sin[c + 2*d*x] + 224
*Sin[2*(c + 2*d*x)] + 894*Sin[3*c + 2*d*x] + 480*Sin[4*c + 2*d*x] - 776*Si
n[c + 3*d*x] - 596*Sin[2*c + 3*d*x] - 596*Sin[4*c + 3*d*x] - 120*Sin[5*c +
 3*d*x] + 149*Sin[3*c + 4*d*x] + 149*Sin[5*c + 4*d*x])))/(7680*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int \csc^6(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 3352$$

$$\int \left(a^2\csc^6(c+dx)+a^2\csc^6(c+dx)\sec^2(c+dx)+2a^2\csc^6(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c +
d*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2
*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d
*x])/d
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.35.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

method	result
norman	$\frac{\frac{a^2}{40d}+\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}+\frac{31a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{12d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln($
derivativedivides	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+}\right.$
default	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+}\right.$
parallelrisch	$\frac{a^2\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+240\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-240\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\right.}{120d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
risch	$-\frac{4ia^2\left(15\,e^{7i(dx+c)}-60\,e^{6i(dx+c)}+85\,e^{5i(dx+c)}-40\,e^{4i(dx+c)}-27\,e^{3i(dx+c)}+108\,e^{2i(dx+c)}-97\,e^{i(dx+c)}+28\right)}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)\left(e^{i(dx+c)}+1\right)}-\frac{2a^2\ln}$

input
```
int(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(1/40/d*a^2+4/15*a^2/d*tan(1/2*d*x+1/2*c)^2+31/12*a^2/d*tan(1/2*d*x+1/2*c)
^4-5*a^2/d*tan(1/2*d*x+1/2*c)^6+1/8/d*a^2*tan(1/2*d*x+1/2*c)^8)/tan(1/2*d*
x+1/2*c)^5/(-1+tan(1/2*d*x+1/2*c)^2)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)+2*a^
2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.60

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{56\,a^2\cos(dx+c)^4-82\,a^2\cos(dx+c)^3-32\,a^2\cos(dx+c)^2+76\,a^2\cos(dx+c)-15\left(a^2\cos(dx+c)^3\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(56*a^2*cos(d*x + c)^4 - 82*a^2*cos(d*x + c)^3 - 32*a^2*cos(d*x + c)
^2 + 76*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 +
 a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a^2*cos(d*x +
c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin
(d*x + c) - 15*a^2)/((d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x +
c))*sin(d*x + c))
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+3\,a^2\left(\frac{15\tan(dx}{15\,d}\right.$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(a^2*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*a^2*((15*tan(d*x
+ c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(
d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{240\,a^2\tan}{\tan\left(\frac{1}{2}\,d\right.}}{120\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/120*(240*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a^2*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 15*a^2*tan(1/2*d*x + 1/2*c) - 240*a^2*tan(1/2*d*x +
 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (345*a^2*tan(1/2*d*x + 1/2*c)^4 + 3
5*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{-39\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{62\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}+\frac{a^2}{5}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

output
```
(4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((32*a^2*tan(c/2 + (d*x)/2)^2)/15 +
(62*a^2*tan(c/2 + (d*x)/2)^4)/3 - 39*a^2*tan(c/2 + (d*x)/2)^6 + a^2/5)/(d*
(8*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (d*x)/2)^7)) + (a^2*tan(c/2 + (d*x)/
2))/(8*d)
```

3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.36.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d}$$

output

```
2*a^2*arctanh(sin(d*x+c))/d-5*a^2*cot(d*x+c)/d-3*a^2*cot(d*x+c)^3/d-7/5*a^
2*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)^7/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x
+c)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

3.36.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 2.96 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-6881280\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c-\right.\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-6881280*Cos[c
+ d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6881280*Cos[c + d*x]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 32*Csc[2*c]*Csc[(c + d*x)/2]^4*Csc
[c + d*x]^3*(-9856*Sin[2*c] + 17288*Sin[d*x] - 29056*Sin[2*d*x] - 7264*Sin
[c - d*x] + 14208*Sin[c + d*x] - 19536*Sin[2*(c + d*x)] + 7104*Sin[3*(c +
d*x)] + 7104*Sin[4*(c + d*x)] - 7104*Sin[5*(c + d*x)] + 1776*Sin[6*(c + d*
x)] + 17288*Sin[2*c + d*x] + 20384*Sin[3*c + d*x] - 23771*Sin[c + 2*d*x] +
 7104*Sin[2*(c + 2*d*x)] - 23771*Sin[3*c + 2*d*x] - 8960*Sin[4*c + 2*d*x]
+ 19984*Sin[c + 3*d*x] + 8644*Sin[2*c + 3*d*x] + 8644*Sin[4*c + 3*d*x] - 6
160*Sin[5*c + 3*d*x] + 8644*Sin[3*c + 4*d*x] + 8644*Sin[5*c + 4*d*x] + 672
0*Sin[6*c + 4*d*x] - 12144*Sin[3*c + 5*d*x] - 8644*Sin[4*c + 5*d*x] - 8644
*Sin[6*c + 5*d*x] - 1680*Sin[7*c + 5*d*x] + 3456*Sin[4*c + 6*d*x] + 2161*S
in[5*c + 6*d*x] + 2161*Sin[7*c + 6*d*x])))/(13762560*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^8(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\downarrow 3042$$

$$\int \frac{\left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^2}{\cos\left(c + dx - \frac{\pi}{2}\right)^8} dx$$

$$\downarrow 4360$$

$$\int \csc^8(c + dx)\sec^2(c + dx)(a(-\cos(c + dx)) - a)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c + dx - \frac{\pi}{2}\right) - a\right)^2}{\sin\left(c + dx - \frac{\pi}{2}\right)^2 \cos\left(c + dx - \frac{\pi}{2}\right)^8} dx$$

$$\downarrow 3352$$

$$\int \left(a^2\csc^8(c + dx) + a^2\csc^8(c + dx)\sec^2(c + dx) + 2a^2\csc^8(c + dx)\sec(c + dx)\right) dx$$

$$\downarrow 2009$$

$$\frac{2a^2\text{arctanh}(\sin(c + dx))}{d} + \frac{a^2\tan(c + dx)}{d} - \frac{2a^2\cot^7(c + dx)}{7d} - \frac{7a^2\cot^5(c + dx)}{5d} - \frac{3a^2\cot^3(c + dx)}{d} - \frac{5a^2\cot(c + dx)}{d} - \frac{2a^2\csc^7(c + dx)}{7d} - \frac{2a^2\csc^5(c + dx)}{5d} - \frac{2a^2\csc^3(c + dx)}{3d} - \frac{2a^2\csc(c + dx)}{d}$$

input
```
Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c +
d*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) -
(2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x
]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.36.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{a^2\left(15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+174\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+910\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-6720\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2$
norman	$\frac{\frac{a^2}{224d}+\frac{29a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{560d}+\frac{163a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{480d}+\frac{67a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24d}+\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{48d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{96d}-\frac{175a^2\tan\left(\frac{dx}{2}\right.}{32d}$ $\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)$
derivativedivides	$\frac{a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a$
default	$\frac{a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a$
risch	$-\frac{4ia^2\left(105\,e^{11i(dx+c)}-420\,e^{10i(dx+c)}+385\,e^{9i(dx+c)}+560\,e^{8i(dx+c)}-1274\,e^{7i(dx+c)}+616\,e^{6i(dx+c)}+454\,e^{5i(dx+c)}-18\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

input
```
int(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(15*cot(1/2*d*x+1/2*c)^7+35*tan(1/2*d*x+1/2*c)^5+174*cot(1/2*d*x+1/2*c
)^5+910*tan(1/2*d*x+1/2*c)^3-6720*ln(tan(1/2*d*x+1/2*c)-1)*tan(1/2*d*x+1/2
*c)^2+6720*ln(tan(1/2*d*x+1/2*c)+1)*tan(1/2*d*x+1/2*c)^2+1141*cot(1/2*d*x+
1/2*c)^3-18375*tan(1/2*d*x+1/2*c)+6720*ln(tan(1/2*d*x+1/2*c)-1)-6720*ln(ta
n(1/2*d*x+1/2*c)+1)+9380*cot(1/2*d*x+1/2*c))/(3360*d*tan(1/2*d*x+1/2*c)^2-
3360*d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.67

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{432\,a^2\cos(dx+c)^6 - 654\,a^2\cos(dx+c)^5 - 636\,a^2\cos(dx+c)^4 + 1226\,a^2\cos(dx+c)^3 + 74\,a^2\cos(dx$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/105*(432*a^2*cos(d*x + c)^6 - 654*a^2*cos(d*x + c)^5 - 636*a^2*cos(d*x
+ c)^4 + 1226*a^2*cos(d*x + c)^3 + 74*a^2*cos(d*x + c)^2 - 562*a^2*cos(d*x
 + c) - 105*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c
)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^2*cos(
d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c
))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 105*a^2)/((d*cos(d*x + c)^5 - 2*d
*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-a^2\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7} - 105\log(\sin(dx+c)+1) + 105\log(\sin(dx+c)-1)\right.$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(a^2*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2
 + 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) -
 1)) + 3*a^2*((140*tan(d*x + c)^6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2
+ 5)/tan(d*x + c)^7 - 35*tan(d*x + c)) + 3*(35*tan(d*x + c)^6 + 35*tan(d*x
 + c)^4 + 21*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 945$$
$$\text{3360}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 6720*a^2*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 6720*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 945*a^2*tan(1/2*
d*x + 1/2*c) - 6720*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
- (10710*a^2*tan(1/2*d*x + 1/2*c)^6 + 1330*a^2*tan(1/2*d*x + 1/2*c)^4 + 18
9*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d} + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32\,d}$$
$$- \frac{-166\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + \frac{268\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3} + \frac{163\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15} + \frac{58\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35} + \frac{a^2}{7}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - 32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\right)}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^8,x)
```

output
```
(a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d +
(9*a^2*tan(c/2 + (d*x)/2))/(32*d) - ((58*a^2*tan(c/2 + (d*x)/2)^2)/35 + (1
63*a^2*tan(c/2 + (d*x)/2)^4)/15 + (268*a^2*tan(c/2 + (d*x)/2)^6)/3 - 166*a
^2*tan(c/2 + (d*x)/2)^8 + a^2/7)/(d*(32*tan(c/2 + (d*x)/2)^7 - 32*tan(c/2
+ (d*x)/2)^9))
```

3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.37.1 Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{9a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d} + \frac{a^2\tan(c+dx)}{d}$$

output

```
2*a^2*arctanh(sin(d*x+c))/d-6*a^2*cot(d*x+c)/d-14/3*a^2*cot(d*x+c)^3/d-16/
5*a^2*cot(d*x+c)^5/d-9/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*c
sc(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+
c)^7/d-2/9*a^2*csc(d*x+c)^9/d+a^2*tan(d*x+c)/d
```

3.37.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1050 vs. $2(201) = 402$.

Time $= 10.92$ (sec) , antiderivative size $= 1050$, normalized size of antiderivative $= 5.22$

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-6899*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^4*(
a + a*Sec[c + d*x])^2)/(80640*d) - (193*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 +
(d*x)/2]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(13440*d) - (71*Co
s[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^4*(a + a*Sec
[c + d*x])^2)/(32256*d) - (Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4608*d) - (Cos[c + d*x]^2*Log[
Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (123041*Cos[c
+ d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d
*x])^2*Sin[(d*x)/2])/(161280*d) + (6899*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 +
(d*x)/2]^3*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(8064
0*d) + (193*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2
]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(13440*d) + (71*Cos[c + d*x]^2*Cs
c[c/2]*Csc[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Si
n[(d*x)/2])/(32256*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[
c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4608*d) + (803*Cos[
c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(a + a*Sec[c + d*x])^2*Sin[(d*x)/
2])/(7680*d) + (49*Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec
[c + d*x])^2*Sin[(d*x)/2])/(7680*d) + (Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 ...
```

3.37.3 Rubi [A] (verified)

Time $= 0.57$ (sec) , antiderivative size $= 201$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4360, 3042, 3352, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{10}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 4360$$

$$\int \csc^{10}(c+dx)\sec^2(c+dx)(a(-\cos(c+dx))-a)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 3352$$

$$\int \left(a^2\csc^{10}(c+dx)+a^2\csc^{10}(c+dx)\sec^2(c+dx)+2a^2\csc^{10}(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
 d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7
*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c
 + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(
7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.37.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{a^2\left(35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^9+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+460\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+1092\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3096\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+16800\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$
derivativedivides	$a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx$
default	$a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx$
risch	$-\frac{4ia^2\left(315\,e^{15i(dx+c)}-1260\,e^{14i(dx+c)}+525\,e^{13i(dx+c)}+4200\,e^{12i(dx+c)}-5817\,e^{11i(dx+c)}-2772\,e^{10i(dx+c)}+10161\,e^{9i(}\right.}{315d\left(e^{i(dx+}\right.}$

input
```
int(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(35*cot(1/2*d*x+1/2*c)^9+63*tan(1/2*d*x+1/2*c)^7+460*cot(1/2*d*x+1/2*c
)^7+1092*tan(1/2*d*x+1/2*c)^5+3096*cot(1/2*d*x+1/2*c)^5+16800*tan(1/2*d*x+
1/2*c)^3-80640*ln(tan(1/2*d*x+1/2*c)-1)*tan(1/2*d*x+1/2*c)^2+80640*ln(tan(
1/2*d*x+1/2*c)+1)*tan(1/2*d*x+1/2*c)^2+16044*cot(1/2*d*x+1/2*c)^3-238140*t
an(1/2*d*x+1/2*c)+80640*ln(tan(1/2*d*x+1/2*c)-1)-80640*ln(tan(1/2*d*x+1/2*
c)+1)+119910*cot(1/2*d*x+1/2*c))/(40320*d*tan(1/2*d*x+1/2*c)^2-40320*d)
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(185) = 370$.

Time $= 0.28$ (sec) , antiderivative size $= 406$, normalized size of antiderivative $= 2.02$

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{1408\,a^2\cos(dx+c)^8 - 2186\,a^2\cos(dx+c)^7 - 3372\,a^2\cos(dx+c)^6 + 6200\,a^2\cos(dx+c)^5 + 2060\,a^2\,c$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/315*(1408*a^2*cos(d*x + c)^8 - 2186*a^2*cos(d*x + c)^7 - 3372*a^2*cos(d
*x + c)^6 + 6200*a^2*cos(d*x + c)^5 + 2060*a^2*cos(d*x + c)^4 - 5784*a^2*c
os(d*x + c)^3 + 268*a^2*cos(d*x + c)^2 + 1756*a^2*cos(d*x + c) - 315*(a^2*
cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x
 + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*lo
g(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x
 + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 -
 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x +
 c) - 315*a^2)/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^6 - d*cos(d*x + c)^5
+ 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x +
 c))*sin(d*x + c))
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{a^2\left(\frac{2\left(315\sin(dx+c)^8+105\sin(dx+c)^6+63\sin(dx+c)^4+45\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(\right.\right.\right.}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/315*(a^2*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^
4 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 3
15*log(sin(d*x + c) - 1)) + 5*a^2*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^
6 + 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*
x + c)) + (315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 +
180*tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80640\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-80640\,a^2\log\left(\right.}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 +
 80640*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 80640*a^2*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) + 17955*a^2*tan(1/2*d*x + 1/2*c) - 80640*a^2*tan(1/2*d*
x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (139545*a^2*tan(1/2*d*x + 1/2*c)
^8 + 19635*a^2*tan(1/2*d*x + 1/2*c)^6 + 3591*a^2*tan(1/2*d*x + 1/2*c)^4 +
495*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{11\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d} + \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{640\,d} + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$
$$- \frac{-699\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \frac{1142\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3} + \frac{764\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{15} + \frac{344\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{35} + \frac{92\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{63} + \frac{a^2}{9}}{d\left(128\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 - 128\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\right)}$$
$$+ \frac{57\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d}$$

input
```
int((a + a/cos(c + d*x))^2/sin(c + d*x)^10,x)
```

output
```
(11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(640*d)
 + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((92*a^2*tan(c/2 + (d*x)/2)^2)/63
 + (344*a^2*tan(c/2 + (d*x)/2)^4)/35 + (764*a^2*tan(c/2 + (d*x)/2)^6)/15 +
 (1142*a^2*tan(c/2 + (d*x)/2)^8)/3 - 699*a^2*tan(c/2 + (d*x)/2)^10 + a^2/9
)/(d*(128*tan(c/2 + (d*x)/2)^9 - 128*tan(c/2 + (d*x)/2)^11)) + (57*a^2*tan
(c/2 + (d*x)/2))/(128*d)
```

3.38 $\int (a + a\sec(c+dx))^3 \sin^9(c+dx)\,dx$

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3.38.1 Optimal result

Integrand size = 21, antiderivative size = 203

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^9(c+dx)\,dx = {} & \frac{11a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} \\ & - \frac{14a^3\cos^3(c+dx)}{3d} - \frac{7a^3\cos^4(c+dx)}{2d} \\ & + \frac{6a^3\cos^5(c+dx)}{5d} + \frac{11a^3\cos^6(c+dx)}{6d} \\ & + \frac{a^3\cos^7(c+dx)}{7d} - \frac{3a^3\cos^8(c+dx)}{8d} \\ & - \frac{a^3\cos^9(c+dx)}{9d} + \frac{a^3\log(\cos(c+dx))}{d} \\ & + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

output

```
11*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-14/3*a^3*cos(d*x+c)^3/d-7/2*a^3*c
os(d*x+c)^4/d+6/5*a^3*cos(d*x+c)^5/d+11/6*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d
*x+c)^7/d-3/8*a^3*cos(d*x+c)^8/d-1/9*a^3*cos(d*x+c)^9/d+a^3*ln(cos(d*x+c))
/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.38.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx$$
$$= \frac{a^3(471450 + 11624760\cos(c + dx) + 2188872\cos(3(c + dx)) + 41160\cos(4(c + dx)) - 204156\cos(5(c +$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

output
```
(a^3*(471450 + 11624760*Cos[c + d*x] + 2188872*Cos[3*(c + d*x)] + 41160*Co
s[4*(c + d*x)] - 204156*Cos[5*(c + d*x)] - 35805*Cos[6*(c + d*x)] + 22972*
Cos[7*(c + d*x)] + 9030*Cos[8*(c + d*x)] - 820*Cos[9*(c + d*x)] - 945*Cos[
10*(c + d*x)] - 140*Cos[11*(c + d*x)] + 645120*Log[Cos[c + d*x]] + 210*Cos
[2*(c + d*x)]*(-413 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(1290240*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^9(c + dx)(a\sec(c + dx) + a)^3\, dx$$
$$\downarrow 3042$$
$$\int \cos\left(c + dx - \frac{\pi}{2}\right)^9 \left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^3 dx$$
$$\downarrow 4360$$
$$\int \sin^6(c + dx)\tan^3(c + dx)\left(-(a(-\cos(c + dx)) - a)^3\right) dx$$
$$\downarrow 25$$
$$-\int -(\cos(c + dx)a + a)^3 \sin^6(c + dx)\tan^3(c + dx) dx$$
$$\downarrow 25$$

$$\int \sin^6(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^9\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^4(\cos(c+dx)a+a)^7\sec^3(c+dx)d(a\cos(c+dx))}{a^9d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^7\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{a^6d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\cos^8(c+dx)a^8+3\cos^7(c+dx)a^8-\cos^6(c+dx)a^8-11\cos^5(c+dx)a^8-6\cos^4(c+dx)a^8+14\cos^3(c+dx)\right.}{}$$

$$\downarrow 2009$$

$$-\frac{1}{9}a^9\cos^9(c+dx)+\frac{3}{8}a^9\cos^8(c+dx)-\frac{1}{7}a^9\cos^7(c+dx)-\frac{11}{6}a^9\cos^6(c+dx)-\frac{6}{5}a^9\cos^5(c+dx)+\frac{7}{2}a^9\cos^4(c+$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

output
```
-((-11*a^9*Cos[c + d*x] - 3*a^9*Cos[c + d*x]^2 + (14*a^9*Cos[c + d*x]^3)/3
 + (7*a^9*Cos[c + d*x]^4)/2 - (6*a^9*Cos[c + d*x]^5)/5 - (11*a^9*Cos[c + d
*x]^6)/6 - (a^9*Cos[c + d*x]^7)/7 + (3*a^9*Cos[c + d*x]^8)/8 + (a^9*Cos[c
 + d*x]^9)/9 - a^9*Log[a*Cos[c + d*x]] - 3*a^9*Sec[c + d*x] - (a^9*Sec[c +
d*x]^2)/2)/(a^6*d))
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.38.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{a^3\left(-645120(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+645120(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+645120(1+\cos(2dx+$
derivativedivides	$a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin(dx+c)^6}{3}+\sin(dx+c)^4+2\sin(dx+c)^2+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\si\right.\right.$
default	$a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin(dx+c)^6}{3}+\sin(dx+c)^4+2\sin(dx+c)^2+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\si\right.\right.$
parts	$-\frac{a^3\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin}\right.}$
risch	$-\frac{2ia^3c}{d}-ia^3x-\frac{a^3\cos(9dx+9c)}{2304d}+\frac{a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{25a^3e^{-3i(dx+c)}}{64d}+\frac{57a^3e^{-2i(dx+c)}}{256d}+\frac{1059a^3e^{-i(}}{256d}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
1/645120*a^3*(-645120*(1+cos(2*d*x+2*c))*ln(sec(1/2*d*x+1/2*c)^2)+645120*(
1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)+645120*(1+cos(2*d*x+2*c))*ln(ta
n(1/2*d*x+1/2*c)+1)-35805*cos(6*d*x+6*c)+22972*cos(7*d*x+7*c)+9030*cos(8*d
*x+8*c)-820*cos(9*d*x+9*c)-945*cos(10*d*x+10*c)-140*cos(11*d*x+11*c)+11624
760*cos(d*x+c)+6529934*cos(2*d*x+2*c)+2188872*cos(3*d*x+3*c)+41160*cos(4*d
*x+4*c)-204156*cos(5*d*x+5*c)+7088114)/d/(1+cos(2*d*x+2*c))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx =$$
$$-\frac{35840\,a^3\cos(dx+c)^{11}+120960\,a^3\cos(dx+c)^{10}-46080\,a^3\cos(dx+c)^9-591360\,a^3\cos(dx+c)^8-}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/322560*(35840*a^3*cos(d*x + c)^11 + 120960*a^3*cos(d*x + c)^10 - 46080*
a^3*cos(d*x + c)^9 - 591360*a^3*cos(d*x + c)^8 - 387072*a^3*cos(d*x + c)^7
 + 1128960*a^3*cos(d*x + c)^6 + 1505280*a^3*cos(d*x + c)^5 - 967680*a^3*co
s(d*x + c)^4 - 3548160*a^3*cos(d*x + c)^3 - 322560*a^3*cos(d*x + c)^2*log(
-cos(d*x + c)) + 212205*a^3*cos(d*x + c)^2 - 967680*a^3*cos(d*x + c) - 161
280*a^3)/(d*cos(d*x + c)^2)
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**9,x)
```

output
```
Timed out
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx =$$

$$-\frac{280\, a^3 \cos(dx + c)^9 + 945\, a^3 \cos(dx + c)^8 - 360\, a^3 \cos(dx + c)^7 - 4620\, a^3 \cos(dx + c)^6 - 3024\, a^3 \cos($$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/2520*(280*a^3*cos(d*x + c)^9 + 945*a^3*cos(d*x + c)^8 - 360*a^3*cos(d*x
 + c)^7 - 4620*a^3*cos(d*x + c)^6 - 3024*a^3*cos(d*x + c)^5 + 8820*a^3*cos
(d*x + c)^4 + 11760*a^3*cos(d*x + c)^3 - 7560*a^3*cos(d*x + c)^2 - 27720*a
^3*cos(d*x + c) - 2520*a^3*log(cos(d*x + c)) - 1260*(6*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^2)/d
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(187) = 374$.

Time $= 0.49$ (sec) , antiderivative size $= 396$, normalized size of antiderivative $= 1.95$

$$\int (a + a\sec(c+dx))^3 \sin^9(c+dx)\,dx =$$

$$-\frac{2520\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{1260\left(9\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^3(\cos(dx+c}{(\cos(dx+c)+}\right.}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="giac")
```

output
```
-1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 1260*(9*a^3
 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^3*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (4
5257*a^3 - 392193*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1467972*a^3*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3001908*a^3*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 + 3232782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 - 2359854*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1190196*a^
3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 397764*a^3*(cos(d*x + c) - 1
)^7/(cos(d*x + c) + 1)^7 + 79281*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) +
1)^8 - 7129*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

3.38.9 Mupad [B] (verification not implemented)

Time $= 12.63$ (sec) , antiderivative size $= 157$, normalized size of antiderivative $= 0.77$

$$\int (a + a\sec(c+dx))^3 \sin^9(c+dx)\,dx$$

$$= \frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2} + 11\,a^3\,\cos(c+d\,x) + 3\,a^3\cos(c+d\,x)^2 - \frac{14\,a^3\cos(c+d\,x)^3}{3} - \frac{7\,a^3\cos(c+d\,x)^4}{2} + \frac{6\,a^3\cos(c+d\,x)^5}{5} +}{d}$$

input
```
int(sin(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

output

```
((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 11*a^3*cos(c + d*x) + 3*a^3
*cos(c + d*x)^2 - (14*a^3*cos(c + d*x)^3)/3 - (7*a^3*cos(c + d*x)^4)/2 + (
6*a^3*cos(c + d*x)^5)/5 + (11*a^3*cos(c + d*x)^6)/6 + (a^3*cos(c + d*x)^7)
/7 - (3*a^3*cos(c + d*x)^8)/8 - (a^3*cos(c + d*x)^9)/9 + a^3*log(cos(c + d
*x)))/d
```

3.39 $\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$

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3.39.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx = {} & \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} \\ & - \frac{2a^3\cos^3(c+dx)}{d} - \frac{2a^3\cos^4(c+dx)}{d} \\ & + \frac{a^3\cos^6(c+dx)}{2d} + \frac{a^3\cos^7(c+dx)}{7d} \\ & + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

output
```
8*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-2*a^3*cos(d*x+c)^3/d-2*a^3*cos(d*x
+c)^4/d+1/2*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x+c)^7/d+3*a^3*sec(d*x+c)/d+1
/2*a^3*sec(d*x+c)^2/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$$
$$= \frac{a^3(427 + 14014\cos(c+dx) - 210\cos(2(c+dx)) + 2548\cos(3(c+dx)) + 196\cos(4(c+dx)) - 188\cos(5$$
$$1792d$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

output
```
(a^3*(427 + 14014*Cos[c + d*x] - 210*Cos[2*(c + d*x)] + 2548*Cos[3*(c + d*
x)] + 196*Cos[4*(c + d*x)] - 188*Cos[5*(c + d*x)] - 56*Cos[6*(c + d*x)] +
9*Cos[7*(c + d*x)] + 7*Cos[8*(c + d*x)] + Cos[9*(c + d*x)])*Sec[c + d*x]^2
)/(1792*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^7\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin^4(c+dx)\tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\sin^4(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin^4(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^7\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^3(\cos(c+dx)a+a)^6\sec^3(c+dx)d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^6\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6-3\cos^5(c+dx)a^6+8\cos^3(c+dx)a^6+\sec^3(c+dx)a^6+6\cos^2(c+dx)a^6+3\sec^2(c+dx)\right.}{a^4d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)-\frac{1}{2}a^7\cos^6(c+dx)+2a^7\cos^4(c+dx)+2a^7\cos^3(c+dx)-3a^7\cos^2(c+dx)-8a^7\cos(c+d}{a^4d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

output
```
-((-8*a^7*Cos[c + d*x] - 3*a^7*Cos[c + d*x]^2 + 2*a^7*Cos[c + d*x]^3 + 2*a
^7*Cos[c + d*x]^4 - (a^7*Cos[c + d*x]^6)/2 - (a^7*Cos[c + d*x]^7)/7 - 3*a^
7*Sec[c + d*x] - (a^7*Sec[c + d*x]^2)/2)/(a^4*d))
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.39.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{a^3(196\cos(4dx+4c)+2548\cos(3dx+3c)+14014\cos(dx+c)+7\cos(8dx+8c)+9\cos(7dx+7c)-56\cos(6dx+6c)-188\cos(5$ $896d(1+\cos(2dx+2c))$
derivativedivides	$a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx+c)^2}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin}{}\right.\right.$
default	$a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx+c)^2}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin}{}\right.\right.$
parts	$-\frac{a^3\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx}{d}\right.\right.$
risch	$-\frac{29a^3e^{3i(dx+c)}}{128d}+\frac{47a^3e^{2i(dx+c)}}{128d}+\frac{421a^3e^{i(dx+c)}}{128d}+\frac{421a^3e^{-i(dx+c)}}{128d}+\frac{47a^3e^{-2i(dx+c)}}{128d}-\frac{29a^3e^{-3i(dx+c)}}{128d}+$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/896/d*a^3*(196*cos(4*d*x+4*c)+2548*cos(3*d*x+3*c)+14014*cos(d*x+c)+7*cos
(8*d*x+8*c)+9*cos(7*d*x+7*c)-56*cos(6*d*x+6*c)-188*cos(5*d*x+5*c)+7800*cos
(2*d*x+2*c)+cos(9*d*x+9*c)+8437)/(1+cos(2*d*x+2*c))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$$
$$= \frac{32\,a^3\cos(dx+c)^9 + 112\,a^3\cos(dx+c)^8 - 448\,a^3\cos(dx+c)^6 - 448\,a^3\cos(dx+c)^5 + 672\,a^3\cos(dx+c}{224\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/224*(32*a^3*cos(d*x + c)^9 + 112*a^3*cos(d*x + c)^8 - 448*a^3*cos(d*x +
c)^6 - 448*a^3*cos(d*x + c)^5 + 672*a^3*cos(d*x + c)^4 + 1792*a^3*cos(d*x
+ c)^3 - 203*a^3*cos(d*x + c)^2 + 672*a^3*cos(d*x + c) + 112*a^3)/(d*cos(d
*x + c)^2)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**7,x)
```

output
```
Timed out
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$$
$$= \frac{2\,a^3\cos(dx+c)^7 + 7\,a^3\cos(dx+c)^6 - 28\,a^3\cos(dx+c)^4 - 28\,a^3\cos(dx+c)^3 + 42\,a^3\cos(dx+c)^2 + 1}{14\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/14*(2*a^3*cos(d*x + c)^7 + 7*a^3*cos(d*x + c)^6 - 28*a^3*cos(d*x + c)^4
- 28*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c)^2 + 112*a^3*cos(d*x + c) + 7
*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.82

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$$
$$= \frac{2\left(\frac{7\left(3a^3+\frac{2a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} - \frac{43a^3-\frac{273a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{672a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{630a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{343a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{105a^3(\cos(}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}\right)}{7d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="giac")
```

output
```
2/7*(7*(3*a^3 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2 - (43*a^3 - 273*a^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 672*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 630
*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 343*a^3*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 105*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)
^5 + 14*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - 1)^7)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^3 \sin^7(c+dx)\,dx$$
$$= \frac{\frac{3a^3\cos(c+dx)+\frac{a^3}{2}}{\cos(c+dx)^2} + 8a^3\cos(c+dx) + 3a^3\cos(c+dx)^2 - 2a^3\cos(c+dx)^3 - 2a^3\cos(c+dx)^4 + \frac{a^3\cos(c}{2}}{d}$$

input
```
int(sin(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

output
```
((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 8*a^3*cos(c + d*x) + 3*a^3*
cos(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 2*a^3*cos(c + d*x)^4 + (a^3*cos(c
+ d*x)^6)/2 + (a^3*cos(c + d*x)^7)/7)/d
```

3.40 $\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx$

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3.40.1 Optimal result

Integrand size = 21, antiderivative size = 134

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx = {} & \frac{5a^3\cos(c+dx)}{d} + \frac{5a^3\cos^2(c+dx)}{2d} \\ & - \frac{a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos^4(c+dx)}{4d} \\ & - \frac{a^3\cos^5(c+dx)}{5d} - \frac{a^3\log(\cos(c+dx))}{d} \\ & + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

output
```
5*a^3*cos(d*x+c)/d+5/2*a^3*cos(d*x+c)^2/d-1/3*a^3*cos(d*x+c)^3/d-3/4*a^3*c
os(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)
/d+1/2*a^3*sec(d*x+c)^2/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$
$$-\frac{a^3(-120 - 12350\cos(c+dx) - 2074\cos(3(c+dx)) - 330\cos(4(c+dx)) + 82\cos(5(c+dx)) + 45\cos($$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

output
```
-1/1920*(a^3*(-120 - 12350*Cos[c + d*x] - 2074*Cos[3*(c + d*x)] - 330*Cos[
4*(c + d*x)] + 82*Cos[5*(c + d*x)] + 45*Cos[6*(c + d*x)] + 6*Cos[7*(c + d*
x)] + 960*Log[Cos[c + d*x]] + 15*Cos[2*(c + d*x)]*(31 + 64*Log[Cos[c + d*x
]]))*Sec[c + d*x]^2)/d
```

3.40.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin^2(c+dx)\tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\sin^2(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin^2(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int (a-a\cos(c+dx))^2(\cos(c+dx)a+a)^5\sec^3(c+dx)d(a\cos(c+dx))}{a^5d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^5\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 99$$

$$-\frac{\int (\cos^4(c+dx)a^4+3\cos^3(c+dx)a^4+\sec^3(c+dx)a^4+\cos^2(c+dx)a^4+3\sec^2(c+dx)a^4-5\cos(c+dx)a^4+}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)+\frac{3}{4}a^5\cos^4(c+dx)+\frac{1}{3}a^5\cos^3(c+dx)-\frac{5}{2}a^5\cos^2(c+dx)-5a^5\cos(c+dx)-\frac{1}{2}a^5\sec^2(c+dx)}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

output
```
-((-5*a^5*Cos[c + d*x] - (5*a^5*Cos[c + d*x]^2)/2 + (a^5*Cos[c + d*x]^3)/3
 + (3*a^5*Cos[c + d*x]^4)/4 + (a^5*Cos[c + d*x]^5)/5 + a^5*Log[a*Cos[c + d
*x]] - 3*a^5*Sec[c + d*x] - (a^5*Sec[c + d*x]^2)/2)/(a^2*d))
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.40.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(d$
default	$\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(d$
parallelrisch	$\frac{a^3\left(960(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-960(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-960(1+\cos(2dx+2c))\ln\left(\tan($
parts	$-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)}{d}+$
norman	$\frac{\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{224a^3}{15d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}+\frac{134a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{15d}+\frac{214a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d}$
risch	$ia^3x-\frac{7a^3e^{3i(dx+c)}}{96d}+\frac{7a^3e^{2i(dx+c)}}{16d}+\frac{37a^3e^{i(dx+c)}}{16d}+\frac{37a^3e^{-i(dx+c)}}{16d}+\frac{7a^3e^{-2i(dx+c)}}{16d}-\frac{7a^3e^{-3i(dx+c)}}{96d}+$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln
(cos(d*x+c)))+3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x
+c)^2)*cos(d*x+c))+3*a^3*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)
))-1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{96\,a^3\cos(dx+c)^7 + 360\,a^3\cos(dx+c)^6 + 160\,a^3\cos(dx+c)^5 - 1200\,a^3\cos(dx+c)^4 - 2400\,a^3\cos(d}{480\,d\cos}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/480*(96*a^3*cos(d*x + c)^7 + 360*a^3*cos(d*x + c)^6 + 160*a^3*cos(d*x +
 c)^5 - 1200*a^3*cos(d*x + c)^4 - 2400*a^3*cos(d*x + c)^3 + 480*a^3*cos(d*
x + c)^2*log(-cos(d*x + c)) + 465*a^3*cos(d*x + c)^2 - 1440*a^3*cos(d*x +
c) - 240*a^3)/(d*cos(d*x + c)^2)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{12\,a^3\cos(dx+c)^5 + 45\,a^3\cos(dx+c)^4 + 20\,a^3\cos(dx+c)^3 - 150\,a^3\cos(dx+c)^2 - 300\,a^3\cos(dx+c}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*a^3*cos(d*x + c)^5 + 45*a^3*cos(d*x + c)^4 + 20*a^3*cos(d*x + c)
^3 - 150*a^3*cos(d*x + c)^2 - 300*a^3*cos(d*x + c) + 60*a^3*log(cos(d*x +
c)) - 30*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(124) = 248.

Time = 0.42 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.22

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx$$

$$= \frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{30\left(15\,a^3+\frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} -}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 30*(15*a^3 + 14*a^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (399*a^3
- 1395*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 390*a^3*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 650*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 565*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 1)^5)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{\frac{a^3\cos(c+dx)^3}{3} - 5a^3\cos(c+dx) - \frac{5a^3\cos(c+dx)^2}{2} - \frac{3a^3\cos(c+dx)+\frac{a^3}{2}}{\cos(c+dx)^2} + \frac{3a^3\cos(c+dx)^4}{4} + \frac{a^3\cos(c+dx)^5}{5} + a^3\ln(}{d}$$

input
```
int(sin(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

output
```
-((a^3*cos(c + d*x)^3)/3 - 5*a^3*cos(c + d*x) - (5*a^3*cos(c + d*x)^2)/2 -
 (3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + (3*a^3*cos(c + d*x)^4)/4 +
(a^3*cos(c + d*x)^5)/5 + a^3*log(cos(c + d*x)))/d
```

3.41 $\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx$

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3.41.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx = \frac{2a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{2d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{2a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output
```
2*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d-2*a^3*ln(
cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx = \frac{a^3(-41 + 226\cos(c+dx) + 29\cos(3(c+dx)) + 9\cos(4(c+dx)) + \cos(5(c+dx)) - 48\log(\cos(c+dx))}{48d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

output
```
(a^3*(-41 + 226*Cos[c + d*x] + 29*Cos[3*(c + d*x)] + 9*Cos[4*(c + d*x)] +
Cos[5*(c + d*x)] - 48*Log[Cos[c + d*x]] - 8*Cos[2*(c + d*x)]*(7 + 6*Log[Co
s[c + d*x]]))*Sec[c + d*x]^2)/(48*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4359$$

$$\int \tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan^3(c+dx)(a\cos(c+dx)+a)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\tan\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3186$$

$$-\frac{\int \frac{(a-a\cos(c+dx))(\cos(c+dx)a+a)^4\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 84$$

$$-\frac{\int \left(a^2\sec^3(c+dx)+3a^2\sec^2(c+dx)+2a^2\sec(c+dx)-2a^2-a^2\cos^2(c+dx)-3a^2\cos(c+dx)\right)d(a\cos(c+d$$

↓ 2009

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)-\frac{3}{2}a^3\cos^2(c+dx)-2a^3\cos(c+dx)-\frac{1}{2}a^3\sec^2(c+dx)-3a^3\sec(c+dx)+2a^3\log(a\cos(c+$$
$$d$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

output
```
-((-2*a^3*Cos[c + d*x] - (3*a^3*Cos[c + d*x]^2)/2 - (a^3*Cos[c + d*x]^3)/3
 + 2*a^3*Log[a*Cos[c + d*x]] - 3*a^3*Sec[c + d*x] - (a^3*Sec[c + d*x]^2)/2
)/d)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.41.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}+\frac{3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{3a^3}{}$
parallelrisch	$\frac{a^3\left(48(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)\right.}{}$
norman	$\frac{\frac{32a^3}{3d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{20a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}+\frac{20a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^3}{}$
risch	$2ia^3x+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{9a^3e^{i(dx+c)}}{8d}+\frac{9a^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{a^3e^{-3i(dx+c)}}{24d}+\frac{4i}{}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+
(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*
a^3*(2+sin(d*x+c)^2)*cos(d*x+c))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{4\,a^3\cos(dx+c)^5+18\,a^3\cos(dx+c)^4+24\,a^3\cos(dx+c)^3-24\,a^3\cos(dx+c)^2\log(-\cos(dx+c))-9}{12\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(4*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^3
 - 24*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 9*a^3*cos(d*x + c)^2 + 36*a^
3*cos(d*x + c) + 6*a^3)/(d*cos(d*x + c)^2)
```

3.41.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx = a^3\left(\int 3\sin^3(c+dx)\sec(c+dx)\,dx + \int 3\sin^3(c+dx)\sec^2(c+dx)\,dx + \int \sin^3(c+dx)\sec^3(c+dx)\,dx + \int \sin^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)**3*sec(c + d*x), x) + Integral(3*sin(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(sin(c + d*x)**3, x))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx = \frac{2\,a^3\cos(dx+c)^3 + 9\,a^3\cos(dx+c)^2 + 12\,a^3\cos(dx+c) - 12\,a^3\log(\cos(dx+c)) + \frac{3\,(6\,a^3\cos(dx+c)+a^3)}{\cos(dx+c)^2}}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(2*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) - 1
2*a^3*log(cos(d*x + c)) + 3*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx$$
$$= -\frac{2\,a^3 \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6\,a^3\cos(dx+c) + a^3}{2\,d\cos(dx+c)^2}$$
$$+ \frac{2\,a^3 d^8 \cos(dx+c)^3 + 9\,a^3 d^8 \cos(dx+c)^2 + 12\,a^3 d^8 \cos(dx+c)}{6\,d^9}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-2*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*cos(d*x + c) + a^3)/(d
*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^3*d^8*cos(d*x + c)^
2 + 12*a^3*d^8*cos(d*x + c))/d^9
```

3.41.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^3 \sin^3(c+dx)\,dx$$
$$= \frac{\frac{3\,a^3\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2} + 2\,a^3\cos(c+d\,x) + \frac{3\,a^3\cos(c+d\,x)^2}{2} + \frac{a^3\cos(c+d\,x)^3}{3} - 2\,a^3\ln(\cos(c+d\,x))}{d}$$

input
```
int(sin(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

output
```
((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 2*a^3*cos(c + d*x) + (3*a^3
*cos(c + d*x)^2)/2 + (a^3*cos(c + d*x)^3)/3 - 2*a^3*log(cos(c + d*x)))/d
```

3.42 $\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx$

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3.42.1 Optimal result

Integrand size = 19, antiderivative size = 62

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = -\frac{a^3\cos(c+dx)}{d} - \frac{3a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output
```
-a^3*cos(d*x+c)/d-3*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*
x+c)^2/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = -\frac{a^3(-4 - 9\cos(c+dx) + \cos(3(c+dx)) + 6\log(\cos(c+dx)) + \cos(2(c+dx))(-2 + 6\log(\cos(c+dx))}{4d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

output
```
-1/4*(a^3*(-4 - 9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*Log[Cos[c + d*x]] +
Cos[2*(c + d*x)]*(-2 + 6*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/d
```

3.42.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \tan(c+dx)\sec^2(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\sec^2(c+dx)\tan(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+a)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3312$$

$$-\frac{\int(\cos(c+dx)a+a)^3\sec^3(c+dx)d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{a^2\int\frac{(\cos(c+dx)a+a)^3\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{a^2 \int \left(\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1\right) d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(a\cos(c+dx)-\frac{1}{2}a\sec^2(c+dx)-3a\sec(c+dx)+3a\log(a\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

output
```
-((a^2*(a*Cos[c + d*x] + 3*a*Log[a*Cos[c + d*x]] - 3*a*Sec[c + d*x] - (a*S
ec[c + d*x]^2)/2))/d)
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.42.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
parts	$-\frac{a^3\cos(dx+c)}{d}+\frac{a^3\sec(dx+c)^2}{2d}+\frac{3a^3\ln(\sec(dx+c))}{d}+\frac{3a^3\sec(dx+c)}{d}$
risch	$3ia^3x-\frac{a^3e^{i(dx+c)}}{2d}-\frac{a^3e^{-i(dx+c)}}{2d}+\frac{6ia^3c}{d}+\frac{2a^3\left(3e^{3i(dx+c)}+e^{2i(dx+c)}+3e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{4a^3}{d}+\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{3a^3\ln\left(1+ta\right.}{}$
parallelrisch	$\frac{a^3\left(-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+6\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(2dx+2c)+9\,co\right.}{2d(1+\cos(2d}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(1/2*sec(d*x+c)^2+3*sec(d*x+c)+3*ln(sec(d*x+c))-1/sec(d*x+c))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx$$
$$=-\frac{2\,a^3\cos(dx+c)^3+6\,a^3\cos(dx+c)^2\log(-\cos(dx+c))-6\,a^3\cos(dx+c)-a^3}{2\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*cos(d*x + c)^3 + 6*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*a
^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

3.42.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = a^3\left(\int 3\sin(c+dx)\sec(c+dx)\,dx + \int 3\sin(c+dx)\sec^2(c+dx)\,dx + \int \sin(c+dx)\sec^3(c+dx)\,dx + \int \sin(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c),x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)*s
ec(c + d*x)**2, x) + Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(
sin(c + d*x), x))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = -\frac{2\,a^3\cos(dx+c) + 6\,a^3\log(\cos(dx+c)) - \frac{6\,a^3}{\cos(dx+c)} - \frac{a^3}{\cos(dx+c)^2}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(2*a^3*cos(d*x + c) + 6*a^3*log(cos(d*x + c)) - 6*a^3/cos(d*x + c) -
a^3/cos(d*x + c)^2)/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = -\frac{a^3\cos(dx+c)}{d} - \frac{3\,a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6\,a^3\cos(dx+c)+a^3}{2\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

output
```
-a^3*cos(d*x + c)/d - 3*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*c
os(d*x + c) + a^3)/(d*cos(d*x + c)^2)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int (a + a\sec(c+dx))^3 \sin(c+dx)\,dx = \frac{a^3\left(3\cos(c+d\,x) - \cos(c+d\,x)^3 - 3\cos(c+d\,x)^2\ln(\cos(c+d\,x)) + \frac{1}{2}\right)}{d\cos(c+d\,x)^2}$$

input
```
int(sin(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*(3*cos(c + d*x) - cos(c + d*x)^3 - 3*cos(c + d*x)^2*log(cos(c + d*x))
 + 1/2))/(d*cos(c + d*x)^2)
```

3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.43.9 Mupad [B] (verification not implemented) . . . 411

3.43.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output
```
4*a^3*ln(1-cos(d*x+c))/d-4*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3
*sec(d*x+c)^2/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(1+6\cos(c+dx)-4\log(\cos(c+dx))-4\cos(2(c+dx))\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{2d}$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + 6*Cos[c + d*x] - 4*Log[Cos[c + d*x]] - 4*Cos[2*(c + d*x)]*(Log[C
os[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) + 8*Log[Sin[(c + d*x)/2]])*Sec[c +
 d*x]^2)/(2*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int \csc(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a\int -\frac{(\cos(c+dx)a+a)^2\sec^3(c+dx)}{a-a\cos(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a\int \frac{(\cos(c+dx)a+a)^2\sec^3(c+dx)}{a-a\cos(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^4 \int \frac{(\cos(c+dx)a+a)^2 \sec^3(c+dx)}{a^3(a-a\cos(c+dx))} d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^4 \int \left(\frac{\sec^3(c+dx)}{a^2} + \frac{3\sec^2(c+dx)}{a^2} + \frac{4\sec(c+dx)}{a^2} + \frac{4}{a(a-a\cos(c+dx))}\right) d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a^4\left(-\frac{\sec^2(c+dx)}{2a} - \frac{3\sec(c+dx)}{a} + \frac{4\log(a\cos(c+dx))}{a} - \frac{4\log(a-a\cos(c+dx))}{a}\right)}{d}$$

input
```
Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^4*((4*Log[a*Cos[c + d*x]])/a - (4*Log[a - a*Cos[c + d*x]])/a - (3*Sec
[c + d*x])/a - Sec[c + d*x]^2/(2*a)))/d)
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.43.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+a^3\ln(-\cot(dx+$
default	$\frac{a^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+a^3\ln(-\cot(dx+$
risch	$\frac{2a^3\left(3\,e^{3i(dx+c)}+e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{8a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{4a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{6a^3}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{8a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
parallelrisch	$\frac{-8a^3(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-a^3\left(-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(2dx+2c)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+$

input
```
int(csc(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a^3*(1/cos(d*x+c)+ln(-cot(d*x
+c)+csc(d*x+c)))+3*a^3*ln(tan(d*x+c))+a^3*ln(-cot(d*x+c)+csc(d*x+c)))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{8\,a^3\cos(dx+c)^2\log(-\cos(dx+c)) - 8\,a^3\cos(dx+c)^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 6\,a^3\cos(dx+c)}{2\,d\cos(dx+c)^2}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(8*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 8*a^3*cos(d*x + c)^2*log(-
1/2*cos(d*x + c) + 1/2) - 6*a^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

3.43.6 Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc(c+dx)\sec(c+dx)\,dx + \int 3\csc(c+dx)\sec^2(c+dx)\,dx + \int \csc(c+dx)\sec^3(c+dx)\,dx + \int \csc(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*csc(c + d*x)*sec(c + d*x), x) + Integral(3*csc(c + d*x)*s
ec(c + d*x)**2, x) + Integral(csc(c + d*x)*sec(c + d*x)**3, x) + Integral(
csc(c + d*x), x))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{8\,a^3\log\left(\cos\left(dx+c\right)-1\right)-8\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{6\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^2}}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(8*a^3*log(cos(d*x + c) - 1) - 8*a^3*log(cos(d*x + c)) + (6*a^3*cos(d*
x + c) + a^3)/cos(d*x + c)^2)/d
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(65) = 130.

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{2\left(2\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{6\,a^3+\frac{8\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
2*(2*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^3*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (6*a^3 + 8*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{d\cos(c+d\,x)^2} - \frac{8\,a^3\,\mathrm{atanh}(2\,\cos(c+d\,x)-1)}{d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x),x)
```

output
```
(3*a^3*cos(c + d*x) + a^3/2)/(d*cos(c + d*x)^2) - (8*a^3*atanh(2*cos(c + d
*x) - 1))/d
```

3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.44.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2a^4}{d(a-a\cos(c+dx))} + \frac{5a^3\log(1-\cos(c+dx))}{d} - \frac{5a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output
```
-2*a^4/d/(a-a*cos(d*x+c))+5*a^3*ln(1-cos(d*x+c))/d-5*a^3*ln(cos(d*x+c))/d+
3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+10\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{16d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/16*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*Csc[(c + d*x)/2]^2 +
 10*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 6*Sec[c + d*x] - Sec[c
 + d*x]^2))/d
```

3.44.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int \csc^3(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^3(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^3(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^3 \int -\frac{(\cos(c+dx)a+a)\sec^3(c+dx)}{(a-a\cos(c+dx))^2} d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^3 \int \frac{(\cos(c+dx)a+a)\sec^3(c+dx)}{(a-a\cos(c+dx))^2} d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^6 \int \frac{(\cos(c+dx)a+a)\sec^3(c+dx)}{a^3(a-a\cos(c+dx))^2} d(a\cos(c+dx))}{d}$$

$$\downarrow 86$$

$$-\frac{a^6 \int \left(\frac{\sec^3(c+dx)}{a^4} + \frac{3\sec^2(c+dx)}{a^4} + \frac{5\sec(c+dx)}{a^4} + \frac{5}{a^3(a-a\cos(c+dx))} + \frac{2}{a^2(a-a\cos(c+dx))^2}\right) d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a^6\left(-\frac{\sec^2(c+dx)}{2a^3} - \frac{3\sec(c+dx)}{a^3} + \frac{5\log(a\cos(c+dx))}{a^3} - \frac{5\log(a-a\cos(c+dx))}{a^3} + \frac{2}{a^2(a-a\cos(c+dx))}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^6*(2/(a^2*(a - a*Cos[c + d*x])) + (5*Log[a*Cos[c + d*x]])/a^3 - (5*Lo
g[a - a*Cos[c + d*x]])/a^3 - (3*Sec[c + d*x])/a^3 - Sec[c + d*x]^2/(2*a^3)
))/d)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.44.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

method	result
risch	$\frac{2a^3\left(5\,e^{5i(dx+c)}-5\,e^{4i(dx+c)}+8\,e^{3i(dx+c)}-5\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}-1\right)^2}+\frac{10a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{5a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a^3}{d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{10a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+$
derivativedivides	$\frac{a^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+$
default	$\frac{a^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+$
parallelrisch	$\frac{a^3\left(9\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos(2dx+2c)-56\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos(dx+c)+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(2dx+2c)-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-$

input
```
int(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2*a^3/d/(exp(2*I*(d*x+c))+1)^2/(exp(I*(d*x+c))-1)^2*(5*exp(5*I*(d*x+c))-5*
exp(4*I*(d*x+c))+8*exp(3*I*(d*x+c))-5*exp(2*I*(d*x+c))+5*exp(I*(d*x+c)))+1
0/d*a^3*ln(exp(I*(d*x+c))-1)-5/d*a^3*ln(exp(2*I*(d*x+c))+1)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{10\,a^3\cos(dx+c)^2 - 5\,a^3\cos(dx+c) - a^3 - 10\left(a^3\cos(dx+c)^3 - a^3\cos(dx+c)^2\right)\log(-\cos(dx+c)) -}{2\left(d\cos(dx+c)^3 - d\cos(dx+c)^2\right)}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(10*a^3*cos(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3 - 10*(a^3*cos(d*x +
c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) + 10*(a^3*cos(d*x + c)^3 - a
^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos
(d*x + c)^2)
```

3.44.6 Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc^3(c+dx)\sec(c+dx)\,dx\right.$$
$$+ \int 3\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$+ \int \csc^3(c+dx)\sec^3(c+dx)\,dx$$
$$\left. + \int \csc^3(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*csc(c + d*x)**3*sec(c + d*x), x) + Integral(3*csc(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(csc(c + d*x)**3, x))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{10\,a^3\log\left(\cos\left(dx+c\right)-1\right)-10\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3}{\cos(dx+c)^3-\cos(dx+c)^2}}{2\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(10*a^3*log(cos(d*x + c) - 1) - 10*a^3*log(cos(d*x + c)) + (10*a^3*cos
(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3)/(cos(d*x + c)^3 - cos(d*x + c)^2))
/d
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(87) = 174.

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{10\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-10\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\left(a^3-\frac{5\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}+\frac{27\,a^3+\frac{38\,a^3(\cos}{\cos(d}}{2\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(10*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 10*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2*(a^3 - 5*a^3*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (2
7*a^3 + 38*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^3*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
^2)/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{-5\,a^3\cos\left(c+d\,x\right)^2+\frac{5\,a^3\,\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\,\left(\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^3\right)} - \frac{10\,a^3\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

output
```
((5*a^3*cos(c + d*x))/2 + a^3/2 - 5*a^3*cos(c + d*x)^2)/(d*(cos(c + d*x)^2
 - cos(c + d*x)^3)) - (10*a^3*atanh(2*cos(c + d*x) - 1))/d
```

3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.45.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^5}{2d(a-a\cos(c+dx))^2} - \frac{3a^4}{d(a-a\cos(c+dx))} + \frac{6a^3\log(1-\cos(c+dx))}{d} - \frac{6a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output
```
-1/2*a^5/d/(a-a*cos(d*x+c))^2-3*a^4/d/(a-a*cos(d*x+c))+6*a^3*ln(1-cos(d*x+
c))/d-6*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log(\cos(c+dx))-\right.\right.}{64d}$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/64*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2
+ Csc[(c + d*x)/2]^4 + 48*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) -
24*Sec[c + d*x] - 4*Sec[c + d*x]^2))/d
```

3.45.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 4360$$

$$\int \csc^5(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^5(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^5(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^5 \int -\frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^3} d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^5 \int \frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^3} d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^8 \int \frac{\sec^3(c+dx)}{a^3(a-a\cos(c+dx))^3} d(a\cos(c+dx))}{d}$$

$$\downarrow 54$$

$$-\frac{a^8 \int \left(\frac{\sec^3(c+dx)}{a^6} + \frac{3\sec^2(c+dx)}{a^6} + \frac{6\sec(c+dx)}{a^6} + \frac{6}{a^5(a-a\cos(c+dx))} + \frac{3}{a^4(a-a\cos(c+dx))^2} + \frac{1}{a^3(a-a\cos(c+dx))^3}\right) d(a\cos(c+}{d}$$

$$\downarrow 2009$$

$$-\frac{a^8\left(-\frac{\sec^2(c+dx)}{2a^5} - \frac{3\sec(c+dx)}{a^5} + \frac{6\log(a\cos(c+dx))}{a^5} - \frac{6\log(a-a\cos(c+dx))}{a^5} + \frac{3}{a^4(a-a\cos(c+dx))} + \frac{1}{2a^3(a-a\cos(c+dx))^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^8*(1/(2*a^3*(a - a*Cos[c + d*x])^2) + 3/(a^4*(a - a*Cos[c + d*x])) +
(6*Log[a*Cos[c + d*x]])/a^5 - (6*Log[a - a*Cos[c + d*x]])/a^5 - (3*Sec[c +
 d*x])/a^5 - Sec[c + d*x]^2/(2*a^5)))/d)
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.45.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

method	result
norman	$\frac{-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}-\frac{23a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d}+\frac{75a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{12a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
parallelrisch	$\frac{12\left(\frac{(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\right.}{d(1+\cos(2dx+2c))}$
risch	$\frac{4a^3\left(3e^{7i(dx+c)}-9e^{6i(dx+c)}+13e^{5i(dx+c)}-16e^{4i(dx+c)}+13e^{3i(dx+c)}-9e^{2i(dx+c)}+3e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)^2}+\frac{12a^3\ln\left(e^{i(dx+c)}-1\right.}{d}$
derivativedivides	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)^2}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\right.}{8}$
default	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)^2}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\right.}{8}$

input
```
int(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8/d*a^3-3/2*a^3/d*tan(1/2*d*x+1/2*c)^2-23/4*a^3/d*tan(1/2*d*x+1/2*c)^6
+75/8*a^3/d*tan(1/2*d*x+1/2*c)^4)/tan(1/2*d*x+1/2*c)^4/(-1+tan(1/2*d*x+1/2
*c)^2)^2+12/d*a^3*ln(tan(1/2*d*x+1/2*c))-6/d*a^3*ln(tan(1/2*d*x+1/2*c)-1)-
6/d*a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{12\,a^3\cos(dx+c)^3 - 18\,a^3\cos(dx+c)^2 + 4\,a^3\cos(dx+c) + a^3 - 12\left(a^3\cos(dx+c)^4 - 2\,a^3\cos(dx+c)^3\right.}{2\left(d\cos(dx+c)\right.}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(12*a^3*cos(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) +
a^3 - 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*
log(-cos(d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*c
os(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d
*x + c)^3 + d*cos(d*x + c)^2)
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{12\,a^3\log\left(\cos\left(dx+c\right)-1\right)-12\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{12\,a^3\cos(dx+c)^3-18\,a^3\cos(dx+c)^2+4\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^4-2\cos(dx+c)^3+\cos(dx+c)^2}}{2\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(12*a^3*log(cos(d*x + c) - 1) - 12*a^3*log(cos(d*x + c)) + (12*a^3*cos
(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a^3)/(cos(d*x +
 c)^4 - 2*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{48\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-48\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{a^3-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{75\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{46\,a^3(\cos(dx+c)-}{(\cos(dx+c)+1)}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)^2}}{8\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(48*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 48*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 - 12*a^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 75*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 46*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)^2)/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{6\,a^3\cos(c+d\,x)^3 - 9\,a^3\cos(c+d\,x)^2 + 2\,a^3\,\cos(c+d\,x) + \frac{a^3}{2}}{d\,\left(\cos(c+d\,x)^4 - 2\cos(c+d\,x)^3 + \cos(c+d\,x)^2\right)}$$
$$- \frac{12\,a^3\,\mathrm{atanh}(2\,\cos(c+d\,x) - 1)}{d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^5,x)
```

output
```
(2*a^3*cos(c + d*x) + a^3/2 - 9*a^3*cos(c + d*x)^2 + 6*a^3*cos(c + d*x)^3)
/(d*(cos(c + d*x)^2 - 2*cos(c + d*x)^3 + cos(c + d*x)^4)) - (12*a^3*atanh(
2*cos(c + d*x) - 1))/d
```

3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.46.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^6}{6d(a-a\cos(c+dx))^3} - \frac{7a^5}{8d(a-a\cos(c+dx))^2} - \frac{31a^4}{8d(a-a\cos(c+dx))} + \frac{111a^3\log(1-\cos(c+dx))}{16d} - \frac{7a^3\log(\cos(c+dx))}{d} + \frac{a^3\log(1+\cos(c+dx))}{16d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

output

```
-1/6*a^6/d/(a-a*cos(d*x+c))^3-7/8*a^5/d/(a-a*cos(d*x+c))^2-31/8*a^4/d/(a-a
*cos(d*x+c))+111/16*a^3*ln(1-cos(d*x+c))/d-7*a^3*ln(cos(d*x+c))/d+1/16*a^3
*ln(1+cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(186\csc^2\left(\frac{1}{2}(c+dx)\right)+21\csc^4\left(\frac{1}{2}(c+dx)\right)+2\csc^6\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

input
```
Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/768*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(186*Csc[(c + d*x)/2]^
2 + 21*Csc[(c + d*x)/2]^4 + 2*Csc[(c + d*x)/2]^6 - 12*(Log[Cos[(c + d*x)/2
]] - 56*Log[Cos[c + d*x]] + 111*Log[Sin[(c + d*x)/2]] + 24*Sec[c + d*x] +
4*Sec[c + d*x]^2)))/d
```

3.46.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^7}dx$$
$$\downarrow 4360$$
$$\int \csc^7(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$
$$\downarrow 25$$
$$-\int -(\cos(c+dx)a+a)^3\csc^7(c+dx)\sec^3(c+dx)dx$$
$$\downarrow 25$$

$$\int \csc^7(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^7\int -\frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^7\int \frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^4(\cos(c+dx)a+a)}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^{10}\int \frac{\sec^3(c+dx)}{a^3(a-a\cos(c+dx))^4(\cos(c+dx)a+a)}d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^{10}\int \left(\frac{\sec^3(c+dx)}{a^8}+\frac{3\sec^2(c+dx)}{a^8}+\frac{7\sec(c+dx)}{a^8}+\frac{111}{16a^7(a-a\cos(c+dx))}-\frac{1}{16a^7(\cos(c+dx)a+a)}+\frac{31}{8a^6(a-a\cos(c+dx))^2}+\frac{}{4a^5(a-}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^{10}\left(-\frac{\sec^2(c+dx)}{2a^7}-\frac{3\sec(c+dx)}{a^7}+\frac{7\log(a\cos(c+dx))}{a^7}-\frac{111\log(a-a\cos(c+dx))}{16a^7}-\frac{\log(a\cos(c+dx)+a)}{16a^7}+\frac{31}{8a^6(a-a\cos(c+dx))}+\frac{}{8a^}\right.}{d}$$

input
```
Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^10*(1/(6*a^4*(a - a*Cos[c + d*x])^3) + 7/(8*a^5*(a - a*Cos[c + d*x])^
2) + 31/(8*a^6*(a - a*Cos[c + d*x])) + (7*Log[a*Cos[c + d*x]])/a^7 - (111*
Log[a - a*Cos[c + d*x]])/(16*a^7) - Log[a + a*Cos[c + d*x]]/(16*a^7) - (3*
Sec[c + d*x])/a^7 - Sec[c + d*x]^2/(2*a^7)))/d)
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.46.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{79\left(\frac{112(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{79}+\frac{112(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{79}+\frac{222(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{79}}{16d(1+\cos(2dx+$
norman	$\frac{-\frac{a^3}{48d}-\frac{23a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}-\frac{91a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{48d}-\frac{103a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16d}+\frac{339a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{111a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-$
risch	$\frac{a^3\left(165\,e^{9i(dx+c)}-822\,e^{8i(dx+c)}+1852\,e^{7i(dx+c)}-2754\,e^{6i(dx+c)}+3182\,e^{5i(dx+c)}-2754\,e^{4i(dx+c)}+1852\,e^{3i(dx+c)}-822\,e\right.}{12d\left(e^{i(dx+c)}-1\right)^6\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)^2}-\frac{1}{3\sin(dx+c)^4\cos(dx+c)^2}+\frac{1}{\sin(dx+c)^2\cos(dx+c)^2}-\frac{2}{\sin(dx+c)^2}+4\ln(\tan(dx+c))\right)+3a^3\left(-\frac{}{6\,si}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)^2}-\frac{1}{3\sin(dx+c)^4\cos(dx+c)^2}+\frac{1}{\sin(dx+c)^2\cos(dx+c)^2}-\frac{2}{\sin(dx+c)^2}+4\ln(\tan(dx+c))\right)+3a^3\left(-\frac{}{6\,si}\right.}{}$

input
```
int(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
79/16*(112/79*(-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+112/79*(-cos(2*
d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)+1)+222/79*(1+cos(2*d*x+2*c))*ln(tan(1/2*
d*x+1/2*c))+cot(1/2*d*x+1/2*c)^2*csc(1/2*d*x+1/2*c)^4*(cos(d*x+c)-161/237*
cos(2*d*x+2*c)+71/237*cos(3*d*x+3*c)-449/7584*cos(4*d*x+4*c)-4319/7584))*a
^3/d/(1+cos(2*d*x+2*c))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(148) = 296$.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.89

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{330\,a^3\cos(dx+c)^4 - 822\,a^3\cos(dx+c)^3 + 596\,a^3\cos(dx+c)^2 - 72\,a^3\cos(dx+c) - 24\,a^3 - 336\,(a^3\cos$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/48*(330*a^3*cos(d*x + c)^4 - 822*a^3*cos(d*x + c)^3 + 596*a^3*cos(d*x +
c)^2 - 72*a^3*cos(d*x + c) - 24*a^3 - 336*(a^3*cos(d*x + c)^5 - 3*a^3*cos(
d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c))
 + 3*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a
^3*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 333*(a^3*cos(d*x + c)^5 -
 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-1/
2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*
x + c)^3 - d*cos(d*x + c)^2)
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+333\,a^3\log\left(\cos\left(dx+c\right)-1\right)-336\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(165\,a^3\cos(dx+c)^4-411\right.}{\cos(dx+c)}}{48\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/48*(3*a^3*log(cos(d*x + c) + 1) + 333*a^3*log(cos(d*x + c) - 1) - 336*a^
3*log(cos(d*x + c)) + 2*(165*a^3*cos(d*x + c)^4 - 411*a^3*cos(d*x + c)^3 +
 298*a^3*cos(d*x + c)^2 - 36*a^3*cos(d*x + c) - 12*a^3)/(cos(d*x + c)^5 -
3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.55

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{666\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 672\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{\left(2\,a^3 - \frac{27\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{234\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{1221\,a^3(\cos($$

input
```
integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/96*(666*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 672*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^3 - 27*a^3*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 234*a^3*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 - 1221*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*
x + c) + 1)^3/(cos(d*x + c) - 1)^3 + 48*(33*a^3 + 50*a^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 21*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{111\,a^3\,\ln(\cos(c+dx)-1)}{16\,d} + \frac{a^3\,\ln(\cos(c+dx)+1)}{16\,d} + \frac{-\frac{55\,a^3\cos(c+dx)^4}{8} + \frac{137\,a^3\cos(c+dx)^3}{8} - \frac{149\,a^3\cos(c+dx)^2}{12} + \frac{3\,a^3\,\cos(c+dx)}{2} + \frac{a^3}{2}}{d\left(-\cos(c+dx)^5 + 3\cos(c+dx)^4 - 3\cos(c+dx)^3 + \cos(c+dx)^2\right)} - \frac{7\,a^3\,\ln(\cos(c+dx))}{d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^7,x)
```

output
```
(111*a^3*log(cos(c + d*x) - 1))/(16*d) + (a^3*log(cos(c + d*x) + 1))/(16*d
) + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (149*a^3*cos(c + d*x)^2)/12 + (137*a
^3*cos(c + d*x)^3)/8 - (55*a^3*cos(c + d*x)^4)/8)/(d*(cos(c + d*x)^2 - 3*c
os(c + d*x)^3 + 3*cos(c + d*x)^4 - cos(c + d*x)^5)) - (7*a^3*log(cos(c + d
*x)))/d
```

3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.47.1 Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = &-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}\\&-\frac{39a^5}{32d(a-a\cos(c+dx))^2}\\&-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a+a\cos(c+dx))}\\&+\frac{501a^3\log(1-\cos(c+dx))}{64d}\\&-\frac{8a^3\log(\cos(c+dx))}{d}+\frac{11a^3\log(1+\cos(c+dx))}{64d}\\&+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

output

```
-1/16*a^7/d/(a-a*cos(d*x+c))^4-1/3*a^6/d/(a-a*cos(d*x+c))^3-39/32*a^5/d/(a
-a*cos(d*x+c))^2-75/16*a^4/d/(a-a*cos(d*x+c))-1/32*a^4/d/(a+a*cos(d*x+c))+
501/64*a^3*ln(1-cos(d*x+c))/d-8*a^3*ln(cos(d*x+c))/d+11/64*a^3*ln(1+cos(d*
x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

3.47.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(1800\csc^2\left(\frac{1}{2}(c+dx)\right)+234\csc^4\left(\frac{1}{2}(c+dx)\right)+32\csc^6\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

input
```
Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/6144*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(1800*Csc[(c + d*x)/2
]^2 + 234*Csc[(c + d*x)/2]^4 + 32*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^
8 - 12*(22*Log[Cos[(c + d*x)/2]] - 512*Log[Cos[c + d*x]] + 1002*Log[Sin[(c
 + d*x)/2]] - Sec[(c + d*x)/2]^2 + 192*Sec[c + d*x] + 32*Sec[c + d*x]^2)))
/d
```

3.47.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^9(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^9}dx$$
$$\downarrow 4360$$
$$\int \csc^9(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$
$$\downarrow 25$$
$$-\int -(\cos(c+dx)a+a)^3\csc^9(c+dx)\sec^3(c+dx)dx$$
$$\downarrow 25$$

$$\int \csc^9(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^9}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^9\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^9\int -\frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^9\int \frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^{12}\int \frac{\sec^3(c+dx)}{a^3(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^{12}\int \left(\frac{\sec^3(c+dx)}{a^{10}}+\frac{3\sec^2(c+dx)}{a^{10}}+\frac{8\sec(c+dx)}{a^{10}}+\frac{501}{64a^9(a-a\cos(c+dx))}-\frac{11}{64a^9(\cos(c+dx)a+a)}+\frac{75}{16a^8(a-a\cos(c+dx))^2}-\frac{}{32a^8(}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^{12}\left(-\frac{\sec^2(c+dx)}{2a^9}-\frac{3\sec(c+dx)}{a^9}+\frac{8\log(a\cos(c+dx))}{a^9}-\frac{501\log(a-a\cos(c+dx))}{64a^9}-\frac{11\log(a\cos(c+dx)+a)}{64a^9}+\frac{75}{16a^8(a-a\cos(c+dx))}+\right.}{d}$$

input
```
Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^12*(1/(16*a^5*(a - a*Cos[c + d*x])^4) + 1/(3*a^6*(a - a*Cos[c + d*x])
^3) + 39/(32*a^7*(a - a*Cos[c + d*x])^2) + 75/(16*a^8*(a - a*Cos[c + d*x])
) + 1/(32*a^8*(a + a*Cos[c + d*x])) + (8*Log[a*Cos[c + d*x]])/a^9 - (501*L
og[a - a*Cos[c + d*x]])/(64*a^9) - (11*Log[a + a*Cos[c + d*x]])/(64*a^9) -
 (3*Sec[c + d*x])/a^9 - Sec[c + d*x]^2/(2*a^9)))/d)
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.47.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-\frac{a^3}{256d}-\frac{19a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{384d}-\frac{263a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{768d}-\frac{431a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{192d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{64d}-\frac{451a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64d}+\frac{749a^3\tan}{6}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
parallelrisch	$-\frac{a^3\left(2048\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2048\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2\ln\right.}{}$
risch	$\frac{a^3\left(735\,e^{13i(dx+c)}-3642\,e^{12i(dx+c)}+6662\,e^{11i(dx+c)}-4650\,e^{10i(dx+c)}-1983\,e^{9i(dx+c)}+8868\,e^{8i(dx+c)}-12748\,e^{7i(dx+c)}\right.}{48d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^2(e^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)^2}-\frac{5}{24\sin(dx+c)^6\cos(dx+c)^2}-\frac{5}{12\sin(dx+c)^4\cos(dx+c)^2}+\frac{5}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)^2}+\right.}{}$
default	$\frac{a^3\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)^2}-\frac{5}{24\sin(dx+c)^6\cos(dx+c)^2}-\frac{5}{12\sin(dx+c)^4\cos(dx+c)^2}+\frac{5}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)^2}+\right.}{}$

input
```
int(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/256/d*a^3-19/384*a^3/d*tan(1/2*d*x+1/2*c)^2-263/768*a^3/d*tan(1/2*d*x+
1/2*c)^4-431/192*a^3/d*tan(1/2*d*x+1/2*c)^6-1/64/d*a^3*tan(1/2*d*x+1/2*c)^
14-451/64*a^3/d*tan(1/2*d*x+1/2*c)^10+749/64*a^3/d*tan(1/2*d*x+1/2*c)^8)/t
an(1/2*d*x+1/2*c)^8/(-1+tan(1/2*d*x+1/2*c)^2)^2+501/32/d*a^3*ln(tan(1/2*d*
x+1/2*c))-8/d*a^3*ln(tan(1/2*d*x+1/2*c)-1)-8/d*a^3*ln(tan(1/2*d*x+1/2*c)+1
)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(190) = 380.

Time = 0.29 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.07

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{1470\,a^3\cos(dx+c)^6 - 3642\,a^3\cos(dx+c)^5 + 1126\,a^3\cos(dx+c)^4 + 3390\,a^3\cos(dx+c)^3 - 2752\,a^3\cos$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/192*(1470*a^3*cos(d*x + c)^6 - 3642*a^3*cos(d*x + c)^5 + 1126*a^3*cos(d*
x + c)^4 + 3390*a^3*cos(d*x + c)^3 - 2752*a^3*cos(d*x + c)^2 + 288*a^3*cos
(d*x + c) + 96*a^3 - 1536*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a
^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(
d*x + c)^2)*log(-cos(d*x + c)) + 33*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x +
c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3
+ a^3*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 1503*(a^3*cos(d*x + c)
^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 -
3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(
d*cos(d*x + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x +
 c)^4 - 3*d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{33\,a^3\log\left(\cos\left(dx+c\right)+1\right)+1503\,a^3\log\left(\cos\left(dx+c\right)-1\right)-1536\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(735\,a^3\cos(dx+c)^6}{192\,d}$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/192*(33*a^3*log(cos(d*x + c) + 1) + 1503*a^3*log(cos(d*x + c) - 1) - 153
6*a^3*log(cos(d*x + c)) + 2*(735*a^3*cos(d*x + c)^6 - 1821*a^3*cos(d*x + c
)^5 + 563*a^3*cos(d*x + c)^4 + 1695*a^3*cos(d*x + c)^3 - 1376*a^3*cos(d*x
+ c)^2 + 144*a^3*cos(d*x + c) + 48*a^3)/(cos(d*x + c)^7 - 3*cos(d*x + c)^6
 + 2*cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 3*cos(d*x + c)^3 + cos(d*x + c)^2
))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{6012\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 6144\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3 - \frac{44\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{34}{768}\right.}{}$$

input
```
integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(6012*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 6144*a
^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 12*a^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - (3*a^3 - 44*a^3*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 348*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2376*a^3*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 12525*a^3*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 1536*(9
*a^3 + 14*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^3*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2
)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{501\,a^3\,\ln(\cos(c+dx)-1)}{64\,d} + \frac{11\,a^3\,\ln(\cos(c+dx)+1)}{64\,d}$$
$$+ \frac{\frac{245\,a^3\cos(c+dx)^6}{32} - \frac{607\,a^3\cos(c+dx)^5}{32} + \frac{563\,a^3\cos(c+dx)^4}{96} + \frac{565\,a^3\cos(c+dx)^3}{32} - \frac{43\,a^3\cos(c+dx)^2}{3} + \frac{3\,a^3\cos(c+dx)}{2} + \frac{a^3}{2}}{d\,\left(\cos(c+dx)^7 - 3\cos(c+dx)^6 + 2\cos(c+dx)^5 + 2\cos(c+dx)^4 - 3\cos(c+dx)^3 + \cos(c+dx)\right.}$$
$$- \frac{8\,a^3\,\ln(\cos(c+dx))}{d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^9,x)
```

output
```
(501*a^3*log(cos(c + d*x) - 1))/(64*d) + (11*a^3*log(cos(c + d*x) + 1))/(6
4*d) + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (43*a^3*cos(c + d*x)^2)/3 + (565*
a^3*cos(c + d*x)^3)/32 + (563*a^3*cos(c + d*x)^4)/96 - (607*a^3*cos(c + d*
x)^5)/32 + (245*a^3*cos(c + d*x)^6)/32)/(d*(cos(c + d*x)^2 - 3*cos(c + d*x
)^3 + 2*cos(c + d*x)^4 + 2*cos(c + d*x)^5 - 3*cos(c + d*x)^6 + cos(c + d*x
)^7)) - (8*a^3*log(cos(c + d*x)))/d
```

3.48 $\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx$

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3.48.1 Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx = &-\frac{805a^3x}{128} - \frac{a^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ &+ \frac{603a^3\cos(c+dx)\sin(c+dx)}{128d} \\ &- \frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d} \\ &- \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} \\ &+ \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^3\sin^3(c+dx)}{3d} \\ &- \frac{2a^3\sin^5(c+dx)}{5d} - \frac{3a^3\sin^7(c+dx)}{7d} \\ &+ \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output

```
-805/128*a^3*x-1/2*a^3*arctanh(sin(d*x+c))/d+603/128*a^3*cos(d*x+c)*sin(d*
x+c)/d-293/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/48*a^3*cos(d*x+c)^5*sin(d*x
+c)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-1/3*a^3*sin(d*x+c)^3/d-2/5*a^3*sin
(d*x+c)^5/d-3/7*a^3*sin(d*x+c)^7/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*t
an(d*x+c)/d
```

3.48.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx$$
$$= \frac{a^3 \sec^2(c+dx)\,(-1352400c - 1352400dx - 215040\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx) - 1352400(c+dx)$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

output
```
(a^3*Sec[c + d*x]^2*(-1352400*c - 1352400*d*x - 215040*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^2 - 1352400*(c + d*x)*Cos[2*(c + d*x)] + 173600*Sin[c + d*
x] + 1052520*Sin[2*(c + d*x)] - 11648*Sin[3*(c + d*x)] + 175280*Sin[4*(c +
 d*x)] + 22784*Sin[5*(c + d*x)] - 18095*Sin[6*(c + d*x)] - 6288*Sin[7*(c +
 d*x)] + 770*Sin[8*(c + d*x)] + 720*Sin[9*(c + d*x)] + 105*Sin[10*(c + d*x
)]))/(430080*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 25, 25, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^8 \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$
$$\downarrow 4360$$
$$\int \sin^5(c+dx)\tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$
$$\downarrow 25$$
$$-\int -(\cos(c+dx)a+a)^3\sin^5(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin^5(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^8\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3351$$

$$\int \left(\cos^8(c+dx)a^{11}+3\cos^7(c+dx)a^{11}-\cos^6(c+dx)a^{11}-11\cos^5(c+dx)a^{11}-6\cos^4(c+dx)a^{11}+14\cos^3(c+$$

$$\downarrow 2009$$

$$\frac{-\frac{a^{11}\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^{11}\sin^7(c+dx)}{7d}-\frac{2a^{11}\sin^5(c+dx)}{5d}-\frac{a^{11}\sin^3(c+dx)}{3d}+\frac{3a^{11}\tan(c+dx)}{d}+\frac{a^{11}\sin(c+dx)\cos^7(c+dx)}{8d}-\frac{a^{11}}{}}{a^8}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

output
```
((-805*a^11*x)/128 - (a^11*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^11*Cos[c
+ d*x]*Sin[c + d*x])/(128*d) - (293*a^11*Cos[c + d*x]^3*Sin[c + d*x])/(192
*d) - (a^11*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^11*Cos[c + d*x]^7*Sin
[c + d*x])/(8*d) - (a^11*Sin[c + d*x]^3)/(3*d) - (2*a^11*Sin[c + d*x]^5)/(
5*d) - (3*a^11*Sin[c + d*x]^7)/(7*d) + (3*a^11*Tan[c + d*x])/d + (a^11*Sec
[c + d*x]*Tan[c + d*x])/(2*d))/a^8
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.48.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{a^3\left(107520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-107520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-1352400dx\cos(2dx+\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}+\frac{7\sin(dx+c)^5}{10}+\frac{7\sin(dx+c)^3}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}+\frac{7\sin(dx+c)^5}{10}+\frac{7\sin(dx+c)^3}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}\right.}{}$
risch	$-\frac{805a^3x}{128}+\frac{127ia^3e^{-2i(dx+c)}}{128d}-\frac{127ia^3e^{2i(dx+c)}}{128d}-\frac{ia^3\left(e^{3i(dx+c)}-6e^{2i(dx+c)}-e^{i(dx+c)}-6\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{67ia^3e^{3i(dx+c)}}{384d}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/215040*a^3*(107520*(1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)-107520*(1
+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)+1)-1352400*d*x*cos(2*d*x+2*c)-13524
00*d*x+22784*sin(5*d*x+5*c)-18095*sin(6*d*x+6*c)-6288*sin(7*d*x+7*c)+770*s
in(8*d*x+8*c)+720*sin(9*d*x+9*c)+105*sin(10*d*x+10*c)+173600*sin(d*x+c)+10
52520*sin(2*d*x+2*c)-11648*sin(3*d*x+3*c)+175280*sin(4*d*x+4*c))/d/(1+cos(
2*d*x+2*c))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx =$$

$$-\frac{84525\,a^3 dx\cos(dx+c)^2 + 3360\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1) - 3360\,a^3\cos(dx+c)^2\log(-\sin$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/13440*(84525*a^3*d*x*cos(d*x + c)^2 + 3360*a^3*cos(d*x + c)^2*log(sin(d
*x + c) + 1) - 3360*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (1680*a^3*
cos(d*x + c)^9 + 5760*a^3*cos(d*x + c)^8 - 280*a^3*cos(d*x + c)^7 - 22656*
a^3*cos(d*x + c)^6 - 20510*a^3*cos(d*x + c)^5 + 32512*a^3*cos(d*x + c)^4 +
 63315*a^3*cos(d*x + c)^3 - 15616*a^3*cos(d*x + c)^2 + 40320*a^3*cos(d*x +
 c) + 6720*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**8,x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.39

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx =$$

$$-\frac{1536\left(30\sin(dx+c)^7 + 42\sin(dx+c)^5 + 70\sin(dx+c)^3 - 105\log(\sin(dx+c)+1) + 105\log(\sin\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="maxima")
```

output
```
-1/107520*(1536*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c
))*a^3 - 1792*(12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 - 30*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1)
+ 180*sin(d*x + c))*a^3 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3
*sin(8*d*x + 8*c) + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3 + 672
0*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x
+ c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(
d*x + c))*a^3)/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int (a + a\sec(c + dx))^3 \sin^8(c + dx)\,dx =$$

$$-\frac{84525\,(dx + c)a^3 + 6720\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 6720\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{13440\left(5}{}}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="giac")
```

output
```
-1/13440*(84525*(d*x + c)*a^3 + 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 13440*(5*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 +
 2*(44205*a^3*tan(1/2*d*x + 1/2*c)^15 + 303065*a^3*tan(1/2*d*x + 1/2*c)^13
 + 841981*a^3*tan(1/2*d*x + 1/2*c)^11 + 1123793*a^3*tan(1/2*d*x + 1/2*c)^9
 + 487983*a^3*tan(1/2*d*x + 1/2*c)^7 - 490749*a^3*tan(1/2*d*x + 1/2*c)^5 -
 267225*a^3*tan(1/2*d*x + 1/2*c)^3 - 44205*a^3*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.52

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx$$

$$= \frac{-\frac{741\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{64} - \frac{12469\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{192} - \frac{5027\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{40} - \frac{19211\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{420} + \frac{199977\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{1120} +}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20} + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} + 13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 28\,\mathrm{ta}\right.}$$
$$\left. - \frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{805\,a^3\,x}{128}\right.$$

input
```
int(sin(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

output
```
((4967*a^3*tan(c/2 + (d*x)/2)^3)/64 + (6243*a^3*tan(c/2 + (d*x)/2)^5)/40 +
 (10233*a^3*tan(c/2 + (d*x)/2)^7)/140 + (877061*a^3*tan(c/2 + (d*x)/2)^9)/
3360 + (199977*a^3*tan(c/2 + (d*x)/2)^11)/1120 - (19211*a^3*tan(c/2 + (d*x
)/2)^13)/420 - (5027*a^3*tan(c/2 + (d*x)/2)^15)/40 - (12469*a^3*tan(c/2 +
(d*x)/2)^17)/192 - (741*a^3*tan(c/2 + (d*x)/2)^19)/64 + (869*a^3*tan(c/2 +
 (d*x)/2))/64)/(d*(6*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)^4 + 8*ta
n(c/2 + (d*x)/2)^6 - 14*tan(c/2 + (d*x)/2)^8 - 28*tan(c/2 + (d*x)/2)^10 -
14*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + 13*tan(c/2 + (d*x)/2)
^16 + 6*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) - (a^3*atanh(t
an(c/2 + (d*x)/2)))/d - (805*a^3*x)/128
```

3.49 $\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx$

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3.49.1 Optimal result

Integrand size = 21, antiderivative size = 182

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx = &-\frac{85a^3x}{16} + \frac{a^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ &- \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx)\sin(c+dx)}{16d} \\ &- \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} \\ &- \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} \\ &- \frac{2a^3\sin^3(c+dx)}{3d} - \frac{3a^3\sin^5(c+dx)}{5d} \\ &+ \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-85/16*a^3*x+1/2*a^3*arctanh(sin(d*x+c))/d-a^3*sin(d*x+c)/d+43/16*a^3*cos(
d*x+c)*sin(d*x+c)/d-5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^
5*sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d-3/5*a^3*sin(d*x+c)^5/d+3*a^3*tan(d*x
+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx =$$
$$-\frac{a^3 \sec^2(c+dx)\,(10200c + 10200dx - 1920\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx) + 10200(c+dx)\cos(2(c+$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

output
```
-1/3840*(a^3*Sec[c + d*x]^2*(10200*c + 10200*d*x - 1920*ArcTanh[Sin[c + d*
x]]*Cos[c + d*x]^2 + 10200*(c + d*x)*Cos[2*(c + d*x)] - 460*Sin[c + d*x] -
 8145*Sin[2*(c + d*x)] + 1156*Sin[3*(c + d*x)] - 1120*Sin[4*(c + d*x)] - 2
68*Sin[5*(c + d*x)] + 55*Sin[6*(c + d*x)] + 36*Sin[7*(c + d*x)] + 5*Sin[8*
(c + d*x)]))/d
```

3.49.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 25, 25, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6 \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$
$$\downarrow 4360$$
$$\int \sin^3(c+dx)\tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$
$$\downarrow 25$$
$$-\int -(\cos(c+dx)a+a)^3\sin^3(c+dx)\tan^3(c+dx)dx$$
$$\downarrow 25$$

$$\int \sin^3(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3351$$

$$\frac{\int\left(-\cos^6(c+dx)a^9-3\cos^5(c+dx)a^9+8\cos^3(c+dx)a^9+\sec^3(c+dx)a^9+6\cos^2(c+dx)a^9+3\sec^2(c+dx)a^9\right.}{a^6}$$

$$\downarrow 2009$$

$$\frac{\frac{a^9\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^9\sin^5(c+dx)}{5d}-\frac{2a^9\sin^3(c+dx)}{3d}-\frac{a^9\sin(c+dx)}{d}+\frac{3a^9\tan(c+dx)}{d}-\frac{a^9\sin(c+dx)\cos^5(c+dx)}{6d}-\frac{5a^9\sin(c+dx)}{2}}{a^6}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

output
```
((-85*a^9*x)/16 + (a^9*ArcTanh[Sin[c + d*x]])/(2*d) - (a^9*Sin[c + d*x])/d
 + (43*a^9*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^9*Cos[c + d*x]^3*Sin[c
 + d*x])/(24*d) - (a^9*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^9*Sin[c +
 d*x]^3)/(3*d) - (3*a^9*Sin[c + d*x]^5)/(5*d) + (3*a^9*Tan[c + d*x])/d + (
a^9*Sec[c + d*x]*Tan[c + d*x])/(2*d))/a^6
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.49.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{a^3\left(-10200dx\cos(2dx+2c)-960\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+960\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-10200dx+4\right.}{}$
derivativedivides	$a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\right.\right.$
default	$a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\right.\right.$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5}{}\right.}{d}$
risch	$-\frac{85a^3x}{16}+\frac{17ia^3e^{-3i(dx+c)}}{96d}+\frac{81ia^3e^{-2i(dx+c)}}{128d}-\frac{81ia^3e^{2i(dx+c)}}{128d}+\frac{15ia^3e^{i(dx+c)}}{16d}-\frac{ia^3\left(e^{3i(dx+c)}-6e^{2i(dx+c}\right.}{d(e^{2i(dx+c)}+}$
norman	$\frac{-\frac{85a^3x}{16}-\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4}-\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{425a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4}-\frac{85}{}}{}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/1920*a^3*(-10200*d*x*cos(2*d*x+2*c)-960*ln(tan(1/2*d*x+1/2*c)-1)*cos(2*d
*x+2*c)+960*ln(tan(1/2*d*x+1/2*c)+1)*cos(2*d*x+2*c)-10200*d*x+460*sin(d*x+
c)+8145*sin(2*d*x+2*c)+1120*sin(4*d*x+4*c)-55*sin(6*d*x+6*c)-5*sin(8*d*x+8
*c)-36*sin(7*d*x+7*c)+268*sin(5*d*x+5*c)-1156*sin(3*d*x+3*c)-960*ln(tan(1/
2*d*x+1/2*c)-1)+960*ln(tan(1/2*d*x+1/2*c)+1))/d/(1+cos(2*d*x+2*c))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx =$$
$$-\frac{1275\,a^3 dx\cos(dx+c)^2 - 60\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1) + 60\,a^3\cos(dx+c)^2\log(-\sin(dx+$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/240*(1275*a^3*d*x*cos(d*x + c)^2 - 60*a^3*cos(d*x + c)^2*log(sin(d*x +
c) + 1) + 60*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + (40*a^3*cos(d*x +
 c)^7 + 144*a^3*cos(d*x + c)^6 + 50*a^3*cos(d*x + c)^5 - 448*a^3*cos(d*x +
 c)^4 - 645*a^3*cos(d*x + c)^3 + 544*a^3*cos(d*x + c)^2 - 720*a^3*cos(d*x
+ c) - 120*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.32

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx =$$

$$-\frac{96\left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 30\sin(dx+c)\right)a^3 - 5\left(4\sin(2dx+2c)^3 + 60dx + 60c + 9\sin(4dx+4c) - 48\sin(2dx+2c)\right)a^3 - 80\left(4\sin(dx+c)^3 - 6\sin(dx+c)\right)/(\sin(dx+c)^2-1) - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 24\sin(dx+c)\right)a^3 + 360\left(15dx + 15c - (9\tan(dx+c)^3 + 7\tan(dx+c))/(\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1) - 8\tan(dx+c)\right)a^3}{d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
-1/960*(96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1
) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c
)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 80*(
4*sin(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
c) + 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^3 + 360*(15*d*x +
15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c
)^2 + 1) - 8*tan(d*x + c))*a^3)/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.16

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx =$$

$$-\frac{1275\,(dx+c)a^3 - 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{240\left(5\,a^3\tan(\right.}{\left(}}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/240*(1275*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
120*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 240*(5*a^3*tan(1/2*d*x + 1/2*
c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(795
*a^3*tan(1/2*d*x + 1/2*c)^11 + 4025*a^3*tan(1/2*d*x + 1/2*c)^9 + 7614*a^3*
tan(1/2*d*x + 1/2*c)^7 + 5634*a^3*tan(1/2*d*x + 1/2*c)^5 - 345*a^3*tan(1/2
*d*x + 1/2*c)^3 - 315*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
1)^6)/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.43

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx = \frac{a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{85\,a^3 x}{16}$$
$$+ \frac{-\frac{93\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{8} - \frac{1039\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{24} - \frac{4319\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{120} + \frac{6169\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{120} + \frac{3933\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{40} + \frac{9}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 4\tan\right.}$$

input
```
int(sin(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*atanh(tan(c/2 + (d*x)/2)))/d - (85*a^3*x)/16 + ((277*a^3*tan(c/2 + (d
*x)/2)^3)/8 + (997*a^3*tan(c/2 + (d*x)/2)^5)/40 + (3933*a^3*tan(c/2 + (d*x
)/2)^7)/40 + (6169*a^3*tan(c/2 + (d*x)/2)^9)/120 - (4319*a^3*tan(c/2 + (d*
x)/2)^11)/120 - (1039*a^3*tan(c/2 + (d*x)/2)^13)/24 - (93*a^3*tan(c/2 + (d
*x)/2)^15)/8 + (77*a^3*tan(c/2 + (d*x)/2))/8)/(d*(4*tan(c/2 + (d*x)/2)^2 +
 4*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - 10*tan(c/2 + (d*x)/2)^8
 - 4*tan(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12 + 4*tan(c/2 + (d*x)/2
)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.50 $\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx$

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3.50.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx = &-\frac{33a^3x}{8} + \frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ &- \frac{2a^3\sin(c+dx)}{d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} \\ &+ \frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\sin^3(c+dx)}{d} \\ &+ \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-33/8*a^3*x+3/2*a^3*arctanh(sin(d*x+c))/d-2*a^3*sin(d*x+c)/d+7/8*a^3*cos(d
*x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+3*
a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx$$
$$= \frac{a^3\sec^2(c+dx)\left(-264c - 264dx + 192\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx) - 264(c+dx)\cos(2(c+dx)) -\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

output
```
(a^3*Sec[c + d*x]^2*(-264*c - 264*d*x + 192*ArcTanh[Sin[c + d*x]]*Cos[c +
d*x]^2 - 264*(c + d*x)*Cos[2*(c + d*x)] - 16*Sin[c + d*x] + 225*Sin[2*(c +
 d*x)] - 72*Sin[3*(c + d*x)] + 18*Sin[4*(c + d*x)] + 8*Sin[5*(c + d*x)] +
Sin[6*(c + d*x)]))/(128*d)
```

3.50.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 25, 25, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin(c+dx)\tan^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\sin(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin(c+dx)\tan^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3351$$

$$\frac{\int \left(\cos^4(c+dx)a^7+3\cos^3(c+dx)a^7+\sec^3(c+dx)a^7+\cos^2(c+dx)a^7+3\sec^2(c+dx)a^7-5\cos(c+dx)a^7+s\right.}{a^4}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{3a^7\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{a^7\sin^3(c+dx)}{d}-\frac{2a^7\sin(c+dx)}{d}+\frac{3a^7\tan(c+dx)}{d}+\frac{a^7\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^7\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^7}{}}{a^4}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

output
```
((-33*a^7*x)/8 + (3*a^7*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^7*Sin[c + d*x]
)/d + (7*a^7*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^7*Cos[c + d*x]^3*Sin[c
+ d*x])/(4*d) - (a^7*Sin[c + d*x]^3)/d + (3*a^7*Tan[c + d*x])/d + (a^7*Sec
[c + d*x]*Tan[c + d*x])/(2*d))/a^4
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.50.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{\left((-12\cos(2dx+2c)-12)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(12\cos(2dx+2c)+12)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-33dx\cos(2dx+2c)-33dx+s\right.}{8d(1+\cos(2dx+2c))}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan\right.}{d}$
risch	$-\frac{33a^3x}{8}-\frac{ia^3e^{3i(dx+c)}}{8d}-\frac{ia^3e^{2i(dx+c)}}{4d}+\frac{11ia^3e^{i(dx+c)}}{8d}-\frac{11ia^3e^{-i(dx+c)}}{8d}+\frac{ia^3e^{-2i(dx+c)}}{4d}+\frac{ia^3e^{-3i(dx+c)}}{8d}$
norman	$\frac{-\frac{33a^3x}{8}-\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}-\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4}-\frac{33a^3}{}}{\left(-1+\tan\left(\frac{dx}{2}+\right.\right.}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*((-12*cos(2*d*x+2*c)-12)*ln(tan(1/2*d*x+1/2*c)-1)+(12*cos(2*d*x+2*c)+1
2)*ln(tan(1/2*d*x+1/2*c)+1)-33*d*x*cos(2*d*x+2*c)-33*d*x+sin(5*d*x+5*c)+1/
8*sin(6*d*x+6*c)-2*sin(d*x+c)+225/8*sin(2*d*x+2*c)-9*sin(3*d*x+3*c)+9/4*si
n(4*d*x+4*c))*a^3/d/(1+cos(2*d*x+2*c))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx =$$
$$-\frac{33\,a^3dx\cos(dx+c)^2-6\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+6\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/8*(33*a^3*d*x*cos(d*x + c)^2 - 6*a^3*cos(d*x + c)^2*log(sin(d*x + c) +
1) + 6*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (2*a^3*cos(d*x + c)^5 +
 8*a^3*cos(d*x + c)^4 + 7*a^3*cos(d*x + c)^3 - 24*a^3*cos(d*x + c)^2 + 24*
a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.50.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx = a^3\left(\int 3\sin^4(c+dx)\sec(c+dx)\,dx + \int 3\sin^4(c+dx)\sec^2(c+dx)\,dx + \int \sin^4(c+dx)\sec^3(c+dx)\,dx + \int \sin^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)**4*sec(c + d*x), x) + Integral(3*sin(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*sec(c + d*x)**3, x) +
Integral(sin(c + d*x)**4, x))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.32

$$\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx = -\frac{16\left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) + 6\sin(dx+c)\right)a^3 - (12\,dx+$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/32*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1) + 6*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a^3 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*
tan(d*x + c))*a^3 + 8*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin
(d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx =$$

$$-\frac{33\,(dx+c)a^3 - 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{8\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{8\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/8*(33*(d*x + c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 12*a^
3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 8*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(25*a^3*tan(
1/2*d*x + 1/2*c)^7 + 81*a^3*tan(1/2*d*x + 1/2*c)^5 + 79*a^3*tan(1/2*d*x +
1/2*c)^3 + 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.48

$$\int (a + a\sec(c+dx))^3 \sin^4(c+dx)\,dx = \frac{3\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{33\,a^3\,x}{8}$$
$$+ \frac{-\frac{45\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{4} - \frac{83\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4} + \frac{25\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{2} + \frac{79\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2} + \frac{27\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4} + \frac{21\,a^3\tan\left(\right.}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(sin(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

output
```
(3*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (33*a^3*x)/8 + ((27*a^3*tan(c/2 + (d
*x)/2)^3)/4 + (79*a^3*tan(c/2 + (d*x)/2)^5)/2 + (25*a^3*tan(c/2 + (d*x)/2)
^7)/2 - (83*a^3*tan(c/2 + (d*x)/2)^9)/4 - (45*a^3*tan(c/2 + (d*x)/2)^11)/4
 + (21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (
d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d
*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.51 $\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx$

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3.51.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx = -\frac{5a^3x}{2} + \frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{3a^3\sin(c+dx)}{d} - \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
-5/2*a^3*x+5/2*a^3*arctanh(sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*cos(d*
x+c)*sin(d*x+c)/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

3.51.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(98) = 196$.

Time $= 2.20$ (sec) , antiderivative size $= 300$, normalized size of antiderivative $= 3.06$

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx$$

$$= \frac{1}{32}a^3(1+\cos(c+dx))^3 \sec^6\left(\frac{1}{2}(c+dx)\right)\left(-10x - \frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{12\cos(dx)\sin(c)}{d} - \frac{\cos(2dx)\sin(2c)}{d} - \frac{12\cos(c)\sin(dx)}{d} - \frac{\cos(2c)\sin(2dx)}{d} + \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input

```
Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

output

```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-10*x - (10*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/d + (10*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/d - (12*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (12*Cos[c]*Sin[d*
x])/d - (Cos[2*c]*Sin[2*d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2) + (12*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*Sin[
(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
)/32
```

3.51.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 25, 25, 3042, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \tan^2(c+dx)\sec(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\sec(c+dx)\tan^2(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan^2(c+dx)\sec(c+dx)(a\cos(c+dx)+a)^3 dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3351$$

$$\frac{\int\left(\sec^3(c+dx)a^5-\cos^2(c+dx)a^5+3\sec^2(c+dx)a^5-3\cos(c+dx)a^5+2\sec(c+dx)a^5-2a^5\right)dx}{a^2}$$

$$\downarrow 2009$$

$$\frac{\frac{5a^5\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^5\sin(c+dx)}{d}+\frac{3a^5\tan(c+dx)}{d}-\frac{a^5\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^5\tan(c+dx)\sec(c+dx)}{2d}-\frac{5a^5x}{2}}{a^2}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

output
```
((-5*a^5*x)/2 + (5*a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^5*Sin[c + d*x])
/d - (a^5*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^5*Tan[c + d*x])/d + (a^5
*Sec[c + d*x]*Tan[c + d*x])/(2*d))/a^2
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.51.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan$
default	$\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan$
parts	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^3(-\sin(dx+c)+\ln$
parallelrisch	$\frac{a^3\left(-20(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-20dx\cos(2dx+2c)+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-20dx+20\ln\left($
norman	$\frac{-\frac{5a^3x}{2}+5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\frac{5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{2}+\frac{18a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{10a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$
risch	$-\frac{5a^3x}{2}+\frac{ia^3e^{2i(dx+c)}}{8d}+\frac{3ia^3e^{i(dx+c)}}{2d}-\frac{3ia^3e^{-i(dx+c)}}{2d}-\frac{ia^3e^{-2i(dx+c)}}{8d}-\frac{ia^3\left(e^{3i(dx+c)}-6e^{2i(dx+c)}-e^{i(dx+}\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$

input
```
int((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+3*a^3*(tan(d*x+c)-d*x-c)+3*a^3*(-sin(d*x+c)+ln(sec(d*x+c)+tan(
d*x+c)))+a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx =$$
$$-\frac{10\,a^3 dx\cos(dx+c)^2-5\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+5\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+}{4\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(10*a^3*d*x*cos(d*x + c)^2 - 5*a^3*cos(d*x + c)^2*log(sin(d*x + c) +
1) + 5*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c)^3 +
 6*a^3*cos(d*x + c)^2 - 6*a^3*cos(d*x + c) - a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2)
```

3.51.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx = a^3\left(\int 3\sin^2(c+dx)\sec(c+dx)\,dx + \int 3\sin^2(c+dx)\sec^2(c+dx)\,dx + \int \sin^2(c+dx)\sec^3(c+dx)\,dx + \int \sin^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*sin(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(sin(c + d*x)**2, x))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx = \frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a^3 - 12\,(dx + c - \tan(dx+c))a^3 - a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \right.}{4\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a^
3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
 - 2*sin(d*x + c)))/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx =$$

$$-\frac{5\,(dx+c)a^3 - 5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 -}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(5*(d*x + c)*a^3 - 5*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 5*a^3*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^
3*tan(1/2*d*x + 1/2*c)^3)/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int (a + a\sec(c+dx))^3 \sin^2(c+dx)\,dx = \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{5\,a^3\,x}{2} + \frac{18\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 1\right)}$$

input
```
int(sin(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

output
```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*x)/2 + (18*a^3*tan(c/2 + (d*x
)/2)^3 - 10*a^3*tan(c/2 + (d*x)/2)^7)/(d*(tan(c/2 + (d*x)/2)^8 - 2*tan(c/2
 + (d*x)/2)^4 + 1))
```

3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.52.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
9/2*a^3*arctanh(sin(d*x+c))/d-4*a^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(
d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(80) = 160.

Time = 1.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.05

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\log\left(\cos\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-18*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 18*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 16*Csc[c/
2]*Csc[(c + d*x)/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(
-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) + (12*Sin[d*x])/((Cos[c/2
] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(32*d)
```

3.52.3 Rubi [A] (verified)

Time $= 0.46$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 0.96$, number of steps used $= 8$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used $= \{3042, 4360, 25, 25, 3042, 25, 3351, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4360$$

$$\int \csc^2(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^2(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3351$$

$$-a^2 \int \left(-a \sec^3(c+dx) - 3a \sec^2(c+dx) - 4a \sec(c+dx) - \frac{4a}{1-\cos(c+dx)}\right) dx$$

$$\downarrow 2009$$

$$-a^2\left(-\frac{9a\text{arctanh}(\sin(c+dx))}{2d} - \frac{3a\tan(c+dx)}{d} + \frac{4a\sin(c+dx)}{d(1-\cos(c+dx))} - \frac{a\tan(c+dx)\sec(c+dx)}{2d}\right)$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
-(a^2*((-9*a*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a*Sin[c + d*x])/(d*(1 - Cos
[c + d*x])) - (3*a*Tan[c + d*x])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)))
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.52.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

method	result
parallelrisch	$\frac{9a^3\left((-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{10\left(\cos(dx+c)-\frac{7\cos(2dx+2c)}{5}-\frac{6}{5}\right.}{9}\right.}{2d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{4a^3}{d}+\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{9a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia^3\left(9\,e^{4i(dx+c)}-7\,e^{3i(dx+c)}+21\,e^{2i(dx+c)}-5\,e^{i(dx+c)}+14\right)}{d\left(e^{i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}+\frac{9a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{9a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\frac{\sin}{}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\frac{\sin}{}\right.}{d}$

input
```
int(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
9/2*a^3*((-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+(1+cos(2*d*x+2*c))*l
n(tan(1/2*d*x+1/2*c)+1)+10/9*(cos(d*x+c)-7/5*cos(2*d*x+2*c)-6/5)*cot(1/2*d
*x+1/2*c))/d/(1+cos(2*d*x+2*c))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$=\frac{9\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)\sin(dx+c)-9\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)\sin(dx+c)}{4\,d\cos(dx+c)^2\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(9*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) - 9*a^3*cos(d
*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 28*a^3*cos(d*x + c)^3 - 18
*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) + 2*a^3)/(d*cos(d*x + c)^2*sin(d
*x + c))
```

3.52.6 Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc^2(c+dx)\sec(c+dx)\,dx + \int 3\csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\sec^3(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*csc(c + d*x)**2*sec(c + d*x), x) + Integral(3*csc(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(csc(c + d*x)**2, x))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + 6a^3\left(\frac{2}{\sin(dx+c)} - \log(\sin($$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(a^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^3*(2/sin(d*x + c) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*(1/tan(d*x + c) - t
an(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(9*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a^3*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 8*a^3/tan(1/2*d*x + 1/2*c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c
)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{9\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{9\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-15\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

output
```
(9*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^3*tan(c/2 + (d*x)/2)^4 - 15*a^3
*tan(c/2 + (d*x)/2)^2 + 4*a^3)/(d*(tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/
2)^3 + tan(c/2 + (d*x)/2)^5))
```

3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.53.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{11a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2} - \frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
11/2*a^3*arctanh(sin(d*x+c))/d-2/3*a^3*sin(d*x+c)/d/(1-cos(d*x+c))^2-17/3*
a^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(
d*x+c)/d
```

3.53.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(110) = 220.

Time = 6.71 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.64

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-4\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right) - 66\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-4*Cot[c/2]*Csc[(c + d*x)/2]
^2 - 66*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 66*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 2*(-19 + 17*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]
^3*Sin[(d*x)/2] + 3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (36*Sin[(d*x
)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 3/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (36*Sin[(d*x)/2])/((Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(96*d)
```

3.53.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4360$$

$$\int \csc^4(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^4(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^4(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3351$$

$$-a^4\int\left(-\frac{\sec^3(c+dx)}{a}-\frac{3\sec^2(c+dx)}{a}-\frac{5\sec(c+dx)}{a}-\frac{5}{a(1-\cos(c+dx))}-\frac{2}{a(1-\cos(c+dx))^2}\right)dx$$

$$\downarrow 2009$$

$$-a^4\left(-\frac{11\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{3\tan(c+dx)}{ad}+\frac{17\sin(c+dx)}{3ad(1-\cos(c+dx))}+\frac{2\sin(c+dx)}{3ad(1-\cos(c+dx))^2}-\frac{\tan(c+dx)\,s}{2a}\right.$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
-(a^4*((-11*ArcTanh[Sin[c + d*x]])/(2*a*d) + (2*Sin[c + d*x])/(3*a*d*(1 -
Cos[c + d*x])^2) + (17*Sin[c + d*x])/(3*a*d*(1 - Cos[c + d*x])) - (3*Tan[c
 + d*x])/(a*d) - (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)))
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.53.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

method	result
parallelrisch	$-\frac{11a^3\left((1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{17\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\cos(dx+c)-\right.}{2d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3}{3d}-\frac{16a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}+\frac{56a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{11a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{11a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d}$
risch	$-\frac{ia^3\left(33\,e^{6i(dx+c)}-99\,e^{5i(dx+c)}+154\,e^{4i(dx+c)}-210\,e^{3i(dx+c)}+161\,e^{2i(dx+c)}-123\,e^{i(dx+c)}+52\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^2}-\frac{11a^3\ln\left(e^{i(dx+c)}-\right.}{2d}$
derivativedivides	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}\right.}{}$

input
```
int(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-11/2*a^3*((1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)+(-cos(2*d*x+2*c)-1)
*ln(tan(1/2*d*x+1/2*c)+1)-17/11*csc(1/2*d*x+1/2*c)^2*(cos(d*x+c)-71/102*co
s(2*d*x+2*c)+13/51*cos(3*d*x+3*c)-65/102)*cot(1/2*d*x+1/2*c))/d/(1+cos(2*d
*x+2*c))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{104\,a^3\cos(dx+c)^4-38\,a^3\cos(dx+c)^3-118\,a^3\cos(dx+c)^2+30\,a^3\cos(dx+c)+6\,a^3-33\left(a^3\cos\right.}{12\,(d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/12*(104*a^3*cos(d*x + c)^4 - 38*a^3*cos(d*x + c)^3 - 118*a^3*cos(d*x +
c)^2 + 30*a^3*cos(d*x + c) + 6*a^3 - 33*(a^3*cos(d*x + c)^3 - a^3*cos(d*x
+ c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 33*(a^3*cos(d*x + c)^3 - a^3*
cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^3 -
d*cos(d*x + c)^2)*sin(d*x + c))
```

3.53.6 Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc^4(c+dx)\sec(c+dx)\,dx + \int 3\csc^4(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\sec^3(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

input
```
integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*csc(c + d*x)**4*sec(c + d*x), x) + Integral(3*csc(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4*sec(c + d*x)**3, x) +
Integral(csc(c + d*x)**4, x))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.71

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + 6a^3\left(\frac{2\left(3\sin}{\sin(}\right.}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(a^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5
- sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) +
 6*a^3*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 12*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(33*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 33*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 6*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 2*(18*a^3*tan(1/2*d*x + 1/2*c)^2 +
 a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{11\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{56\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{16\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

output
```
(11*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a^3*tan(c/2 + (d*x)/2)^2)/3 -
(56*a^3*tan(c/2 + (d*x)/2)^4)/3 + 11*a^3*tan(c/2 + (d*x)/2)^6 + a^3/3)/(d*
(tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^7))
```

3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.54.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\begin{aligned}\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = {}& \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} \\ & + \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} \\ & - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} \\ & - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} \\ & - \frac{76a^6\sec(c+dx)\tan(c+dx)}{15d\,(a^3-a^3\cos(c+dx))}\end{aligned}$$

output

```
13/2*a^3*arctanh(sin(d*x+c))/d+152/15*a^3*tan(d*x+c)/d+13/2*a^3*sec(d*x+c)
*tan(d*x+c)/d-1/5*a^6*sec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^3-11/15*a^5
*sec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^2-76/15*a^6*sec(d*x+c)*tan(d*x+c
)/d/(a^3-a^3*cos(d*x+c))
```

3.54.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(165) = 330.

Time = 2.01 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.14

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(24960\cos^2(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.-\sin\left(\frac{1}{2}(c$$

input
```
Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/30720*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(2496
0*Cos[c + d*x]^2*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c]*(-1235*S
in[(d*x)/2] + 3805*Sin[(3*d*x)/2] + 4329*Sin[c - (d*x)/2] - 1989*Sin[c + (
d*x)/2] - 3575*Sin[2*c + (d*x)/2] + 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c
+ (3*d*x)/2] + 2275*Sin[3*c + (3*d*x)/2] - 2673*Sin[c + (5*d*x)/2] + 105*S
in[2*c + (5*d*x)/2] - 1593*Sin[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2]
 + 1325*Sin[2*c + (7*d*x)/2] - 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7
*d*x)/2] + 195*Sin[5*c + (7*d*x)/2] - 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*
c + (9*d*x)/2] - 214*Sin[5*c + (9*d*x)/2])))/d
```

3.54.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.048$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3348, 25, 3042, 3245, 3042, 3457, 3042, 3457, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int \csc^6(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^6(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^6(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3348$$

$$-a^6\int -\frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$a^6\int \frac{\sec^3(c+dx)}{(a-a\cos(c+dx))^3}dx$$

$$\downarrow 3042$$

$$a^6\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 3245$$

$$a^6\left(\frac{\int \frac{(4\cos(c+dx)a+7a)\sec^3(c+dx)}{(a-a\cos(c+dx))^2}dx}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3042$$

$$a^6\left(\frac{\int \frac{4\sin\left(c+dx+\frac{\pi}{2}\right)a+7a}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3457$$

$$a^6\left(\frac{\frac{\int \frac{\left(33\cos(c+dx)a^2+43a^2\right)\sec^3(c+dx)}{a-a\cos(c+dx)}dx}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3042$$

$$a^6\left(\frac{\frac{\int \frac{33\sin\left(c+dx+\frac{\pi}{2}\right)a^2+43a^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3457$$

$$a^6\left(\frac{\frac{\frac{\int\left(152\cos(c+dx)a^3+195a^3\right)\sec^3(c+dx)dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3042$$

$$a^6\left(\frac{\frac{\frac{\int \frac{152\sin\left(c+dx+\frac{\pi}{2}\right)a^3+195a^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3227$$

$$a^6\left(\frac{\frac{\frac{195a^3\int\sec^3(c+dx)dx+152a^3\int\sec^2(c+dx)dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 3042$$

$$a^6\left(\frac{\frac{\frac{152a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+195a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 4254$$

$$a^6\left(\frac{\frac{195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{152a^3\int 1d(-\tan(c+dx))}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 24$$

$$a^6\left(\frac{\frac{\frac{195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}\right)$$

$$\downarrow 4255$$

$$a^6\left(\frac{\frac{\frac{195a^3\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)\sec}{5d(a-a\cos(c}\right)$$

$$\downarrow 3042$$

$$a^6\left(\frac{\frac{\frac{195a^3\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a-a\,co}\right)$$

$$\downarrow 4257$$

$$a^6\left(\frac{\frac{\frac{195a^3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{152a^3\tan(c+dx)}{d}}{a^2}-\frac{76a^2\tan(c+dx)\sec(c+dx)}{d(a-a\cos(c+dx))}}{3a^2}-\frac{11a\tan(c+dx)\sec(c+dx)}{3d(a-a\cos(c+dx))^2}}{5a^2}-\frac{\tan(c+d}{5d(a-a}\right)$$

input
```
Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
a^6*(-1/5*(Sec[c + d*x]*Tan[c + d*x])/(d*(a - a*Cos[c + d*x])^3) + ((-11*a
*Sec[c + d*x]*Tan[c + d*x])/(3*d*(a - a*Cos[c + d*x])^2) + ((-76*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(d*(a - a*Cos[c + d*x])) + ((152*a^3*Tan[c + d*x])/d
 + 195*a^3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d
)))/a^2)/(3*a^2))/(5*a^2))
```

3.54.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3348
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a^(2*m)   Int[(d*
Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.54.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{777a^3\left(\frac{520(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{777}+\frac{520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{777}+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\cos\right.\right.}{80d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3}{20d}-\frac{17a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{30d}-\frac{97a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{15d}+\frac{131a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{6d}-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{13a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$
risch	$-\frac{ia^3\left(195\,e^{8i(dx+c)}-975\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}-3575\,e^{5i(dx+c)}+4329\,e^{4i(dx+c)}-3805\,e^{3i(dx+c)}+2673\,e^{2i(dx+c)}-1\right.}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{}$
default	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{}$

input
```
int(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
777/80*a^3*(520/777*(-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+520/777*(
1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)+1)+cot(1/2*d*x+1/2*c)*csc(1/2*d*x+
1/2*c)^4*(cos(d*x+c)-174/259*cos(2*d*x+2*c)+239/777*cos(3*d*x+3*c)-152/233
1*cos(4*d*x+4*c)-1354/2331))/d/(1+cos(2*d*x+2*c))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{608\,a^3\cos(dx+c)^5-826\,a^3\cos(dx+c)^4-476\,a^3\cos(dx+c)^3+868\,a^3\cos(dx+c)^2-120\,a^3\cos(dx}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(608*a^3*cos(d*x + c)^5 - 826*a^3*cos(d*x + c)^4 - 476*a^3*cos(d*x +
 c)^3 + 868*a^3*cos(d*x + c)^2 - 120*a^3*cos(d*x + c) - 30*a^3 - 195*(a^3*
cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(sin(d*x +
c) + 1)*sin(d*x + c) + 195*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^
3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(a^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2
- 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*l
og(sin(d*x + c) - 1)) + 6*a^3*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3
)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) +
36*a^3*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(
d*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)
^5)/d
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{60\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{60\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 390*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 60*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (465*a^3*tan(1/2*d*x + 1/2*c)^
4 + 40*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{51\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-\frac{262\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{388\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}+\frac{34\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}+\frac{a^3}{5}}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

output
```
(13*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/15 +
 (388*a^3*tan(c/2 + (d*x)/2)^4)/15 - (262*a^3*tan(c/2 + (d*x)/2)^6)/3 + 51
*a^3*tan(c/2 + (d*x)/2)^8 + a^3/5)/(d*(4*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2
+ (d*x)/2)^7 + 4*tan(c/2 + (d*x)/2)^9))
```

3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.55.1 Optimal result

Integrand size = 21, antiderivative size = 192

$$\begin{aligned}\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = {}& \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ & - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} \\ & - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d} \\ & - \frac{15a^3\csc(c+dx)}{2d} - \frac{5a^3\csc^3(c+dx)}{2d} \\ & - \frac{3a^3\csc^5(c+dx)}{2d} - \frac{15a^3\csc^7(c+dx)}{14d} \\ & + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}\end{aligned}$$

output

```
15/2*a^3*arctanh(sin(d*x+c))/d-13*a^3*cot(d*x+c)/d-7*a^3*cot(d*x+c)^3/d-3*
a^3*cot(d*x+c)^5/d-4/7*a^3*cot(d*x+c)^7/d-15/2*a^3*csc(d*x+c)/d-5/2*a^3*cs
c(d*x+c)^3/d-3/2*a^3*csc(d*x+c)^5/d-15/14*a^3*csc(d*x+c)^7/d+1/2*a^3*csc(d
*x+c)^7*sec(d*x+c)^2/d+3*a^3*tan(d*x+c)/d
```

3.55.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 430 vs. $2(192) = 384$.

Time $= 3.79$ (sec) , antiderivative size $= 430$, normalized size of antiderivative $= 2.24$

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\cos(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-860160\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c-\right.\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*Cos[c + d*x]*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(-860160*Cos[c +
 d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 860160*Cos[c + d*x]^2*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 8*Csc[2*c]*Csc[(c + d*x)/2]^6*Cs
c[c + d*x]*(5264*Sin[2*c] - 9580*Sin[d*x] + 8480*Sin[2*d*x] + 2776*Sin[c -
 d*x] - 6080*Sin[c + d*x] + 8816*Sin[2*(c + d*x)] - 7904*Sin[3*(c + d*x)]
+ 4864*Sin[4*(c + d*x)] - 1824*Sin[5*(c + d*x)] + 304*Sin[6*(c + d*x)] - 9
580*Sin[2*c + d*x] - 10024*Sin[3*c + d*x] + 13891*Sin[c + 2*d*x] + 7720*Si
n[2*(c + 2*d*x)] + 13891*Sin[3*c + 2*d*x] + 10080*Sin[4*c + 2*d*x] - 10060
*Sin[c + 3*d*x] - 12454*Sin[2*c + 3*d*x] - 12454*Sin[4*c + 3*d*x] - 6580*S
in[5*c + 3*d*x] + 7664*Sin[3*c + 4*d*x] + 7664*Sin[5*c + 4*d*x] + 2520*Sin
[6*c + 4*d*x] - 3420*Sin[3*c + 5*d*x] - 2874*Sin[4*c + 5*d*x] - 2874*Sin[6
*c + 5*d*x] - 420*Sin[7*c + 5*d*x] + 640*Sin[4*c + 6*d*x] + 479*Sin[5*c +
6*d*x] + 479*Sin[7*c + 6*d*x]))/(917504*d)
```

3.55.3 Rubi [A] (verified)

Time $= 0.97$ (sec) , antiderivative size $= 192$, normalized size of antiderivative $= 1.00$, number of steps used $= 8$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used $= \{3042, 4360, 25, 25, 3042, 25, 3352, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^8(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$

$$\int \frac{\left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^3}{\cos\left(c + dx - \frac{\pi}{2}\right)^8} dx$$

$$\downarrow 4360$$

$$\int \csc^8(c + dx)\sec^3(c + dx)\left(-(a(-\cos(c + dx)) - a)^3\right) dx$$

$$\downarrow 25$$

$$-\int -(\cos(c + dx)a + a)^3 \csc^8(c + dx)\sec^3(c + dx) dx$$

$$\downarrow 25$$

$$\int \csc^8(c + dx)\sec^3(c + dx)(a\cos(c + dx) + a)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a - a\sin\left(c + dx - \frac{\pi}{2}\right)\right)^3}{\sin\left(c + dx - \frac{\pi}{2}\right)^3 \cos\left(c + dx - \frac{\pi}{2}\right)^8} dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a - a\sin\left(\frac{1}{2}(2c - \pi) + dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c - \pi) + dx\right)^8 \sin\left(\frac{1}{2}(2c - \pi) + dx\right)^3} dx$$

$$\downarrow 3352$$

$$-\int \left(-a^3\sec^8\left(\frac{1}{2}(2c - \pi) + dx\right) - a^3\sec^3(c + dx)\sec^8\left(\frac{1}{2}(2c - \pi) + dx\right) - 3a^3\sec^2(c + dx)\sec^8\left(\frac{1}{2}(2c - \pi) +\right.\right.$$

$$\downarrow 2009$$

$$\frac{15a^3\text{arctanh}(\sin(c + dx))}{2d} + \frac{3a^3\tan(c + dx)}{d} - \frac{4a^3\cot^7(c + dx)}{7d} - \frac{3a^3\cot^5(c + dx)}{d} -$$
$$\frac{7a^3\cot^3(c + dx)}{d} - \frac{13a^3\cot(c + dx)}{d} - \frac{15a^3\csc^7(c + dx)}{14d} - \frac{3a^3\csc^5(c + dx)}{2d} - \frac{5a^3\csc^3(c + dx)}{2d} -$$
$$\frac{15a^3\csc(c + dx)}{2d} + \frac{a^3\csc^7(c + dx)\sec^2(c + dx)}{2d}$$

input
```
Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Co
t[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d)
- (15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[
c + d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*S
ec[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.55.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00

method	result
norman	$-\frac{\frac{a^3}{112d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{28d}-\frac{85a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{112d}-\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2d}+\frac{395a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16d}-\frac{57a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
risch	$-\frac{ia^3\left(105\,e^{11i(dx+c)}-630\,e^{10i(dx+c)}+1645\,e^{9i(dx+c)}-2520\,e^{8i(dx+c)}+2506\,e^{7i(dx+c)}-1316\,e^{6i(dx+c)}-694\,e^{5i(dx+c)}+\right.}{7d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)^2}$
parallelrisch	$\frac{5\sec\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\left(-\frac{329}{40}+\frac{21\left(\frac{\sin(6dx+6c)}{16}+\frac{29\sin(2dx+2c)}{16}-\frac{3\sin(5dx+5c)}{8}-\frac{5\sin(dx+c)}{4}+\sin(4dx+4c)-\frac{13\sin(3dx+3c)}{8}\right)}{2}\right)\ln\left(\tan\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln}{}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln}{}\right.}{}$

input
```
int(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/112/d*a^3-3/28*a^3/d*tan(1/2*d*x+1/2*c)^2-85/112*a^3/d*tan(1/2*d*x+1/2
*c)^4-15/2*a^3/d*tan(1/2*d*x+1/2*c)^6+395/16*a^3/d*tan(1/2*d*x+1/2*c)^8-57
/4*a^3/d*tan(1/2*d*x+1/2*c)^10-1/16*a^3/d*tan(1/2*d*x+1/2*c)^12)/tan(1/2*d
*x+1/2*c)^7/(-1+tan(1/2*d*x+1/2*c)^2)^2-15/2/d*a^3*ln(tan(1/2*d*x+1/2*c)-1
)+15/2/d*a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.45

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{320\,a^3\cos(dx+c)^6-750\,a^3\cos(dx+c)^5+170\,a^3\cos(dx+c)^4+720\,a^3\cos(dx+c)^3-520\,a^3\cos(dx}{}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/28*(320*a^3*cos(d*x + c)^6 - 750*a^3*cos(d*x + c)^5 + 170*a^3*cos(d*x +
 c)^4 + 720*a^3*cos(d*x + c)^3 - 520*a^3*cos(d*x + c)^2 + 42*a^3*cos(d*x +
 c) + 14*a^3 - 105*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(
d*x + c)^3 - a^3*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*
(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*
d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - d*cos(d*x + c)^2)*sin(d*x + c))
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.40

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3\left(\frac{2\left(315\sin(dx+c)^8-210\sin(dx+c)^6-42\sin(dx+c)^4-18\sin(dx+c)^2-10\right)}{\sin(dx+c)^9-\sin(dx+c)^7}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(\right.\right.\right.}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/140*(a^3*(2*(315*sin(d*x + c)^8 - 210*sin(d*x + c)^6 - 42*sin(d*x + c)^
4 - 18*sin(d*x + c)^2 - 10)/(sin(d*x + c)^9 - sin(d*x + c)^7) - 315*log(si
n(d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*a^3*(2*(105*sin(d*x + c)^
6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 + 15)/sin(d*x + c)^7 - 105*log(s
in(d*x + c) + 1) + 105*log(sin(d*x + c) - 1)) + 12*a^3*((140*tan(d*x + c)^
6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5)/tan(d*x + c)^7 - 35*tan(d*x
 + c)) + 4*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)
*a^3/tan(d*x + c)^7)/d
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{112\left(5\,a^3\,t}{}}{112\,d}$$

input
```
integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/112*(840*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 840*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 7*a^3*tan(1/2*d*x + 1/2*c) - 112*(5*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 -
 (1050*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^4 + 14*a^
3*tan(1/2*d*x + 1/2*c)^2 + a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

$$-\frac{230\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-396\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+120\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{85\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{7}+\frac{12\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{7}}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}$$

$$-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^8,x)
```

output
```
(15*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((12*a^3*tan(c/2 + (d*x)/2)^2)/7 +
(85*a^3*tan(c/2 + (d*x)/2)^4)/7 + 120*a^3*tan(c/2 + (d*x)/2)^6 - 396*a^3*t
an(c/2 + (d*x)/2)^8 + 230*a^3*tan(c/2 + (d*x)/2)^10 + a^3/7)/(d*(16*tan(c/
2 + (d*x)/2)^7 - 32*tan(c/2 + (d*x)/2)^9 + 16*tan(c/2 + (d*x)/2)^11)) - (a
^3*tan(c/2 + (d*x)/2))/(16*d)
```

3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.56.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\begin{aligned}\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = {}& \frac{17a^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ & - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} \\ & - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a^3\cot^7(c+dx)}{7d} \\ & - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{17a^3\csc(c+dx)}{2d} \\ & - \frac{17a^3\csc^3(c+dx)}{6d} - \frac{17a^3\csc^5(c+dx)}{10d} \\ & - \frac{17a^3\csc^7(c+dx)}{14d} - \frac{17a^3\csc^9(c+dx)}{18d} \\ & + \frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}\end{aligned}$$

output

```
17/2*a^3*arctanh(sin(d*x+c))/d-16*a^3*cot(d*x+c)/d-34/3*a^3*cot(d*x+c)^3/d
-36/5*a^3*cot(d*x+c)^5/d-19/7*a^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-17
/2*a^3*csc(d*x+c)/d-17/6*a^3*csc(d*x+c)^3/d-17/10*a^3*csc(d*x+c)^5/d-17/14
*a^3*csc(d*x+c)^7/d-17/18*a^3*csc(d*x+c)^9/d+1/2*a^3*csc(d*x+c)^9*sec(d*x+
c)^2/d+3*a^3*tan(d*x+c)/d
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1000 vs. $2(232) = 464$.

Time $= 12.60$ (sec) , antiderivative size $= 1000$, normalized size of antiderivative $= 4.31$

$$
\begin{aligned}
&\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= -\frac{9833\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{80640d} \\
&\quad -\frac{979\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{53760d} \\
&\quad -\frac{5\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{2016d} \\
&\quad -\frac{\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^8\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{4608d} \\
&\quad -\frac{17\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{16d} \\
&\quad +\frac{17\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{16d} \\
&\quad +\frac{197147\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{161280d} \\
&\quad +\frac{9833\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{80640d} \\
&\quad +\frac{979\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{53760d} \\
&\quad +\frac{5\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{2016d} \\
&\quad +\frac{\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^9\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{4608d} \\
&\quad -\frac{35\cos^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{1536d} \\
&\quad -\frac{\cos^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{1536d} \\
&\quad +\frac{\cos(c+dx)\sec(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin(dx)}{16d} \\
&\quad +\frac{\cos^2(c+dx)\sec(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3(\sin(c)+6\sin(dx))}{16d} \\
&\quad -\frac{\cos^3(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\tan\left(\frac{c}{2}\right)}{1536d}
\end{aligned}
$$

input
```
Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-9833*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(
a + a*Sec[c + d*x])^3)/(80640*d) - (979*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 +
(d*x)/2]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(53760*d) - (5*Cos
[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[
c + d*x])^3)/(2016*d) - (Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[
c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(4608*d) - (17*Cos[c + d*x]^3*Log
[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[
c + d*x])^3)/(16*d) + (17*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (197147*
Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec
[c + d*x])^3*Sin[(d*x)/2])/(161280*d) + (9833*Cos[c + d*x]^3*Csc[c/2]*Csc[
c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])
/(80640*d) + (979*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (
d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(53760*d) + (5*Cos[c + d*x]
^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sin[(d*x)/2])/(2016*d) + (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*
Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(4608*d) - (35*C
os[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^3*Sin[(d*
x)/2])/(1536*d) - (Cos[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec
[c + d*x])^3*Sin[(d*x)/2])/(1536*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d...
```

3.56.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{10}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 4360$$

$$\int \csc^{10}(c+dx)\sec^3(c+dx)\left(-(a(-\cos(c+dx))-a)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(\cos(c+dx)a+a)^3\csc^{10}(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^{10}(c+dx)\sec^3(c+dx)(a\cos(c+dx)+a)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{10}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3352$$

$$-\int \left(-a^3\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)-a^3\sec^3(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)-3a^3\sec^2(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi\right)\right.$$

$$\downarrow 2009$$

$$\frac{17a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}-\frac{17a^3\csc^9(c+dx)}{18d}-\frac{17a^3\csc^7(c+dx)}{14d}-\frac{17a^3\csc^5(c+dx)}{10d}-\frac{17a^3\csc^3(c+dx)}{6d}-\frac{17a^3\csc(c+dx)}{2d}+\frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d}$$

input

```
Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

output

```
(17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*C
ot[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x
]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) -
(17*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*C
sc[c + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]
^9*Sec[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.56.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{ia^3\left(5355\,e^{15i(dx+c)}-32130\,e^{14i(dx+c)}+73185\,e^{13i(dx+c)}-64260\,e^{12i(dx+c)}-34629\,e^{11i(dx+c)}+157794\,e^{10i(dx+c)}-20\right.}{3}$
parallelrisch	$-\frac{1673\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\frac{31314}{11711}+\frac{765\left(\frac{\sin(8dx+8c)}{14}-\frac{3\sin(7dx+7c)}{7}-\sin(5dx+5c)+\sin(6dx+6c)-\frac{\sin(4dx+4c)}{7}+\frac{13\sin(3dx+3c)}{7}+\frac{17}{478}\right.}{1673}$
derivativedivides	$\frac{a^3\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)^2}-\frac{11}{63\sin(dx+c)^7\cos(dx+c)^2}-\frac{11}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{11}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{11}{6\sin(dx+c)\cos}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)^2}-\frac{11}{63\sin(dx+c)^7\cos(dx+c)^2}-\frac{11}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{11}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{11}{6\sin(dx+c)\cos}\right.}{}$

input
```
int(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/315*I*a^3*(5355*exp(15*I*(d*x+c))-32130*exp(14*I*(d*x+c))+73185*exp(13*
I*(d*x+c))-64260*exp(12*I*(d*x+c))-34629*exp(11*I*(d*x+c))+157794*exp(10*I
*(d*x+c))-207111*exp(9*I*(d*x+c))+125256*exp(8*I*(d*x+c))+62713*exp(7*I*(d
*x+c))-175518*exp(6*I*(d*x+c))+171707*exp(5*I*(d*x+c))-80132*exp(4*I*(d*x+
c))-37919*exp(3*I*(d*x+c))+78974*exp(2*I*(d*x+c))-42261*exp(I*(d*x+c))+793
6)/d/(exp(I*(d*x+c))-1)^9/(exp(I*(d*x+c))+1)^3/(exp(2*I*(d*x+c))+1)^2-17/2
*a^3/d*ln(exp(I*(d*x+c))-I)+17/2*a^3/d*ln(exp(I*(d*x+c))+I)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.62

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{15872\,a^3\cos(dx+c)^8 - 36906\,a^3\cos(dx+c)^7 - 8322\,a^3\cos(dx+c)^6 + 73402\,a^3\cos(dx+c)^5 - 3334}{}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/1260*(15872*a^3*cos(d*x + c)^8 - 36906*a^3*cos(d*x + c)^7 - 8322*a^3*co
s(d*x + c)^6 + 73402*a^3*cos(d*x + c)^5 - 33342*a^3*cos(d*x + c)^4 - 34746
*a^3*cos(d*x + c)^3 + 26702*a^3*cos(d*x + c)^2 - 1890*a^3*cos(d*x + c) - 6
30*a^3 - 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x +
 c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*
log(sin(d*x + c) + 1)*sin(d*x + c) + 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(
d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x +
 c)^3 + a^3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d
*x + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 -
 3*d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.33

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3\left(\frac{2\left(3465\sin(dx+c)^{10}-2310\sin(dx+c)^8-462\sin(dx+c)^6-198\sin(dx+c)^4-110\sin(dx+c)^2-70\right)}{\sin(dx+c)^{11}-\sin(dx+c)^9}-3465\log\left(\sin\left(dx+c\right)+\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/1260*(a^3*(2*(3465*sin(d*x + c)^10 - 2310*sin(d*x + c)^8 - 462*sin(d*x
+ c)^6 - 198*sin(d*x + c)^4 - 110*sin(d*x + c)^2 - 70)/(sin(d*x + c)^11 -
sin(d*x + c)^9) - 3465*log(sin(d*x + c) + 1) + 3465*log(sin(d*x + c) - 1))
 + 6*a^3*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 +
 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*
log(sin(d*x + c) - 1)) + 60*a^3*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6
+ 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x
+ c)) + 4*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 +
180*tan(d*x + c)^2 + 35)*a^3/tan(d*x + c)^9)/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 - 171360*a^3*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) + 171360*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3780*a^3*
tan(1/2*d*x + 1/2*c) + 20160*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + (220185*a^3*tan(1/2*d*x +
1/2*c)^8 + 26880*a^3*tan(1/2*d*x + 1/2*c)^6 + 4347*a^3*tan(1/2*d*x + 1/2*c
)^4 + 540*a^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{17\,a^3\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192\,d} - \frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}$$

$$- \frac{1019\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - \frac{5282\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{3} + \frac{8132\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{15} + \frac{6242\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{105} + \frac{3302\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{315} +}{d\left(64\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} - 128\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 64\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\right)}$$

input
```
int((a + a/cos(c + d*x))^3/sin(c + d*x)^10,x)
```

output
```
(17*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(192*d)
- (3*a^3*tan(c/2 + (d*x)/2))/(16*d) - ((94*a^3*tan(c/2 + (d*x)/2)^2)/63 +
(3302*a^3*tan(c/2 + (d*x)/2)^4)/315 + (6242*a^3*tan(c/2 + (d*x)/2)^6)/105
+ (8132*a^3*tan(c/2 + (d*x)/2)^8)/15 - (5282*a^3*tan(c/2 + (d*x)/2)^10)/3
+ 1019*a^3*tan(c/2 + (d*x)/2)^12 + a^3/9)/(d*(64*tan(c/2 + (d*x)/2)^9 - 12
8*tan(c/2 + (d*x)/2)^11 + 64*tan(c/2 + (d*x)/2)^13))
```

3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.57.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}$$

output
```
1/3*cos(d*x+c)^3/a/d-3/5*cos(d*x+c)^5/a/d+3/7*cos(d*x+c)^7/a/d-1/9*cos(d*x
+c)^9/a/d+1/8*sin(d*x+c)^8/a/d
```

3.57.2 Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{(4258+6995\cos(c+dx)+3650\cos(2(c+dx))+1085\cos(3(c+dx))+140\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+\right.}{315ad}$$

input
```
Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

output
```
((4258 + 6995*Cos[c + d*x] + 3650*Cos[2*(c + d*x)] + 1085*Cos[3*(c + d*x)]
 + 140*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 3044, 15, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^9}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^9(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^9(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^9(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^9}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 3314$$

$$\frac{\int -\cos^2(c+dx)\sin^7(c+dx)dx}{a}-\frac{\int -\cos(c+dx)\sin^7(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cos(c+dx)\sin^7(c+dx)dx}{a}-\frac{\int \cos^2(c+dx)\sin^7(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^7 dx}{a} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^7 dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^7(c+dx) d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^7 dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^8(c+dx)}{8ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^7 dx}{a}$$

$$\downarrow 3045$$

$$\frac{\int \cos^2(c+dx)\left(1-\cos^2(c+dx)\right)^3 d\cos(c+dx)}{ad} + \frac{\sin^8(c+dx)}{8ad}$$

$$\downarrow 244$$

$$\frac{\int \left(-\cos^8(c+dx)+3\cos^6(c+dx)-3\cos^4(c+dx)+\cos^2(c+dx)\right) d\cos(c+dx)}{ad} + \frac{\sin^8(c+dx)}{8ad}$$

$$\downarrow 2009$$

$$\frac{\sin^8(c+dx)}{8ad} + \frac{-\frac{1}{9}\cos^9(c+dx)+\frac{3}{7}\cos^7(c+dx)-\frac{3}{5}\cos^5(c+dx)+\frac{1}{3}\cos^3(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3/3 - (3*Cos[c + d*x]^5)/5 + (3*Cos[c + d*x]^7)/7 - Cos[c +
d*x]^9/9)/(a*d) + Sin[c + d*x]^8/(8*a*d)
```

3.57.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.57.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{8}+\frac{3\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}-\frac{3\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{8}+\frac{3\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}-\frac{3\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
parallelrisch	$\frac{8820\cos(4dx+4c)+17640\cos(dx+c)+27409+315\cos(8dx+8c)+900\cos(7dx+7c)-2520\cos(6dx+6c)-2016\cos(5dx+5c}{322560da}$
norman	$\frac{\frac{32}{315ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{35da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{35da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{15da}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9}$
risch	$\frac{7\cos(dx+c)}{128ad}-\frac{\cos(9dx+9c)}{2304ad}+\frac{\cos(8dx+8c)}{1024ad}+\frac{5\cos(7dx+7c)}{1792ad}-\frac{\cos(6dx+6c)}{128ad}-\frac{\cos(5dx+5c)}{160ad}+\frac{7\cos(4dx+4c}{256ad}$

input
```
int(sin(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/9*cos(d*x+c)^9+1/8*cos(d*x+c)^8+3/7*cos(d*x+c)^7-1/2*cos(d*x+c)^
6-3/5*cos(d*x+c)^5+3/4*cos(d*x+c)^4+1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{280\cos(dx+c)^9-315\cos(dx+c)^8-1080\cos(dx+c)^7+1260\cos(dx+c)^6+1512\cos(dx+c)^5-}{2520\,ad}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 1
260*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d
*x + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{280\cos(dx+c)^9 - 315\cos(dx+c)^8 - 1080\cos(dx+c)^7 + 1260\cos(dx+c)^6 + 1512\cos(dx+c)^5 -}{2520\,ad}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 1
260*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d
*x + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{32\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{315\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^9}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
32/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(c+dx)^2}{2a}-\frac{\cos(c+dx)^3}{3a}-\frac{3\cos(c+dx)^4}{4a}+\frac{3\cos(c+dx)^5}{5a}+\frac{\cos(c+dx)^6}{2a}-\frac{3\cos(c+dx)^7}{7a}-\frac{\cos(c+dx)^8}{8a}+\frac{\cos(c+dx)^9}{9a}}{d}$$

input
```
int(sin(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

output
```
-(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - (3*cos(c + d*x)^4)/(4*a) +
 (3*cos(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(2*a) - (3*cos(c + d*x)^7)/(7*a
) - cos(c + d*x)^8/(8*a) + cos(c + d*x)^9/(9*a))/d
```

3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.58.8 Giac [A] (verification not implemented) . . . 517
3.58.9 Mupad [B] (verification not implemented) . . . 518

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\sin^6(c+dx)}{6ad}$$

output
```
1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/6*sin(d*x
+c)^6/a/d
```

3.58.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{4(123+197\cos(c+dx)+85\cos(2(c+dx))+15\cos(3(c+dx)))\sin^8\left(\frac{1}{2}(c+dx)\right)}{105ad}$$

input
```
Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

output
```
(4*(123 + 197*Cos[c + d*x] + 85*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Si
n[(c + d*x)/2]^8)/(105*a*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 3044, 15, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^7(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^7(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^7(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^7}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 3314$$

$$\frac{\int -\cos^2(c+dx)\sin^5(c+dx)dx}{a}-\frac{\int -\cos(c+dx)\sin^5(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cos(c+dx)\sin^5(c+dx)dx}{a}-\frac{\int \cos^2(c+dx)\sin^5(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^5 dx}{a} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^5 dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^5(c+dx) d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^5 dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^6(c+dx)}{6ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^5 dx}{a}$$

$$\downarrow 3045$$

$$\frac{\int \cos^2(c+dx)\left(1-\cos^2(c+dx)\right)^2 d\cos(c+dx)}{ad} + \frac{\sin^6(c+dx)}{6ad}$$

$$\downarrow 244$$

$$\frac{\int \left(\cos^6(c+dx) - 2\cos^4(c+dx) + \cos^2(c+dx)\right) d\cos(c+dx)}{ad} + \frac{\sin^6(c+dx)}{6ad}$$

$$\downarrow 2009$$

$$\frac{\sin^6(c+dx)}{6ad} + \frac{\frac{1}{7}\cos^7(c+dx) - \frac{2}{5}\cos^5(c+dx) + \frac{1}{3}\cos^3(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3/3 - (2*Cos[c + d*x]^5)/5 + Cos[c + d*x]^7/7)/(a*d) + Sin[c
 + d*x]^6/(6*a*d)
```

3.58.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.58.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}-\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}-\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
parallelrisch	$\frac{-525\cos(2dx+2c)+862+210\cos(4dx+4c)+525\cos(dx+c)+35\cos(3dx+3c)-63\cos(5dx+5c)-35\cos(6dx+6c)+15\cos($
norman	$\frac{\frac{16}{105ad}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15da}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5da}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3da}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$
risch	$\frac{5\cos(dx+c)}{64ad}+\frac{\cos(7dx+7c)}{448ad}-\frac{\cos(6dx+6c)}{192ad}-\frac{3\cos(5dx+5c)}{320ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{192ad}-\frac{5\cos(2dx+2c}{64ad}$

input
```
int(sin(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/7*cos(d*x+c)^7-1/6*cos(d*x+c)^6-2/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4
+1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{30\cos(dx+c)^7 - 35\cos(dx+c)^6 - 84\cos(dx+c)^5 + 105\cos(dx+c)^4 + 70\cos(dx+c)^3 - 105\cos($$
$$210\,ad$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos
(d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{30\cos(dx+c)^7 - 35\cos(dx+c)^6 - 84\cos(dx+c)^5 + 105\cos(dx+c)^4 + 70\cos(dx+c)^3 - 105\cos(}{210\,ad}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos
(d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{16\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{140(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 1\right)}{105\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
16/105*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 140*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1)/(a*d*((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1)^7)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(c+dx)^2}{2a} - \frac{\cos(c+dx)^3}{3a} - \frac{\cos(c+dx)^4}{2a} + \frac{2\cos(c+dx)^5}{5a} + \frac{\cos(c+dx)^6}{6a} - \frac{\cos(c+dx)^7}{7a}}{d}$$

input
```
int(sin(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

output
```
-(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(2*a) + (2*
cos(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(6*a) - cos(c + d*x)^7/(7*a))/d
```

3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.59.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\sin^4(c+dx)}{4ad}$$

output
```
1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/4*sin(d*x+c)^4/a/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2(13+21\cos(c+dx)+6\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15ad}$$

input
```
Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
(2*(13 + 21*Cos[c + d*x] + 6*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a*d
)
```

3.59.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 3044, 15, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^5(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^5(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^5(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^5}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 3314$$

$$\frac{\int -\cos^2(c+dx)\sin^3(c+dx)dx}{a}-\frac{\int -\cos(c+dx)\sin^3(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cos(c+dx)\sin^3(c+dx)dx}{a}-\frac{\int \cos^2(c+dx)\sin^3(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^3dx}{a} - \frac{\int \cos(c+dx)^2\sin(c+dx)^3dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^3(c+dx)d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^3dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^4(c+dx)}{4ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^3dx}{a}$$

$$\downarrow 3045$$

$$\frac{\int \cos^2(c+dx)\left(1-\cos^2(c+dx)\right)d\cos(c+dx)}{ad} + \frac{\sin^4(c+dx)}{4ad}$$

$$\downarrow 244$$

$$\frac{\int \left(\cos^2(c+dx)-\cos^4(c+dx)\right)d\cos(c+dx)}{ad} + \frac{\sin^4(c+dx)}{4ad}$$

$$\downarrow 2009$$

$$\frac{\sin^4(c+dx)}{4ad} + \frac{\frac{1}{3}\cos^3(c+dx)-\frac{1}{5}\cos^5(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3/3 - Cos[c + d*x]^5/5)/(a*d) + Sin[c + d*x]^4/(4*a*d)
```

3.59.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.59.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	49
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	49
parallelrisch	$\frac{-60\cos(2dx+2c)+109+15\cos(4dx+4c)+60\cos(dx+c)+10\cos(3dx+3c)-6\cos(5dx+5c)}{480da}$	63
norman	$\frac{\frac{4}{15ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$	83
risch	$\frac{\cos(dx+c)}{8ad}-\frac{\cos(5dx+5c)}{80ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{48ad}-\frac{\cos(2dx+2c)}{8ad}$	84

input
```
int(sin(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/5*cos(d*x+c)^5+1/4*cos(d*x+c)^4+1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^
2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{12\cos(dx+c)^5-15\cos(dx+c)^4-20\cos(dx+c)^3+30\cos(dx+c)^2}{60\,ad}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(
d*x + c)^2)/(a*d)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{12\cos(dx+c)^5 - 15\cos(dx+c)^4 - 20\cos(dx+c)^3 + 30\cos(dx+c)^2}{60\,ad}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(
d*x + c)^2)/(a*d)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{4\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{10(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{30(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - 1\right)}{15\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^5}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
4/15*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 1)/(a*
d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(c+dx)^2}{2a} - \frac{\cos(c+dx)^3}{3a} - \frac{\cos(c+dx)^4}{4a} + \frac{\cos(c+dx)^5}{5a}}{d}$$

input
```
int(sin(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

output
```
-(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(4*a) + cos
(c + d*x)^5/(5*a))/d
```

3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.60.9 Mupad [B] (verification not implemented) . . . 531

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} + \frac{\sin^2(c+dx)}{2ad}$$

output
```
1/3*cos(d*x+c)^3/a/d+1/2*sin(d*x+c)^2/a/d
```

3.60.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2(1+2\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)}{3ad}$$

input
```
Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(2*(1 + 2*Cos[c + d*x])*Sin[(c + d*x)/2]^4)/(3*a*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 3044, 15, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^3(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^3(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^3(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^3}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 3314$$

$$\frac{\int -\cos^2(c+dx)\sin(c+dx)dx}{a}-\frac{\int -\cos(c+dx)\sin(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cos(c+dx)\sin(c+dx)dx}{a}-\frac{\int \cos^2(c+dx)\sin(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)dx}{a} - \frac{\int \cos(c+dx)^2\sin(c+dx)dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin(c+dx)d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^2(c+dx)}{2ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)dx}{a}$$

$$\downarrow 3045$$

$$\frac{\int \cos^2(c+dx)d\cos(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad}$$

$$\downarrow 15$$

$$\frac{\sin^2(c+dx)}{2ad} + \frac{\cos^3(c+dx)}{3ad}$$

input
```
Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

3.60.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.60.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	29
default	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	29
parallelrisch	$\frac{7+3\cos(dx+c)+\cos(3dx+3c)-3\cos(2dx+2c)}{12da}$	39
risch	$\frac{\cos(dx+c)}{4ad}+\frac{\cos(3dx+3c)}{12ad}-\frac{\cos(2dx+2c)}{4ad}$	50
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}+\frac{2}{3ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	64

input
```
int(sin(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\cos(dx+c)^3 - 3\cos(dx+c)^2}{6\,ad}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\cos(dx+c)^3 - 3\cos(dx+c)^2}{6\,ad}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\cos(dx+c)^3}{d} - \frac{3\cos(dx+c)^2}{d}}{6\,a}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(2*cos(d*x + c)^3/d - 3*cos(d*x + c)^2/d)/a
```

3.60.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos(c+d\,x)^2\,(2\,\cos(c+d\,x)-3)}{6\,a\,d}$$

input
```
int(sin(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

output
```
(cos(c + d*x)^2*(2*cos(c + d*x) - 3))/(6*a*d)
```

3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.61.8 Giac [A] (verification not implemented) 536
3.61.9 Mupad [B] (verification not implemented) 537

3.61.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(c+dx)}{ad} + \frac{\log(1+\cos(c+dx))}{ad}$$

output
```
-cos(d*x+c)/a/d+ln(1+cos(d*x+c))/a/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(c+dx) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x] - 2*Log[Cos[(c + d*x)/2]])/(a*d))
```

3.61.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos(c+dx)}{\cos(c+dx)a+a}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a\cos(c+dx)}{\cos(c+dx)a+a}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 49$$

$$-\frac{\int \left(1-\frac{a}{\cos(c+dx)a+a}\right) d(a\cos(c+dx))}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{a\cos(c+dx)-a\log(a\cos(c+dx)+a)}{a^2 d}$$

input
```
Int[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x] - a*Log[a + a*Cos[c + d*x]])/(a^2*d))
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.61.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+1-\cos(dx+c)}{ad}$	32
derivativedivides	$\frac{\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))}{da}$	37
default	$\frac{\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))}{da}$	37
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{ad}$	58
risch	$-\frac{ix}{a}-\frac{e^{i(dx+c)}}{2da}-\frac{e^{-i(dx+c)}}{2da}-\frac{2ic}{da}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{da}$	73

input
```
int(sin(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(-ln(sec(1/2*d*x+1/2*c)^2)+1-cos(d*x+c))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(dx+c)-\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{ad}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(cos(d*x + c) - log(1/2*cos(d*x + c) + 1/2))/(a*d)
```

3.61.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(dx+c)}{a} - \frac{\log(\cos(dx+c)+1)}{a}}{d}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(cos(d*x + c)/a - log(cos(d*x + c) + 1)/a)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(dx+c)}{ad} + \frac{\log(|-\cos(dx+c)-1|)}{ad}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a*d) + log(abs(-cos(d*x + c) - 1))/(a*d)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)-\cos\left(c+d\,x\right)}{a\,d}$$

input
```
int(sin(c + d*x)/(a + a/cos(c + d*x)),x)
```

output
```
(log(cos(c + d*x) + 1) - cos(c + d*x))/(a*d)
```

3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.62.2 Mathematica [A] (verified) 538
3.62.3 Rubi [A] (verified) 539
3.62.4 Maple [A] (verified) 542
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3.62.7 Maxima [A] (verification not implemented) 543
3.62.8 Giac [A] (verification not implemented) 543
3.62.9 Mupad [B] (verification not implemented) 544

3.62.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\csc^2(c+dx)}{2ad}$$

output
```
-1/2*arctanh(cos(d*x+c))/a/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d-1/2*csc(d*x+c)^
2/a/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}$$

input
```
Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*((1 + 2*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)
/2]]))*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

3.62.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3185, 25, 3042, 25, 3086, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4359$$

$$\int -\frac{\cot(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)}{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)}{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3185$$

$$-\frac{\int \cot^2(c+dx)\csc(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^2(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^2 \tan\left(c+dx-\frac{\pi}{2}\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right) \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2 \tan\left(\frac{1}{2}(2c-\pi)+dx\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right) \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc(c+dx) d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right) \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right) \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^2(c+dx)}{2ad}$$

$$\downarrow 3091$$

$$-\frac{-\frac{1}{2}\int \csc(c+dx) dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}}{a} - \frac{\csc^2(c+dx)}{2ad}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{2}\int \csc(c+dx) dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}}{a} - \frac{\csc^2(c+dx)}{2ad}$$

$$\downarrow 4257$$

$$-\frac{\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{\cot(c+dx)\csc(c+dx)}{2d}}{a} - \frac{\csc^2(c+dx)}{2ad}$$

input
```
Int[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*Csc[c + d*x]^2/(a*d) - (ArcTanh[Cos[c + d*x]]/(2*d) - (Cot[c + d*x]*C
sc[c + d*x])/(2*d))/a
```

3.62.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.62.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$	35
norman	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	39
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}-\frac{1}{2(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{4}}{da}$	43
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}-\frac{1}{2(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{4}}{da}$	43
risch	$-\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}$	72

input
```
int(csc(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-tan(1/2*d*x+1/2*c)^2+2*ln(tan(1/2*d*x+1/2*c)))/d/a
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{4\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c) + 1)*
log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

3.62.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} + \frac{2}{a\cos(dx+c)+a}}{4\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c
) + a))/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (cos(d*x + c) -
 1)/(a*(cos(d*x + c) + 1)))/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{2\,d\,(a+a\,\cos(c+d\,x))} - \frac{\operatorname{atanh}(\cos(c+d\,x))}{2\,a\,d}$$

input
```
int(1/(sin(c + d*x)*(a + a/cos(c + d*x))),x)
```

output
```
- 1/(2*d*(a + a*cos(c + d*x))) - atanh(cos(c + d*x))/(2*a*d)
```

3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.63.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\csc^4(c+dx)}{4ad}$$

output
```
-1/8*arctanh(cos(d*x+c))/a/d-1/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*
csc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

3.63.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(2\cot^2\left(\frac{1}{2}(c+dx)\right)+4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec^2\left(\frac{1}{2}(c+d\right.\right.}{16ad(1+\sec(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-1/16*((2*Cot[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]]
 - Log[Sin[(c + d*x)/2]]) + Sec[(c + d*x)/2]^2)*Sec[c + d*x])/(a*d*(1 + Se
c[c + d*x]))
```

3.63.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 25, 3086, 15, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^2(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^2(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3314$$

$$-\frac{\int \cot^2(c+dx)\csc^3(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^4(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^4(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc^3(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^4 \tan\left(c+dx-\frac{\pi}{2}\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^3 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^4 \tan\left(\frac{1}{2}(2c-\pi)+dx\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^3 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^3(c+dx) d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^3 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^3 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^4(c+dx)}{4ad}$$

$$\downarrow 3091$$

$$-\frac{-\frac{1}{4}\int \csc^3(c+dx)dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}}{a} - \frac{\csc^4(c+dx)}{4ad}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{4}\int \csc(c+dx)^3 dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}}{a} - \frac{\csc^4(c+dx)}{4ad}$$

$$\downarrow 4255$$

$$-\frac{\frac{1}{4}\left(\frac{\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}\int \csc(c+dx)dx\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}}{a} - \frac{\csc^4(c+dx)}{4ad}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{4}\left(\frac{\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}\int \csc(c+dx)dx\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}}{a} - \frac{\csc^4(c+dx)}{4ad}$$

$$\downarrow 4257$$

$$-\frac{\frac{1}{4}\left(\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} + \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}}{a} - \frac{\csc^4(c+dx)}{4ad}$$

input
```
Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-1/4*Csc[c + d*x]^4/(a*d) - (-1/4*(Cot[c + d*x]*Csc[c + d*x]^3)/d + (ArcTa
nh[Cos[c + d*x]]/(2*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*d))/4)/a
```

3.63.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.63.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{16}+\frac{1}{8\cos(dx+c)-8}+\frac{\ln(\cos(dx+c)-1)}{16}}{da}$	55
default	$\frac{-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{16}+\frac{1}{8\cos(dx+c)-8}+\frac{\ln(\cos(dx+c)-1)}{16}}{da}$	55
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}$	61
norman	$\frac{-\frac{1}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}$	79
risch	$\frac{e^{5i(dx+c)}+2e^{4i(dx+c)}+10e^{3i(dx+c)}+2e^{2i(dx+c)}+e^{i(dx+c)}}{4ad\left(e^{i(dx+c)}+1\right)^4\left(e^{i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8da}$	128

input
```
int(csc(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/8/(cos(d*x+c)+1)^2-1/16*ln(cos(d*x+c)+1)+1/8/(cos(d*x+c)-1)+1/16
*ln(cos(d*x+c)-1))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\cos(dx+\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-\right.}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) -
 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d
*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*cos(d*x + c) + 4)/(a*d*cos(d
*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

3.63.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\cos(dx+c)^2+\cos(dx+c)+2\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a} - \frac{\log(\cos(dx+c)+1)}{a} + \frac{\log(\cos(dx+c)-1)}{a}}{16\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(d*x + c)^2 + cos(d*x + c) + 2)/(a*cos(d*x + c)^3 + a*cos(d*x
+ c)^2 - a*cos(d*x + c) - a) - log(cos(d*x + c) + 1)/a + log(cos(d*x + c)
- 1)/a)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.57

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)} - \frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} - \frac{\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/(a
*(cos(d*x + c) - 1)) - 2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a - (2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)/a^2)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{atanh}(\cos(c+d\,x))}{8\,a\,d} - \frac{\frac{\cos(c+d\,x)^2}{8}+\frac{\cos(c+d\,x)}{8}+\frac{1}{4}}{d\left(-a\cos(c+d\,x)^3 - a\cos(c+d\,x)^2 + a\cos(c+d\,x) + a\right)}$$

input
```
int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

output
```
- atanh(cos(c + d*x))/(8*a*d) - (cos(c + d*x)/8 + cos(c + d*x)^2/8 + 1/4)/
(d*(a + a*cos(c + d*x) - a*cos(c + d*x)^2 - a*cos(c + d*x)^3))
```

3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.64.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{16ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad}$$

output
```
-1/16*arctanh(cos(d*x+c))/a/d-1/16*cot(d*x+c)*csc(d*x+c)/a/d-1/24*cot(d*x+
c)*csc(d*x+c)^3/a/d+1/6*cot(d*x+c)*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d
```

3.64.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)+24\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{192ad(1+\sec(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-1/192*(Cos[(c + d*x)/2]^2*(12*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 +
 24*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 3*Sec[(c + d*x)/2]^4
 + 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

3.64.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3314, 25, 3042, 25, 3086, 15, 3091, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^4(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^4(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^4(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3314$$

$$-\frac{\int \cot^2(c+dx)\csc^5(c+dx)dx}{a} - \frac{\int -\cot(c+dx)\csc^6(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^6(c+dx)dx}{a} - \frac{\int \cot^2(c+dx)\csc^5(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^6 \tan\left(c+dx-\frac{\pi}{2}\right)dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^5 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^6 \tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^5 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^5(c+dx)d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^5 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^5 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 3091$$

$$-\frac{-\frac{1}{6}\int \csc^5(c+dx)dx - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{a} - \frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{6}\int \csc(c+dx)^5 dx - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{a} - \frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 4255$$

$$-\frac{\frac{1}{6}\left(\frac{\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3}{4}\int \csc^3(c+dx)dx\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{a} - \frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{6}\left(\frac{\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3}{4}\int \csc(c+dx)^3 dx\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{a} - \frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 4255$$

$$-\frac{\frac{1}{6}\left(\frac{\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3}{4}\left(\frac{1}{2}\int\csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)\right)-\frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{\frac{a}{6ad}}-\frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{6}\left(\frac{\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3}{4}\left(\frac{1}{2}\int\csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)\right)-\frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{\frac{a}{6ad}}-\frac{\csc^6(c+dx)}{6ad}$$

$$\downarrow 4257$$

$$-\frac{\frac{1}{6}\left(\frac{\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3}{4}\left(-\frac{\operatorname{arctanh}(\cos(c+dx))}{2d}-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)\right)-\frac{\cot(c+dx)\csc^5(c+dx)}{6d}}{\frac{a}{6ad}}-\frac{\csc^6(c+dx)}{6ad}$$

input
```
Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-1/6*Csc[c + d*x]^6/(a*d) - (-1/6*(Cot[c + d*x]*Csc[c + d*x]^5)/d + ((Cot[
c + d*x]*Csc[c + d*x]^3)/(4*d) - (3*(-1/2*ArcTanh[Cos[c + d*x]]/d - (Cot[c
 + d*x]*Csc[c + d*x])/(2*d)))/4)/6)/a
```

3.64.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.64.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}+\frac{1}{16\cos(dx+c)-16}+\frac{\ln(\cos(dx+c)-1)}{32}-\frac{1}{24(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{32}}{da}$
default	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}+\frac{1}{16\cos(dx+c)-16}+\frac{\ln(\cos(dx+c)-1)}{32}-\frac{1}{24(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{32}}{da}$
parallelrisch	$\frac{-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-18\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384da}$
norman	$\frac{-\frac{1}{128ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{64da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{192da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}$
risch	$\frac{3e^{9i(dx+c)}+6e^{8i(dx+c)}-8e^{7i(dx+c)}-22e^{6i(dx+c)}-150e^{5i(dx+c)}-22e^{4i(dx+c)}-8e^{3i(dx+c)}+6e^{2i(dx+c)}+3e^{i(dx+c)}}{24ad\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^4}-$

input
```
int(csc(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/32/(cos(d*x+c)-1)^2+1/16/(cos(d*x+c)-1)+1/32*ln(cos(d*x+c)-1)-1/
24/(cos(d*x+c)+1)^3-1/32/(cos(d*x+c)+1)^2-1/32*ln(cos(d*x+c)+1))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(96) = 192$.

Time $= 0.26$ (sec) , antiderivative size $= 217$, normalized size of antiderivative $= 2.05$

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{6\cos(dx+c)^4+6\cos(dx+c)^3-10\cos(dx+c)^2-3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-\right.}{96\left(}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(6*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 3*(cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
 c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + cos(d*x + c)^4
- 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*x
 + c) + 1/2) - 10*cos(d*x + c) - 16)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c
)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d
)
```

3.64.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{2\left(3\cos(dx+c)^4+3\cos(dx+c)^3-5\cos(dx+c)^2-5\cos(dx+c)-8\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a} - \frac{3\log(\cos(dx+c)+1)}{a} + \frac{3\log(\cos(dx+c)-1)}{a}}{96\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(2*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 5*cos(d*
x + c) - 8)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*
a*cos(d*x + c)^2 + a*cos(d*x + c) + a) - 3*log(cos(d*x + c) + 1)/a + 3*log
(cos(d*x + c) - 1)/a)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2} + \frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{9a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{2a^2(\cos(dx+c}{(\cos(dx+c}}{a^3}}{384\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/384*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) +
 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (12*a^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 13.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{atanh}(\cos(c+dx))}{16\,a\,d} - \frac{-\frac{\cos(c+dx)^4}{16} - \frac{\cos(c+dx)^3}{16} + \frac{5\cos(c+dx)^2}{48} + \frac{5\cos(c+dx)}{48} + \frac{1}{6}}{d\left(a\cos(c+dx)^5 + a\cos(c+dx)^4 - 2a\cos(c+dx)^3 - 2a\cos(c+dx)^2 + a\cos(c+dx) + a\right)}$$

input
```
int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

output
```
- atanh(cos(c + d*x))/(16*a*d) - ((5*cos(c + d*x))/48 + (5*cos(c + d*x)^2)
/48 - cos(c + d*x)^3/16 - cos(c + d*x)^4/16 + 1/6)/(d*(a + a*cos(c + d*x)
- 2*a*cos(c + d*x)^2 - 2*a*cos(c + d*x)^3 + a*cos(c + d*x)^4 + a*cos(c + d
*x)^5))
```

3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.65.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{5x}{128a} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} + \frac{\sin^7(c+dx)}{7ad}$$

output
```
-5/128*x/a-5/128*cos(d*x+c)*sin(d*x+c)/a/d+5/64*cos(d*x+c)^3*sin(d*x+c)/a/
d+5/48*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a/d+1/7
*sin(d*x+c)^7/a/d
```

3.65.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1176c - 840dx + 1680\sin(c+dx) + 336\sin(2(c+dx)) - 1008\sin(3(c+dx)\right.}{1075}$$

input
```
Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1176*c - 840*d*x + 1680*Sin[c + d*x] + 3
36*Sin[2*(c + d*x)] - 1008*Sin[3*(c + d*x)] + 168*Sin[4*(c + d*x)] + 336*S
in[5*(c + d*x)] - 112*Sin[6*(c + d*x)] - 48*Sin[7*(c + d*x)] + 21*Sin[8*(c
 + d*x)] - 1176*Tan[c/2]))/(10752*a*d*(1 + Sec[c + d*x]))
```

3.65.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^8}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^8(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^8(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^8(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^8}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{\int \cos(c+dx)\sin^6(c+dx)dx}{a} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^6 dx}{a} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^6 dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^6(c+dx) d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^6 dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^6 dx}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\int \cos^2(c+dx)\sin^4(c+dx)dx - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\int \cos(c+dx)^2 \sin(c+dx)^4 dx - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\left(\frac{1}{2}\int \cos^2(c+dx)\sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\left(\frac{1}{2}\int \cos(c+dx)^2 \sin(c+dx)^2 dx - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \cos^2(c+dx)dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^7(c+dx)}{7ad} - \frac{\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 3115$$

$$\frac{\frac{\sin^7(c+dx)}{7ad} - \frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

$$\downarrow 24$$

$$\frac{\frac{\sin^7(c+dx)}{7ad} - \frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d} + \frac{x}{2}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}\right) - \frac{\sin^5(c+dx)\cos^3(c+dx)}{8d}}{a}$$

input
```
Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output
```
Sin[c + d*x]^7/(7*a*d) - (-1/8*(Cos[c + d*x]^3*Sin[c + d*x]^5)/d + (5*(-1/
6*(Cos[c + d*x]^3*Sin[c + d*x]^3)/d + (-1/4*(Cos[c + d*x]^3*Sin[c + d*x])/
d + (x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))/4)/2))/8)/a
```

3.65.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.65.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{-840dx+1680\sin(dx+c)-48\sin(7dx+7c)+336\sin(5dx+5c)-1008\sin(3dx+3c)+21\sin(8dx+8c)-112\sin(6dx+6c)+16}{21504da}$
risch	$-\frac{5x}{128a}+\frac{5\sin(dx+c)}{64ad}+\frac{\sin(8dx+8c)}{1024da}-\frac{\sin(7dx+7c)}{448da}-\frac{\sin(6dx+6c)}{192da}+\frac{\sin(5dx+5c)}{64da}+\frac{\sin(4dx+4c)}{128da}-\frac{3\sin}{}$
derivativedivides	$-\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{49152}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{49152}-\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{344064}-\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{344064}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{49152}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{da}$
default	$-\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{49152}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{49152}-\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{344064}-\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{344064}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{49152}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{da}$
norman	$-\frac{5x}{128a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192ad}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{192ad}+\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{1344ad}+\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{1344ad}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{192ad}-$

input
```
int(sin(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/21504*(-840*d*x+1680*sin(d*x+c)-48*sin(7*d*x+7*c)+336*sin(5*d*x+5*c)-100
8*sin(3*d*x+3*c)+21*sin(8*d*x+8*c)-112*sin(6*d*x+6*c)+168*sin(4*d*x+4*c)+3
36*sin(2*d*x+2*c))/d/a
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{105\,dx-\left(336\cos(dx+c)^7-384\cos(dx+c)^6-952\cos(dx+c)^5+1152\cos(dx+c)^4+826\cos(d\right.}{2688\,ad}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2688*(105*d*x - (336*cos(d*x + c)^7 - 384*cos(d*x + c)^6 - 952*cos(d*x
+ c)^5 + 1152*cos(d*x + c)^4 + 826*cos(d*x + c)^3 - 1152*cos(d*x + c)^2 -
105*cos(d*x + c) + 384)*sin(d*x + c))/(a*d)
```

3.65.6 Sympy [F]

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(113) = 226$.

Time $= 0.29$ (sec) , antiderivative size $= 360$, normalized size of antiderivative $= 2.88$

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} + \frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2681\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5053\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{44099\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{2681\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a + \frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}} - \frac{}{1344\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 2681*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5053*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 44099*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 268
1*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 805*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a + 8*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*
a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin
(d*x + c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2681\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-44099\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-5053\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-26\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*
x + 1/2*c)^13 + 2681*tan(1/2*d*x + 1/2*c)^11 - 44099*tan(1/2*d*x + 1/2*c)^
9 - 5053*tan(1/2*d*x + 1/2*c)^7 - 2681*tan(1/2*d*x + 1/2*c)^5 - 805*tan(1/
2*d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)
^8*a))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$=\frac{-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}-\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192}+\frac{44099\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{1344}+\frac{5053\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{1344}+\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}+\frac{1}{}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)^{8}}$$

$$-\frac{5\,x}{128\,a}$$

input
```
int(sin(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

output
```
((5*tan(c/2 + (d*x)/2))/64 + (115*tan(c/2 + (d*x)/2)^3)/192 + (383*tan(c/2
 + (d*x)/2)^5)/192 + (5053*tan(c/2 + (d*x)/2)^7)/1344 + (44099*tan(c/2 + (
d*x)/2)^9)/1344 - (383*tan(c/2 + (d*x)/2)^11)/192 - (115*tan(c/2 + (d*x)/2
)^13)/192 - (5*tan(c/2 + (d*x)/2)^15)/64)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^
8) - (5*x)/(128*a)
```

3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.66.6 Sympy [F] . . . 573
3.66.7 Maxima [B] (verification not implemented) . . . 573
3.66.8 Giac [A] (verification not implemented) . . . 574
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3.66.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{16a} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad}$$

output
```
-1/16*x/a-1/16*cos(d*x+c)*sin(d*x+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1
/6*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

3.66.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(75c-60dx+120\sin(c+dx)+15\sin(2(c+dx))-60\sin(3(c+dx))+15\right.}{480ad(1+\sec(c+dx))}$$

input
```
Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(75*c - 60*d*x + 120*Sin[c + d*x] + 15*Si
n[2*(c + d*x)] - 60*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)] + 12*Sin[5*(c +
 d*x)] - 5*Sin[6*(c + d*x)] - 75*Tan[c/2]))/(480*a*d*(1 + Sec[c + d*x]))
```

3.66.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3048, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^6(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^6(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^6(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^6}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{\int \cos(c+dx)\sin^4(c+dx)dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^4dx}{a} - \frac{\int \cos(c+dx)^2\sin(c+dx)^4dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^4(c+dx)d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^4dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^4 dx}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\int \cos^2(c+dx)\sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\int \cos(c+dx)^2 \sin(c+dx)^2 dx - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\left(\frac{1}{4}\int \cos^2(c+dx)dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

$$\downarrow 3115$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

$$\downarrow 24$$

$$\frac{\sin^5(c+dx)}{5ad} - \frac{\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d} + \frac{x}{2}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right) - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6d}}{a}$$

input
```
Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
Sin[c + d*x]^5/(5*a*d) - (-1/6*(Cos[c + d*x]^3*Sin[c + d*x]^3)/d + (-1/4*(
Cos[c + d*x]^3*Sin[c + d*x])/d + (x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
/4)/2)/a
```

3.66.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.66.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{-60dx+120\sin(dx+c)+12\sin(5dx+5c)-60\sin(3dx+3c)-5\sin(6dx+6c)+15\sin(4dx+4c)+15\sin(2dx+2c)}{960da}$
risch	$-\frac{x}{16a}+\frac{\sin(dx+c)}{8ad}-\frac{\sin(6dx+6c)}{192da}+\frac{\sin(5dx+5c)}{80da}+\frac{\sin(4dx+4c)}{64da}-\frac{\sin(3dx+3c)}{16da}+\frac{\sin(2dx+2c)}{64da}$
derivativedivides	$\frac{-\frac{64\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{512}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1536}-\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{1280}-\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1280}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{6}}-\frac{\arctan\left(\tan\left($
default	$\frac{-\frac{64\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{512}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1536}-\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{1280}-\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1280}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{6}}-\frac{\arctan\left(\tan\left($
norman	$\frac{-\frac{x}{16a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}+\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}+\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}-\frac{3x\tan\left(\frac{dx}{2}+}{8a}$

input
```
int(sin(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/960*(-60*d*x+120*sin(d*x+c)+12*sin(5*d*x+5*c)-60*sin(3*d*x+3*c)-5*sin(6*
d*x+6*c)+15*sin(4*d*x+4*c)+15*sin(2*d*x+2*c))/d/a
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{15\,dx+\left(40\,\cos(dx+c)^5-48\,\cos(dx+c)^4-70\,\cos(dx+c)^3+96\,\cos(dx+c)^2+15\,\cos(dx+c)-\right.}{240\,ad}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/240*(15*d*x + (40*cos(d*x + c)^5 - 48*cos(d*x + c)^4 - 70*cos(d*x + c)^
3 + 96*cos(d*x + c)^2 + 15*cos(d*x + c) - 48)*sin(d*x + c))/(a*d)
```

3.66.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(89) = 178$.

Time $= 0.30$ (sec) , antiderivative size $= 278$, normalized size of antiderivative $= 2.81$

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{198\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1338\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} - \frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 85*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 198*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1338*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x
 + c)^11/(cos(d*x + c) + 1)^11)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 1
5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1338\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-198\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\tan\left(\frac{1}{2}\,dx-\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x +
1/2*c)^9 - 1338*tan(1/2*d*x + 1/2*c)^7 - 198*tan(1/2*d*x + 1/2*c)^5 - 85*t
an(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^6*a))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8}-\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{24}+\frac{223\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{20}+\frac{33\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6} - \frac{x}{16\,a}$$

input
```
int(sin(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)/8 + (17*tan(c/2 + (d*x)/2)^3)/24 + (33*tan(c/2 + (d*x)
/2)^5)/20 + (223*tan(c/2 + (d*x)/2)^7)/20 - (17*tan(c/2 + (d*x)/2)^9)/24 -
 tan(c/2 + (d*x)/2)^11/8)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) - x/(16*a)
```

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.67.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{8a} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad}$$

output
```
-1/8*x/a-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d+1/3
*sin(d*x+c)^3/a/d
```

3.67.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24\sin(c+dx) - 8\sin(3(c+dx)) + 3\left(4c - 4dx + \sin(4(c+dx)) - 4\tan\left(\frac{c}{2}\right)\right)\right.}{48ad(1+\sec(c+dx))}$$

input
```
Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(24*Sin[c + d*x] - 8*Sin[3*(c + d*x)] + 3
*(4*c - 4*d*x + Sin[4*(c + d*x)] - 4*Tan[c/2])))/(48*a*d*(1 + Sec[c + d*x]
))
```

3.67.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^4(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^4(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^4(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^4}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{\int \cos(c+dx)\sin^2(c+dx)dx}{a} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \cos(c+dx)\sin(c+dx)^2dx}{a} - \frac{\int \cos(c+dx)^2\sin(c+dx)^2dx}{a}$$

$$\downarrow 3044$$

$$\frac{\int \sin^2(c+dx)d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^2dx}{a}$$

$$\downarrow 15$$

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\int \cos(c+dx)^2 \sin(c+dx)^2 dx}{a}$$

$$\downarrow 3048$$

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\frac{1}{4}\int \cos^2(c+dx)dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a}$$

$$\downarrow 3042$$

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a}$$

$$\downarrow 3115$$

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\frac{1}{4}\left(\frac{\int 1dx}{2} + \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a}$$

$$\downarrow 24$$

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\frac{1}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d} + \frac{x}{2}\right) - \frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a}$$

input
```
Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
Sin[c + d*x]^3/(3*a*d) - (-1/4*(Cos[c + d*x]^3*Sin[c + d*x])/d + (x/2 + (C
os[c + d*x]*Sin[c + d*x])/(2*d))/4)/a
```

3.67.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.67.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{-12dx+24\sin(dx+c)-8\sin(3dx+3c)+3\sin(4dx+4c)}{96da}$
risch	$-\frac{x}{8a}+\frac{\sin(dx+c)}{4ad}+\frac{\sin(4dx+4c)}{32da}-\frac{\sin(3dx+3c)}{12da}$
derivativedivides	$\frac{-\frac{16\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{64}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{192}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\frac{16\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{64}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{192}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
norman	$\frac{-\frac{x}{8a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}+\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(sin(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(-12*d*x+24*sin(d*x+c)-8*sin(3*d*x+3*c)+3*sin(4*d*x+4*c))/d/a
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\,dx-\left(6\cos(dx+c)^3-8\cos(dx+c)^2-3\cos(dx+c)+8\right)\sin(dx+c)}{24\,ad}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/24*(3*d*x - (6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 3*cos(d*x + c) + 8)*
sin(d*x + c))/(a*d)
```

3.67.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(65) = 130$.

Time $= 0.29$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 2.68$

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{53\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 53*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a)/d
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-53\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 - 53*tan(1/2*d*x + 1/2*
c)^5 - 11*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^4*a))/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin(c+d\,x)}{4\,a\,d}-\frac{x}{8\,a}-\frac{\sin(3\,c+3\,d\,x)}{12\,a\,d}+\frac{\sin(4\,c+4\,d\,x)}{32\,a\,d}$$

input
```
int(sin(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

output
```
sin(c + d*x)/(4*a*d) - x/(8*a) - sin(3*c + 3*d*x)/(12*a*d) + sin(4*c + 4*d
*x)/(32*a*d)
```

3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.68.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{2a} + \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

output
```
-1/2*x/a+sin(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

3.68.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-c+2dx-4\sin(c+dx)+\sin(2(c+dx))+\tan\left(\frac{c}{2}\right)\right)}{2ad(1+\sec(c+dx))}$$

input
```
Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-c + 2*d*x - 4*Sin[c + d*x] + Sin[2
*(c + d*x)] + Tan[c/2]))/(a*d*(1 + Sec[c + d*x]))
```

3.68.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^2(c+dx)\cos(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^2(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^2(c+dx)\cos(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^2}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{\int \cos(c+dx)dx}{a} - \frac{\int \cos^2(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a} - \frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3115$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a} - \frac{\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}}{a}$$

$$\downarrow 24$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a} - \frac{\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}}{a}$$

$$\downarrow 3117$$

$$\frac{\sin(c+dx)}{ad} - \frac{\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}}{a}$$

input
```
Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
Sin[c + d*x]/(a*d) - (x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))/a
```

3.68.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.68.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\frac{-2dx+4\sin(dx+c)-\sin(2dx+2c)}{4da}$	33
risch	$-\frac{x}{2a}+\frac{\sin(dx+c)}{ad}-\frac{\sin(2dx+2c)}{4da}$	38
derivativedivides	$\frac{-\frac{4\left(-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	64
default	$\frac{-\frac{4\left(-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	64
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{2a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	93

input
```
int(sin(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*d*x+4*sin(d*x+c)-sin(2*d*x+2*c))/d/a
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{dx+(\cos(dx+c)-2)\sin(dx+c)}{2\,ad}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d*x + (cos(d*x + c) - 2)*sin(d*x + c))/(a*d)
```

3.68.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sin(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} + \frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a + \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4) - arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*((d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sin(2c+2dx) - 4\sin(c+dx) + 2dx}{4ad}$$

input
```
int(sin(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

output
```
-(sin(2*c + 2*d*x) - 4*sin(c + d*x) + 2*d*x)/(4*a*d)
```

3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.69.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} - \frac{\csc^3(c+dx)}{3ad}$$

output
```
1/3*cot(d*x+c)^3/a/d-1/3*csc(d*x+c)^3/a/d
```

3.69.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\csc(c)\csc(2(c+dx))(-6\sin(c)+4\sin(dx)+2\sin(c+dx)+\sin(2(c+dx))+2\sin(c+2dx))}{6ad(1+\sec(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(Csc[c]*Csc[2*(c + d*x)]*(-6*Sin[c] + 4*Sin[d*x] + 2*Sin[c + d*x] + Sin[2*
(c + d*x)] + 2*Sin[c + 2*d*x]))/(6*a*d*(1 + Sec[c + d*x]))
```

3.69.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{\int \cot^2(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^3(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^3(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc^2(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^3 \tan\left(c+dx-\frac{\pi}{2}\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^3 \tan\left(\frac{1}{2}(2c-\pi)+dx\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^2(c+dx) d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^3(c+dx)}{3ad}$$

$$\downarrow 3087$$

$$-\frac{\int \cot^2(c+dx) d(-\cot(c+dx))}{ad} - \frac{\csc^3(c+dx)}{3ad}$$

$$\downarrow 15$$

$$\frac{\cot^3(c+dx)}{3ad} - \frac{\csc^3(c+dx)}{3ad}$$

input
```
Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

3.69.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.69.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-3}{12d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}$	35
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36
norman	$\frac{-\frac{1}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{12da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	41
risch	$-\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}+1\right)}{3ad\left(\mathrm{e}^{i(dx+c)}+1\right)^3\left(\mathrm{e}^{i(dx+c)}-1\right)}$	60

input
```
int(csc(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-tan(1/2*d*x+1/2*c)^4-3)/d/tan(1/2*d*x+1/2*c)/a
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(dx+c)^2+\cos(dx+c)+1}{3\,(ad\cos(dx+c)+ad)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(cos(d*x + c)^2 + cos(d*x + c) + 1)/((a*d*cos(d*x + c) + a*d)*sin(d*x
 + c))
```

3.69.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\,(\cos(dx+c)+1)}{a\sin(dx+c)}+\frac{\sin(dx+c)^3}{a(\cos(dx+c)+1)^3}}{12\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*(cos(d*x + c) + 1)/(a*sin(d*x + c)) + sin(d*x + c)^3/(a*(cos(d*x
+ c) + 1)^3))/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(tan(1/2*d*x + 1/2*c)^3/a + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3}{12\,a\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

output
```
-(tan(c/2 + (d*x)/2)^4 + 3)/(12*a*d*tan(c/2 + (d*x)/2))
```

3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.70.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} - \frac{\csc^5(c+dx)}{5ad}$$

output
```
1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d-1/5*csc(d*x+c)^5/a/d
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\csc(c)\csc^3(c+dx)\sec(c+dx)(240\sin(c) - 96\sin(dx) - 54\sin(c+dx) - 18\sin(2(c+dx)) + 18\sin(3($$
$$960ad(1+\sec(c+d$$

input
```
Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-1/960*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]*(240*Sin[c] - 96*Sin[d*x] - 54*
Sin[c + d*x] - 18*Sin[2*(c + d*x)] + 18*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*
x)] - 32*Sin[c + 2*d*x] + 32*Sin[2*c + 3*d*x] + 16*Sin[3*c + 4*d*x]))/(a*d
*(1 + Sec[c + d*x]))
```

3.70.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^3(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^3(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{\int \cot^2(c+dx)\csc^4(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^5(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^5(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc^4(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^5 \tan\left(c+dx-\frac{\pi}{2}\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^4 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^5 \tan\left(\frac{1}{2}(2c-\pi)+dx\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^4 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^4(c+dx) d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^4 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^4 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^5(c+dx)}{5ad}$$

$$\downarrow 3087$$

$$-\frac{\int \cot^2(c+dx)\left(\cot^2(c+dx)+1\right) d(-\cot(c+dx))}{ad} - \frac{\csc^5(c+dx)}{5ad}$$

$$\downarrow 244$$

$$-\frac{\int \left(\cot^4(c+dx)+\cot^2(c+dx)\right) d(-\cot(c+dx))}{ad} - \frac{\csc^5(c+dx)}{5ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{5}\cot^5(c+dx)-\frac{1}{3}\cot^3(c+dx)}{ad} - \frac{\csc^5(c+dx)}{5ad}$$

input
```
Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/3*Cot[c + d*x]^3 - Cot[c + d*x]^5/5)/(a*d)) - Csc[c + d*x]^5/(5*a*d)
```

3.70.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.70.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-5\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-30\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{240da}$	60
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{16da}$	62
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{16da}$	62
norman	$\frac{-\frac{1}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{80da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	79
risch	$\frac{4i\left(15\,e^{4i(dx+c)}+6\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{15ad\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)^3}$	82

input
```
int(csc(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/240*(-3*tan(1/2*d*x+1/2*c)^5-5*cot(1/2*d*x+1/2*c)^3-10*tan(1/2*d*x+1/2*c
)^3-30*cot(1/2*d*x+1/2*c))/d/a
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\cos(dx+c)^4+2\cos(dx+c)^3-3\cos(dx+c)^2-3\cos(dx+c)-3}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(2*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 3*cos(d*x
+ c) - 3)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a
*d)*sin(d*x + c))
```

3.70.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a} + \frac{5\left(\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/240*((10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/a + 5*(6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x
+ c) + 1)^3/(a*sin(d*x + c)^3))/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{5\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3} + \frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^5}}{240\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/240*(5*(6*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*tan(1/2*d*x + 1/2*c)^3) + (3*a
^4*tan(1/2*d*x + 1/2*c)^5 + 10*a^4*tan(1/2*d*x + 1/2*c)^3)/a^5)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+5}{240\,a\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

output
```
-(30*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^6 + 3*tan(c/2 + (d*x)/2)
^8 + 5)/(240*a*d*tan(c/2 + (d*x)/2)^3)
```

3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.71.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} + \frac{2\cot^5(c+dx)}{5ad} + \frac{\cot^7(c+dx)}{7ad} - \frac{\csc^7(c+dx)}{7ad}$$

output
```
1/3*cot(d*x+c)^3/a/d+2/5*cot(d*x+c)^5/a/d+1/7*cot(d*x+c)^7/a/d-1/7*csc(d*x
+c)^7/a/d
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(73) = 146.

Time = 0.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\csc(c)\csc^5(c+dx)\sec(c+dx)(-8960\sin(c)+2560\sin(dx)+1500\sin(c+dx)+375\sin(2(c+dx))-75}{}$$

input
```
Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
(Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]*(-8960*Sin[c] + 2560*Sin[d*x] + 1500*S
in[c + d*x] + 375*Sin[2*(c + d*x)] - 750*Sin[3*(c + d*x)] - 300*Sin[4*(c +
 d*x)] + 150*Sin[5*(c + d*x)] + 75*Sin[6*(c + d*x)] + 640*Sin[c + 2*d*x] -
 1280*Sin[2*c + 3*d*x] - 512*Sin[3*c + 4*d*x] + 256*Sin[4*c + 5*d*x] + 128
*Sin[5*c + 6*d*x]))/(53760*a*d*(1 + Sec[c + d*x]))
```

3.71.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^5(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^5(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^5(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{\int \cot^2(c+dx)\csc^6(c+dx)dx}{a} - \frac{\int -\cot(c+dx)\csc^7(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^7(c+dx)dx}{a} - \frac{\int \cot^2(c+dx)\csc^6(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^7 \tan\left(c+dx-\frac{\pi}{2}\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^6 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^7 \tan\left(\frac{1}{2}(2c-\pi)+dx\right) dx}{a} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^6 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^6(c+dx)d\csc(c+dx)}{ad} - \frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^6 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^6 \tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a} - \frac{\csc^7(c+dx)}{7ad}$$

$$\downarrow 3087$$

$$-\frac{\int \cot^2(c+dx)\left(\cot^2(c+dx)+1\right)^2 d(-\cot(c+dx))}{ad} - \frac{\csc^7(c+dx)}{7ad}$$

$$\downarrow 244$$

$$-\frac{\int \left(\cot^6(c+dx)+2\cot^4(c+dx)+\cot^2(c+dx)\right) d(-\cot(c+dx))}{ad} - \frac{\csc^7(c+dx)}{7ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}\cot^7(c+dx)-\frac{2}{5}\cot^5(c+dx)-\frac{1}{3}\cot^3(c+dx)}{ad} - \frac{\csc^7(c+dx)}{7ad}$$

input
```
Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/3*Cot[c + d*x]^3 - (2*Cot[c + d*x]^5)/5 - Cot[c + d*x]^7/7)/(a*d)) -
 Csc[c + d*x]^7/(7*a*d)
```

3.71.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.71.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

method	result	siz
parallelrisch	$\frac{-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-84\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-21\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-175\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-140\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3-525\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{6720da}$	86
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64da}$	88
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64da}$	88
risch	$-\frac{16i\left(70\,e^{6i(dx+c)}+20\,e^{5i(dx+c)}+5\,e^{4i(dx+c)}-10\,e^{3i(dx+c)}-4\,e^{2i(dx+c)}+2\,e^{i(dx+c)}+1\right)}{105ad\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^5}$	10
norman	$\frac{-\frac{1}{320ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{448ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{48da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{64da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{192da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{80da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$	11

input
```
int(csc(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/6720*(-15*tan(1/2*d*x+1/2*c)^7-84*tan(1/2*d*x+1/2*c)^5-21*cot(1/2*d*x+1/
2*c)^5-175*tan(1/2*d*x+1/2*c)^3-140*cot(1/2*d*x+1/2*c)^3-525*cot(1/2*d*x+1
/2*c))/d/a
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{8\cos(dx+c)^6+8\cos(dx+c)^5-20\cos(dx+c)^4-20\cos(dx+c)^3+15\cos(dx+c)^2+15\cos(d}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2\,ad\cos(dx+c)^3-2\,ad\cos(dx+c)^2+ad\cos(dx+c)+ad}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/105*(8*cos(d*x + c)^6 + 8*cos(d*x + c)^5 - 20*cos(d*x + c)^4 - 20*cos(d
*x + c)^3 + 15*cos(d*x + c)^2 + 15*cos(d*x + c) + 15)/((a*d*cos(d*x + c)^5
 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*
cos(d*x + c) + a*d)*sin(d*x + c))
```

3.71.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{175\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}+\frac{7\left(\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{6720\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6720*((175*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a + 7*(20*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{7\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{15a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+84a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+175a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^7}}{6720\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6720*(7*(75*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^2 + 3)/(a*
tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7 + 84*a^6*tan(1/2*
d*x + 1/2*c)^5 + 175*a^6*tan(1/2*d*x + 1/2*c)^3)/a^7)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.10

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+525\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+175

3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.72.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot^5(c+dx)}{5ad} + \frac{3\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} - \frac{\csc^9(c+dx)}{9ad}$$

output
```
1/3*cot(d*x+c)^3/a/d+3/5*cot(d*x+c)^5/a/d+3/7*cot(d*x+c)^7/a/d+1/9*cot(d*x
+c)^9/a/d-1/9*csc(d*x+c)^9/a/d
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(91) = 182.

Time = 2.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.20

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\csc(c)\csc^7(c+dx)\sec(c+dx)(645120\sin(c) - 143360\sin(dx) - 85750\sin(c+dx) - 17150\sin(2(c+d$$

input
```
Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output
```
-1/5160960*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]*(645120*Sin[c] - 143360*Sin
[d*x] - 85750*Sin[c + d*x] - 17150*Sin[2*(c + d*x)] + 51450*Sin[3*(c + d*x
)] + 17150*Sin[4*(c + d*x)] - 17150*Sin[5*(c + d*x)] - 7350*Sin[6*(c + d*x
)] + 2450*Sin[7*(c + d*x)] + 1225*Sin[8*(c + d*x)] - 28672*Sin[c + 2*d*x]
+ 86016*Sin[2*c + 3*d*x] + 28672*Sin[3*c + 4*d*x] - 28672*Sin[4*c + 5*d*x]
 - 12288*Sin[5*c + 6*d*x] + 4096*Sin[6*c + 7*d*x] + 2048*Sin[7*c + 8*d*x])
)/(a*d*(1 + Sec[c + d*x]))
```

3.72.3 Rubi [A] (verified)

Time $= 0.53$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 0.84$, number of steps used $= 16$, number of rules used $= 15$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used $= \{3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 244, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^8(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^7(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^7(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^7(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^8\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{\int \cot^2(c+dx)\csc^8(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^9(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^9(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc^8(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^9\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^8\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^9\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^8\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^8(c+dx)d\csc(c+dx)}{ad}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^8\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^8\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}-\frac{\csc^9(c+dx)}{9ad}$$

$$\downarrow 3087$$

$$-\frac{\int \cot^2(c+dx)\left(\cot^2(c+dx)+1\right)^3d(-\cot(c+dx))}{ad}-\frac{\csc^9(c+dx)}{9ad}$$

$$\downarrow 244$$

$$-\frac{\int \left(\cot^8(c+dx)+3\cot^6(c+dx)+3\cot^4(c+dx)+\cot^2(c+dx)\right)d(-\cot(c+dx))}{ad}-\frac{\csc^9(c+dx)}{9ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{9}\cot^9(c+dx)-\frac{3}{7}\cot^7(c+dx)-\frac{3}{5}\cot^5(c+dx)-\frac{1}{3}\cot^3(c+dx)}{ad}-\frac{\csc^9(c+dx)}{9ad}$$

input
```
Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/3*Cot[c + d*x]^3 - (3*Cot[c + d*x]^5)/5 - (3*Cot[c + d*x]^7)/7 - Cot
[c + d*x]^9/9)/(a*d)) - Csc[c + d*x]^9/(9*a*d)
```

3.72.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.72.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-45\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7-882\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-378\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1470\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-14}{80640da}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}-\frac{}{3\tan}}{256da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}-\frac{}{3\tan}}{256da}$
risch	$\frac{32i\left(315\,e^{8i(dx+c)}+70\,e^{7i(dx+c)}+14\,e^{6i(dx+c)}-42\,e^{5i(dx+c)}-14\,e^{4i(dx+c)}+14\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{315ad\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^7}$
norman	$\frac{-\frac{1}{1792ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{640ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{896ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{2304ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{640da}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{384da}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128da}-\frac{7\tan\left(\frac{dx}{2}\right.}{384}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

input
```
int(csc(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/80640*(-35*tan(1/2*d*x+1/2*c)^9-270*tan(1/2*d*x+1/2*c)^7-45*cot(1/2*d*x+
1/2*c)^7-882*tan(1/2*d*x+1/2*c)^5-378*cot(1/2*d*x+1/2*c)^5-1470*tan(1/2*d*
x+1/2*c)^3-1470*cot(1/2*d*x+1/2*c)^3-4410*cot(1/2*d*x+1/2*c))/d/a
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.95

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{16\cos(dx+c)^8+16\cos(dx+c)^7-56\cos(dx+c)^6-56\cos(dx+c)^5+70\cos(dx+c)^4+70\cos}{315\left(ad\cos(dx+c)^7+ad\cos(dx+c)^6-3\,ad\cos(dx+c)^5-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^3+\right.}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/315*(16*cos(d*x + c)^8 + 16*cos(d*x + c)^7 - 56*cos(d*x + c)^6 - 56*cos
(d*x + c)^5 + 70*cos(d*x + c)^4 + 70*cos(d*x + c)^3 - 35*cos(d*x + c)^2 -
35*cos(d*x + c) - 35)/((a*d*cos(d*x + c)^7 + a*d*cos(d*x + c)^6 - 3*a*d*co
s(d*x + c)^5 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^3 + 3*a*d*cos(d*x
 + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

3.72.6 Sympy [F]

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.93

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{1470\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{882\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a} + \frac{3\left(\frac{126\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1470\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15\right)(\cos(dx+c)-}{a\sin(dx+c)^7}$$
$$80640\,d$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80640*((1470*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 882*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x
 + c)^9/(cos(d*x + c) + 1)^9)/a + 3*(126*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 490*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1470*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 15)*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{3\left(1470\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+126\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}+\frac{35a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+882a^8\tan\left(\frac{1}{2}\right.}{a^9}}{80640\,d}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/80640*(3*(1470*tan(1/2*d*x + 1/2*c)^6 + 490*tan(1/2*d*x + 1/2*c)^4 + 12
6*tan(1/2*d*x + 1/2*c)^2 + 15)/(a*tan(1/2*d*x + 1/2*c)^7) + (35*a^8*tan(1/
2*d*x + 1/2*c)^9 + 270*a^8*tan(1/2*d*x + 1/2*c)^7 + 882*a^8*tan(1/2*d*x +
1/2*c)^5 + 1470*a^8*tan(1/2*d*x + 1/2*c)^3)/a^9)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+378\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1470\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4410\cos\left(\frac{c}{2}\right.}$$

input
```
int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))),x)
```

output
```
-(45*cos(c/2 + (d*x)/2)^16 + 35*sin(c/2 + (d*x)/2)^16 + 270*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^14 + 882*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^12 + 1470*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 4410*cos(c/2 + (d*
x)/2)^10*sin(c/2 + (d*x)/2)^6 + 1470*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)
/2)^4 + 378*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(80640*a*d*cos(c/2
 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7)
```

3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.73.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} + \frac{4\cot^5(c+dx)}{5ad} + \frac{6\cot^7(c+dx)}{7ad} + \frac{4\cot^9(c+dx)}{9ad} + \frac{\cot^{11}(c+dx)}{11ad} - \frac{\csc^{11}(c+dx)}{11ad}$$

output

```
1/3*cot(d*x+c)^3/a/d+4/5*cot(d*x+c)^5/a/d+6/7*cot(d*x+c)^7/a/d+4/9*cot(d*x
+c)^9/a/d+1/11*cot(d*x+c)^11/a/d-1/11*csc(d*x+c)^11/a/d
```

3.73.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(109) = 218.

Time = 3.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.22

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\csc(c)\csc^9(c+dx)\sec(c+dx)(-45416448\sin(c)+8257536\sin(dx)+5000940\sin(c+dx)+833490\sin(2$$

input

```
Integrate[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

output

```
(Csc[c]*Csc[c + d*x]^9*Sec[c + d*x]*(-45416448*Sin[c] + 8257536*Sin[d*x] +
 5000940*Sin[c + d*x] + 833490*Sin[2*(c + d*x)] - 3333960*Sin[3*(c + d*x)]
 - 952560*Sin[4*(c + d*x)] + 1428840*Sin[5*(c + d*x)] + 535815*Sin[6*(c +
d*x)] - 357210*Sin[7*(c + d*x)] - 158760*Sin[8*(c + d*x)] + 39690*Sin[9*(c
 + d*x)] + 19845*Sin[10*(c + d*x)] + 1376256*Sin[c + 2*d*x] - 5505024*Sin[
2*c + 3*d*x] - 1572864*Sin[3*c + 4*d*x] + 2359296*Sin[4*c + 5*d*x] + 88473
6*Sin[5*c + 6*d*x] - 589824*Sin[6*c + 7*d*x] - 262144*Sin[7*c + 8*d*x] + 6
5536*Sin[8*c + 9*d*x] + 32768*Sin[9*c + 10*d*x]))/(454164480*a*d*(1 + Sec[
c + d*x]))
```

3.73.3 Rubi [A] (verified)

Time $= 0.54$ (sec) , antiderivative size $= 88$, normalized size of antiderivative $= 0.81$, number of steps used $= 16$, number of rules used $= 15$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used $= \{3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 244, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{10}(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^{9}(c+dx)}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^{9}(c+dx)}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^{9}(c+dx)}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{10}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)dx}{a}-\frac{\int -\cot(c+dx)\csc^{11}(c+dx)dx}{a}$$

$$\downarrow 25$$

$$\frac{\int \cot(c+dx)\csc^{11}(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^{11}\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^{10}\tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 25$$

$$-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^{11}\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^{10}\tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 3086$$

$$-\frac{\int \csc^{10}(c+dx)d\csc(c+dx)}{ad}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^{10}\tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\downarrow 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^{10}\tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a}-\frac{\csc^{11}(c+dx)}{11ad}$$

$$\downarrow 3087$$

$$-\frac{\int \cot^2(c+dx)\left(\cot^2(c+dx)+1\right)^4 d(-\cot(c+dx))}{ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

$$\downarrow 244$$

$$-\frac{\int \left(\cot^{10}(c+dx)+4\cot^8(c+dx)+6\cot^6(c+dx)+4\cot^4(c+dx)+\cot^2(c+dx)\right)d(-\cot(c+dx))}{ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{11}\cot^{11}(c+dx)-\frac{4}{9}\cot^9(c+dx)-\frac{6}{7}\cot^7(c+dx)-\frac{4}{5}\cot^5(c+dx)-\frac{1}{3}\cot^3(c+dx)}{ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

input
```
Int[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/3*Cot[c + d*x]^3 - (4*Cot[c + d*x]^5)/5 - (6*Cot[c + d*x]^7)/7 - (4*
Cot[c + d*x]^9)/9 - Cot[c + d*x]^11/11)/(a*d)) - Csc[c + d*x]^11/(11*a*d)
```

3.73.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.73.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-385\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-3080\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-3960\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-13365\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-18711\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-33264\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-55440\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-48510\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-145530\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{3548160da}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{1024da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{1024da}$
risch	$-\frac{256i\left(1386\,e^{10i(dx+c)}+252\,e^{9i(dx+c)}+42\,e^{8i(dx+c)}-168\,e^{7i(dx+c)}-48\,e^{6i(dx+c)}+72\,e^{5i(dx+c)}+27\,e^{4i(dx+c)}-18\,e^{3i(dx}{3465ad\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{9}}$

input
```
int(csc(d*x+c)^10/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3548160*(-315*tan(1/2*d*x+1/2*c)^11-385*cot(1/2*d*x+1/2*c)^9-3080*tan(1/
2*d*x+1/2*c)^9-3960*cot(1/2*d*x+1/2*c)^7-13365*tan(1/2*d*x+1/2*c)^7-18711*
cot(1/2*d*x+1/2*c)^5-33264*tan(1/2*d*x+1/2*c)^5-55440*cot(1/2*d*x+1/2*c)^3
-48510*tan(1/2*d*x+1/2*c)^3-145530*cot(1/2*d*x+1/2*c))/d/a
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{128\cos(dx+c)^{10}+128\cos(dx+c)^{9}-576\cos(dx+c)^{8}-576\cos(dx+c)^{7}+1008\cos(dx+c)^{6}+}{3465\left(ad\cos(dx+c)^{9}+ad\cos(dx+c)^{8}-4\,ad\cos(dx+c)^{7}-4\,ad\cos(dx+c)^{6}+6\,ad\cos(dx+\right.}$$

input
```
integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3465*(128*cos(d*x + c)^10 + 128*cos(d*x + c)^9 - 576*cos(d*x + c)^8 - 5
76*cos(d*x + c)^7 + 1008*cos(d*x + c)^6 + 1008*cos(d*x + c)^5 - 840*cos(d*
x + c)^4 - 840*cos(d*x + c)^3 + 315*cos(d*x + c)^2 + 315*cos(d*x + c) + 31
5)/((a*d*cos(d*x + c)^9 + a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^7 - 4*a*
d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^5 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos
(d*x + c)^3 - 4*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**10/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(97) = 194.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.98

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{48510\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{33264\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{13365\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3080\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}+\frac{11\left(\frac{360\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1701\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5040\sin(d}{a\sin(d}\right.$$
$$3548160\,d$$

input
```
integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3548160*((48510*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 33264*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 + 13365*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 308
0*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c)
+ 1)^11)/a + 11*(360*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1701*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 5040*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 132
30*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)*(cos(d*x + c) + 1)^9/(a*sin(d
*x + c)^9))/d
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.48

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{11\left(13230\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+5040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1701\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+35\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9}+\frac{315a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3080a^1}{3548160\,d}$$

input
```
integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/3548160*(11*(13230*tan(1/2*d*x + 1/2*c)^8 + 5040*tan(1/2*d*x + 1/2*c)^6
 + 1701*tan(1/2*d*x + 1/2*c)^4 + 360*tan(1/2*d*x + 1/2*c)^2 + 35)/(a*tan(1
/2*d*x + 1/2*c)^9) + (315*a^10*tan(1/2*d*x + 1/2*c)^11 + 3080*a^10*tan(1/2
*d*x + 1/2*c)^9 + 13365*a^10*tan(1/2*d*x + 1/2*c)^7 + 33264*a^10*tan(1/2*d
*x + 1/2*c)^5 + 48510*a^10*tan(1/2*d*x + 1/2*c)^3)/a^11)/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{63\cos(c+dx)+\frac{21\cos(2c+2dx)}{2}-42\cos(3c+3dx)-12\cos(4c+4dx)+18\cos(5c+5dx)+\frac{27\,co}{3548160\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}\right.}$$

input
```
int(1/(sin(c + d*x)^10*(a + a/cos(c + d*x))),x)
```

output
```
-(63*cos(c + d*x) + (21*cos(2*c + 2*d*x))/2 - 42*cos(3*c + 3*d*x) - 12*cos
(4*c + 4*d*x) + 18*cos(5*c + 5*d*x) + (27*cos(6*c + 6*d*x))/4 - (9*cos(7*c
 + 7*d*x))/2 - 2*cos(8*c + 8*d*x) + cos(9*c + 9*d*x)/2 + cos(10*c + 10*d*x
)/4 + 693/2)/(3548160*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9)
```

3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.74.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(a-a\cos(c+dx))^6}{3a^8d} - \frac{4(a-a\cos(c+dx))^7}{a^9d} + \frac{19(a-a\cos(c+dx))^8}{4a^{10}d} - \frac{25(a-a\cos(c+dx))^9}{9a^{11}d} + \frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d} - \frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}$$

output
```
4/3*(a-a*cos(d*x+c))^6/a^8/d-4*(a-a*cos(d*x+c))^7/a^9/d+19/4*(a-a*cos(d*x+
c))^8/a^10/d-25/9*(a-a*cos(d*x+c))^9/a^11/d+4/5*(a-a*cos(d*x+c))^10/a^12/d
-1/11*(a-a*cos(d*x+c))^11/a^13/d
```

3.74.2 Mathematica [A] (verified)

Time = 3.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(2360+4038\cos(c+dx)+2586\cos(2(c+dx))+1189\cos(3(c+dx))+342\cos(4(c+dx))+45\cos(5(c$$

input
```
Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

output
```
(4*(2360 + 4038*Cos[c + d*x] + 2586*Cos[2*(c + d*x)] + 1189*Cos[3*(c + d*x
)] + 342*Cos[4*(c + d*x)] + 45*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]^12)/(495
*a^2*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{11}(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{11}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^{11}(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^{11}}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^{11}\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^3 d(a\cos(c+dx))}{a^{11}d}$$

$$\downarrow 27$$

$$-\frac{\int a^2\cos^2(c+dx)(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^3 d(a\cos(c+dx))}{a^{13}d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-(a-a\cos(c+dx))^{10}+8a(a-a\cos(c+dx))^{9}-25a^2(a-a\cos(c+dx))^{8}+38a^3(a-a\cos(c+dx))^{7}-28\right.}{a^{13}d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4}{3}a^5(a-a\cos(c+dx))^{6}+4a^4(a-a\cos(c+dx))^{7}-\frac{19}{4}a^3(a-a\cos(c+dx))^{8}+\frac{25}{9}a^2(a-a\cos(c+dx))^{9}+\frac{1}{11}}{a^{13}d}$$

input
```
Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

output
```
-(((-4*a^5*(a - a*Cos[c + d*x])^6)/3 + 4*a^4*(a - a*Cos[c + d*x])^7 - (19*
a^3*(a - a*Cos[c + d*x])^8)/4 + (25*a^2*(a - a*Cos[c + d*x])^9)/9 - (4*a*(
a - a*Cos[c + d*x])^10)/5 + (a - a*Cos[c + d*x])^11/11)/(a^13*d))
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.74.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{\cos(dx+c)^{10}}{5}-\frac{2\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{4}-\cos(dx+c)^6+\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
default	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{\cos(dx+c)^{10}}{5}-\frac{2\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{4}-\cos(dx+c)^6+\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
parallelrisch	$\frac{-42470-3960\cos(4dx+4c)-990\cos(3dx+3c)+45\cos(11dx+11c)-34650\cos(dx+c)+990\cos(8dx+8c)-1485\cos(7dx+$
risch	$-\frac{35\cos(dx+c)}{512a^2d}+\frac{\cos(11dx+11c)}{11264d\,a^2}-\frac{\cos(10dx+10c)}{2560d\,a^2}+\frac{\cos(9dx+9c)}{9216d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1024d\,a^2}-\frac{\cos(6}{51}$

input
```
int(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/11*cos(d*x+c)^11-1/5*cos(d*x+c)^10-2/9*cos(d*x+c)^9+3/4*cos(d*x
+c)^8-cos(d*x+c)^6+2/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{180\cos(dx+c)^{11}-396\cos(dx+c)^{10}-440\cos(dx+c)^9+1485\cos(dx+c)^8-1980\cos(dx+c)^6+}{1980\,a^2d}$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 1
485*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*
x + c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{180\cos(dx+c)^{11} - 396\cos(dx+c)^{10} - 440\cos(dx+c)^{9} + 1485\cos(dx+c)^{8} - 1980\cos(dx+c)^{6} + }{1980\,a^2 d}$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 1
485*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*
x + c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{64\left(\frac{11(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{55(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{330(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{462(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{198(\cos(dx+c)+c)}{(\cos(dx+c)}\right.}{495\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}}$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-64/495*(11*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 55*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 165*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 3
30*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 462*(cos(d*x + c) - 1)^5/(c
os(d*x + c) + 1)^5 + 198*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 990*(
cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1)/(a^2*d*((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1)^11)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{\cos(c+dx)^3}{3a^2}-\frac{\cos(c+dx)^4}{2a^2}-\frac{2\cos(c+dx)^5}{5a^2}+\frac{\cos(c+dx)^6}{a^2}-\frac{3\cos(c+dx)^8}{4a^2}+\frac{2\cos(c+dx)^9}{9a^2}+\frac{\cos(c+dx)^{10}}{5a^2}-\frac{\cos(c+dx)^{11}}{11a^2}}{d}$$

input
```
int(sin(c + d*x)^11/(a + a/cos(c + d*x))^2,x)
```

output
```
-(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) - (2*cos(c + d*x)^5)/(5*
a^2) + cos(c + d*x)^6/a^2 - (3*cos(c + d*x)^8)/(4*a^2) + (2*cos(c + d*x)^9
)/(9*a^2) + cos(c + d*x)^10/(5*a^2) - cos(c + d*x)^11/(11*a^2))/d
```

3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.75.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(a-a\cos(c+dx))^5}{5a^7d} - \frac{2(a-a\cos(c+dx))^6}{a^8d} + \frac{13(a-a\cos(c+dx))^7}{7a^9d} - \frac{3(a-a\cos(c+dx))^8}{4a^{10}d} + \frac{(a-a\cos(c+dx))^9}{9a^{11}d}$$

output
```
4/5*(a-a*cos(d*x+c))^5/a^7/d-2*(a-a*cos(d*x+c))^6/a^8/d+13/7*(a-a*cos(d*x+
c))^7/a^9/d-3/4*(a-a*cos(d*x+c))^8/a^10/d+1/9*(a-a*cos(d*x+c))^9/a^11/d
```

3.75.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2(992+1615\cos(c+dx)+970\cos(2(c+dx))+385\cos(3(c+dx))+70\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+dx)\right)}{315a^2d}$$

input
```
Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*(992 + 1615*Cos[c + d*x] + 970*Cos[2*(c + d*x)] + 385*Cos[3*(c + d*x)]
+ 70*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a^2*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^9}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^9(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^9}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^2 d(a\cos(c+dx))}{a^9 d}$$

$$\downarrow 27$$

$$-\frac{\int a^2\cos^2(c+dx)(a-a\cos(c+dx))^4(\cos(c+dx)a+a)^2 d(a\cos(c+dx))}{a^{11}d}$$

$$\downarrow 99$$

$$-\frac{\int \left((a-a\cos(c+dx))^8-6a(a-a\cos(c+dx))^7+13a^2(a-a\cos(c+dx))^6-12a^3(a-a\cos(c+dx))^5+4a^4(a\right.}{a^{11}d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4}{5}a^4(a-a\cos(c+dx))^5+2a^3(a-a\cos(c+dx))^6-\frac{13}{7}a^2(a-a\cos(c+dx))^7-\frac{1}{9}(a-a\cos(c+dx))^9+\frac{3}{4}a(a}{a^{11}d}$$

input
```
Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

output
```
-(((-4*a^4*(a - a*Cos[c + d*x])^5)/5 + 2*a^3*(a - a*Cos[c + d*x])^6 - (13*
a^2*(a - a*Cos[c + d*x])^7)/7 + (3*a*(a - a*Cos[c + d*x])^8)/4 - (a - a*Co
s[c + d*x])^9/9)/(a^11*d))
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.75.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{4}+\frac{\cos(dx+c)^7}{7}-\frac{2\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{4}+\frac{\cos(dx+c)^7}{7}-\frac{2\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
parallelrisch	$\frac{-\frac{64}{315}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3}-\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{35}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{15}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{35}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9 a^2}$
norman	$\frac{-\frac{64}{315ad}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3ad}-\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{35da}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{35da}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{15da}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9 a}$
risch	$-\frac{13\cos(dx+c)}{128a^2d}-\frac{\cos(9dx+9c)}{2304d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1792d\,a^2}-\frac{\cos(6dx+6c)}{192d\,a^2}+\frac{\cos(5dx+5c)}{80d\,a^2}-\frac{\cos(4dx+$

input
```
int(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/9*cos(d*x+c)^9+1/4*cos(d*x+c)^8+1/7*cos(d*x+c)^7-2/3*cos(d*x+c
)^6+1/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{140\cos(dx+c)^9-315\cos(dx+c)^8-180\cos(dx+c)^7+840\cos(dx+c)^6-252\cos(dx+c)^5-63}{1260\,a^2d}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 84
0*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x +
 c)^3)/(a^2*d)
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{140\cos(dx+c)^9 - 315\cos(dx+c)^8 - 180\cos(dx+c)^7 + 840\cos(dx+c)^6 - 252\cos(dx+c)^5 - 63}{1260\,a^2 d}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 84
0*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x +
 c)^3)/(a^2*d)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{64\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{210(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-1\right)}{315\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-64/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 210*(cos(d*x + c) - 1)^6/(cos
(d*x + c) + 1)^6 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9
)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\cos(c+dx)^4}{2a^2}-\frac{\cos(c+dx)^3}{3a^2}+\frac{\cos(c+dx)^5}{5a^2}-\frac{2\cos(c+dx)^6}{3a^2}+\frac{\cos(c+dx)^7}{7a^2}+\frac{\cos(c+dx)^8}{4a^2}-\frac{\cos(c+dx)^9}{9a^2}}{d}$$

input
```
int(sin(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

output
```
(cos(c + d*x)^4/(2*a^2) - cos(c + d*x)^3/(3*a^2) + cos(c + d*x)^5/(5*a^2)
- (2*cos(c + d*x)^6)/(3*a^2) + cos(c + d*x)^7/(7*a^2) + cos(c + d*x)^8/(4*
a^2) - cos(c + d*x)^9/(9*a^2))/d
```

3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.76.1 Optimal result 635
3.76.2 Mathematica [A] (verified) 635
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3.76.8 Giac [B] (verification not implemented) 639
3.76.9 Mupad [B] (verification not implemented) 640

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)}{3a^2d} + \frac{\cos^4(c+dx)}{2a^2d} - \frac{\cos^6(c+dx)}{3a^2d} + \frac{\cos^7(c+dx)}{7a^2d}$$

output
```
-1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/3*cos(d*x+c)^6/a^2/d+1/7*
cos(d*x+c)^7/a^2/d
```

3.76.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(17\cos(c+dx)+10\cos(2(c+dx))+3(4+\cos(3(c+dx))))\sin^8\left(\frac{1}{2}(c+dx)\right)}{21a^2d}$$

input
```
Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

output
```
(4*(17*Cos[c + d*x] + 10*Cos[2*(c + d*x)] + 3*(4 + Cos[3*(c + d*x)]))*Sin[
(c + d*x)/2]^8)/(21*a^2*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^7(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^3(\cos(c+dx)a+a)d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 27$$

$$-\frac{\int a^2\cos^2(c+dx)(a-a\cos(c+dx))^3(\cos(c+dx)a+a)d(a\cos(c+dx))}{a^9 d}$$

$$\downarrow 84$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6+2\cos^5(c+dx)a^6-2\cos^3(c+dx)a^6+\cos^2(c+dx)a^6\right)d(a\cos(c+dx))}{a^9 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{1}{3}a^7\cos^6(c+dx)-\frac{1}{2}a^7\cos^4(c+dx)+\frac{1}{3}a^7\cos^3(c+dx)}{a^9 d}$$

input
```
Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

output
```
-(((a^7*Cos[c + d*x]^3)/3 - (a^7*Cos[c + d*x]^4)/2 + (a^7*Cos[c + d*x]^6)/
3 - (a^7*Cos[c + d*x]^7)/7)/(a^9*d))
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.76.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	49
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	49
parallelrisch	$\frac{126\cos(2dx+2c)-368-14\cos(6dx+6c)-231\cos(dx+c)-49\cos(3dx+3c)+21\cos(5dx+5c)+3\cos(7dx+7c)}{1344a^2d}$	74
risch	$-\frac{11\cos(dx+c)}{64a^2d}+\frac{\cos(7dx+7c)}{448d\,a^2}-\frac{\cos(6dx+6c)}{96d\,a^2}+\frac{\cos(5dx+5c)}{64d\,a^2}-\frac{7\cos(3dx+3c)}{192d\,a^2}+\frac{3\cos(2dx+2c)}{32d\,a^2}$	101
norman	$\frac{-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}-\frac{8}{21ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}-\frac{40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3da}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7 a}$	124

input
```
int(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/7*cos(d*x+c)^7-1/3*cos(d*x+c)^6+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)
^3)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\cos(dx+c)^7-14\cos(dx+c)^6+21\cos(dx+c)^4-14\cos(dx+c)^3}{42\,a^2d}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*
x + c)^3)/(a^2*d)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\cos(dx+c)^7 - 14\cos(dx+c)^6 + 21\cos(dx+c)^4 - 14\cos(dx+c)^3}{42\,a^2 d}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*
x + c)^3)/(a^2*d)
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{8\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{14(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{42(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{21\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-8/21*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 14*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 42*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\cos(c+dx)^3}{3a^2} - \frac{\cos(c+dx)^4}{2a^2} + \frac{\cos(c+dx)^6}{3a^2} - \frac{\cos(c+dx)^7}{7a^2}}{d}$$

input
```
int(sin(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

output
```
-(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) + cos(c + d*x)^6/(3*a^2)
 - cos(c + d*x)^7/(7*a^2))/d
```

3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.77.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)}{3a^2d} + \frac{\cos^4(c+dx)}{2a^2d} - \frac{\cos^5(c+dx)}{5a^2d}$$

output
```
-1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a^2/d
```

3.77.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(4+3\cos(c+dx)+3\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15a^2d}$$

input
```
Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
(4*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a^2*d
)
```

3.77.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^5(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^2 d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int a^2\cos^2(c+dx)(a-a\cos(c+dx))^2 d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\cos^4(c+dx)a^4-2\cos^3(c+dx)a^4+\cos^2(c+dx)a^4\right)d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)-\frac{1}{2}a^5\cos^4(c+dx)+\frac{1}{3}a^5\cos^3(c+dx)}{a^7 d}$$

input
```
Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-(((a^5*Cos[c + d*x]^3)/3 - (a^5*Cos[c + d*x]^4)/2 + (a^5*Cos[c + d*x]^5)/
5)/(a^7*d))
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.77.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	39
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	39
parallelrisch	$\frac{60\cos(2dx+2c)-203-35\cos(3dx+3c)-3\cos(5dx+5c)-90\cos(dx+c)+15\cos(4dx+4c)}{240a^2d}$	63
risch	$-\frac{3\cos(dx+c)}{8a^2d}-\frac{\cos(5dx+5c)}{80d\,a^2}+\frac{\cos(4dx+4c)}{16d\,a^2}-\frac{7\cos(3dx+3c)}{48d\,a^2}+\frac{\cos(2dx+2c)}{4d\,a^2}$	84
norman	$\frac{-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}-\frac{16}{15ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5 a}$	105

input
```
int(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\cos(dx+c)^5-15\cos(dx+c)^4+10\cos(dx+c)^3}{30\,a^2d}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\cos(dx+c)^5 - 15\cos(dx+c)^4 + 10\cos(dx+c)^3}{30\,a^2 d}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(49) = 98.

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{8\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{15(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{15(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 2\right)}{15\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^5}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-8/15*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 15*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2)/(a^2*d*((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1)^5)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)^3\left(6\cos(c+dx)^2 - 15\cos(c+dx) + 10\right)}{30\,a^2\,d}$$

input
```
int(sin(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

output
```
-(cos(c + d*x)^3*(6*cos(c + d*x)^2 - 15*cos(c + d*x) + 10))/(30*a^2*d)
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.78.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos(c+dx)}{a^2 d} - \frac{\cos^2(c+dx)}{a^2 d} + \frac{\cos^3(c+dx)}{3a^2 d} - \frac{2\log(1+\cos(c+dx))}{a^2 d}$$

output
```
2*cos(d*x+c)/a^2/d-cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a^2/d-2*ln(1+cos(d*
x+c))/a^2/d
```

3.78.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-22+27\cos(c+dx)-6\cos(2(c+dx))+\cos(3(c+dx))-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{12a^2 d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-22 + 27*Cos[c + d*x] - 6*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] - 48*Log[Co
s[(c + d*x)/2]])/(12*a^2*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3315, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^3(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3315$$

$$-\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))}{\cos(c+dx)a+a}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)(a-a\cos(c+dx))}{\cos(c+dx)a+a}d(a\cos(c+dx))}{a^5d}$$

$$\downarrow 86$$

$$-\frac{\int \left(\frac{2a^3}{\cos(c+dx)a+a}-\cos^2(c+dx)a^2+2\cos(c+dx)a^2-2a^2\right)d(a\cos(c+dx))}{a^5d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a^3\cos^2(c+dx)-2a^3\cos(c+dx)+2a^3\log(a\cos(c+dx)+a)}{a^5d}$$

input
```
Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((-2*a^3*Cos[c + d*x] + a^3*Cos[c + d*x]^2 - (a^3*Cos[c + d*x]^3)/3 + 2*a
^3*Log[a + a*Cos[c + d*x]])/(a^5*d))
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.78.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\cos(dx+c)^2+2\cos(dx+c)-2\ln(\cos(dx+c)+1)}{d\,a^2}$	48
default	$\frac{\frac{\cos(dx+c)^3}{3}-\cos(dx+c)^2+2\cos(dx+c)-2\ln(\cos(dx+c)+1)}{d\,a^2}$	48
parallelrisch	$\frac{24\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+34+\cos(3dx+3c)-6\cos(2dx+2c)+27\cos(dx+c)}{12a^2d}$	53
norman	$\frac{\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}+\frac{14}{3ad}+\frac{12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3 a}+\frac{2\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^2d}$	90
risch	$\frac{2ix}{a^2}+\frac{9\,e^{i(dx+c)}}{8a^2d}+\frac{9\,e^{-i(dx+c)}}{8a^2d}+\frac{4ic}{a^2d}-\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\cos(3dx+3c)}{12d\,a^2}-\frac{\cos(2dx+2c)}{2d\,a^2}$	107

input
```
int(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*cos(d*x+c)^3-cos(d*x+c)^2+2*cos(d*x+c)-2*ln(cos(d*x+c)+1))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos(dx+c)^3 - 3\cos(dx+c)^2 + 6\cos(dx+c) - 6\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{3\,a^2d}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c) - 6*log(1/2*cos(d*
x + c) + 1/2))/(a^2*d)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\cos(dx+c)^3-3\cos(dx+c)^2+6\cos(dx+c)}{a^2} - \frac{6\log(\cos(dx+c)+1)}{a^2}}{3\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*((cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c))/a^2 - 6*log(cos(
d*x + c) + 1)/a^2)/d
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\log\left(\left|-\cos(dx+c)-1\right|\right)}{a^2 d} + \frac{a^4d^2\cos(dx+c)^3 - 3\,a^4d^2\cos(dx+c)^2 + 6\,a^4d^2\cos(dx+c)}{3\,a^6d^3}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/3*(a^4*d^2*cos(d*x + c)^3 - 3*a
^4*d^2*cos(d*x + c)^2 + 6*a^4*d^2*cos(d*x + c))/(a^6*d^3)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\ln(\cos(c+dx)+1)}{a^2} - \frac{2\cos(c+dx)}{a^2} + \frac{\cos(c+dx)^2}{a^2} - \frac{\cos(c+dx)^3}{3a^2}}{d}$$

input
```
int(sin(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

output
```
-((2*log(cos(c + d*x) + 1))/a^2 - (2*cos(c + d*x))/a^2 + cos(c + d*x)^2/a^
2 - cos(c + d*x)^3/(3*a^2))/d
```

3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.79.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{a^2 d} + \frac{1}{d\left(a^2+a^2\cos(c+dx)\right)} + \frac{2\log(1+\cos(c+dx))}{a^2 d}$$

output
```
-cos(d*x+c)/a^2/d+1/d/(a^2+a^2*cos(d*x+c))+2*ln(1+cos(d*x+c))/a^2/d
```

3.79.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(-3+\cos(2(c+dx))-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^2 d}$$

input
```
Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/4*((-3 + Cos[2*(c + d*x)] - 8*Log[Cos[(c + d*x)/2]] - 8*Cos[c + d*x]*Lo
g[Cos[(c + d*x)/2]])*Sec[(c + d*x)/2]^2)/(a^2*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4360, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos^2(c+dx)}{(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)}{(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{a^2}{(\cos(c+dx)a+a)^2}-\frac{2a}{\cos(c+dx)a+a}+1\right)d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a^2}{a\cos(c+dx)+a}+a\cos(c+dx)-2a\log(a\cos(c+dx)+a)}{a^3d}$$

input
```
Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a*Cos[c + d*x] - a^2/(a + a*Cos[c + d*x]) - 2*a*Log[a + a*Cos[c + d*x]]
)/(a^3*d))
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.79.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{1-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-2\cos(dx+c)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2d}$	44
derivativedivides	$\frac{-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))}{d\,a^2}$	51
default	$\frac{-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))}{d\,a^2}$	51
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2da}-\frac{5}{2ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^2d}$	71
risch	$-\frac{2ix}{a^2}-\frac{e^{i(dx+c)}}{2a^2d}-\frac{e^{-i(dx+c)}}{2a^2d}-\frac{4ic}{a^2d}+\frac{2e^{i(dx+c)}}{a^2d\left(e^{i(dx+c)}+1\right)^2}+\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	103

input
```
int(sin(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(1-4*ln(sec(1/2*d*x+1/2*c)^2)-2*cos(d*x+c)+sec(1/2*d*x+1/2*c)^2)/a^2/d
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(dx+c)^2-2(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\cos(dx+c)-1}{a^2d\cos(dx+c)+a^2d}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(cos(d*x + c)^2 - 2*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + cos(
d*x + c) - 1)/(a^2*d*cos(d*x + c) + a^2*d)
```

3.79.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{1}{a^2\cos(dx+c)+a^2} - \frac{\cos(dx+c)}{a^2} + \frac{2\log(\cos(dx+c)+1)}{a^2}}{d}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(1/(a^2*cos(d*x + c) + a^2) - cos(d*x + c)/a^2 + 2*log(cos(d*x + c) + 1)/a
^2)/d
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(dx+c)}{a^2 d} + \frac{2\log(|-\cos(dx+c)-1|)}{a^2 d} + \frac{1}{a^2 d(\cos(dx+c)+1)}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a^2*d) + 2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/(a^2*d*(
cos(d*x + c) + 1))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^2\,d} - \frac{\cos\left(c+d\,x\right)^2-2}{a^2\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int(sin(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

output
```
(2*log(cos(c + d*x) + 1))/(a^2*d) - (cos(c + d*x)^2 - 2)/(a^2*d*(cos(c + d
*x) + 1))
```

3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.80.2 Mathematica [A] (verified) . . . 659
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3.80.7 Maxima [A] (verification not implemented) . . . 663
3.80.8 Giac [A] (verification not implemented) . . . 663
3.80.9 Mupad [B] (verification not implemented) . . . 664

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{4a^2d} + \frac{1}{4d(a+a\cos(c+dx))^2} - \frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}$$

output
```
-1/4*arctanh(cos(d*x+c))/a^2/d+1/4/d/(a+a*cos(d*x+c))^2-3/4/d/(a^2+a^2*cos
(d*x+c))
```

3.80.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(-1+6\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^2(c+}{4a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/4*((-1 + 6*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)
/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2
)
```

3.80.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4360, 3042, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3315$$

$$\frac{a\int \frac{\cos^2(c+dx)}{(a-a\cos(c+dx))(\cos(c+dx)a+a)^3}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^2\cos^2(c+dx)}{(a-a\cos(c+dx))(\cos(c+dx)a+a)^3}d(-a\cos(c+dx))}{ad}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{a}{2(\cos(c+dx)a+a)^3}+\frac{1}{4(a^2-a^2\cos^2(c+dx))}-\frac{3}{4(\cos(c+dx)a+a)^2}\right)d(-a\cos(c+dx))}{ad}$$

$$\downarrow 2009$$

$$\frac{-\frac{\operatorname{arctanh}(\cos(c+dx))}{4a}+\frac{a}{4(a\cos(c+dx)+a)^2}-\frac{3}{4(a\cos(c+dx)+a)}}{ad}$$

input
```
Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/4*ArcTanh[Cos[c + d*x]]/a + a/(4*(a + a*Cos[c + d*x])^2) - 3/(4*(a + a
*Cos[c + d*x])))/(a*d)
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.80.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^2d}$	46
derivativedivides	$\frac{\frac{1}{4(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{8}+\frac{\ln(\cos(dx+c)-1)}{8}}{da^2}$	55
default	$\frac{\frac{1}{4(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{8}+\frac{\ln(\cos(dx+c)-1)}{8}}{da^2}$	55
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}}{a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d}$	63
risch	$-\frac{3e^{3i(dx+c)}+4e^{2i(dx+c)}+3e^{i(dx+c)}}{2a^2d\left(e^{i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{4a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{4a^2d}$	97

input
```
int(csc(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(tan(1/2*d*x+1/2*c)^4-4*tan(1/2*d*x+1/2*c)^2+4*ln(tan(1/2*d*x+1/2*c))
)/a^2/d
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.77

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*((cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) -
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 6*cos
(d*x + c) + 4)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.80.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\,(3\cos(dx+c)+2)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2} + \frac{\log(\cos(dx+c)+1)}{a^2} - \frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/8*(2*(3*cos(d*x + c) + 2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^
2) + log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c) - 1)/a^2)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{\frac{4a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (4*a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^4)/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{3\cos(c+dx)}{4}+\frac{1}{2}}{d\left(a^2\cos\left(c+dx\right)^2+2a^2\cos\left(c+dx\right)+a^2\right)} - \frac{\operatorname{atanh}(\cos(c+dx))}{4a^2d}$$

input
```
int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

output
```
- ((3*cos(c + d*x))/4 + 1/2)/(d*(2*a^2*cos(c + d*x) + a^2 + a^2*cos(c + d*
x)^2)) - atanh(cos(c + d*x))/(4*a^2*d)
```

3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.81.2 Mathematica [A] (verified) . . . 665
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3.81.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}$$

output
```
1/6*(-a-2*a*cos(d*x+c))/d/(1-cos(d*x+c))/(a+a*cos(d*x+c))^3
```

3.81.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(1+2\cos(c+dx))\csc^2(c+dx)}{6a^2d(1+\cos(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6*((1 + 2*Cos[c + d*x])*Csc[c + d*x]^2)/(a^2*d*(1 + Cos[c + d*x])^2)
```

3.81.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4360, 3042, 3315, 27, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3315$$

$$\frac{a^3\int \frac{\cos^2(c+dx)}{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^4}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a\int \frac{a^2\cos^2(c+dx)}{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^4}d(-a\cos(c+dx))}{d}$$

$$\downarrow 91$$

$$-\frac{a(2a\cos(c+dx)+a)}{6d(a-a\cos(c+dx))(a\cos(c+dx)+a)^3}$$

input
```
Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6*(a*(a + 2*a*Cos[c + d*x]))/(d*(a - a*Cos[c + d*x])*(a + a*Cos[c + d*x
])^3)
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p + 
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.81.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-3-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{96d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	46
derivativedivides	$\frac{\frac{1}{12(\cos(dx+c)+1)^3}-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{1}{16\cos(dx+c)-16}}{d\,a^2}$	57
default	$\frac{\frac{1}{12(\cos(dx+c)+1)^3}-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{1}{16\cos(dx+c)-16}}{d\,a^2}$	57
norman	$\frac{-\frac{1}{32ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a}$	63
risch	$\frac{\frac{8\,e^{5i(dx+c)}}{3}+\frac{8\,e^{4i(dx+c)}}{3}+\frac{8\,e^{3i(dx+c)}}{3}}{a^2d\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^2}$	63

input
```
int(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*(tan(1/2*d*x+1/2*c)^8-3-6*tan(1/2*d*x+1/2*c)^4)/d/a^2/tan(1/2*d*x+1/2
*c)^2
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos(dx+c)+1}{6\left(a^2d\cos(dx+c)^4+2\,a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)-a^2d\right)}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*cos(d*x + c) + 1)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 -
2*a^2*d*cos(d*x + c) - a^2*d)
```

3.81.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos(dx+c)+1}{6\left(a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2\right)d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*cos(d*x + c) + 1)/((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a
^2*cos(d*x + c) - a^2)*d)
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)} + \frac{\frac{6a^4(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/96*(3*(cos(d*x + c) + 1)/(a^2*(cos(d*x + c) - 1)) + (6*a^4*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/
a^6)/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{\frac{\cos(c+dx)}{3}+\frac{1}{6}}{d\left(-a^2\cos(c+dx)^4-2a^2\cos(c+dx)^3+2a^2\cos(c+dx)+a^2\right)}$$

input
```
int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

output
```
-(cos(c + d*x)/3 + 1/6)/(d*(2*a^2*cos(c + d*x) + a^2 - 2*a^2*cos(c + d*x)^
3 - a^2*cos(c + d*x)^4))
```

3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.82.7 Maxima [A] (verification not implemented) . . . 675
3.82.8 Giac [A] (verification not implemented) . . . 676
3.82.9 Mupad [B] (verification not implemented) . . . 676

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} - \frac{1}{32d(a+a\cos(c+dx))^2} - \frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)} - \frac{1}{32d\left(a^2+a^2\cos(c+dx)\right)}$$

output
```
1/64*arctanh(cos(d*x+c))/a^2/d-1/64/d/(a-a*cos(d*x+c))^2+1/32*a^2/d/(a+a*c
os(d*x+c))^4-1/48*a/d/(a+a*cos(d*x+c))^3-1/32/d/(a+a*cos(d*x+c))^2-1/64/d/
(a^2-a^2*cos(d*x+c))-1/32/d/(a^2+a^2*cos(d*x+c))
```

3.82.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)+24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{384a^2d(1}$$

input
```
Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/384*(Cos[(c + d*x)/2]^4*(12*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 +
 24*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]
^2 + 12*Sec[(c + d*x)/2]^4 + 4*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*
Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.82.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3315$$

$$\frac{a^5\int \frac{\cos^2(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^5}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^3\int \frac{a^2\cos^2(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^5}d(-a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$\frac{a^3 \int \left(\frac{1}{8(\cos(c+dx)a+a)^5 a} - \frac{1}{16(\cos(c+dx)a+a)^4 a^2} + \frac{1}{32(a-a\cos(c+dx))^3 a^3} - \frac{1}{16(\cos(c+dx)a+a)^3 a^3} - \frac{1}{64(a^2-a^2\cos^2(c+dx))a^4} + \frac{1}{64(}}{d}$$

$$\downarrow 2009$$

$$\frac{a^3 \left(\frac{\operatorname{arctanh}(\cos(c+dx))}{64a^5} - \frac{1}{64a^4(a-a\cos(c+dx))} - \frac{1}{32a^4(a\cos(c+dx)+a)} - \frac{1}{64a^3(a-a\cos(c+dx))^2} - \frac{1}{32a^3(a\cos(c+dx)+a)^2} - \frac{1}{48a^2(a}}{d}$$

input
```
Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^3*(ArcTanh[Cos[c + d*x]]/(64*a^5) - 1/(64*a^3*(a - a*Cos[c + d*x])^2) -
 1/(64*a^4*(a - a*Cos[c + d*x])) + 1/(32*a*(a + a*Cos[c + d*x])^4) - 1/(48
*a^2*(a + a*Cos[c + d*x])^3) - 1/(32*a^3*(a + a*Cos[c + d*x])^2) - 1/(32*a
^4*(a + a*Cos[c + d*x]))))/d
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.82.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-6\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-24\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-24\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{1536a^2d}$
derivativedivides	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}+\frac{1}{64\cos(dx+c)-64}-\frac{\ln(\cos(dx+c)-1)}{128}+\frac{1}{32(\cos(dx+c)+1)^4}-\frac{1}{48(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{1}{32(\cos(}{d\,a^2}$
default	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}+\frac{1}{64\cos(dx+c)-64}-\frac{\ln(\cos(dx+c)-1)}{128}+\frac{1}{32(\cos(dx+c)+1)^4}-\frac{1}{48(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{1}{32(\cos(}{d\,a^2}$
norman	$\frac{-\frac{1}{256ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{512ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{64da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{256da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{192da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^2d}$
risch	$-\frac{3e^{11i(dx+c)}+12e^{10i(dx+c)}+7e^{9i(dx+c)}-32e^{8i(dx+c)}+566e^{7i(dx+c)}+424e^{6i(dx+c)}+566e^{5i(dx+c)}-32e^{4i(dx+c)}+7e}{96a^2d\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^4}$

input
```
int(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/1536*(3*tan(1/2*d*x+1/2*c)^8+8*tan(1/2*d*x+1/2*c)^6-6*cot(1/2*d*x+1/2*c)
^4-6*tan(1/2*d*x+1/2*c)^4-24*cot(1/2*d*x+1/2*c)^2-48*tan(1/2*d*x+1/2*c)^2-
24*ln(tan(1/2*d*x+1/2*c)))/a^2/d
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(134) = 268.

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.94

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{6\cos(dx+c)^5+12\cos(dx+c)^4-4\cos(dx+c)^3-20\cos(dx+c)^2-3\left(\cos(dx+c)^6+2\cos(dx+\right.}{}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/384*(6*cos(d*x + c)^5 + 12*cos(d*x + c)^4 - 4*cos(d*x + c)^3 - 20*cos(d
*x + c)^2 - 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(
d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1
/2) + 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x +
c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) +
 70*cos(d*x + c) + 32)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^
2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)
```

3.82.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(3\cos(dx+c)^5+6\cos(dx+c)^4-2\cos(dx+c)^3-10\cos(dx+c)^2+35\cos(dx+c)+16\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2} - \frac{3\log(\cos(dx+c)+1)}{a^2} + \frac{3\log(\cos}{384\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/384*(2*(3*cos(d*x + c)^5 + 6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 10*cos
(d*x + c)^2 + 35*cos(d*x + c) + 16)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d*x +
c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 + 2*
a^2*cos(d*x + c) + a^2) - 3*log(cos(d*x + c) + 1)/a^2 + 3*log(cos(d*x + c)
 - 1)/a^2)/d
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{6\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2} - \frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{\frac{48a^6(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{6a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{8a^6(\cos(dx+c}{(\cos(dx+c}}{a^8}}{1536\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1536*(6*(4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^2
) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (48*a^6*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^6*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 8*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(
d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\operatorname{atanh}(\cos(c+d\,x))}{64\,a^2\,d}$$
$$- \frac{\frac{\cos(c+d\,x)^5}{64} + \frac{\cos(c+d\,x)^4}{32} - \frac{\cos(c+d\,x)^3}{96} - \frac{5\cos(c+d\,x)^2}{96} + \frac{35\cos(c+d\,x)}{192} + \frac{1}{12}}{d\,\left(a^2\cos(c+d\,x)^6 + 2\,a^2\cos(c+d\,x)^5 - a^2\cos(c+d\,x)^4 - 4\,a^2\cos(c+d\,x)^3 - a^2\cos(c+d\,x)^2 + 2\,a}$$

input
```
int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

output
```
atanh(cos(c + d*x))/(64*a^2*d) - ((35*cos(c + d*x))/192 - (5*cos(c + d*x)^
2)/96 - cos(c + d*x)^3/96 + cos(c + d*x)^4/32 + cos(c + d*x)^5/64 + 1/12)/
(d*(2*a^2*cos(c + d*x) + a^2 - a^2*cos(c + d*x)^2 - 4*a^2*cos(c + d*x)^3 -
 a^2*cos(c + d*x)^4 + 2*a^2*cos(c + d*x)^5 + a^2*cos(c + d*x)^6))
```

3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.83.1 Optimal result . . . 677
3.83.2 Mathematica [A] (verified) . . . 677
3.83.3 Rubi [A] (verified) . . . 678
3.83.4 Maple [A] (verified) . . . 680
3.83.5 Fricas [A] (verification not implemented) . . . 680
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3.83.7 Maxima [B] (verification not implemented) . . . 681
3.83.8 Giac [A] (verification not implemented) . . . 682
3.83.9 Mupad [B] (verification not implemented) . . . 682

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{2\sin^7(c+dx)}{7a^2d}$$

output
```
11/128*x/a^2+11/128*cos(d*x+c)*sin(d*x+c)/a^2/d-7/64*cos(d*x+c)^3*sin(d*x+
c)/a^2/d-1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/6*cos(d*x+c)^3*sin(d*x+c)^3/
a^2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-2/5*sin(d*x+c)^5/a^2/d+2/7*sin(d
*x+c)^7/a^2/d
```

3.83.2 Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(9240dx - 10080\sin(c+dx) - 1680\sin(2(c+dx)) + 3360\sin(3(c+dx)) -}{26880}$$

input
```
Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(9240*d*x - 10080*Sin[c + d*x] - 1680*S
in[2*(c + d*x)] + 3360*Sin[3*(c + d*x)] - 2520*Sin[4*(c + d*x)] + 672*Sin[
5*(c + d*x)] + 560*Sin[6*(c + d*x)] - 480*Sin[7*(c + d*x)] + 105*Sin[8*(c
+ d*x)] + 980*Tan[c/2]))/(26880*a^2*d*(1 + Sec[c + d*x])^2)
```

3.83.3 Rubi [A] (verified)

Time $= 0.75$ (sec) , antiderivative size $= 171$, normalized size of antiderivative $= 1.02$, number of steps used $= 7$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used $= \{3042, 4360, 3042, 3354, 3042, 3352, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^8}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^8(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^8}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^2\sin^4(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \cos\left(c+dx+\frac{\pi}{2}\right)^4\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{\int \left(a^2\cos^4(c+dx)\sin^4(c+dx) - 2a^2\cos^3(c+dx)\sin^4(c+dx) + a^2\cos^2(c+dx)\sin^4(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^2\sin^7(c+dx)}{7d} - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d} - \frac{a^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{7a^2\sin(c+dx)\cos}{64d}}{a^4}$$

input
```
Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
((11*a^2*x)/128 + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) - (7*a^2*Cos[
c + d*x]^3*Sin[c + d*x])/(64*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(16*d)
 - (a^2*Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*d) - (a^2*Cos[c + d*x]^5*Sin[c +
 d*x]^3)/(8*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (2*a^2*Sin[c + d*x]^7)/(7*
d))/a^4
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.83.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{9240dx-10080\sin(dx+c)+105\sin(8dx+8c)+560\sin(6dx+6c)-2520\sin(4dx+4c)-1680\sin(2dx+2c)-480\sin(7dx+7c)}{107520a^2d}$
derivativedivides	$\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24576}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{122880}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{860160}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{860160}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{122880}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^2d}$
default	$\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24576}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{122880}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{860160}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{860160}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{122880}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^2d}$
risch	$\frac{11x}{128a^2}-\frac{3\sin(dx+c)}{32a^2d}+\frac{\sin(8dx+8c)}{1024a^2d}-\frac{\sin(7dx+7c)}{224a^2d}+\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{160a^2d}-\frac{3\sin(4dx+4c)}{128a^2d}+\frac{\sin(3}{3}$
norman	$\frac{11x}{128a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192ad}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{960ad}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6720ad}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6720ad}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{960ad}$

input
```
int(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/107520*(9240*d*x-10080*sin(d*x+c)+105*sin(8*d*x+8*c)+560*sin(6*d*x+6*c)-
2520*sin(4*d*x+4*c)-1680*sin(2*d*x+2*c)-480*sin(7*d*x+7*c)+672*sin(5*d*x+5
*c)+3360*sin(3*d*x+3*c))/a^2/d
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{1155\,dx + (1680\cos(dx+c)^7 - 3840\cos(dx+c)^6 - 280\cos(dx+c)^5 + 6144\cos(dx+c)^4 - 3710\cos}{13440\,a^2d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/13440*(1155*d*x + (1680*cos(d*x + c)^7 - 3840*cos(d*x + c)^6 - 280*cos(d
*x + c)^5 + 6144*cos(d*x + c)^4 - 3710*cos(d*x + c)^3 - 768*cos(d*x + c)^2
 + 1155*cos(d*x + c) - 1536)*sin(d*x + c))/(a^2*d)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(151) = 302$.

Time $= 0.29$ (sec) , antiderivative size $= 378$, normalized size of antiderivative $= 2.26$

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{a^2+\frac{8\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56\,a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28\,a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8\,a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)}{(\cos(dx+c)+1}}{6720\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 8855*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 29491*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 55583*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 - 31007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
+ 142541*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8855*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^2 + 8*a
^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x +
c)^8/(cos(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 28*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos
(d*x + c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{1155\,(dx+c)}{a^2} + \frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+8855\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-142541\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+31007\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-55583\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^8 a^2}}{13440\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 + 8855*tan(1
/2*d*x + 1/2*c)^13 - 142541*tan(1/2*d*x + 1/2*c)^11 + 31007*tan(1/2*d*x +
1/2*c)^9 - 55583*tan(1/2*d*x + 1/2*c)^7 - 29491*tan(1/2*d*x + 1/2*c)^5 - 8
855*tan(1/2*d*x + 1/2*c)^3 - 1155*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^8*a^2))/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{11\,x}{128\,a^2}$$
$$- \frac{-\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} - \frac{253\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192} + \frac{20363\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{960} - \frac{31007\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{6720} + \frac{55583\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{6720} + \frac{4213\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{}}{960}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}$$

input
```
int(sin(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

output
```
(11*x)/(128*a^2) - ((11*tan(c/2 + (d*x)/2))/64 + (253*tan(c/2 + (d*x)/2)^3
)/192 + (4213*tan(c/2 + (d*x)/2)^5)/960 + (55583*tan(c/2 + (d*x)/2)^7)/672
0 - (31007*tan(c/2 + (d*x)/2)^9)/6720 + (20363*tan(c/2 + (d*x)/2)^11)/960
- (253*tan(c/2 + (d*x)/2)^13)/192 - (11*tan(c/2 + (d*x)/2)^15)/64)/(a^2*d*
(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.84.1 Optimal result 684
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3.84.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d}$$

output
```
3/16*x/a^2-3/16*cos(d*x+c)*sin(d*x+c)/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)^3/a^
2/d-1/6*(a-a*cos(d*x+c))^3*sin(d*x+c)^3/a^5/d-1/10*sin(d*x+c)^5/a^2/d
```

3.84.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(360dx - 480\sin(c+dx) + 30\sin(2(c+dx)) + 80\sin(3(c+dx)) - 90\sin(4\right.}{480a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(360*d*x - 480*Sin[c + d*x] + 30*Sin[2*
(c + d*x)] + 80*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x
)] - 10*Sin[6*(c + d*x)] + 25*Tan[c/2]))/(480*a^2*d*(1 + Sec[c + d*x])^2)
```

3.84.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 3042, 3354, 3042, 3349, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^6(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^2\sin^2(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \cos\left(c+dx+\frac{\pi}{2}\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx}{a^4}$$

$$\downarrow 3349$$

$$\frac{\frac{1}{2}a\int(a-a\cos(c+dx))\sin^4(c+dx)dx-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}a\int\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)dx-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3148$$

$$\frac{\frac{1}{2}a\left(a\int\sin^4(c+dx)dx-\frac{a\sin^5(c+dx)}{5d}\right)-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}a\left(a\int \sin(c+dx)^4dx - \frac{a\sin^5(c+dx)}{5d}\right) - \frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3115$$

$$\frac{\frac{1}{2}a\left(a\left(\frac{3}{4}\int \sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{a\sin^5(c+dx)}{5d}\right) - \frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}a\left(a\left(\frac{3}{4}\int \sin(c+dx)^2dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{a\sin^5(c+dx)}{5d}\right) - \frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 3115$$

$$\frac{\frac{1}{2}a\left(a\left(\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{a\sin^5(c+dx)}{5d}\right) - \frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

$$\downarrow 24$$

$$\frac{\frac{1}{2}a\left(a\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{a\sin^5(c+dx)}{5d}\right) - \frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6ad}}{a^4}$$

input
```
Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/6*((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(a*d) + (a*(-1/5*(a*Sin[c +
d*x]^5)/d + a*(-1/4*(Cos[c + d*x]*Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d
*x]*Sin[c + d*x])/(2*d)))/4)))/2)/a^4
```

3.84.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3349
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Simp[a/(2*g^
2)   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; F
reeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.84.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{180dx-240\sin(dx+c)-5\sin(6dx+6c)-45\sin(4dx+4c)+15\sin(2dx+2c)+24\sin(5dx+5c)+40\sin(3dx+3c)}{960a^2d}$
risch	$\frac{3x}{16a^2}-\frac{\sin(dx+c)}{4a^2d}-\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{40a^2d}-\frac{3\sin(4dx+4c)}{64a^2d}+\frac{\sin(3dx+3c)}{24a^2d}+\frac{\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{32\left(\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{256}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{768}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{640}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{640}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}+\frac{3\arctan(\tan}{a^2d}$
default	$\frac{32\left(\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{256}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{768}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{640}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{640}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}+\frac{3\arctan(\tan}{a^2d}$
norman	$\frac{\frac{3x}{16a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8ad}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}+\frac{9x\tan\left(\frac{dx}{2}\right.}{8a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}$

input
```
int(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/960*(180*d*x-240*sin(d*x+c)-5*sin(6*d*x+6*c)-45*sin(4*d*x+4*c)+15*sin(2*
d*x+2*c)+24*sin(5*d*x+5*c)+40*sin(3*d*x+3*c))/a^2/d
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{45\,dx-\left(40\cos(dx+c)^5-96\cos(dx+c)^4+50\cos(dx+c)^3+32\cos(dx+c)^2-45\cos(dx+c)+64\right.}{240\,a^2d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(45*d*x - (40*cos(d*x + c)^5 - 96*cos(d*x + c)^4 + 50*cos(d*x + c)^3
 + 32*cos(d*x + c)^2 - 45*cos(d*x + c) + 64)*sin(d*x + c))/(a^2*d)
```

3.84.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.

Time $= 0.29$ (sec) , antiderivative size $= 292$, normalized size of antiderivative $= 2.81$

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{594\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{174\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1025\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} - \frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 594*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 174*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 1025*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6
*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x
+ c) + 1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{45\,(dx+c)}{a^2} + \frac{2\left(45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 1025\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 174\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 594\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 255\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 45\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6 a^2}}{240\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 1025*tan(1/2*d*x
 + 1/2*c)^9 - 174*tan(1/2*d*x + 1/2*c)^7 - 594*tan(1/2*d*x + 1/2*c)^5 - 25
5*tan(1/2*d*x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 + 1)^6*a^2))/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{3\,x}{16\,a^2}$$
$$- \frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{205\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8} + \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(sin(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

output
```
(3*x)/(16*a^2) - ((3*tan(c/2 + (d*x)/2))/8 + (17*tan(c/2 + (d*x)/2)^3)/8 +
 (99*tan(c/2 + (d*x)/2)^5)/20 + (29*tan(c/2 + (d*x)/2)^7)/20 + (205*tan(c/
2 + (d*x)/2)^9)/24 - (3*tan(c/2 + (d*x)/2)^11)/8)/(a^2*d*(tan(c/2 + (d*x)/
2)^2 + 1)^6)
```

3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.85.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7x}{8a^2} - \frac{2\sin(c+dx)}{a^2 d} + \frac{7\cos(c+dx)\sin(c+dx)}{8a^2 d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2 d} + \frac{2\sin^3(c+dx)}{3a^2 d}$$

output
```
7/8*x/a^2-2*sin(d*x+c)/a^2/d+7/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c
)^3*sin(d*x+c)/a^2/d+2/3*sin(d*x+c)^3/a^2/d
```

3.85.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(84dx - 144\sin(c+dx) + 48\sin(2(c+dx)) - 16\sin(3(c+dx)) + 3\sin(4(c\right.}{24a^2 d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(84*d*x - 144*Sin[c + d*x] + 48*Sin[2*(
c + d*x)] - 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)] + 2*Tan[c/2]))/(24*a^
2*d*(1 + Sec[c + d*x])^2)
```

3.85.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3348, 3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^4(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3348$$

$$\frac{\int \cos^2(c+dx)(a-a\cos(c+dx))^2dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2dx}{a^4}$$

$$\downarrow 3236$$

$$\frac{\int \left(a^2\cos^4(c+dx)-2a^2\cos^3(c+dx)+a^2\cos^2(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}}{a^4}$$

input
```
Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
((7*a^2*x)/8 - (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (2*a^2*Sin[c + d*x]^3)/(
3*d))/a^4
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

rule 3348
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a^(2*m)   Int[(d*
Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.85.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{84dx-144\sin(dx+c)-16\sin(3dx+3c)+3\sin(4dx+4c)+48\sin(2dx+2c)}{96a^2d}$
risch	$\frac{7x}{8a^2}-\frac{3\sin(dx+c)}{2a^2d}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{\sin(3dx+3c)}{6a^2d}+\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{8\left(-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{96}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
default	$\frac{8\left(-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{96}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
norman	$\frac{\frac{7x}{8a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12ad}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{21x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4 a}$

input
```
int(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*(84*d*x-144*sin(d*x+c)-16*sin(3*d*x+3*c)+3*sin(4*d*x+4*c)+48*sin(2*d*
x+2*c))/a^2/d
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{21\,dx+\left(6\cos(dx+c)^3-16\cos(dx+c)^2+21\cos(dx+c)-32\right)\sin(dx+c)}{24\,a^2d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(21*d*x + (6*cos(d*x + c)^3 - 16*cos(d*x + c)^2 + 21*cos(d*x + c) - 3
2)*sin(d*x + c))/(a^2*d)
```

3.85.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.37

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{83\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 83*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^
2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin(d*x + c
)/(cos(d*x + c) + 1))/a^2)/d
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21\,(dx+c)}{a^2} - \frac{2\left(75\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+83\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^2}}{24\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(21*(d*x + c)/a^2 - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 83*tan(1/2*d*x + 1
/2*c)^5 + 77*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^4*a^2))/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 17.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7\,x}{8\,a^2} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} + \frac{83\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12} + \frac{77\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12} + \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4}$$

input
```
int(sin(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

output
```
(7*x)/(8*a^2) - ((7*tan(c/2 + (d*x)/2))/4 + (77*tan(c/2 + (d*x)/2)^3)/12 +
 (83*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d*x)/2)^7)/4)/(a^2*d*(tan(c
/2 + (d*x)/2)^2 + 1)^4)
```

3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.86.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5x}{2a^2} + \frac{2\sin(c+dx)}{a^2 d} - \frac{\cos(c+dx)\sin(c+dx)}{2a^2 d} + \frac{2\sin(c+dx)}{a^2 d(1+\cos(c+dx))}$$

output
```
-5/2*x/a^2+2*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d+2*sin(d*x+c)
/a^2/d/(1+cos(d*x+c))
```

3.86.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(60dx\cos\left(\frac{dx}{2}\right)+60dx\cos\left(c+\frac{dx}{2}\right)-119\sin\left(\frac{dx}{2}\right)-25\sin\left(c+\frac{dx}{2}\right)-21\sin\left(c+\right.\right.}{48a^2 d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/48*(Sec[c/2]*Sec[(c + d*x)/2]*(60*d*x*Cos[(d*x)/2] + 60*d*x*Cos[c + (d*
x)/2] - 119*Sin[(d*x)/2] - 25*Sin[c + (d*x)/2] - 21*Sin[c + (3*d*x)/2] - 2
1*Sin[2*c + (3*d*x)/2] + 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))
/(a^2*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 3042, 3353, 3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^2(c+dx)\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3353$$

$$\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))}{\cos(c+dx)a+a}dx}{a^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}$$

$$\downarrow 3429$$

$$\frac{\int (a-a\cos(c+dx))^2\cot^2(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^2 dx}{a^4}$$

$$\downarrow 3188$$

$$\frac{\int \left(-\cos^2(c+dx)+2\cos(c+dx)+\frac{2}{\cos(c+dx)+1}-2\right)dx}{a^2}$$

$$\downarrow 2009$$

$$\frac{\frac{2\sin(c+dx)}{d}-\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{2\sin(c+dx)}{d(\cos(c+dx)+1)}-\frac{5x}{2}}{a^2}$$

input
```
Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
((-5*x)/2 + (2*Sin[c + d*x])/d - (Cos[c + d*x]*Sin[c + d*x])/(2*d) + (2*Si
n[c + d*x])/(d*(1 + Cos[c + d*x])))/a^2
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3353
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/b^2   Int[(d*Sin[e
 + f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[
n, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.86.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{(15-\cos(2dx+2c)+6\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-10dx}{4a^2d}$	45
derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73
risch	$-\frac{5x}{2a^2}-\frac{ie^{i(dx+c)}}{a^2d}+\frac{ie^{-i(dx+c)}}{a^2d}+\frac{4i}{a^2d\left(e^{i(dx+c)}+1\right)}-\frac{\sin(2dx+2c)}{4a^2d}$	83
norman	$\frac{-\frac{5x}{2a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a}$	116

input
```
int(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((15-cos(2*d*x+2*c)+6*cos(d*x+c))*tan(1/2*d*x+1/2*c)-10*d*x)/a^2/d
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{5\,dx\cos(dx+c)+5\,dx+\left(\cos(dx+c)^2-3\cos(dx+c)-8\right)\sin(dx+c)}{2\left(a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(5*d*x*cos(d*x + c) + 5*d*x + (cos(d*x + c)^2 - 3*cos(d*x + c) - 8)*s
in(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

3.86.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time $= 0.28$ (sec) , antiderivative size $= 140$, normalized size of antiderivative $= 2.03$

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{5\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{2\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
((3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 2*s
in(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{5\,(dx+c)}{a^2} - \frac{4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} - \frac{2\left(5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(5*(d*x + c)/a^2 - 4*tan(1/2*d*x + 1/2*c)/a^2 - 2*(5*tan(1/2*d*x + 1/
2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.86.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(c+dx) + 10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(sin(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

output
```
(4*sin(c/2 + (d*x)/2) - 5*cos(c/2 + (d*x)/2)*(c + d*x) + 10*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2) - 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(2*
a^2*d*cos(c/2 + (d*x)/2))
```

3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.87.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

output
```
-1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*
csc(d*x+c)^5/a^2/d
```

3.87.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\csc(c)\csc(c+dx)\sec^2(c+dx)(-80\sin(c)+80\sin(dx)+55\sin(c+dx)+44\sin(2(c+dx))+11\sin(3(c+$$
$$240a^2d(1+\sec(c+dx))^2$$

input
```
Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Csc[c]*Csc[c + d*x]*Sec[c + d*x]^2*(-80*Sin[c] + 80*Sin[d*x] + 55*Sin[c +
 d*x] + 44*Sin[2*(c + d*x)] + 11*Sin[3*(c + d*x)] - 60*Sin[2*c + d*x] + 16
*Sin[c + 2*d*x] + 4*Sin[2*c + 3*d*x]))/(240*a^2*d*(1 + Sec[c + d*x])^2)
```

3.87.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4359, 3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4359$$

$$\int \frac{\cot^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\tan\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3190$$

$$\frac{\int \left(a^2\csc^2(c+dx)\cot^4(c+dx)-2a^2\csc^3(c+dx)\cot^3(c+dx)+a^2\csc^4(c+dx)\cot^2(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}}{a^4}$$

input
```
Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/3*(a^2*Cot[c + d*x]^3)/d - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c
 + d*x]^3)/(3*d) + (2*a^2*Csc[c + d*x]^5)/(5*d))/a^4
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.87.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{120a^2d}$	58
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8da^2}$	60
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8da^2}$	60
norman	$\frac{-\frac{1}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{24da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{40da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}$	82
risch	$-\frac{2i\left(15\,e^{4i(dx+c)}+20\,e^{3i(dx+c)}+20\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+1\right)}{15a^2d\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)}$	82

input
```
int(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/120*(3*tan(1/2*d*x+1/2*c)^5-5*tan(1/2*d*x+1/2*c)^3-15*cot(1/2*d*x+1/2*c)
-15*tan(1/2*d*x+1/2*c))/a^2/d
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(dx+c)^3+2\cos(dx+c)^2+8\cos(dx+c)+4}{15\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + 8*cos(d*x + c) + 4)/((a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

3.87.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{120\,d} + \frac{15\left(\cos(dx+c)+1\right)}{a^2\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 + 15*(cos(d*x + c)
+ 1)/(a^2*sin(d*x + c)))/d
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/120*(15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 5*
a^8*tan(1/2*d*x + 1/2*c)^3 - 15*a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 14\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 3}{120\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

output
```
-(14*cos(c/2 + (d*x)/2)^2 - 4*cos(c/2 + (d*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^
6 - 3)/(120*a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))
```

3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.88.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5a^2d} - \frac{2\cot^7(c+dx)}{7a^2d} - \frac{2\csc^5(c+dx)}{5a^2d} + \frac{2\csc^7(c+dx)}{7a^2d}$$

output
```
-1/3*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a^2/d-2/7*cot(d*x+c)^7/a^2/d-2/5*
csc(d*x+c)^5/a^2/d+2/7*csc(d*x+c)^7/a^2/d
```

3.88.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.64

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\csc(c)\csc^3(c+dx)\sec^2(c+dx)(1344\sin(c) - 1456\sin(dx) - 714\sin(c+dx) - 408\sin(2(c+dx)) + 15$$

input
```
Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/13440*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]^2*(1344*Sin[c] - 1456*Sin[d*x
] - 714*Sin[c + d*x] - 408*Sin[2*(c + d*x)] + 153*Sin[3*(c + d*x)] + 204*S
in[4*(c + d*x)] + 51*Sin[5*(c + d*x)] + 1680*Sin[2*c + d*x] + 128*Sin[c +
2*d*x] - 48*Sin[2*c + 3*d*x] - 64*Sin[3*c + 4*d*x] - 16*Sin[4*c + 5*d*x]))
/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.88.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\int (a-a\cos(c+dx))^2\cot^2(c+dx)\csc^6(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^8}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{\int \left(a^2\cot^2(c+dx)\csc^6(c+dx)-2a^2\cot^3(c+dx)\csc^5(c+dx)+a^2\cot^4(c+dx)\csc^4(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{3a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^5(c+dx)}{5d}}{a^4}$$

input
```
Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/3*(a^2*Cot[c + d*x]^3)/d - (3*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c
 + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Csc[c + d*x]^7)/(
7*d))/a^4
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.88.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-70\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3-210\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-105\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{3360a^2d}$	84
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{32da^2}$	86
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{32da^2}$	86
risch	$\frac{4i\left(105\,e^{6i(dx+c)}+84\,e^{5i(dx+c)}+91\,e^{4i(dx+c)}-8\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+1\right)}{105a^2d\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^3}$	104
norman	$\frac{-\frac{1}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{48da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{160da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{224da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}$	120

input
```
int(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3360*(15*tan(1/2*d*x+1/2*c)^7+21*tan(1/2*d*x+1/2*c)^5-70*tan(1/2*d*x+1/2
*c)^3-35*cot(1/2*d*x+1/2*c)^3-210*tan(1/2*d*x+1/2*c)-105*cot(1/2*d*x+1/2*c
))/a^2/d
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\cos(dx+c)^5+4\cos(dx+c)^4-\cos(dx+c)^3-6\cos(dx+c)^2+24\cos(dx+c)+12}{105\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(2*cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 6*cos(d*x +
c)^2 + 24*cos(d*x + c) + 12)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)
^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

3.88.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{210\sin(dx+c)}{\cos(dx+c)+1}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}+\frac{35\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3360*((210*sin(d*x + c)/(cos(d*x + c) + 1) + 70*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^2 + 35*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)
*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{15a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-70a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-210a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{14}}}{3360\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3360*(35*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) -
(15*a^12*tan(1/2*d*x + 1/2*c)^7 + 21*a^12*tan(1/2*d*x + 1/2*c)^5 - 70*a^12
*tan(1/2*d*x + 1/2*c)^3 - 210*a^12*tan(1/2*d*x + 1/2*c))/a^14)/d
```

3.88.9 Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 96\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} + 54\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} - 15}{3360\,a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

output
```
-(54*cos(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^4 + 24*cos(c/2 + (d*x)/2)
^6 - 96*cos(c/2 + (d*x)/2)^8 + 64*cos(c/2 + (d*x)/2)^10 - 15)/(3360*a^2*d*
(cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)^9*sin(c/2 +
(d*x)/2)))
```

3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.89.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{4\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{2\cot^9(c+dx)}{9a^2d} - \frac{2\csc^7(c+dx)}{7a^2d} + \frac{2\csc^9(c+dx)}{9a^2d}$$

output
```
-1/3*cot(d*x+c)^3/a^2/d-4/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2/d-2/9*
cot(d*x+c)^9/a^2/d-2/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2/d
```

3.89.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.75

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\csc(c)\csc^5(c+dx)\sec^2(c+dx)(-61440\sin(c)+84480\sin(dx)+25875\sin(c+dx)+11500\sin(2(c+dx)$$

input
```
Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^2*(-61440*Sin[c] + 84480*Sin[d*x] + 25
875*Sin[c + d*x] + 11500*Sin[2*(c + d*x)] - 10925*Sin[3*(c + d*x)] - 9200*
Sin[4*(c + d*x)] + 575*Sin[5*(c + d*x)] + 2300*Sin[6*(c + d*x)] + 575*Sin[
7*(c + d*x)] - 107520*Sin[2*c + d*x] - 10240*Sin[c + 2*d*x] + 9728*Sin[2*c
 + 3*d*x] + 8192*Sin[3*c + 4*d*x] - 512*Sin[4*c + 5*d*x] - 2048*Sin[5*c +
6*d*x] - 512*Sin[6*c + 7*d*x]))/(1290240*a^2*d*(1 + Sec[c + d*x])^2)
```

3.89.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\int (a-a\cos(c+dx))^2\cot^2(c+dx)\csc^8(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{\int \left(a^2 \cot^2(c+dx)\csc^8(c+dx) - 2a^2\cot^3(c+dx)\csc^7(c+dx) + a^2\cot^4(c+dx)\csc^6(c+dx)\right)dx}{a^4}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{2a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc^9(c+dx)}{9d} - \frac{2a^2\csc^7(c+dx)}{7d}}{a^4}$$

input
```
Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/3*(a^2*Cot[c + d*x]^3)/d - (4*a^2*Cot[c + d*x]^5)/(5*d) - (5*a^2*Cot[c
 + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x]^7)/(
7*d) + (2*a^2*Csc[c + d*x]^9)/(9*d))/a^4
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.89.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+135\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-63\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-525\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-315\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3-1575\,t}{40320a^2d}$
derivativedivides	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-5\tan\left(\frac{dx}{2}\right.}{128da^2}$
default	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-5\tan\left(\frac{dx}{2}\right.}{128da^2}$
risch	$-\frac{16i\left(210\,e^{8i(dx+c)}+120\,e^{7i(dx+c)}+165\,e^{6i(dx+c)}-20\,e^{5i(dx+c)}+19\,e^{4i(dx+c)}+16\,e^{3i(dx+c)}-e^{2i(dx+c)}-4\,e^{i(dx+c)}-1\right)}{315a^2d\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{640ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{896ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1152ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{128da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{128da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{384da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{640da}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

input
```
int(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/40320*(35*tan(1/2*d*x+1/2*c)^9+135*tan(1/2*d*x+1/2*c)^7+63*tan(1/2*d*x+1
/2*c)^5-63*cot(1/2*d*x+1/2*c)^5-525*tan(1/2*d*x+1/2*c)^3-315*cot(1/2*d*x+1
/2*c)^3-1575*tan(1/2*d*x+1/2*c)-315*cot(1/2*d*x+1/2*c))/a^2/d
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{8\cos(dx+c)^7+16\cos(dx+c)^6-12\cos(dx+c)^5-40\cos(dx+c)^4-5\cos(dx+c)^3+30\cos(}{315\left(a^2d\cos(dx+c)^6+2a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^3-a^2d\cos(dx+c)^2+\right.}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/315*(8*cos(d*x + c)^7 + 16*cos(d*x + c)^6 - 12*cos(d*x + c)^5 - 40*cos(d
*x + c)^4 - 5*cos(d*x + c)^3 + 30*cos(d*x + c)^2 - 40*cos(d*x + c) - 20)/(
(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*
a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)*sin(d*x + c))
```

3.89.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.60

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{1575\sin(dx+c)}{\cos(dx+c)+1}+\frac{525\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{135\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2} + \frac{63\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)(\cos(dx+c)+1}{a^2\sin(dx+c)^5}}{40320\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/40320*((1575*sin(d*x + c)/(cos(d*x + c) + 1) + 525*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 135*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 + 6
3*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{63\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} - \frac{35a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+135a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-525a^{16}\tan\left(\frac{1}{2}\right.}{a^{18}}}{40320\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/40320*(63*(5*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^
2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x + 1/2*c)^9 + 135*a^16*tan
(1/2*d*x + 1/2*c)^7 + 63*a^16*tan(1/2*d*x + 1/2*c)^5 - 525*a^16*tan(1/2*d*
x + 1/2*c)^3 - 1575*a^16*tan(1/2*d*x + 1/2*c))/a^18)/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{375\cos(c+dx)}{8} - \frac{5\cos(2c+2dx)}{2} + \frac{19\cos(3c+3dx)}{8} + 2\cos(4c+4dx) - \frac{\cos(5c+5dx)}{8} - \frac{\cos(6c+6dx)}{2} - \frac{\cos(7c+7dx)}{8}}{40320\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

input
```
int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^2),x)
```

output
```
-((375*cos(c + d*x))/8 - (5*cos(2*c + 2*d*x))/2 + (19*cos(3*c + 3*d*x))/8
+ 2*cos(4*c + 4*d*x) - cos(5*c + 5*d*x)/8 - cos(6*c + 6*d*x)/2 - cos(7*c +
 7*d*x)/8 + 15)/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)
```

3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.90.1 Optimal result 720
3.90.2 Mathematica [A] (verified) 720
3.90.3 Rubi [A] (verified) 721
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3.90.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{a^2d} - \frac{9\cot^7(c+dx)}{7a^2d} - \frac{7\cot^9(c+dx)}{9a^2d} - \frac{2\cot^{11}(c+dx)}{11a^2d} - \frac{2\csc^9(c+dx)}{9a^2d} + \frac{2\csc^{11}(c+dx)}{11a^2d}$$

output
```
-1/3*cot(d*x+c)^3/a^2/d-cot(d*x+c)^5/a^2/d-9/7*cot(d*x+c)^7/a^2/d-7/9*cot(
d*x+c)^9/a^2/d-2/11*cot(d*x+c)^11/a^2/d-2/9*csc(d*x+c)^9/a^2/d+2/11*csc(d*
x+c)^11/a^2/d
```

3.90.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.86

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\csc(c)\csc^7(c+dx)\sec^2(c+dx)(630784\sin(c) - 1103872\sin(dx) - 218834\sin(c+dx) - 79576\sin(2(c$$

input
```
Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/22708224*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^2*(630784*Sin[c] - 1103872
*Sin[d*x] - 218834*Sin[c + d*x] - 79576*Sin[2*(c + d*x)] + 119364*Sin[3*(c
 + d*x)] + 79576*Sin[4*(c + d*x)] - 28420*Sin[5*(c + d*x)] - 34104*Sin[6*(
c + d*x)] - 1421*Sin[7*(c + d*x)] + 5684*Sin[8*(c + d*x)] + 1421*Sin[9*(c
+ d*x)] + 1419264*Sin[2*c + d*x] + 114688*Sin[c + 2*d*x] - 172032*Sin[2*c
+ 3*d*x] - 114688*Sin[3*c + 4*d*x] + 40960*Sin[4*c + 5*d*x] + 49152*Sin[5*
c + 6*d*x] + 2048*Sin[6*c + 7*d*x] - 8192*Sin[7*c + 8*d*x] - 2048*Sin[8*c
+ 9*d*x]))/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^8(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc^6(c+dx)}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\int (a-a\cos(c+dx))^2\cot^2(c+dx)\csc^{10}(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{12}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{\int \left(a^2 \cot^2(c+dx)\csc^{10}(c+dx) - 2a^2\cot^3(c+dx)\csc^9(c+dx) + a^2\cot^4(c+dx)\csc^8(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^{11}(c+dx)}{11d} - \frac{7a^2\cot^9(c+dx)}{9d} - \frac{9a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^5(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc^{11}(c+dx)}{11d} - \frac{2a^2\csc^9(c+dx)}{9d}}{a^4}$$

input
```
Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-1/3*(a^2*Cot[c + d*x]^3)/d - (a^2*Cot[c + d*x]^5)/d - (9*a^2*Cot[c + d*x
]^7)/(7*d) - (7*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Cot[c + d*x]^11)/(11*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (2*a^2*Csc[c + d*x]^11)/(11*d))/a^4
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.90.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan}$ / $512da^2$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan}$ / $512da^2$
parallelrisch	$\frac{63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+385\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+792\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-99\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-693\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-3234\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-18}{354816a^2d}$
risch	$\frac{32i\left(693\,\mathrm{e}^{10i(dx+c)}+308\,\mathrm{e}^{9i(dx+c)}+539\,\mathrm{e}^{8i(dx+c)}-56\,\mathrm{e}^{7i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+56\,\mathrm{e}^{5i(dx+c)}-20\,\mathrm{e}^{4i(dx+c)}-24\,\mathrm{e}^{3i(dx+c)}-\right.}{693a^2d\left(\mathrm{e}^{i(dx+c)}+1\right)^{11}\left(\mathrm{e}^{i(dx+c)}-1\right)^{7}}$

input
```
int(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/512/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1
/2*d*x+1/2*c)^7-14/3*tan(1/2*d*x+1/2*c)^3-14*tan(1/2*d*x+1/2*c)-1/7/tan(1/
2*d*x+1/2*c)^7-1/tan(1/2*d*x+1/2*c)^5-8/3/tan(1/2*d*x+1/2*c)^3)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.63

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{16\cos(dx+c)^9+32\cos(dx+c)^8-40\cos(dx+c)^7-112\cos(dx+c)^6+14\cos(dx+c)^5+140\cos(}{693\left(a^2d\cos(dx+c)^8+2a^2d\cos(dx+c)^7-2a^2d\cos(dx+c)^6-6a^2d\cos(dx+c)^5+6a^2d\cos(dx+\right.}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/693*(16*cos(d*x + c)^9 + 32*cos(d*x + c)^8 - 40*cos(d*x + c)^7 - 112*cos
(d*x + c)^6 + 14*cos(d*x + c)^5 + 140*cos(d*x + c)^4 + 35*cos(d*x + c)^3 -
 70*cos(d*x + c)^2 + 56*cos(d*x + c) + 28)/((a^2*d*cos(d*x + c)^8 + 2*a^2*
d*cos(d*x + c)^7 - 2*a^2*d*cos(d*x + c)^6 - 6*a^2*d*cos(d*x + c)^5 + 6*a^2
*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - 2*a^2*d*cos(d*x + c) - a^2*d)
*sin(d*x + c))
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{9702\sin(dx+c)}{\cos(dx+c)+1}+\frac{3234\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{792\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2} + \frac{33\left(\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{56\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)-}{a^2\sin(dx+c)^7}$$
$$354816\,d$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/354816*((9702*sin(d*x + c)/(cos(d*x + c) + 1) + 3234*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - 792*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x
+ c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^
2 + 33*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 56*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{33\left(56\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7} - \frac{63a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+385a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+792a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3234a^{20}t}{a^{22}}}{354816\,d}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/354816*(33*(56*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^2 + 3)/
(a^2*tan(1/2*d*x + 1/2*c)^7) - (63*a^20*tan(1/2*d*x + 1/2*c)^11 + 385*a^20
*tan(1/2*d*x + 1/2*c)^9 + 792*a^20*tan(1/2*d*x + 1/2*c)^7 - 3234*a^20*tan(
1/2*d*x + 1/2*c)^3 - 9702*a^20*tan(1/2*d*x + 1/2*c))/a^22)/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.61

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{99\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1848\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+9702\cos\left(\frac{c}{2}\right.}{}$$

input
```
int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^2),x)
```

output
```
-(99*cos(c/2 + (d*x)/2)^18 - 63*sin(c/2 + (d*x)/2)^18 - 385*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^16 - 792*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^14 + 3234*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 9702*cos(c/2 + (d*
x)/2)^10*sin(c/2 + (d*x)/2)^8 + 1848*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)
/2)^4 + 693*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2)/(354816*a^2*d*cos(
c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^7)
```

3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.91.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2(a-a\cos(c+dx))^6}{3a^9d} - \frac{16(a-a\cos(c+dx))^7}{7a^{10}d} + \frac{25(a-a\cos(c+dx))^8}{8a^{11}d} - \frac{19(a-a\cos(c+dx))^9}{9a^{12}d} + \frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d} - \frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}$$

output
```
2/3*(a-a*cos(d*x+c))^6/a^9/d-16/7*(a-a*cos(d*x+c))^7/a^10/d+25/8*(a-a*cos(
d*x+c))^8/a^11/d-19/9*(a-a*cos(d*x+c))^9/a^12/d+7/10*(a-a*cos(d*x+c))^10/a
^13/d-1/11*(a-a*cos(d*x+c))^11/a^14/d
```

3.91.2 Mathematica [A] (verified)

Time = 3.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-1615571 + 2273040\cos(c+dx) - 1496880\cos(2(c+dx)) + 535920\cos(3(c+dx)) + 110880\cos(4(c+$$

input
```
Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

output
```
(-1615571 + 2273040*Cos[c + d*x] - 1496880*Cos[2*(c + d*x)] + 535920*Cos[3
*(c + d*x)] + 110880*Cos[4*(c + d*x)] - 293832*Cos[5*(c + d*x)] + 212520*C
os[6*(c + d*x)] - 67320*Cos[7*(c + d*x)] - 27720*Cos[8*(c + d*x)] + 40040*
Cos[9*(c + d*x)] - 16632*Cos[10*(c + d*x)] + 2520*Cos[11*(c + d*x)])/(2838
5280*a^3*d)
```

3.91.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{11}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{11}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^{11}(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^{11}(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^{11}(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^{11}}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^{11}\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^3(c+dx)(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^2 d(a\cos(c+dx))}{a^{11}d}$$

$$\downarrow 27$$

$$-\frac{\int a^3\cos^3(c+dx)(a-a\cos(c+dx))^5(\cos(c+dx)a+a)^2 d(a\cos(c+dx))}{a^{14}d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-(a-a\cos(c+dx))^{10}+7a(a-a\cos(c+dx))^9-19a^2(a-a\cos(c+dx))^8+25a^3(a-a\cos(c+dx))^7-16\right.}{a^{14}d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{2}{3}a^5(a-a\cos(c+dx))^6+\frac{16}{7}a^4(a-a\cos(c+dx))^7-\frac{25}{8}a^3(a-a\cos(c+dx))^8+\frac{19}{9}a^2(a-a\cos(c+dx))^9+}{a^{14}d}$$

input
```
Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

output
```
-(((-2*a^5*(a - a*Cos[c + d*x])^6)/3 + (16*a^4*(a - a*Cos[c + d*x])^7)/7 -
 (25*a^3*(a - a*Cos[c + d*x])^8)/8 + (19*a^2*(a - a*Cos[c + d*x])^9)/9 - (
7*a*(a - a*Cos[c + d*x])^10)/10 + (a - a*Cos[c + d*x])^11/11)/(a^14*d))
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.91.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{3\cos(dx+c)^{10}}{10}+\frac{\cos(dx+c)^9}{9}+\frac{5\cos(dx+c)^8}{8}-\frac{5\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{3\cos(dx+c)^{10}}{10}+\frac{\cos(dx+c)^9}{9}+\frac{5\cos(dx+c)^8}{8}-\frac{5\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
parallelrisch	$\frac{13860\cos(4dx+4c)+66990\cos(3dx+3c)+315\cos(11dx+11c)+284130\cos(dx+c)-3465\cos(8dx+8c)-8415\cos(7dx+7c)}{3548160}$
risch	$\frac{41\cos(dx+c)}{512a^3d}+\frac{\cos(11dx+11c)}{11264d\,a^3}-\frac{3\cos(10dx+10c)}{5120d\,a^3}+\frac{13\cos(9dx+9c)}{9216d\,a^3}-\frac{\cos(8dx+8c)}{1024d\,a^3}-\frac{17\cos(7dx+7c)}{7168d\,a^3}+\frac{23}{}$

input
```
int(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/11*cos(d*x+c)^11-3/10*cos(d*x+c)^10+1/9*cos(d*x+c)^9+5/8*cos(d*
x+c)^8-5/7*cos(d*x+c)^7-1/6*cos(d*x+c)^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)^4
)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2520\cos(dx+c)^{11} - 8316\cos(dx+c)^{10} + 3080\cos(dx+c)^9 + 17325\cos(dx+c)^8 - 19800\cos(dx+$$
$$27720\,a^3 d$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
 + 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 166
32*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2520\cos(dx+c)^{11} - 8316\cos(dx+c)^{10} + 3080\cos(dx+c)^9 + 17325\cos(dx+c)^8 - 19800\cos(dx+$$
$$27720\,a^3 d$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
 + 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 166
32*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.49

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{32\left(\frac{209\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{1045\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{3135\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{6270\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{8778\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{13398\,(\cos}{(\cos(dx}$$
$$3465\,a^3 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}$$

input
```
integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
32/3465*(209*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1045*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 3135*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 - 6270*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8778*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 - 13398*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
 - 2310*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 9240*(cos(d*x + c) - 1
)^8/(cos(d*x + c) + 1)^8 - 19)/(a^3*d*((cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 1)^11)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{\cos(c+dx)^4}{4a^3} - \frac{3\cos(c+dx)^5}{5a^3} + \frac{\cos(c+dx)^6}{6a^3} + \frac{5\cos(c+dx)^7}{7a^3} - \frac{5\cos(c+dx)^8}{8a^3} - \frac{\cos(c+dx)^9}{9a^3} + \frac{3\cos(c+dx)^{10}}{10a^3} - \frac{\cos(c+dx)^{11}}{11a^3}}{d}$$

input
```
int(sin(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

output
```
-(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(6*
a^3) + (5*cos(c + d*x)^7)/(7*a^3) - (5*cos(c + d*x)^8)/(8*a^3) - cos(c + d
*x)^9/(9*a^3) + (3*cos(c + d*x)^10)/(10*a^3) - cos(c + d*x)^11/(11*a^3))/d
```

3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.92.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4(c+dx)}{4a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^6(c+dx)}{3a^3d} - \frac{2\cos^7(c+dx)}{7a^3d} + \frac{3\cos^8(c+dx)}{8a^3d} - \frac{\cos^9(c+dx)}{9a^3d}$$

output
```
-1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/3*cos(d*x+c)^6/a^3/d-2/7*
cos(d*x+c)^7/a^3/d+3/8*cos(d*x+c)^8/a^3/d-1/9*cos(d*x+c)^9/a^3/d
```

3.92.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{34771 - 52920\cos(c+dx) + 37800\cos(2(c+dx)) - 18480\cos(3(c+dx)) + 3780\cos(4(c+dx)) + 302}{3225}$$

input
```
Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/322560*(34771 - 52920*Cos[c + d*x] + 37800*Cos[2*(c + d*x)] - 18480*Cos
[3*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 3024*Cos[5*(c + d*x)] - 4200*Cos[6
*(c + d*x)] + 2700*Cos[7*(c + d*x)] - 945*Cos[8*(c + d*x)] + 140*Cos[9*(c
+ d*x)])/(a^3*d)
```

3.92.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^9}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^9(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^9(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^9(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^9}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^9\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^3(c+dx)(a-a\cos(c+dx))^4(\cos(c+dx)a+a)d(a\cos(c+dx))}{a^9 d}$$

$$\downarrow 27$$

$$-\frac{\int a^3\cos^3(c+dx)(a-a\cos(c+dx))^4(\cos(c+dx)a+a)d(a\cos(c+dx))}{a^{12}d}$$

$$\downarrow 84$$

$$-\frac{\int \left(\cos^8(c+dx)a^8 - 3\cos^7(c+dx)a^8 + 2\cos^6(c+dx)a^8 + 2\cos^5(c+dx)a^8 - 3\cos^4(c+dx)a^8 + \cos^3(c+dx)a^8\right.}{a^{12}d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{9}a^9\cos^9(c+dx) - \frac{3}{8}a^9\cos^8(c+dx) + \frac{2}{7}a^9\cos^7(c+dx) + \frac{1}{3}a^9\cos^6(c+dx) - \frac{3}{5}a^9\cos^5(c+dx) + \frac{1}{4}a^9\cos^4(c+d}{a^{12}d}$$

input
```
Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

output
```
-(((a^9*Cos[c + d*x]^4)/4 - (3*a^9*Cos[c + d*x]^5)/5 + (a^9*Cos[c + d*x]^6
)/3 + (2*a^9*Cos[c + d*x]^7)/7 - (3*a^9*Cos[c + d*x]^8)/8 + (a^9*Cos[c + d
*x]^9)/9)/(a^12*d))
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.92.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{8}-\frac{2\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{8}-\frac{2\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
parallelrisch	$\frac{-3780\cos(4dx+4c)+18480\cos(3dx+3c)+52920\cos(dx+c)+945\cos(8dx+8c)-2700\cos(7dx+7c)+4200\cos(6dx+6c)-3}{322560a^3d}$
risch	$\frac{21\cos(dx+c)}{128a^3d}-\frac{\cos(9dx+9c)}{2304d\,a^3}+\frac{3\cos(8dx+8c)}{1024d\,a^3}-\frac{15\cos(7dx+7c)}{1792d\,a^3}+\frac{5\cos(6dx+6c)}{384d\,a^3}-\frac{3\cos(5dx+5c)}{320d\,a^3}-\frac{3\cos(4}{256}$
norman	$\frac{\frac{128}{315ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{35da}+\frac{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{35da}+\frac{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{15da}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9 a^2}$

input
```
int(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/9*cos(d*x+c)^9+3/8*cos(d*x+c)^8-2/7*cos(d*x+c)^7-1/3*cos(d*x+c
)^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)^4)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{280\cos(dx+c)^9 - 945\cos(dx+c)^8 + 720\cos(dx+c)^7 + 840\cos(dx+c)^6 - 1512\cos(dx+c)^5 + 6}{2520\,a^3 d}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 84
0*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{280\cos(dx+c)^9 - 945\cos(dx+c)^8 + 720\cos(dx+c)^7 + 840\cos(dx+c)^6 - 1512\cos(dx+c)^5 + 6}{2520\,a^3 d}$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 84
0*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.70

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{32\left(\frac{36\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{144\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{336\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{504\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{630\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{105\,(\cos(dx+c)-}{(\cos(dx+c)+1}$$
$$315\,a^3 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9$$

input
```
integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
32/315*(36*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 144*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 336*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5
04*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(c
os(d*x + c) + 1)^5 - 105*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 315*(
cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4)/(a^3*d*((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1)^9)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.77

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{\cos(c+dx)^4}{4a^3} - \frac{3\cos(c+dx)^5}{5a^3} + \frac{\cos(c+dx)^6}{3a^3} + \frac{2\cos(c+dx)^7}{7a^3} - \frac{3\cos(c+dx)^8}{8a^3} + \frac{\cos(c+dx)^9}{9a^3}}{d}$$

input
```
int(sin(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

output
```
-(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(3*
a^3) + (2*cos(c + d*x)^7)/(7*a^3) - (3*cos(c + d*x)^8)/(8*a^3) + cos(c + d
*x)^9/(9*a^3))/d
```

3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.93.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4(c+dx)}{4a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{\cos^7(c+dx)}{7a^3d}$$

output

```
-1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/2*cos(d*x+c)^6/a^3/d+1/7*
cos(d*x+c)^7/a^3/d
```

3.93.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-2421 + 4060\cos(c+dx) - 3220\cos(2(c+dx)) + 2100\cos(3(c+dx)) - 1120\cos(4(c+dx)) + 476\cos(}{8960a^3d}$$

input

```
Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

output

```
(-2421 + 4060*Cos[c + d*x] - 3220*Cos[2*(c + d*x)] + 2100*Cos[3*(c + d*x)]
 - 1120*Cos[4*(c + d*x)] + 476*Cos[5*(c + d*x)] - 140*Cos[6*(c + d*x)] + 2
0*Cos[7*(c + d*x)])/(8960*a^3*d)
```

3.93.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^7(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^7(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^7(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int \cos^3(c+dx)(a-a\cos(c+dx))^3 d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 27$$

$$-\frac{\int a^3\cos^3(c+dx)(a-a\cos(c+dx))^3 d(a\cos(c+dx))}{a^{10}d}$$

$$\downarrow 49$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6+3\cos^5(c+dx)a^6-3\cos^4(c+dx)a^6+\cos^3(c+dx)a^6\right) d(a\cos(c+dx))}{a^{10}d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{1}{2}a^7\cos^6(c+dx)-\frac{3}{5}a^7\cos^5(c+dx)+\frac{1}{4}a^7\cos^4(c+dx)}{a^{10}d}$$

input
```
Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

output
```
-(((a^7*Cos[c + d*x]^4)/4 - (3*a^7*Cos[c + d*x]^5)/5 + (a^7*Cos[c + d*x]^6
)/2 - (a^7*Cos[c + d*x]^7)/7)/(a^10*d))
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.93.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
parallelrisch	$\frac{-805\cos(2dx+2c)+2784-280\cos(4dx+4c)-35\cos(6dx+6c)+1015\cos(dx+c)+525\cos(3dx+3c)+119\cos(5dx+5c)+5\,c}{2240a^3d}$
risch	$\frac{29\cos(dx+c)}{64a^3d}+\frac{\cos(7dx+7c)}{448d\,a^3}-\frac{\cos(6dx+6c)}{64d\,a^3}+\frac{17\cos(5dx+5c)}{320d\,a^3}-\frac{\cos(4dx+4c)}{8d\,a^3}+\frac{15\cos(3dx+3c)}{64d\,a^3}-\frac{23\cos(2c}{64d}$
norman	$\frac{\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{ad}+\frac{52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}+\frac{52}{35ad}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}+\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5da}+\frac{156\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7a^2}$

input
```
int(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/7*cos(d*x+c)^7-1/2*cos(d*x+c)^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)
^4)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{20\cos(dx+c)^7-70\cos(dx+c)^6+84\cos(dx+c)^5-35\cos(dx+c)^4}{140\,a^3d}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(
d*x + c)^4)/(a^3*d)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{20\cos(dx+c)^7 - 70\cos(dx+c)^6 + 84\cos(dx+c)^5 - 35\cos(dx+c)^4}{140\,a^3 d}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(
d*x + c)^4)/(a^3*d)
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(65) = 130.

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.23

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{4\left(\frac{91(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{273(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{455(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{490(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{210(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{140(\cos(dx+c)-1}{(\cos(dx+c)+1)^6}\right.}{35\,a^3 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}$$

input
```
integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
4/35*(91*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 273*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 455*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 490
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 210*(cos(d*x + c) - 1)^5/(cos
(d*x + c) + 1)^5 - 140*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 13)/(a^
3*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\cos(c+dx)^4}{4a^3} - \frac{3\cos(c+dx)^5}{5a^3} + \frac{\cos(c+dx)^6}{2a^3} - \frac{\cos(c+dx)^7}{7a^3}}{d}$$

input
```
int(sin(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

output
```
-(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(2*
a^3) - cos(c + d*x)^7/(7*a^3))/d
```

3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.94.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{4\cos(c+dx)}{a^3d} + \frac{2\cos^2(c+dx)}{a^3d} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{3\cos^4(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)}{5a^3d} + \frac{4\log(1+\cos(c+dx))}{a^3d}$$

output
```
-4*cos(d*x+c)/a^3/d+2*cos(d*x+c)^2/a^3/d-4/3*cos(d*x+c)^3/a^3/d+3/4*cos(d*
x+c)^4/a^3/d-1/5*cos(d*x+c)^5/a^3/d+4*ln(1+cos(d*x+c))/a^3/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3857 - 4920\cos(c+dx) + 1320\cos(2(c+dx)) - 380\cos(3(c+dx)) + 90\cos(4(c+dx)) - 12\cos(5(c+d}{960a^3d}$$

input
```
Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
(3857 - 4920*Cos[c + d*x] + 1320*Cos[2*(c + d*x)] - 380*Cos[3*(c + d*x)] +
 90*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 7680*Log[Cos[(c + d*x)/2]])/(
960*a^3*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^5(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^5(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^5(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int \frac{\cos^3(c+dx)(a-a\cos(c+dx))^2}{\cos(c+dx)a+a}d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3 \cos^3(c+dx)(a-a\cos(c+dx))^2}{\cos(c+dx)a+a} d(a\cos(c+dx))}{a^8 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{4a^5}{\cos(c+dx)a+a} + \cos^4(c+dx)a^4 - 3\cos^3(c+dx)a^4 + 4\cos^2(c+dx)a^4 - 4\cos(c+dx)a^4 + 4a^4\right) d(a\cos(c+d$$
$$a^8 d$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx) - \frac{3}{4}a^5\cos^4(c+dx) + \frac{4}{3}a^5\cos^3(c+dx) - 2a^5\cos^2(c+dx) + 4a^5\cos(c+dx) - 4a^5\log(a\cos(c+$$
$$a^8 d$$

input
```
Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((4*a^5*Cos[c + d*x] - 2*a^5*Cos[c + d*x]^2 + (4*a^5*Cos[c + d*x]^3)/3 -
(3*a^5*Cos[c + d*x]^4)/4 + (a^5*Cos[c + d*x]^5)/5 - 4*a^5*Log[a + a*Cos[c
+ d*x]])/(a^8*d))
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.94.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}-\frac{4\cos(dx+c)^3}{3}+2\cos(dx+c)^2-4\cos(dx+c)+4\ln(\cos(dx+c)+1)}{d\,a^3}$
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}-\frac{4\cos(dx+c)^3}{3}+2\cos(dx+c)^2-4\cos(dx+c)+4\ln(\cos(dx+c)+1)}{d\,a^3}$
parallelrisch	$\frac{-1920\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-3361-6\cos(5dx+5c)+45\cos(4dx+4c)-190\cos(3dx+3c)+660\cos(2dx+2c)-2460\cos(dx+c}{480a^3d}$
norman	$\frac{-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}-\frac{166}{15ad}-\frac{78\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}-\frac{154\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}-\frac{278\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5a^2}-\frac{4\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$
risch	$-\frac{4ix}{a^3}-\frac{41\,e^{i(dx+c)}}{16a^3d}-\frac{41\,e^{-i(dx+c)}}{16a^3d}-\frac{8ic}{a^3d}+\frac{8\ln\left(e^{i(dx+c)}+1\right)}{a^3d}-\frac{\cos(5dx+5c)}{80d\,a^3}+\frac{3\cos(4dx+4c)}{32d\,a^3}-\frac{19\cos(3d}{48d}$

input
```
int(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/5*cos(d*x+c)^5+3/4*cos(d*x+c)^4-4/3*cos(d*x+c)^3+2*cos(d*x+c)^
2-4*cos(d*x+c)+4*ln(cos(d*x+c)+1))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{12\cos(dx+c)^5 - 45\cos(dx+c)^4 + 80\cos(dx+c)^3 - 120\cos(dx+c)^2 + 240\cos(dx+c) - 240\log}{60\,a^3 d}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos
(d*x + c)^2 + 240*cos(d*x + c) - 240*log(1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{12\cos(dx+c)^5 - 45\cos(dx+c)^4 + 80\cos(dx+c)^3 - 120\cos(dx+c)^2 + 240\cos(dx+c)}{a^3} - \frac{240\log(\cos(dx+c)+1)}{a^3}}{60\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*co
s(d*x + c)^2 + 240*cos(d*x + c))/a^3 - 240*log(cos(d*x + c) + 1)/a^3)/d
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.69

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{85(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{200(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{205(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/15*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (85*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 - 200*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 205*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
 1)^5 - 29)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.94.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{4\ln(\cos(c+dx)+1)}{a^3}-\frac{4\cos(c+dx)}{a^3}+\frac{2\cos(c+dx)^2}{a^3}-\frac{4\cos(c+dx)^3}{3a^3}+\frac{3\cos(c+dx)^4}{4a^3}-\frac{\cos(c+dx)^5}{5a^3}}{d}$$

input
```
int(sin(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

output
```
((4*log(cos(c + d*x) + 1))/a^3 - (4*cos(c + d*x))/a^3 + (2*cos(c + d*x)^2)
/a^3 - (4*cos(c + d*x)^3)/(3*a^3) + (3*cos(c + d*x)^4)/(4*a^3) - cos(c + d
*x)^5/(5*a^3))/d
```

3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.95.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{5\cos(c+dx)}{a^3d} - \frac{3\cos^2(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{2}{d\,(a^3+a^3\cos(c+dx))} - \frac{7\log(1+\cos(c+dx))}{a^3d}$$

output
```
5*cos(d*x+c)/a^3/d-3/2*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^3/d-2/d/(a^3+
a^3*cos(d*x+c))-7*ln(1+cos(d*x+c))/a^3/d
```

3.95.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(389-184\cos(2(c+dx))+28\cos(3(c+dx))-4\cos(4(c+dx))+1344\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{24a^3d(1+\cos(c+dx))^3}$$

input
```
Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/24*(Cos[(c + d*x)/2]^4*(389 - 184*Cos[2*(c + d*x)] + 28*Cos[3*(c + d*x)
] - 4*Cos[4*(c + d*x)] + 1344*Log[Cos[(c + d*x)/2]] + Cos[c + d*x]*(-19 +
1344*Log[Cos[(c + d*x)/2]])))/(a^3*d*(1 + Cos[c + d*x])^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^3(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^3(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^3(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3315$$

$$-\frac{\int \frac{\cos^3(c+dx)(a-a\cos(c+dx))}{(\cos(c+dx)a+a)^2}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3 \cos^3(c+dx)(a-a\cos(c+dx))}{(\cos(c+dx)a+a)^2} d(a\cos(c+dx))}{a^6 d}$$

$$\downarrow 86$$

$$-\frac{\int \left(-\frac{2a^4}{(\cos(c+dx)a+a)^2} + \frac{7a^3}{\cos(c+dx)a+a} - \cos^2(c+dx)a^2 + 3\cos(c+dx)a^2 - 5a^2\right) d(a\cos(c+dx))}{a^6 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{2a^4}{a\cos(c+dx)+a} - \frac{1}{3}a^3\cos^3(c+dx) + \frac{3}{2}a^3\cos^2(c+dx) - 5a^3\cos(c+dx) + 7a^3\log(a\cos(c+dx)+a)}{a^6 d}$$

input
```
Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-5*a^3*Cos[c + d*x] + (3*a^3*Cos[c + d*x]^2)/2 - (a^3*Cos[c + d*x]^3)/3
 + (2*a^4)/(a + a*Cos[c + d*x]) + 7*a^3*Log[a + a*Cos[c + d*x]])/(a^6*d))
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.95.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{3\cos(dx+c)^2}{2}+5\cos(dx+c)-7\ln(\cos(dx+c)+1)-\frac{2}{\cos(dx+c)+1}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{3\cos(dx+c)^2}{2}+5\cos(dx+c)-7\ln(\cos(dx+c)+1)-\frac{2}{\cos(dx+c)+1}}{d\,a^3}$
parallelrisch	$\frac{-12\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2+63\cos(dx+c)+\cos(3dx+3c)-9\cos(2dx+2c)+84\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+121}{12a^3d}$
norman	$\frac{\frac{34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}+\frac{41}{3ad}+\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3a^2}+\frac{7\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$
risch	$\frac{7ix}{a^3}+\frac{21\,e^{i(dx+c)}}{8a^3d}+\frac{21\,e^{-i(dx+c)}}{8a^3d}+\frac{14ic}{a^3d}-\frac{4\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^2}-\frac{14\ln\left(e^{i(dx+c)}+1\right)}{a^3d}+\frac{\cos(3dx+3c)}{12d\,a^3}-\frac{3\cos($

input
```
int(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/3*cos(d*x+c)^3-3/2*cos(d*x+c)^2+5*cos(d*x+c)-7*ln(cos(d*x+c)+1)
-2/(cos(d*x+c)+1))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{4\cos(dx+c)^4 - 14\cos(dx+c)^3 + 42\cos(dx+c)^2 - 84(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 69}{12(a^3 d\cos(dx+c)+a^3 d)}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*cos(d*x + c)^4 - 14*cos(d*x + c)^3 + 42*cos(d*x + c)^2 - 84*(cos(d
*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 69*cos(d*x + c) - 15)/(a^3*d*co
s(d*x + c) + a^3*d)
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{12}{a^3\cos(dx+c)+a^3} - \frac{2\cos(dx+c)^3 - 9\cos(dx+c)^2 + 30\cos(dx+c)}{a^3} + \frac{42\log(\cos(dx+c)+1)}{a^3}}{6d}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(12/(a^3*cos(d*x + c) + a^3) - (2*cos(d*x + c)^3 - 9*cos(d*x + c)^2 +
 30*cos(d*x + c))/a^3 + 42*log(cos(d*x + c) + 1)/a^3)/d
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{7\log\left(\left|-\cos\left(dx+c\right)-1\right|\right)}{a^3 d} - \frac{2}{a^3 d(\cos\left(dx+c\right)+1)}$$
$$+ \frac{2\,a^6 d^5\cos\left(dx+c\right)^3 - 9\,a^6 d^5\cos\left(dx+c\right)^2 + 30\,a^6 d^5\cos\left(dx+c\right)}{6\,a^9 d^6}$$

input
```
integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-7*log(abs(-cos(d*x + c) - 1))/(a^3*d) - 2/(a^3*d*(cos(d*x + c) + 1)) + 1/
6*(2*a^6*d^5*cos(d*x + c)^3 - 9*a^6*d^5*cos(d*x + c)^2 + 30*a^6*d^5*cos(d*
x + c))/(a^9*d^6)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{2}{a^3\cos(c+dx)+a^3} + \frac{7\ln(\cos(c+dx)+1)}{a^3} - \frac{5\cos(c+dx)}{a^3} + \frac{3\cos(c+dx)^2}{2\,a^3} - \frac{\cos(c+dx)^3}{3\,a^3}}{d}$$

input
```
int(sin(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

output
```
-(2/(a^3*cos(c + d*x) + a^3) + (7*log(cos(c + d*x) + 1))/a^3 - (5*cos(c +
d*x))/a^3 + (3*cos(c + d*x)^2)/(2*a^3) - cos(c + d*x)^3/(3*a^3))/d
```

3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.96.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{a^3 d} - \frac{1}{2ad(a+a\cos(c+dx))^2} + \frac{3}{d\left(a^3+a^3\cos(c+dx)\right)} + \frac{3\log(1+\cos(c+dx))}{a^3 d}$$

output
```
-cos(d*x+c)/a^3/d-1/2/a/d/(a+a*cos(d*x+c))^2+3/d/(a^3+a^3*cos(d*x+c))+3*ln
(1+cos(d*x+c))/a^3/d
```

3.96.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(21-2\cos(3(c+dx))+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(-5+24\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{4a^3 d(1+\cos(c+dx))^3}$$

input
```
Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]^2*(21 - 2*Cos[3*(c + d*x)] + 72*Log[Cos[(c + d*x)/2]] +
Cos[2*(c + d*x)]*(-5 + 24*Log[Cos[(c + d*x)/2]]) + Cos[c + d*x]*(22 + 96*L
og[Cos[(c + d*x)/2]])))/(4*a^3*d*(1 + Cos[c + d*x])^3)
```

3.96.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\sin\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos^3(c+dx)}{(\cos(c+dx)a+a)^3}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(\cos(c+dx)a+a)^3}d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 49$$

$$-\frac{\int \left(-\frac{a^3}{(\cos(c+dx)a+a)^3}+\frac{3a^2}{(\cos(c+dx)a+a)^2}-\frac{3a}{\cos(c+dx)a+a}+1\right) d(a\cos(c+dx))}{a^4 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{a^3}{2(a\cos(c+dx)+a)^2}-\frac{3a^2}{a\cos(c+dx)+a}+a\cos(c+dx)-3a\log(a\cos(c+dx)+a)}{a^4 d}$$

input
```
Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a*Cos[c + d*x] + a^3/(2*(a + a*Cos[c + d*x])^2) - (3*a^2)/(a + a*Cos[c
+ d*x]) - 3*a*Log[a + a*Cos[c + d*x]])/(a^4*d))
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.96.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{-29-24\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4-8\cos(dx+c)+12\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3d}$	59
derivativedivides	$\frac{-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))}{d\,a^3}$	63
default	$\frac{-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))}{d\,a^3}$	63
norman	$\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8da}-\frac{13}{4ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2}-\frac{3\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$	90
risch	$-\frac{3ix}{a^3}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{6ic}{a^3d}+\frac{6e^{3i(dx+c)}+10e^{2i(dx+c)}+6e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^4}+\frac{6\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	128

input
```
int(sin(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-29-24*ln(sec(1/2*d*x+1/2*c)^2)-sec(1/2*d*x+1/2*c)^4-8*cos(d*x+c)+12*
sec(1/2*d*x+1/2*c)^2)/a^3/d
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\cos(dx+c)^3+4\cos(dx+c)^2-6\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\,(\text{illegible})}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c) - 5)/(a^3*d*cos(d*x
 + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

3.96.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\cos(dx+c)+5}{a^3\cos(dx+c)^2+2a^3\cos(dx+c)+a^3} - \frac{2\cos(dx+c)}{a^3} + \frac{6\log(\cos(dx+c)+1)}{a^3}}{2d}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((6*cos(d*x + c) + 5)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
- 2*cos(d*x + c)/a^3 + 6*log(cos(d*x + c) + 1)/a^3)/d
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos(dx+c)}{a^3 d} + \frac{3\log(|-\cos(dx+c)-1|)}{a^3 d} + \frac{6\cos(dx+c)+5}{2a^3 d(\cos(dx+c)+1)^2}$$

input
```
integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a^3*d) + 3*log(abs(-cos(d*x + c) - 1))/(a^3*d) + 1/2*(6*cos
(d*x + c) + 5)/(a^3*d*(cos(d*x + c) + 1)^2)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\ln(\cos(c+dx)+1)}{a^3 d} - \frac{\cos(c+dx)}{a^3 d} + \frac{3\cos(c+dx)+\frac{5}{2}}{a^3 d(\cos(c+dx)+1)^2}$$

input
```
int(sin(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

output
```
(3*log(cos(c + d*x) + 1))/(a^3*d) - cos(c + d*x)/(a^3*d) + (3*cos(c + d*x)
 + 5/2)/(a^3*d*(cos(c + d*x) + 1)^2)
```

3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.97.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{8a^3d} - \frac{1}{6d(a+a\cos(c+dx))^3} + \frac{5}{8ad(a+a\cos(c+dx))^2} - \frac{7}{8d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
-1/8*arctanh(cos(d*x+c))/a^3/d-1/6/d/(a+a*cos(d*x+c))^3+5/8/a/d/(a+a*cos(d
*x+c))^2-7/8/d/(a^3+a^3*cos(d*x+c))
```

3.97.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\left(2-15\cos^2\left(\frac{1}{2}(c+dx)\right)+42\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin(\right.\right.\right.}{12a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/12*((2 - 15*Cos[(c + d*x)/2]^2 + 42*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*
x)/2]^6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(
a^3*d*(1 + Sec[c + d*x])^3)
```

3.97.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos^2(c+dx)\cot(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^2(c+dx)\cot(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a\int -\frac{\cos^3(c+dx)}{(a-a\cos(c+dx))(\cos(c+dx)a+a)^4}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a\int \frac{\cos^3(c+dx)}{(a-a\cos(c+dx))(\cos(c+dx)a+a)^4}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(a-a\cos(c+dx))(\cos(c+dx)a+a)^4}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{a^2}{2(\cos(c+dx)a+a)^4}+\frac{5a}{4(\cos(c+dx)a+a)^3}+\frac{1}{8(a^2-a^2\cos^2(c+dx))}-\frac{7}{8(\cos(c+dx)a+a)^2}\right)d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{a^2}{6(a\cos(c+dx)+a)^3}+\frac{\text{arctanh}(\cos(c+dx))}{8a}-\frac{5a}{8(a\cos(c+dx)+a)^2}+\frac{7}{8(a\cos(c+dx)+a)}}{a^2d}$$

input
```
Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((ArcTanh[Cos[c + d*x]]/(8*a) + a^2/(6*(a + a*Cos[c + d*x])^3) - (5*a)/(8
*(a + a*Cos[c + d*x])^2) + 7/(8*(a + a*Cos[c + d*x])))/(a^2*d))
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.97.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96a^3d}$	61
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}-\frac{1}{6(\cos(dx+c)+1)^3}+\frac{5}{8(\cos(dx+c)+1)^2}-\frac{7}{8(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$	67
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}-\frac{1}{6(\cos(dx+c)+1)^3}+\frac{5}{8(\cos(dx+c)+1)^2}-\frac{7}{8(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$	67
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{48da}}{a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d}$	82
risch	$-\frac{21e^{5i(dx+c)}+54e^{4i(dx+c)}+82e^{3i(dx+c)}+54e^{2i(dx+c)}+21e^{i(dx+c)}}{12a^3d\left(e^{i(dx+c)}+1\right)^6}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8a^3d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8a^3d}$	119

input
```
int(csc(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/96*(-2*tan(1/2*d*x+1/2*c)^6+9*tan(1/2*d*x+1/2*c)^4-18*tan(1/2*d*x+1/2*c)
^2+12*ln(tan(1/2*d*x+1/2*c)))/a^3/d
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{42\cos(dx+c)^2+3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3}{48\left(a^3d\cos(dx+c)^3+3a^3d\cos(d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/48*(42*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 54*cos(d*x + c)
+ 20)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c
) + a^3*d)
```

3.97.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{2\left(21\cos(dx+c)^2+27\cos(dx+c)+10\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3\log(\cos(dx+c)+1)}{a^3}-\frac{3\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/48*(2*(21*cos(d*x + c)^2 + 27*cos(d*x + c) + 10)/(a^3*cos(d*x + c)^3 +
3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3*log(cos(d*x + c) + 1)
/a^3 - 3*log(cos(d*x + c) - 1)/a^3)/d
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{\frac{18a^6(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{9a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96\,d}$$

input
```
integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (18*a^6*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{7\cos(c+dx)^2}{8} + \frac{9\cos(c+dx)}{8} + \frac{5}{12}}{d\left(a^3\cos(c+dx)^3 + 3a^3\cos(c+dx)^2 + 3a^3\cos(c+dx) + a^3\right)} - \frac{\operatorname{atanh}(\cos(c+dx))}{8a^3d}$$

input
```
int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

output
```
- ((9*cos(c + d*x))/8 + (7*cos(c + d*x)^2)/8 + 5/12)/(d*(3*a^3*cos(c + d*x
) + a^3 + 3*a^3*cos(c + d*x)^2 + a^3*cos(c + d*x)^3)) - atanh(cos(c + d*x)
)/(8*a^3*d)
```

3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.98.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\cos(c+dx))}{32a^3d} - \frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c+dx))^2} - \frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)} - \frac{1}{16d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
1/32*arctanh(cos(d*x+c))/a^3/d-1/16*a/d/(a+a*cos(d*x+c))^4+1/6/d/(a+a*cos(
d*x+c))^3-3/32/a/d/(a+a*cos(d*x+c))^2-1/32/d/(a^3-a^3*cos(d*x+c))-1/16/d/(
a^3+a^3*cos(d*x+c))
```

3.98.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+24\sec^2\left(\right.\right.}{96a^3d(1+\sec(c+}$$

input
```
Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(-Log[Cos[(c + d*x)/
2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 18*Sec[(c + d*x)/2]
^4 - 16*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3*d*
(1 + Sec[c + d*x])^3)
```

3.98.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4359$$
$$\int -\frac{\cot^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cot^3(c+dx)}{(\cos(c+dx)a+a)^3}dx$$
$$\downarrow 25$$
$$\int \frac{\cot^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)^3}{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3186$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(a-a\cos(c+dx))^2(\cos(c+dx)a+a)^5}d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{a}{4(\cos(c+dx)a+a)^5}+\frac{1}{2(\cos(c+dx)a+a)^4}-\frac{3}{16(\cos(c+dx)a+a)^3a}-\frac{1}{32(a^2-a^2\cos^2(c+dx))a^2}+\frac{1}{32(a-a\cos(c+dx))^2a^2}-\frac{1}{16(\cos(c\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\text{arctanh}(\cos(c+dx))}{32a^3}+\frac{1}{32a^2(a-a\cos(c+dx))}+\frac{1}{16a^2(a\cos(c+dx)+a)}+\frac{a}{16(a\cos(c+dx)+a)^4}-\frac{1}{6(a\cos(c+dx)+a)^3}+\frac{3}{32a(a\cos(c+d}{d}$$

input
```
Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-1/32*ArcTanh[Cos[c + d*x]]/a^3 + 1/(32*a^2*(a - a*Cos[c + d*x])) + a/(
16*(a + a*Cos[c + d*x])^4) - 1/(6*(a + a*Cos[c + d*x])^3) + 3/(32*a*(a + a
*Cos[c + d*x])^2) + 1/(16*a^2*(a + a*Cos[c + d*x])))/d)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.98.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768a^3d}$
derivativedivides	$\frac{\frac{1}{32\cos(dx+c)-32}-\frac{\ln(\cos(dx+c)-1)}{64}-\frac{1}{16(\cos(dx+c)+1)^4}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{3}{32(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{\ln(\cos(dx+c}{d\,a^3}$
default	$\frac{\frac{1}{32\cos(dx+c)-32}-\frac{\ln(\cos(dx+c)-1)}{64}-\frac{1}{16(\cos(dx+c)+1)^4}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{3}{32(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{\ln(\cos(dx+c}{d\,a^3}$
norman	$\frac{-\frac{1}{64ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{64da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{192da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{256da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^3d}$
risch	$-\frac{3e^{9i(dx+c)}+18e^{8i(dx+c)}-88e^{7i(dx+c)}-162e^{6i(dx+c)}-310e^{5i(dx+c)}-162e^{4i(dx+c)}-88e^{3i(dx+c)}+18e^{2i(dx+c)}+3e^{i}}{48a^3d\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

input
```
int(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/768*(-3*tan(1/2*d*x+1/2*c)^8+4*tan(1/2*d*x+1/2*c)^6+12*tan(1/2*d*x+1/2*c
)^4-24*tan(1/2*d*x+1/2*c)^2-12*cot(1/2*d*x+1/2*c)^2-24*ln(tan(1/2*d*x+1/2*
c)))/a^3/d
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.

Time $= 0.27$ (sec) , antiderivative size $= 240$, normalized size of antiderivative $= 1.90$

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{6\cos(dx+c)^4+18\cos(dx+c)^3-50\cos(dx+c)^2-3\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+\right.}{19}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/192*(6*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - 50*cos(d*x + c)^2 - 3*(cos(
d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*co
s(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + 3*cos(d*
x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*log(-
1/2*cos(d*x + c) + 1/2) - 54*cos(d*x + c) - 16)/(a^3*d*cos(d*x + c)^5 + 3*
a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3
*a^3*d*cos(d*x + c) - a^3*d)
```

3.98.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(3\cos(dx+c)^4+9\cos(dx+c)^3-25\cos(dx+c)^2-27\cos(dx+c)-8\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}-\frac{3\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/192*(2*(3*cos(d*x + c)^4 + 9*cos(d*x + c)^3 - 25*cos(d*x + c)^2 - 27*co
s(d*x + c) - 8)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*cos(d*x
 + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) - 3*log(cos(d*x
 + c) + 1)/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{24a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4a^9(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{3a^9(\cos(dx+}{(\cos(dx+c)}}{a^{12}}}{768\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(12*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(
a^3*(cos(d*x + c) - 1)) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^3 + (24*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^9*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^9*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 - 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{atanh}(\cos(c+d\,x))}{32\,a^3\,d} - \frac{-\frac{\cos(c+d\,x)^4}{32} - \frac{3\cos(c+d\,x)^3}{32} + \frac{25\cos(c+d\,x)^2}{96} + \frac{9\cos(c+d\,x)}{32} + \frac{1}{12}}{d\,\left(-a^3\cos(c+d\,x)^5 - 3\,a^3\cos(c+d\,x)^4 - 2\,a^3\cos(c+d\,x)^3 + 2\,a^3\cos(c+d\,x)^2 + 3\,a^3\cos(c+d\,x) +$$

input
```
int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

output
```
atanh(cos(c + d*x))/(32*a^3*d) - ((9*cos(c + d*x))/32 + (25*cos(c + d*x)^2
)/96 - (3*cos(c + d*x)^3)/32 - cos(c + d*x)^4/32 + 1/12)/(d*(3*a^3*cos(c +
 d*x) + a^3 + 2*a^3*cos(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 3*a^3*cos(c +
d*x)^4 - a^3*cos(c + d*x)^5))
```

3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.99.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\operatorname{arctanh}(\cos(c+dx))}{128a^3d} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{a^2}{40d(a+a\cos(c+dx))^5} + \frac{3a}{64d(a+a\cos(c+dx))^4} - \frac{1}{64ad(a+a\cos(c+dx))^2} - \frac{3}{128d\left(a^3+a^3\cos(c+dx)\right)}$$

output
```
3/128*arctanh(cos(d*x+c))/a^3/d-1/128/a/d/(a-a*cos(d*x+c))^2-1/40*a^2/d/(a
+a*cos(d*x+c))^5+3/64*a/d/(a+a*cos(d*x+c))^4-1/64/a/d/(a+a*cos(d*x+c))^2-3
/128/d/(a^3+a^3*cos(d*x+c))
```

3.99.2 Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\left(4-15\cos^2\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)+10\cos^6\left(\frac{1}{2}(c+dx)\right)\left(2+\cot^4\left(\frac{1}{2}(c+dx)\right)\right)-120\cos^{1}\right.}{640a^3d(1+\sec(c+$$

input
```
Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/640*((4 - 15*Cos[(c + d*x)/2]^2 + 60*Cos[(c + d*x)/2]^8 + 10*Cos[(c + d
*x)/2]^6*(2 + Cot[(c + d*x)/2]^4) - 120*Cos[(c + d*x)/2]^10*(Log[Cos[(c +
d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(a^3
*d*(1 + Sec[c + d*x])^3)
```

3.99.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3315, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc^2(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc^2(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3315$$

$$\frac{a^5 \int -\frac{\cos^3(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^6}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^5 \int \frac{\cos^3(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^6}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^2 \int \frac{a^3\cos^3(c+dx)}{(a-a\cos(c+dx))^3(\cos(c+dx)a+a)^6}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 99$$

$$-\frac{a^2 \int \left(-\frac{1}{8(\cos(c+dx)a+a)^6}+\frac{3}{16(\cos(c+dx)a+a)^5a}+\frac{1}{64(a-a\cos(c+dx))^3a^3}-\frac{1}{32(\cos(c+dx)a+a)^3a^3}-\frac{3}{128(a^2-a^2\cos^2(c+dx))a^4}-\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{3\operatorname{arctanh}(\cos(c+dx))}{128a^5}+\frac{3}{128a^4(a\cos(c+dx)+a)}+\frac{1}{128a^3(a-a\cos(c+dx))^2}+\frac{1}{64a^3(a\cos(c+dx)+a)^2}-\frac{3}{64a(a\cos(c+dx)+a)^4}+\right.}{d}$$

input
```
Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^2*((-3*ArcTanh[Cos[c + d*x]])/(128*a^5) + 1/(128*a^3*(a - a*Cos[c + d
*x])^2) + 1/(40*(a + a*Cos[c + d*x])^5) - 3/(64*a*(a + a*Cos[c + d*x])^4)
+ 1/(64*a^3*(a + a*Cos[c + d*x])^2) + 3/(128*a^4*(a + a*Cos[c + d*x])))))/d
)
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3315
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
 x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Intege
rQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.99.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{40(\cos(dx+c)+1)^5}+\frac{3}{64(\cos(dx+c)+1)^4}-\frac{1}{64(\cos(dx+c)+1)^2}-\frac{3}{128(\cos(dx+c)+1)}+\frac{3\ln(\cos(dx+c)+1)}{256}-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{3\ln}{}}{d\,a^3}$
default	$\frac{-\frac{1}{40(\cos(dx+c)+1)^5}+\frac{3}{64(\cos(dx+c)+1)^4}-\frac{1}{64(\cos(dx+c)+1)^2}-\frac{3}{128(\cos(dx+c)+1)}+\frac{3\ln(\cos(dx+c)+1)}{256}-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{3\ln}{}}{d\,a^3}$
parallelrisch	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-60\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-20\cot\left(\frac{d}{}\right.}{5120a^3d}$
norman	$\frac{-\frac{1}{512ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{1024ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1280ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{256da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{256da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{512da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{256da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2}-\frac{3\ln\left(\tan\left(\right.\right.}{128a}$
risch	$-\frac{15\,e^{13i(dx+c)}+90\,e^{12i(dx+c)}+170\,e^{11i(dx+c)}-30\,e^{10i(dx+c)}+1521\,e^{9i(dx+c)}+1476\,e^{8i(dx+c)}+3756\,e^{7i(dx+c)}+1476\,e^{6}}{320a^3d\left(e^{i(dx+c)}+1\right)^{10}\left(e^{i(dx+c)}-1\right)^4}$

input
```
int(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/40/(cos(d*x+c)+1)^5+3/64/(cos(d*x+c)+1)^4-1/64/(cos(d*x+c)+1)^
2-3/128/(cos(d*x+c)+1)+3/256*ln(cos(d*x+c)+1)-1/128/(cos(d*x+c)-1)^2-3/256
*ln(cos(d*x+c)-1))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(117) = 234$.

Time $= 0.27$ (sec) , antiderivative size $= 317$, normalized size of antiderivative $= 2.48$

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{30\cos(dx+c)^6+90\cos(dx+c)^5+40\cos(dx+c)^4-120\cos(dx+c)^3+122\cos(dx+c)^2-15(c}{}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/1280*(30*cos(d*x + c)^6 + 90*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 120*c
os(d*x + c)^3 + 122*cos(d*x + c)^2 - 15*(cos(d*x + c)^7 + 3*cos(d*x + c)^6
 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 +
 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^7 + 3*
cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + co
s(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 126*cos(
d*x + c) + 32)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(
d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.99.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.47

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{2\left(15\cos(dx+c)^6+45\cos(dx+c)^5+20\cos(dx+c)^4-60\cos(dx+c)^3+61\cos(dx+c)^2+63\cos(dx+c)+16\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} - \frac{15\log(\cos(dx+c)}{a^3}}{1280\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/1280*(2*(15*cos(d*x + c)^6 + 45*cos(d*x + c)^5 + 20*cos(d*x + c)^4 - 60
*cos(d*x + c)^3 + 61*cos(d*x + c)^2 + 63*cos(d*x + c) + 16)/(a^3*cos(d*x +
 c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 -
 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) - 1
5*log(cos(d*x + c) + 1)/a^3 + 15*log(cos(d*x + c) - 1)/a^3)/d
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.81

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{10\left(\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2} - \frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{\frac{60\,a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{30\,a^{12}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{20\,a^{12}(c}{(\cos(}}{5120\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/5120*(10*(2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^
2) - 60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (60*a^12*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 30*a^12*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 20*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*a^1
2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 4*a^12*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5)/a^15)/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\mathrm{atanh}(\cos(c+d\,x))}{128\,a^3\,d} - \frac{\frac{3\cos(c+d\,x)^6}{128} + \frac{9\cos(c+d\,x)^5}{128} + \frac{\cos(c+d\,x)^4}{32} - \frac{3\cos(c+d\,x)^3}{32} + \frac{61\cos(c+d\,x)^2}{640} + \frac{63\cos(c}{64}}{d\,\left(a^3\cos(c+d\,x)^7 + 3\,a^3\cos(c+d\,x)^6 + a^3\cos(c+d\,x)^5 - 5\,a^3\cos(c+d\,x)^4 - 5\,a^3\cos(c+d\,x)^3 + a\right.}$$

input
```
int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

output
```
(3*atanh(cos(c + d*x)))/(128*a^3*d) - ((63*cos(c + d*x))/640 + (61*cos(c +
 d*x)^2)/640 - (3*cos(c + d*x)^3)/32 + cos(c + d*x)^4/32 + (9*cos(c + d*x)
^5)/128 + (3*cos(c + d*x)^6)/128 + 1/40)/(d*(3*a^3*cos(c + d*x) + a^3 + a^
3*cos(c + d*x)^2 - 5*a^3*cos(c + d*x)^3 - 5*a^3*cos(c + d*x)^4 + a^3*cos(c
 + d*x)^5 + 3*a^3*cos(c + d*x)^6 + a^3*cos(c + d*x)^7))
```

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.100.1 Optimal result . . . 782
3.100.2 Mathematica [A] (verified) . . . 782
3.100.3 Rubi [A] (verified) . . . 783
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3.100.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{29x}{128a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{3\sin^7(c+dx)}{7a^3d}$$

output

```
-29/128*x/a^3-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d-29/192*cos(d*x+c)^3*sin(d
*x+c)/a^3/d+23/48*cos(d*x+c)^5*sin(d*x+c)/a^3/d+1/8*cos(d*x+c)^7*sin(d*x+c
)/a^3/d+4/3*sin(d*x+c)^3/a^3/d-7/5*sin(d*x+c)^5/a^3/d+3/7*sin(d*x+c)^7/a^3
/d
```

3.100.2 Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-24360dx + 38640\sin(c+dx) - 6720\sin(2(c+dx)) - 3920\sin(3(c+dx)\right.}{134}$$

input
```
Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-24360*d*x + 38640*Sin[c + d*x] - 6720
*Sin[2*(c + d*x)] - 3920*Sin[3*(c + d*x)] + 5880*Sin[4*(c + d*x)] - 4368*S
in[5*(c + d*x)] + 2240*Sin[6*(c + d*x)] - 720*Sin[7*(c + d*x)] + 105*Sin[8
*(c + d*x)] + 294*Tan[c/2]))/(13440*a^3*d*(1 + Sec[c + d*x])^3)
```

3.100.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 25, 25, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^8}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^8(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos^3(c+dx)\sin^8(c+dx)}{(\cos(c+dx)a+a)^3}dx$$
$$\downarrow 25$$
$$\int \frac{\sin^8(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^8}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 3354$$

$$\frac{\int \cos^3(c+dx)(a-a\cos(c+dx))^3\sin^2(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int \cos\left(c+dx+\frac{\pi}{2}\right)^2 \sin\left(c+dx+\frac{\pi}{2}\right)^3 \left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$

$$\downarrow 3352$$

$$\frac{\int \left(-a^3\sin^2(c+dx)\cos^6(c+dx)+3a^3\sin^2(c+dx)\cos^5(c+dx)-3a^3\sin^2(c+dx)\cos^4(c+dx)+a^3\sin^2(c+dx)\right.}{a^6}$$

$$\downarrow 2009$$

$$\frac{\frac{3a^3\sin^7(c+dx)}{7d}-\frac{7a^3\sin^5(c+dx)}{5d}+\frac{4a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{23a^3\sin(c+dx)\cos^5(c+dx)}{48d}-\frac{29a^3\sin(c+dx)\cos^3(c+dx)}{192d}}{a^6}$$

input
```
Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output
```
((-29*a^3*x)/128 - (29*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d) - (29*a^3*Co
s[c + d*x]^3*Sin[c + d*x])/(192*d) + (23*a^3*Cos[c + d*x]^5*Sin[c + d*x])/
(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) + (4*a^3*Sin[c + d*x]^3)/
(3*d) - (7*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Sin[c + d*x]^7)/(7*d))/a^6
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.100.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{-24360dx+38640\sin(dx+c)+105\sin(8dx+8c)+2240\sin(6dx+6c)+5880\sin(4dx+4c)-6720\sin(2dx+2c)-720\sin(7dx-}{107520a^3d}$
risch	$-\frac{29x}{128a^3}+\frac{23\sin(dx+c)}{64a^3d}+\frac{\sin(8dx+8c)}{1024a^3d}-\frac{3\sin(7dx+7c)}{448a^3d}+\frac{\sin(6dx+6c)}{48a^3d}-\frac{13\sin(5dx+5c)}{320a^3d}+\frac{7\sin(4dx+4c)}{128a^3d}$
derivativedivides	$-\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12288}-\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{61440}-\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{430080}-\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{143360}-\frac{1759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^3d}$
default	$-\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12288}-\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{61440}-\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{430080}-\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{143360}-\frac{1759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^3d}$
norman	$-\frac{29x}{128a}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192ad}+\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{960ad}+\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6720ad}+\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2240ad}+\frac{1759\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{320ad}$

input
```
int(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/107520*(-24360*d*x+38640*sin(d*x+c)+105*sin(8*d*x+8*c)+2240*sin(6*d*x+6*
c)+5880*sin(4*d*x+4*c)-6720*sin(2*d*x+2*c)-720*sin(7*d*x+7*c)-4368*sin(5*d
*x+5*c)-3920*sin(3*d*x+3*c))/a^3/d
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3045\,dx - (1680\cos(dx+c)^7 - 5760\cos(dx+c)^6 + 6440\cos(dx+c)^5 - 1536\cos(dx+c)^4 - 2030}{13440\,a^3 d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/13440*(3045*d*x - (1680*cos(d*x + c)^7 - 5760*cos(d*x + c)^6 + 6440*cos
(d*x + c)^5 - 1536*cos(d*x + c)^4 - 2030*cos(d*x + c)^3 + 2432*cos(d*x + c
)^2 - 3045*cos(d*x + c) + 4864)*sin(d*x + c))/(a^3*d)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(141) = 282.

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.41

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{3045\sin(dx+c)}{\cos(dx+c)+1} + \frac{23345\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{77749\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{146537\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{218007\sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{36939\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{120015\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{3045\sin(dx+c)}{(\cos(dx+c)+1)}}{a^3 + \frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{1}}}}{6720\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) + 23345*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 77749*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 146537*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 + 218007*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + 36939*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 120015*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 +
8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x
 + c)^8/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(
cos(d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{3045\,(dx+c)}{a^3} + \frac{2\left(3045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15} - 120015\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13} - 36939\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 218007\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} - 146537\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^8 a^3}}{13440\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 - 120015*ta
n(1/2*d*x + 1/2*c)^13 - 36939*tan(1/2*d*x + 1/2*c)^11 - 218007*tan(1/2*d*x
 + 1/2*c)^9 - 146537*tan(1/2*d*x + 1/2*c)^7 - 77749*tan(1/2*d*x + 1/2*c)^5
 - 23345*tan(1/2*d*x + 1/2*c)^3 - 3045*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^8*a^3))/d
```

3.100.9 Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{-\frac{29\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{1143\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{1759\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{320}+\frac{72669\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{2240}+\frac{146537\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{6720}+\frac{11107\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{960}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}$$

$$-\frac{29\,x}{128\,a^3}$$

input
```
int(sin(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

output
```
((29*tan(c/2 + (d*x)/2))/64 + (667*tan(c/2 + (d*x)/2)^3)/192 + (11107*tan(
c/2 + (d*x)/2)^5)/960 + (146537*tan(c/2 + (d*x)/2)^7)/6720 + (72669*tan(c/
2 + (d*x)/2)^9)/2240 + (1759*tan(c/2 + (d*x)/2)^11)/320 + (1143*tan(c/2 +
(d*x)/2)^13)/64 - (29*tan(c/2 + (d*x)/2)^15)/64)/(a^3*d*(tan(c/2 + (d*x)/2
)^2 + 1)^8) - (29*x)/(128*a^3)
```

3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.101.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{23x}{16a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} - \frac{7\sin^3(c+dx)}{3a^3d} + \frac{3\sin^5(c+dx)}{5a^3d}$$

output
```
-23/16*x/a^3+4*sin(d*x+c)/a^3/d-23/16*cos(d*x+c)*sin(d*x+c)/a^3/d-23/24*co
s(d*x+c)^3*sin(d*x+c)/a^3/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d-7/3*sin(d*x+
c)^3/a^3/d+3/5*sin(d*x+c)^5/a^3/d
```

3.101.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-2760dx + 5040\sin(c+dx) - 1890\sin(2(c+dx)) + 760\sin(3(c+dx)) - \right.}{240a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-2760*d*x + 5040*Sin[c + d*x] - 1890*S
in[2*(c + d*x)] + 760*Sin[3*(c + d*x)] - 270*Sin[4*(c + d*x)] + 72*Sin[5*(
c + d*x)] - 10*Sin[6*(c + d*x)] + 9*Tan[c/2]))/(240*a^3*d*(1 + Sec[c + d*x
])^3)
```

3.101.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 25, 25, 3042, 3348, 3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^6(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos^3(c+dx)\sin^6(c+dx)}{(\cos(c+dx)a+a)^3}dx$$
$$\downarrow 25$$
$$\int \frac{\sin^6(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 3348$$
$$\frac{\int \cos^3(c+dx)(a-a\cos(c+dx))^3dx}{a^6}$$
$$\downarrow 3042$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^3 \left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$

$$\downarrow 3236$$

$$\frac{\int \left(-a^3\cos^6(c+dx)+3a^3\cos^5(c+dx)-3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\right) dx}{a^6}$$

$$\downarrow 2009$$

$$\frac{\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}-\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{23a^3\sin(c+dx)\cos(c+dx)}{16d}}{a^6}$$

input
```
Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
((-23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d - (23*a^3*Cos[c + d*x]*Sin[c + d*
x])/(16*d) - (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^3*Cos[c + d*
x]^5*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d
*x]^5)/(5*d))/a^6
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

rule 3348
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a^(2*m)   Int[(d*
Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.101.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{-1380dx+2520\sin(dx+c)+36\sin(5dx+5c)+380\sin(3dx+3c)-5\sin(6dx+6c)-135\sin(4dx+4c)-945\sin(2dx+2c)}{960a^3d}$
risch	$-\frac{23x}{16a^3}+\frac{21\sin(dx+c)}{8a^3d}-\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(5dx+5c)}{80a^3d}-\frac{9\sin(4dx+4c)}{64a^3d}+\frac{19\sin(3dx+3c)}{48a^3d}-\frac{63\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{16\left(-\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{128}-\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{128}-\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{320}-\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{320}-\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{384}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}-\frac{2}{a^3d}$
default	$\frac{16\left(-\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{128}-\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{128}-\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{320}-\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{320}-\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{384}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}-\frac{2}{a^3d}$
norman	$\frac{-\frac{23x}{16a}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24ad}+\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}+\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}+\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{8ad}+\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}-\frac{69}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

input
```
int(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/960*(-1380*d*x+2520*sin(d*x+c)+36*sin(5*d*x+5*c)+380*sin(3*d*x+3*c)-5*si
n(6*d*x+6*c)-135*sin(4*d*x+4*c)-945*sin(2*d*x+2*c))/a^3/d
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{345\,dx+\left(40\cos(dx+c)^5-144\cos(dx+c)^4+230\cos(dx+c)^3-272\cos(dx+c)^2+345\cos(dx+\right.}{240\,a^3d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/240*(345*d*x + (40*cos(d*x + c)^5 - 144*cos(d*x + c)^4 + 230*cos(d*x +
c)^3 - 272*cos(d*x + c)^2 + 345*cos(d*x + c) - 544)*sin(d*x + c))/(a^3*d)
```

3.101.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(117) = 234$.

Time $= 0.28$ (sec) , antiderivative size $= 292$, normalized size of antiderivative $= 2.26$

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5814\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3165\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1575\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} - \frac{345\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) + 1955*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 4554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5814*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 3165*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1575
*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{\frac{345\,(dx+c)}{a^3} - \frac{2\left(1575\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3165\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5814\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4554\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1955\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+345\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6 a^3}}{240\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/240*(345*(d*x + c)/a^3 - 2*(1575*tan(1/2*d*x + 1/2*c)^11 + 3165*tan(1/2
*d*x + 1/2*c)^9 + 5814*tan(1/2*d*x + 1/2*c)^7 + 4554*tan(1/2*d*x + 1/2*c)^
5 + 1955*tan(1/2*d*x + 1/2*c)^3 + 345*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^6*a^3))/d
```

3.101.9 Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{211\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{8} + \frac{969\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{20} + \frac{759\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20} + \frac{391\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{24} + \frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6}$$

$$-\frac{23\,x}{16\,a^3}$$

input
```
int(sin(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

output
```
((23*tan(c/2 + (d*x)/2))/8 + (391*tan(c/2 + (d*x)/2)^3)/24 + (759*tan(c/2
+ (d*x)/2)^5)/20 + (969*tan(c/2 + (d*x)/2)^7)/20 + (211*tan(c/2 + (d*x)/2)
^9)/8 + (105*tan(c/2 + (d*x)/2)^11)/8)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6
) - (23*x)/(16*a^3)
```

3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.102.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{51x}{8a^3} - \frac{7\sin(c+dx)}{a^3d} + \frac{19\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))} + \frac{\sin^3(c+dx)}{a^3d}$$

output

```
51/8*x/a^3-7*sin(d*x+c)/a^3/d+19/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x
+c)^3*sin(d*x+c)/a^3/d-4*sin(d*x+c)/a^3/d/(1+cos(d*x+c))+sin(d*x+c)^3/a^3/
d
```

3.102.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(2040dx\cos\left(\frac{dx}{2}\right)+2040dx\cos\left(c+\frac{dx}{2}\right)-3563\sin\left(\frac{dx}{2}\right)-997\sin\left(c+\frac{dx}{2}\right)-800\,\mathrm{s}\right.}{}$$

input

```
Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(2040*d*x*Cos[(d*x)/2] + 2040*d*x*Cos[c + (d*x)
/2] - 3563*Sin[(d*x)/2] - 997*Sin[c + (d*x)/2] - 800*Sin[c + (3*d*x)/2] -
800*Sin[2*c + (3*d*x)/2] + 160*Sin[2*c + (5*d*x)/2] + 160*Sin[3*c + (5*d*x
)/2] - 35*Sin[3*c + (7*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*
d*x)/2] + 5*Sin[5*c + (9*d*x)/2]))/(640*a^3*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 3354, 3042, 25, 3351, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^4(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}\,dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^4(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^4(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 3354$$

$$\frac{\int \cos(c+dx)(a-a\cos(c+dx))^3\cot^2(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c-\pi)+dx\right)a+a\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2}dx}{a^6}$$

$$\downarrow 3351$$

$$-\frac{\int \left(-a\cos^4(c+dx)+3a\cos^3(c+dx)-4a\cos^2(c+dx)+4a\cos(c+dx)-4a+\frac{4a}{\cos(c+dx)+1}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a\sin^3(c+dx)}{d}+\frac{7a\sin(c+dx)}{d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{19a\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a\sin(c+dx)}{d(\cos(c+dx)+1)}-\frac{51ax}{8}}{a^4}$$

input
```
Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-(((-51*a*x)/8 + (7*a*Sin[c + d*x])/d - (19*a*Cos[c + d*x]*Sin[c + d*x])/(
8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (4*a*Sin[c + d*x])/(d*(1 +
Cos[c + d*x])) - (a*Sin[c + d*x]^3)/d)/a^4)
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3351
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/a^p   Int[Expan
dTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
 + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (G
tQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.102.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{204dx+(-295+\cos(4dx+4c)-6\cos(3dx+3c)+26\cos(2dx+2c)-134\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^3d}$
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16}-\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16}-\frac{123\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16}-\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16}-\frac{123\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$
risch	$\frac{51x}{8a^3}+\frac{25ie^{i(dx+c)}}{8a^3d}-\frac{25ie^{-i(dx+c)}}{8a^3d}-\frac{8i}{a^3d\left(e^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32a^3d}-\frac{\sin(3dx+3c)}{4a^3d}+\frac{5\sin(2dx+2c)}{4a^3d}$
norman	$\frac{\frac{51x}{8a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{187\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{245\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4ad}-\frac{141\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{ad}+\frac{51x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{153x\,\text{ta}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a^2}$

input
```
int(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(204*d*x+(-295+cos(4*d*x+4*c)-6*cos(3*d*x+3*c)+26*cos(2*d*x+2*c)-134*
cos(d*x+c))*tan(1/2*d*x+1/2*c))/a^3/d
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{51\,dx\cos(dx+c)+51\,dx+\left(2\cos(dx+c)^4-6\cos(dx+c)^3+11\cos(dx+c)^2-29\cos(dx+c)-80\right.}{8\left(a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(51*d*x*cos(d*x + c) + 51*d*x + (2*cos(d*x + c)^4 - 6*cos(d*x + c)^3 +
 11*cos(d*x + c)^2 - 29*cos(d*x + c) - 80)*sin(d*x + c))/(a^3*d*cos(d*x +
c) + a^3*d)
```

3.102.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.10

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{149\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{77\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{51\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{16\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{4d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*((35*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 149*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 77*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 51*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + 16*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{51\,(dx+c)}{a^3}-\frac{32\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{2\left(77\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+149\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+123\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8d}$$

input
```
integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(51*(d*x + c)/a^3 - 32*tan(1/2*d*x + 1/2*c)/a^3 - 2*(77*tan(1/2*d*x +
1/2*c)^7 + 149*tan(1/2*d*x + 1/2*c)^5 + 123*tan(1/2*d*x + 1/2*c)^3 + 35*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

3.102.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{51\,x}{8\,a^3} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3\,d} - \frac{\frac{77\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} + \frac{149\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} + \frac{123\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4} + \frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4}$$

input
```
int(sin(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

output
```
(51*x)/(8*a^3) - (4*tan(c/2 + (d*x)/2))/(a^3*d) - ((35*tan(c/2 + (d*x)/2))
/4 + (123*tan(c/2 + (d*x)/2)^3)/4 + (149*tan(c/2 + (d*x)/2)^5)/4 + (77*tan
(c/2 + (d*x)/2)^7)/4)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.103.1 Optimal result 802
3.103.2 Mathematica [A] (verified) 802
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3.103.8 Giac [A] (verification not implemented) 807
3.103.9 Mupad [B] (verification not implemented) 807

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{11x}{2a^3} + \frac{3\sin(c+dx)}{a^3 d} - \frac{\cos(c+dx)\sin(c+dx)}{2a^3 d} - \frac{2\sin(c+dx)}{3a^3 d(1+\cos(c+dx))^2} + \frac{19\sin(c+dx)}{3a^3 d(1+\cos(c+dx))}$$

output
```
-11/2*x/a^3+3*sin(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d-2/3*sin(d*x
+c)/a^3/d/(1+cos(d*x+c))^2+19/3*sin(d*x+c)/a^3/d/(1+cos(d*x+c))
```

3.103.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(1980dx\cos\left(\frac{dx}{2}\right)+1980dx\cos\left(c+\frac{dx}{2}\right)+660dx\cos\left(c+\frac{3dx}{2}\right)+660dx\cos\left(2c\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/960*(Sec[c/2]*Sec[(c + d*x)/2]^3*(1980*d*x*Cos[(d*x)/2] + 1980*d*x*Cos[
c + (d*x)/2] + 660*d*x*Cos[c + (3*d*x)/2] + 660*d*x*Cos[2*c + (3*d*x)/2] -
 3216*Sin[(d*x)/2] + 1326*Sin[c + (d*x)/2] - 2012*Sin[c + (3*d*x)/2] - 498
*Sin[2*c + (3*d*x)/2] - 135*Sin[2*c + (5*d*x)/2] - 135*Sin[3*c + (5*d*x)/2
] + 15*Sin[3*c + (7*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(a^3*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 25, 25, 3042, 3353, 3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^2(c+dx)\cos^3(c+dx)}{(a(-\cos(c+dx))-a)^3}\,dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^2(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^2(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 3353$$

$$\frac{\int \frac{\cos^3(c+dx)(a-a\cos(c+dx))}{(\cos(c+dx)a+a)^2}dx}{a^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3 (a-a\sin\left(c+dx+\frac{\pi}{2}\right))}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}$$

$$\downarrow 3445$$

$$\frac{\int \left(-\frac{\cos^2(c+dx)}{a}+\frac{3\cos(c+dx)}{a}-\frac{5}{a}+\frac{7}{a(\cos(c+dx)+1)}-\frac{2}{a(\cos(c+dx)+1)^2}\right)dx}{a^2}$$

$$\downarrow 2009$$

$$\frac{\frac{3\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{19\sin(c+dx)}{3ad(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3ad(\cos(c+dx)+1)^2}-\frac{11x}{2a}}{a^2}$$

input
```
Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
((-11*x)/(2*a) + (3*Sin[c + d*x])/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a
*d) - (2*Sin[c + d*x])/(3*a*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x])/(3
*a*d*(1 + Cos[c + d*x])))/a^2
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3353
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/b^2   Int[(d*Sin[e
 + f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[
n, 0])
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.103.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\frac{275\left(\cos(dx+c)+\frac{24\cos(2dx+2c)}{275}-\frac{3\cos(3dx+3c)}{275}+\frac{232}{275}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{48}-\frac{11dx}{2}}{a^3d}$	66
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da^3}$	87
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da^3}$	87
risch	$-\frac{11x}{2a^3}+\frac{ie^{2i(dx+c)}}{8a^3d}-\frac{3ie^{i(dx+c)}}{2a^3d}+\frac{3ie^{-i(dx+c)}}{2a^3d}-\frac{ie^{-2i(dx+c)}}{8a^3d}+\frac{2i\left(21e^{2i(dx+c)}+36e^{i(dx+c)}+19\right)}{3a^3d\left(e^{i(dx+c)}+1\right)^3}$	126
norman	$\frac{-\frac{11x}{2a}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3ad}-\frac{11x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{11x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^2}$	135

input
```
int(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
11/48*(25*(cos(d*x+c)+24/275*cos(2*d*x+2*c)-3/275*cos(3*d*x+3*c)+232/275)*
tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-24*d*x)/a^3/d
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{33\,dx\cos(dx+c)^2+66\,dx\cos(dx+c)+33\,dx+(3\cos(dx+c)^3-12\cos(dx+c)^2-71\cos(dx+c)}{6\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/6*(33*d*x*cos(d*x + c)^2 + 66*d*x*cos(d*x + c) + 33*d*x + (3*cos(d*x +
c)^3 - 12*cos(d*x + c)^2 - 71*cos(d*x + c) - 52)*sin(d*x + c))/(a^3*d*cos(
d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

3.103.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.69

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{3\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{18\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(3*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4) + (18*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 33*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^3)/d
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{33\,(dx+c)}{a^3} - \frac{6\left(7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} + \frac{2\left(a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-18\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^9}}{6\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(33*(d*x + c)/a^3 - 6*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 2*(a^6*tan(1/2*d*x + 1/2*c)^3
- 18*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 38\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin}{6\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(sin(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

output

```
-(2*sin(c/2 + (d*x)/2) - 38*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 42*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 +
 (d*x)/2) + 33*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^3*d*cos(c/2 + (d*x)/2)
^3)
```

3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.104.2 Mathematica [A] (verified) . . . 809
3.104.3 Rubi [F] . . . 810
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3.104.9 Mupad [B] (verification not implemented) . . . 814

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{4\cot^7(c+dx)}{7a^3d} - \frac{\csc^3(c+dx)}{a^3d} + \frac{7\csc^5(c+dx)}{5a^3d} - \frac{4\csc^7(c+dx)}{7a^3d}$$

output
```
3/5*cot(d*x+c)^5/a^3/d+4/7*cot(d*x+c)^7/a^3/d-csc(d*x+c)^3/a^3/d+7/5*csc(d
*x+c)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

3.104.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\csc(c)\csc(c+dx)\sec^3(c+dx)(-840\sin(c)+448\sin(dx)+602\sin(c+dx)+602\sin(2(c+dx))+258\sin$$

input
```
Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Csc[c]*Csc[c + d*x]*Sec[c + d*x]^3*(-840*Sin[c] + 448*Sin[d*x] + 602*Sin[
c + d*x] + 602*Sin[2*(c + d*x)] + 258*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x
)] - 560*Sin[2*c + d*x] + 168*Sin[c + 2*d*x] - 280*Sin[3*c + 2*d*x] - 48*S
in[2*c + 3*d*x] - 8*Sin[3*c + 4*d*x]))/(2240*a^3*d*(1 + Sec[c + d*x])^3)
```

3.104.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)\cot^2(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\cot^2(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3354$$

$$-\frac{\int -(a-a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$\frac{\int (a-a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^8}dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c-\pi)+dx\right)a+a\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^8}dx}{a^6}$$

$$\downarrow 3352$$

$$-\frac{\int \left(a^3\cos^6(c+dx)\sec^8\left(\frac{1}{2}(2c-\pi)+dx\right)-3a^3\cos^5(c+dx)\sec^8\left(\frac{1}{2}(2c-\pi)+dx\right)+3a^3\cos^4(c+dx)\sec^8\left(\frac{1}{2}(2c\right.\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{a^3\int \cos^6(c+dx)\sec^8\left(\frac{1}{2}(2c-\pi)+dx\right)dx-3a^3\int \cos^5(c+dx)\sec^8\left(\frac{1}{2}(2c-\pi)+dx\right)dx+3a^3\int \cos^4(c+dx)\se}{a^6}$$

input
```
Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
$Aborted
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.104.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-70\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{560a^3d}$	58
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da^3}$	60
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da^3}$	60
norman	$\frac{-\frac{1}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{40da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{112da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}$	82
risch	$-\frac{2i\left(35\,e^{6i(dx+c)}+70\,e^{5i(dx+c)}+105\,e^{4i(dx+c)}+56\,e^{3i(dx+c)}+21\,e^{2i(dx+c)}-6\,e^{i(dx+c)}-1\right)}{35a^3d\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)}$	104

input
```
int(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/560*(-5*tan(1/2*d*x+1/2*c)^7+14*tan(1/2*d*x+1/2*c)^5-70*tan(1/2*d*x+1/2*
c)-35*cot(1/2*d*x+1/2*c))/a^3/d
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos(dx+c)^4+3\cos(dx+c)^3-15\cos(dx+c)^2-18\cos(dx+c)-6}{35\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/35*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 15*cos(d*x + c)^2 - 18*cos(d*x +
 c) - 6)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)*sin(d*x + c))
```

3.104.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{70\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}+\frac{35(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{560\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/560*((70*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 + 35*(cos(d*x + c)
 + 1)/(a^3*sin(d*x + c)))/d
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{35}{a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)} + \frac{5a^{18}\tan(\frac{1}{2}dx+\frac{1}{2}c)^7 - 14a^{18}\tan(\frac{1}{2}dx+\frac{1}{2}c)^5 + 70a^{18}\tan(\frac{1}{2}dx+\frac{1}{2}c)}{a^{21}}}{560\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/560*(35/(a^3*tan(1/2*d*x + 1/2*c)) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 1
4*a^18*tan(1/2*d*x + 1/2*c)^5 + 70*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.104.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{-16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 72\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 34\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 5}{560\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

output
```
-(72*cos(c/2 + (d*x)/2)^4 - 34*cos(c/2 + (d*x)/2)^2 + 8*cos(c/2 + (d*x)/2)
^6 - 16*cos(c/2 + (d*x)/2)^8 + 5)/(560*a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2
+ (d*x)/2))
```

3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.105.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{\cot^7(c+dx)}{a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc^5(c+dx)}{5a^3d} + \frac{\csc^7(c+dx)}{a^3d} - \frac{4\csc^9(c+dx)}{9a^3d}$$

output
```
3/5*cot(d*x+c)^5/a^3/d+cot(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3/5*csc(d
*x+c)^5/a^3/d+csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^3/d
```

3.105.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\csc(c)\csc^3(2(c+dx))(5376\sin(c) - 1152\sin(dx) - 1764\sin(c+dx) - 1323\sin(2(c+dx)) + 98\sin(3(c$$

input
```
Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5760*(Csc[c]*Csc[2*(c + d*x)]^3*(5376*Sin[c] - 1152*Sin[d*x] - 1764*Sin
[c + d*x] - 1323*Sin[2*(c + d*x)] + 98*Sin[3*(c + d*x)] + 588*Sin[4*(c + d
*x)] + 294*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)] + 3456*Sin[2*c + d*x] -
1152*Sin[c + 2*d*x] + 2880*Sin[3*c + 2*d*x] - 128*Sin[2*c + 3*d*x] - 768*S
in[3*c + 4*d*x] - 384*Sin[4*c + 5*d*x] - 64*Sin[5*c + 6*d*x]))/(a^3*d*(1 +
 Sec[c + d*x])^3)
```

3.105.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3354$$

$$-\frac{\int -(a-a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$\frac{\int (a-a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^{10}}dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c-\pi)+dx\right)a+a\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{10}}dx}{a^6}$$

$$\downarrow 3352$$

$$-\frac{\int \left(a^3\cos^6(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)-3a^3\cos^5(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)+3a^3\cos^4(c+dx)\sec^{10}\left(\frac{1}{2}(\right.\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{a^3\int \cos^6(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)dx-3a^3\int \cos^5(c+dx)\sec^{10}\left(\frac{1}{2}(2c-\pi)+dx\right)dx+3a^3\int \cos^4(c+dx)}{a^6}$$

input
```
Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
$Aborted
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.105.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

method	result	si
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64da^3}$	60
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64da^3}$	60
parallelrisch	$\frac{-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-135\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-15}{2880da^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	61
norman	$\frac{-\frac{1}{192ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{576ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{64da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{320da}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	82
risch	$\frac{4i\left(45\,e^{8i(dx+c)}+54\,e^{7i(dx+c)}+84\,e^{6i(dx+c)}+18\,e^{5i(dx+c)}+18\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}+12\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+1\right)}{45a^3d\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^3}$	12

input
```
int(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+3/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d
*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\cos(dx+c)^6 + 6\cos(dx+c)^5 + 3\cos(dx+c)^4 - 7\cos(dx+c)^3 + 3\cos(dx+c)^2 + 6\cos(d}{45\left(a^3 d\cos(dx+c)^5 + 3a^3 d\cos(dx+c)^4 + 2a^3 d\cos(dx+c)^3 - 2a^3 d\cos(dx+c)^2 - 3a^3 d\cos(dx+c)\right.}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/45*(2*cos(d*x + c)^6 + 6*cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 7*cos(d*x +
 c)^3 + 3*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)/((a^3*d*cos(d*x + c)^5 + 3*
a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3
*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

3.105.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1} - \frac{27\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{2880\,d} + \frac{15\left(\cos(dx+c)+1\right)^3}{a^3\sin(dx+c)^3}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2880*((135*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 + 15*(cos(d*x +
c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{15}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-27\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+135\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{27}}}{2880\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2880*(15/(a^3*tan(1/2*d*x + 1/2*c)^3) + (5*a^24*tan(1/2*d*x + 1/2*c)^9
- 27*a^24*tan(1/2*d*x + 1/2*c)^5 + 135*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

3.105.9 Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+135\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-27\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{2880\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

output
```
-(15*cos(c/2 + (d*x)/2)^12 + 5*sin(c/2 + (d*x)/2)^12 - 27*cos(c/2 + (d*x)/
2)^4*sin(c/2 + (d*x)/2)^8 + 135*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)
/(2880*a^3*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)
```

3.106 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.106.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{10\cot^7(c+dx)}{7a^3d} + \frac{11\cot^9(c+dx)}{9a^3d} + \frac{4\cot^{11}(c+dx)}{11a^3d} - \frac{3\csc^7(c+dx)}{7a^3d} + \frac{7\csc^9(c+dx)}{9a^3d} - \frac{4\csc^{11}(c+dx)}{11a^3d}$$

output
```
3/5*cot(d*x+c)^5/a^3/d+10/7*cot(d*x+c)^7/a^3/d+11/9*cot(d*x+c)^9/a^3/d+4/1
1*cot(d*x+c)^11/a^3/d-3/7*csc(d*x+c)^7/a^3/d+7/9*csc(d*x+c)^9/a^3/d-4/11*c
sc(d*x+c)^11/a^3/d
```

3.106.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\csc(c)\csc^5(c+dx)\sec^3(c+dx)(-3886080\sin(c)+563200\sin(dx)+524150\sin(c+dx)+314490\sin(2(c$$

input
```
Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^3*(-3886080*Sin[c] + 563200*Sin[d*x] +
 524150*Sin[c + d*x] + 314490*Sin[2*(c + d*x)] - 162010*Sin[3*(c + d*x)] -
 238250*Sin[4*(c + d*x)] - 47650*Sin[5*(c + d*x)] + 47650*Sin[6*(c + d*x)]
 + 28590*Sin[7*(c + d*x)] + 4765*Sin[8*(c + d*x)] - 2027520*Sin[2*c + d*x]
 + 1486848*Sin[c + 2*d*x] - 2365440*Sin[3*c + 2*d*x] + 452608*Sin[2*c + 3*
d*x] + 665600*Sin[3*c + 4*d*x] + 133120*Sin[4*c + 5*d*x] - 133120*Sin[5*c
+ 6*d*x] - 79872*Sin[6*c + 7*d*x] - 13312*Sin[7*c + 8*d*x]))/(56770560*a^3
*d*(1 + Sec[c + d*x])^3)
```

3.106.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc^3(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc^3(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc^3(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3354$$

$$-\frac{\int -(a-a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$\frac{\int (a-a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^{12}}dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c-\pi)+dx\right)a+a\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{12}}dx}{a^6}$$

$$\downarrow 3352$$

$$-\frac{\int \left(a^3\cos^6(c+dx)\sec^{12}\left(\frac{1}{2}(2c-\pi)+dx\right)-3a^3\cos^5(c+dx)\sec^{12}\left(\frac{1}{2}(2c-\pi)+dx\right)+3a^3\cos^4(c+dx)\sec^{12}\left(\frac{1}{2}(\right.\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{a^3\int \cos^6(c+dx)\sec^{12}\left(\frac{1}{2}(2c-\pi)+dx\right)dx-3a^3\int \cos^5(c+dx)\sec^{12}\left(\frac{1}{2}(2c-\pi)+dx\right)dx+3a^3\int \cos^4(c+dx)}{a^6}$$

input
```
Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
$Aborted
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.106.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-770\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+990\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+4158\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-693\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-2310\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{887040a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{3\tan(}{}}{256da^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{3\tan(}{}}{256da^3}$
risch	$-\frac{16i\left(2310\,e^{10i(dx+c)}+1980\,e^{9i(dx+c)}+3795\,e^{8i(dx+c)}+550\,e^{7i(dx+c)}+1452\,e^{6i(dx+c)}+442\,e^{5i(dx+c)}+650\,e^{4i(dx+c)}+1\right.}{3465a^3d\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{5}}$
norman	$\frac{-\frac{1}{1280ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{896ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1152ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{2816ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{384da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{128da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{128da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{640da}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}$

input
```
int(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/887040*(-315*tan(1/2*d*x+1/2*c)^11-770*tan(1/2*d*x+1/2*c)^9+990*tan(1/2*
d*x+1/2*c)^7+4158*tan(1/2*d*x+1/2*c)^5-693*cot(1/2*d*x+1/2*c)^5-2310*cot(1
/2*d*x+1/2*c)^3-20790*tan(1/2*d*x+1/2*c)+6930*cot(1/2*d*x+1/2*c))/a^3/d
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{104\cos(dx+c)^8 + 312\cos(dx+c)^7 + 52\cos(dx+c)^6 - 676\cos(dx+c)^5 - 585\cos(dx+c)^4 + 325}{3465\left(a^3d\cos(dx+c)^7 + 3\,a^3d\cos(dx+c)^6 + a^3d\cos(dx+c)^5 - 5\,a^3d\cos(dx+c)^4 - 5\,a^3d\cos(dx+c)\right.}$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3465*(104*cos(d*x + c)^8 + 312*cos(d*x + c)^7 + 52*cos(d*x + c)^6 - 676*
cos(d*x + c)^5 - 585*cos(d*x + c)^4 + 325*cos(d*x + c)^3 - 25*cos(d*x + c)
^2 - 150*cos(d*x + c) - 50)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^
6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3
 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

3.106.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.37

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{\frac{20790\sin(dx+c)}{\cos(dx+c)+1}-\frac{4158\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{990\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3}+\frac{231\left(\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c$$
$$887040\,d$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/887040*((20790*sin(d*x + c)/(cos(d*x + c) + 1) - 4158*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 990*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x
 + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/
a^3 + 231*(10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 30*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{231\left(30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{315a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+770a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-990a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-4158a^{30}\tan$$
$$887040\,d$$

input
```
integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/887040*(231*(30*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*x + 1/2*c)^2 - 3)/
(a^3*tan(1/2*d*x + 1/2*c)^5) - (315*a^30*tan(1/2*d*x + 1/2*c)^11 + 770*a^3
0*tan(1/2*d*x + 1/2*c)^9 - 990*a^30*tan(1/2*d*x + 1/2*c)^7 - 4158*a^30*tan
(1/2*d*x + 1/2*c)^5 + 20790*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

3.106.9 Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.58

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+2310\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+20790\cos$$

input
```
int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

output
```
-(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 + 770*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 - 990*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^12 - 4158*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 20790*cos(c/2 +
(d*x)/2)^10*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d
*x)/2)^4 + 2310*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(887040*a^3*d*
cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5)
```

3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.107.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{13\cot^7(c+dx)}{7a^3d} + \frac{7\cot^9(c+dx)}{3a^3d} + \frac{15\cot^{11}(c+dx)}{11a^3d} + \frac{4\cot^{13}(c+dx)}{13a^3d} - \frac{\csc^9(c+dx)}{3a^3d} + \frac{7\csc^{11}(c+dx)}{11a^3d} - \frac{4\csc^{13}(c+dx)}{13a^3d}$$

output
```
3/5*cot(d*x+c)^5/a^3/d+13/7*cot(d*x+c)^7/a^3/d+7/3*cot(d*x+c)^9/a^3/d+15/1
1*cot(d*x+c)^11/a^3/d+4/13*cot(d*x+c)^13/a^3/d-1/3*csc(d*x+c)^9/a^3/d+7/11
*csc(d*x+c)^11/a^3/d-4/13*csc(d*x+c)^13/a^3/d
```

3.107.2 Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.83

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\csc(c)\csc^7(c+dx)\sec^3(c+dx)(49201152\sin(c) - 6336512\sin(dx) - 2764580\sin(c+dx) - 1382290\,\mathrm{si}$$

input
```
Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output

```
-1/984023040*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^3*(49201152*Sin[c] - 6336
512*Sin[d*x] - 2764580*Sin[c + d*x] - 1382290*Sin[2*(c + d*x)] + 1275960*S
in[3*(c + d*x)] + 1336720*Sin[4*(c + d*x)] - 60760*Sin[5*(c + d*x)] - 5240
55*Sin[6*(c + d*x)] - 167090*Sin[7*(c + d*x)] + 60760*Sin[8*(c + d*x)] + 4
5570*Sin[9*(c + d*x)] + 7595*Sin[10*(c + d*x)] + 20500480*Sin[2*c + d*x] -
 23668736*Sin[c + 2*d*x] + 30750720*Sin[3*c + 2*d*x] - 6537216*Sin[2*c + 3
*d*x] - 6848512*Sin[3*c + 4*d*x] + 311296*Sin[4*c + 5*d*x] + 2684928*Sin[5
*c + 6*d*x] + 856064*Sin[6*c + 7*d*x] - 311296*Sin[7*c + 8*d*x] - 233472*S
in[8*c + 9*d*x] - 38912*Sin[9*c + 10*d*x]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

3.107.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^8(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc^5(c+dx)}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc^5(c+dx)}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc^5(c+dx)}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^8\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^8\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3354$$

$$-\frac{\int -(a-a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$\frac{\int (a-a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^{14}}dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(\sin\left(\frac{1}{2}(2c-\pi)+dx\right)a+a\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{14}}dx}{a^6}$$

$$\downarrow 3352$$

$$-\frac{\int \left(a^3\cos^6(c+dx)\sec^{14}\left(\frac{1}{2}(2c-\pi)+dx\right)-3a^3\cos^5(c+dx)\sec^{14}\left(\frac{1}{2}(2c-\pi)+dx\right)+3a^3\cos^4(c+dx)\sec^{14}\left(\frac{1}{2}(\right.\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{a^3\int \cos^6(c+dx)\sec^{14}\left(\frac{1}{2}(2c-\pi)+dx\right)dx-3a^3\int \cos^5(c+dx)\sec^{14}\left(\frac{1}{2}(2c-\pi)+dx\right)dx+3a^3\int \cos^4(c+dx)}{a^6}$$

input
```
Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output
```
$Aborted
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.107.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{-1155\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-5460\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-5005\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+17160\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+42042\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-210210\tan}{15375360a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{13}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{3}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-}{1024da^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{13}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{3}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-}{1024da^3}$
risch	$\frac{32i\left(15015\,\mathrm{e}^{12i(dx+c)}+10010\,\mathrm{e}^{11i(dx+c)}+24024\,\mathrm{e}^{10i(dx+c)}+3094\,\mathrm{e}^{9i(dx+c)}+11557\,\mathrm{e}^{8i(dx+c)}+3192\,\mathrm{e}^{7i(dx+c)}+3344\,\mathrm{e}^{6i(d}{15015a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^{13}\left(\mathrm{e}^{i(dx+c)}-1\right.}$

input
```
int(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/15375360*(-1155*tan(1/2*d*x+1/2*c)^13-5460*tan(1/2*d*x+1/2*c)^11-5005*ta
n(1/2*d*x+1/2*c)^9+17160*tan(1/2*d*x+1/2*c)^7+42042*tan(1/2*d*x+1/2*c)^5-2
10210*tan(1/2*d*x+1/2*c)-2145*cot(1/2*d*x+1/2*c)*(cot(1/2*d*x+1/2*c)^6+28/
5*cot(1/2*d*x+1/2*c)^4+7*cot(1/2*d*x+1/2*c)^2-56))/a^3/d
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{304\cos(dx+c)^{10} + 912\cos(dx+c)^9 - 152\cos(dx+c)^8 - 2888\cos(dx+c)^7 - 1862\cos(dx+c)^6 + 2}{15015\left(a^3 d\cos(dx+c)^9 + 3\,a^3 d\cos(dx+c)^8 - 8\,a^3 d\cos(dx+c)^6 - 6\,a^3 d\cos(dx+c)\right.}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15015*(304*cos(d*x + c)^10 + 912*cos(d*x + c)^9 - 152*cos(d*x + c)^8 - 2
888*cos(d*x + c)^7 - 1862*cos(d*x + c)^6 + 2926*cos(d*x + c)^5 + 3325*cos(
d*x + c)^4 - 665*cos(d*x + c)^3 - 35*cos(d*x + c)^2 + 210*cos(d*x + c) + 7
0)/((a^3*d*cos(d*x + c)^9 + 3*a^3*d*cos(d*x + c)^8 - 8*a^3*d*cos(d*x + c)^
6 - 6*a^3*d*cos(d*x + c)^5 + 6*a^3*d*cos(d*x + c)^4 + 8*a^3*d*cos(d*x + c)
^3 - 3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{210210\sin(dx+c)}{\cos(dx+c)+1}-\frac{42042\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17160\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5005\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{5460\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1155\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}{a^3}+\frac{429\left(\frac{28\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35\sin(}{(\cos(dx+c)}\right.}{15375360\,d}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15375360*((210210*sin(d*x + c)/(cos(d*x + c) + 1) - 42042*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 - 17160*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5005*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 5460*sin(d*x + c)^11/(cos(d*x + c) +
 1)^11 + 1155*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)/a^3 + 429*(28*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 35*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 28
0*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5)*(cos(d*x + c) + 1)^7/(a^3*sin(d
*x + c)^7))/d
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{429\left(280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}-\frac{1155a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+5460a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+5005a^{36}\tan}{}}{15375360\,d}$$

input
```
integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/15375360*(429*(280*tan(1/2*d*x + 1/2*c)^6 - 35*tan(1/2*d*x + 1/2*c)^4 -
28*tan(1/2*d*x + 1/2*c)^2 - 5)/(a^3*tan(1/2*d*x + 1/2*c)^7) - (1155*a^36*t
an(1/2*d*x + 1/2*c)^13 + 5460*a^36*tan(1/2*d*x + 1/2*c)^11 + 5005*a^36*tan
(1/2*d*x + 1/2*c)^9 - 17160*a^36*tan(1/2*d*x + 1/2*c)^7 - 42042*a^36*tan(1
/2*d*x + 1/2*c)^5 + 210210*a^36*tan(1/2*d*x + 1/2*c))/a^39)/d
```

3.107.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.72

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{2145\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+12012\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+15015\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12012}{}$$

input
```
int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^3),x)
```

output
```
-(2145*cos(c/2 + (d*x)/2)^20 + 1155*sin(c/2 + (d*x)/2)^20 + 5460*cos(c/2 +
 (d*x)/2)^2*sin(c/2 + (d*x)/2)^18 + 5005*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d
*x)/2)^16 - 17160*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 - 42042*cos(c
/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 + 210210*cos(c/2 + (d*x)/2)^12*sin(c
/2 + (d*x)/2)^8 - 120120*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 + 1501
5*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 + 12012*cos(c/2 + (d*x)/2)^18
*sin(c/2 + (d*x)/2)^2)/(15375360*a^3*d*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*
x)/2)^7)
```

3.108 $\int (a + a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx$

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3.108.8 Giac [F] . . . 843
3.108.9 Mupad [F(-1)] . . . 843

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\frac{ae^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}$$

output

```
-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(5/2)*arctanh((e*sin
(d*x+c))^(1/2)/e^(1/2))/d-2/3*a*e*(e*sin(d*x+c))^(3/2)/d-2/5*a*e*cos(d*x+c
)*(e*sin(d*x+c))^(3/2)/d-6/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin
(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\frac{a(e\sin(c+dx))^{5/2}\left(15\arctan\left(\sqrt{\sin(c+dx)}\right) - 15\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right) + 18E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right.\right.}{15d\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/15*(a*(e*Sin[c + d*x])^(5/2)*(15*ArcTan[Sqrt[Sin[c + d*x]]] - 15*ArcTan
h[Sqrt[Sin[c + d*x]]] + 18*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + 10*Sin[c
+ d*x]^(3/2) + 3*Sqrt[Sin[c + d*x]]*Sin[2*(c + d*x)]))/(d*Sin[c + d*x]^(5/
2))
```

3.108.3 Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 27, 262, 266, 827, 216, 219, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\sin(c+dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\downarrow 4360$$

$$\int -\Big(\sec(c+dx)(a(-\cos(c+dx))-a)(e\sin(c+dx))^{5/2}\Big)\,dx$$

$$\downarrow 25$$

$$-\int -\Big((\cos(c+dx)a+a)\sec(c+dx)(e\sin(c+dx))^{5/2}\Big)\,dx$$

$$\downarrow 25$$

$$\int \sec(c+dx)(a\cos(c+dx)+a)(e\sin(c+dx))^{5/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int (e\sin(c+dx))^{5/2}dx + a\int \sec(c+dx)(e\sin(c+dx))^{5/2}dx$$

$$\downarrow 3042$$

$$a\int (e\sin(c+dx))^{5/2}dx + a\int \frac{(e\sin(c+dx))^{5/2}}{\cos(c+dx)}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2(e\sin(c+dx))^{5/2}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{de} + a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{(e\sin(c+dx))^{5/2}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d} + a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 262$$

$$\frac{ae\left(e^2\int \frac{\sqrt{e\sin(c+dx)}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx)) - \frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d} + a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 266$$

$$\frac{ae\left(2e^2\int \frac{e^2\sin^2(c+dx)}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)} - \frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d} + a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 827$$

$$\frac{ae\left(2e^2\left(\frac{1}{2}\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)} - \frac{1}{2}\int \frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}\right) - \frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d} +$$
$$a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 216$$

$$\frac{ae\left(2e^2\left(\frac{1}{2}\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)} - \frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right) - \frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d} +$$
$$a\int (e\sin(c+dx))^{5/2}dx$$

$$\downarrow 219$$

$$a\int (e\sin(c+dx))^{5/2}dx +$$
$$\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}} - \frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right) - \frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)-\frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)-\frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}$$

$$\downarrow 3121$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)-\frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)-\frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}$$

$$\downarrow 3119$$

$$\frac{ae\left(2e^2\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)-\frac{2}{3}(e\sin(c+dx))^{3/2}\right)}{d}+a\left(\frac{6e^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(a*e*(2*e^2*(-1/2*ArcTan[Sqrt[e]*Sin[c + d*x]]/Sqrt[e] + ArcTanh[Sqrt[e]*S
in[c + d*x]]/(2*Sqrt[e])) - (2*(e*Sin[c + d*x])^(3/2))/3))/d + a*((6*e^2*E
llipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*
x]]) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d))
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.108.4 Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

method	result
default	$-\frac{2ae(e\sin(dx+c))^{\frac{3}{2}}}{3}-a\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+a\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{a\,e^{3}\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\mathrm{El}\right.}{d}$
parts	$-\frac{a\,e^{3}\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{}\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*a*e*(e*sin(d*x+c))^(3/2)-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/
2))+a*e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-1/5*a*e^3*(6*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+
2*sin(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.108.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 649, normalized size of antiderivative = 4.13

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = \left[\frac{72i\sqrt{2}a\sqrt{-ie}e^{2}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
[1/120*(72*I*sqrt(2)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 72*I*sqrt(2)*a*sqrt(I*e)*e^2
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
+ c))) - 30*a*sqrt(-e)*e^2*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2
)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e)) +
 15*a*sqrt(-e)*e^2*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(
d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*s
qrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4
 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(3*a*
e^2*cos(d*x + c) + 5*a*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/d, 1/120*(7
2*I*sqrt(2)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c))) - 72*I*sqrt(2)*a*sqrt(I*e)*e^2*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) -
30*a*e^(5/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d
*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 15*a*e^(5/2)*l
og((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*
x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d
*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 +
 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(3*a*e^2*cos(d*x + c) + 5*
a*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/d]
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.108.7 Maxima [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

3.108.8 Giac [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (e\,\sin(c + d\,x))^{5/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.109 $\int (a + a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx$

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3.109.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{ae^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}$$

output

```
a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*
x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2*a*e*(e*sin(d*x+c))^(1/2)/d-2/3*a*e*cos
(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

3.109.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{a(e\sin(c+dx))^{3/2}\left(12\arctan\left(\sqrt{\sin(c+dx)}\right) + 6\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right) - 8\,\text{EllipticF}\left(\frac{1}{4}(-\right.\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(a*(e*Sin[c + d*x])^(3/2)*(12*ArcTan[Sqrt[Sin[c + d*x]]] + 6*ArcTanh[Sqrt[
Sin[c + d*x]]] - 8*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 3*Log[1 - Sqrt[Si
n[c + d*x]]] + 3*Log[1 + Sqrt[Sin[c + d*x]]] - 24*Sqrt[Sin[c + d*x]] - 8*C
os[c + d*x]*Sec[2*(c + d*x)]*Sqrt[Sin[c + d*x]] + 16*Cos[c + d*x]*Sec[2*(c
 + d*x)]*Sin[c + d*x]^(5/2)))/(12*d*Sin[c + d*x]^(3/2))
```

3.109.3 Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 27, 262, 266, 756, 216, 219, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\sin(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\downarrow 4360$$

$$\int -\Big(\sec(c+dx)(a(-\cos(c+dx))-a)(e\sin(c+dx))^{3/2}\Big)\,dx$$

$$\downarrow 25$$

$$-\int -\Big((\cos(c+dx)a+a)\sec(c+dx)(e\sin(c+dx))^{3/2}\Big)\,dx$$

$$\downarrow 25$$

$$\int \sec(c+dx)(a\cos(c+dx)+a)(e\sin(c+dx))^{3/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int (e\sin(c+dx))^{3/2}dx + a\int \sec(c+dx)(e\sin(c+dx))^{3/2}dx$$

$$\downarrow 3042$$

$$a\int (e\sin(c+dx))^{3/2}dx + a\int \frac{(e\sin(c+dx))^{3/2}}{\cos(c+dx)}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2(e\sin(c+dx))^{3/2}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{de} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{(e\sin(c+dx))^{3/2}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 262$$

$$\frac{ae\left(e^2\int \frac{1}{\sqrt{e\sin(c+dx)}(e^2-e^2\sin^2(c+dx))}d(e\sin(c+dx)) - 2\sqrt{e\sin(c+dx)}\right)}{d} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 266$$

$$\frac{ae\left(2e^2\int \frac{1}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)} - 2\sqrt{e\sin(c+dx)}\right)}{d} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 756$$

$$\frac{ae\left(2e^2\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e} + \frac{\int \frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}}{2e}\right) - 2\sqrt{e\sin(c+dx)}\right)}{d} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 216$$

$$\frac{ae\left(2e^2\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e} + \frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right) - 2\sqrt{e\sin(c+dx)}\right)}{d} + a\int (e\sin(c+dx))^{3/2}dx$$

$$\downarrow 219$$

$$a\int (e\sin(c+dx))^{3/2}dx + \frac{ae\left(2e^2\left(\frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}} + \frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right) - 2\sqrt{e\sin(c+dx)}\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{ae\left(2e^2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)-2\sqrt{e\sin(c+dx)}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{ae\left(2e^2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)-2\sqrt{e\sin(c+dx)}\right)}{d}$$

$$\downarrow 3121$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{ae\left(2e^2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)-2\sqrt{e\sin(c+dx)}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{ae\left(2e^2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)-2\sqrt{e\sin(c+dx)}\right)}{d}$$

$$\downarrow 3120$$

$$\frac{ae\left(2e^2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)-2\sqrt{e\sin(c+dx)}\right)}{d}+a\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(a*e*(2*e^2*(ArcTan[Sqrt[e]*Sin[c + d*x]]/(2*e^(3/2)) + ArcTanh[Sqrt[e]*Si
n[c + d*x]]/(2*e^(3/2))) - 2*Sqrt[e*Sin[c + d*x]]))/d + a*((2*e^2*Elliptic
F[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) -
(2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d))
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.109.4 Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

method	result
default	$\frac{a e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right) + a e^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right) - 2ae\sqrt{e \sin(dx+c)} - \frac{a e^2 \left(\sqrt{-\sin(dx+c)+1}\, \sqrt{2\sin(dx+c)+2}\, \sqrt{\sin(dx+c)}\, \text{Ellipt}}{3\cos(dx+c)\sqrt{}}}{d}$
parts	$-\frac{a e^2 \left(\sqrt{-\sin(dx+c)+1}\, \sqrt{2\sin(dx+c)+2}\, \sqrt{\sin(dx+c)}\, \text{EllipticF}\left(\sqrt{-\sin(dx+c)+1}, \frac{\sqrt{2}}{2}\right) - 2\sin(dx+c)^3 + 2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e \sin(dx+c)}\, d} + \frac{a\left(e^{\frac{3}{2}} a\right.}{}$

input
```
int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+a*e^(3/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))-2*a*e*(e*sin(d*x+c))^(1/2)-1/3*a*e^2*((-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^
(1/2))/d
```

3.109.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.92

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx = \left[\frac{8\sqrt{2}a\sqrt{-ie}e\text{weierstrassPInverse}(4, 0, \cos(dx + c) + i\sin(dx + c)) + 8\sqrt{2}a\sqrt{ie}e\text{weierstras}}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(8*sqrt(2)*a*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 8*sqrt(2)*a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x
 + c) - I*sin(d*x + c)) - 6*a*sqrt(-e)*e*arctan(1/4*(cos(d*x + c)^2 - 6*si
n(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*
x + c) - e)) + 3*a*sqrt(-e)*e*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2
+ 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(
d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos
(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
 - 16*(a*e*cos(d*x + c) + 3*a*e)*sqrt(e*sin(d*x + c)))/d, 1/24*(8*sqrt(2)*
a*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) +
8*sqrt(2)*a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 6*a*e^(3/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt
(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 3*a*e^
(3/2)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 +
(cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(
e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
 c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a*e*cos(d*x + c) +
 3*a*e)*sqrt(e*sin(d*x + c)))/d]
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

3.109.8 Giac [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx = \int (e\,\sin(c + d\,x))^{3/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.110 $\int (a + a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$

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3.110.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int (a + a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = -\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

output
```
-a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d+a*arctanh((e*sin(d*x+c))
^(1/2)/e^(1/2))*e^(1/2)/d-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+
c))^(1/2)/d/sin(d*x+c)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int (a + a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{a\left(-\arctan\left(\sqrt{\sin(c+dx)}\right) + \operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right) - 2E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(a*(-ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]] - 2*Elliptic
E[(-2*c + Pi - 2*d*x)/4, 2])*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

3.110.3 Rubi [A] (warning: unable to verify)

Time $= 0.58$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 0.93$, number of steps used $= 17$, number of rules used $= 16$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used $=$ {3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 27, 266, 827, 216, 219, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\sqrt{e\sin(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\left(\sec(c+dx)(a(-\cos(c+dx))-a)\sqrt{e\sin(c+dx)}\right)dx$$

$$\downarrow 25$$

$$-\int -\left((\cos(c+dx)a+a)\sec(c+dx)\sqrt{e\sin(c+dx)}\right)dx$$

$$\downarrow 25$$

$$\int \sec(c+dx)(a\cos(c+dx)+a)\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sqrt{e\sin(c+dx)}dx + a\int \sec(c+dx)\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 3042$$

$$a\int\sqrt{e\sin(c+dx)}dx+a\int\frac{\sqrt{e\sin(c+dx)}}{\cos(c+dx)}dx$$

$$\downarrow 3044$$

$$\frac{a\int\frac{e^2\sqrt{e\sin(c+dx)}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{de}+a\int\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 27$$

$$\frac{ae\int\frac{\sqrt{e\sin(c+dx)}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}+a\int\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 266$$

$$\frac{2ae\int\frac{e^2\sin^2(c+dx)}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}+a\int\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 827$$

$$\frac{2ae\left(\frac{1}{2}\int\frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}-\frac{1}{2}\int\frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}\right)}{d}+$$
$$a\int\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 216$$

$$\frac{2ae\left(\frac{1}{2}\int\frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)}{d}+a\int\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 219$$

$$a\int\sqrt{e\sin(c+dx)}dx+\frac{2ae\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)}{d}$$

$$\downarrow 3121$$

$$\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ae\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)}{d}$$

$$\downarrow 3042$$

$$\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ae\left(\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)}{d}$$

$$\downarrow 3119$$

$$\frac{2ae\left(\frac{\text{arctanh}(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}-\frac{\arctan(\sqrt{e}\sin(c+dx))}{2\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*e*(-1/2*ArcTan[Sqrt[e]*Sin[c + d*x]]/Sqrt[e] + ArcTanh[Sqrt[e]*Sin[c
+ d*x]]/(2*Sqrt[e])))/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin
[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.110.4 Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

method	result
default	$\frac{-a\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{ae\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sqrt{\sin(dx+c)}\left(2\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)}-\right.\right.}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{ae\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sqrt{\sin(dx+c)}\left(2\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$ _

input
```
int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*e^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+a*e^(1/2)*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))-a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.110.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 555, normalized size of antiderivative = 5.34

$$\int (a + a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$$

$$= \left[\frac{8i\sqrt{2}a\sqrt{-ie}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))) - 8i\sqrt{2}a\sqrt{ie}\right.$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(8*I*sqrt(2)*a*sqrt(-I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 8*I*sqrt(2)*a*sqrt(I*e)*weierstrass
Zeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a
*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x
+ c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e)) + a*sqrt(-e)*log((
e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x +
c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x
+ c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*
(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, 1/8*(8*I*sqrt(2)*a*sqrt(-I*e)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
 c))) - 8*I*sqrt(2)*a*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a*sqrt(e)*arctan(1/4*(cos(d*x +
c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2
+ e*sin(d*x + c) - e)) + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x +
c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e
*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/
(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) +
 8)))/d]
```

3.110.6 Sympy [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = a\left(\int \sqrt{e\sin(c + dx)}\,dx + \int \sqrt{e\sin(c + dx)}\sec(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

output
```
a*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(sqrt(e*sin(c + d*x))*sec(c
 + d*x), x))
```

3.110.7 Maxima [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int (a\sec(dx + c) + a)\sqrt{e\sin(dx + c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

3.110.8 Giac [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int (a\sec(dx + c) + a)\sqrt{e\sin(dx + c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int \sqrt{e\,\sin(c + d\,x)}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

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3.111.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

output
```
a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(1/2)-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(
1/2)/d/(e*sin(d*x+c))^(1/2)
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.95 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.19

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{9a\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\cot^2\left(\frac{1}{2}(c\right.\right.}{5d\left(4\left(-2\,\text{AppellF1}\left(\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\cot^2\left(\frac{1}{2}(c+dx)\right),\cot^2\left(\frac{1}{2}(c+dx)\right)\right)+\text{AppellF1}\left(\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\cot^2\left(\frac{1}{2}(c\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(9*a*AppellF1[5/4, 1/2, 1, 9/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x]^3)/(5*d*(4*(-2*AppellF1[
9/4, 1/2, 2, 13/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2] + AppellF1[9/4
, 3/2, 1, 13/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2])*Cos[(c + d*x)/2]
^2 + 9*AppellF1[5/4, 1/2, 1, 9/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2]
*(-1 + Cos[c + d*x]))*Sqrt[e*Sin[c + d*x]])
```

3.111.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 27, 266, 756, 216, 219, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{e\sin(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sec(c+dx)(a(-\cos(c+dx))-a)}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$-\int -\frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\int \frac{\sec(c+dx)(a\cos(c+dx)+a)}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{a - a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx - a\int -\frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx + a\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx + a\int \frac{1}{\cos(c+dx)\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2}{\sqrt{e\sin(c+dx)}(e^2-e^2\sin^2(c+dx))}d(e\sin(c+dx))}{de} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}(e^2-e^2\sin^2(c+dx))}d(e\sin(c+dx))}{d} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 266$$

$$\frac{2ae\int \frac{1}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 756$$

$$\frac{2ae\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e} + \frac{\int \frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}}{2e}\right)}{d} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 216$$

$$\frac{2ae\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e} + \frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{d} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 219$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2ae\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{d}$$

$$\downarrow 3121$$

$$\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{d}$$

$$\downarrow 3042$$

$$\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{d}$$

$$\downarrow 3120$$

$$\frac{2ae\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{d}+\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*e*(ArcTan[Sqrt[e]*Sin[c + d*x]]/(2*e^(3/2)) + ArcTanh[Sqrt[e]*Sin[c +
 d*x]]/(2*e^(3/2))))/d + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.111.4 Maple [A] (verified)

Time = 7.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result
parts	$-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a\left(\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+\text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\right)}{\sqrt{e}\,d}$
default	$\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

input
```
int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d+a*(a
rctan((e*sin(d*x+c))^(1/2)/e^(1/2))+arctanh((e*sin(d*x+c))^(1/2)/e^(1/2)))
/e^(1/2)/d
```

3.111.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 550, normalized size of antiderivative = 5.34

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \left[\frac{8\sqrt{2}a\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+8\sqrt{2}a\sqrt{i\,e}\,\text{weierstrassPInverse}(4,0}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(8*sqrt(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 8*sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 2*a*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x +
 c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c)
- e)) - a*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(
d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*s
qrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4
 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/(d*e), 1/
8*(8*sqrt(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d
*x + c)) + 8*sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c)
- 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))
 + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x +
c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e)
 - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*co
s(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/(d*e)]
```

3.111.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
a*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*sin(
c + d*x)), x))
```

3.111.7 Maxima [F]

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{a\sec(dx+c) + a}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

3.111.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{e\sin(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\sqrt{e\,\sin(c + d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2), x)
```

3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

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3.112.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

output

```
-a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+a*arctanh((e*sin(d*x+c))
^(1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/
(e*sin(d*x+c))^(1/2)+2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(
1/2)/d/e^2/sin(d*x+c)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{a(1+\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\left(\arctan\left(\sqrt{\sin(c+dx)}\right)\sec\left(\frac{1}{2}(c+dx)\right) - \operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sec\right.}{2d(e\sin(c$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
-1/2*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]*(ArcTan[Sqrt[Sin[c + d*x]]]*Se
c[(c + d*x)/2] - ArcTanh[Sqrt[Sin[c + d*x]]]*Sec[(c + d*x)/2] - 2*Elliptic
E[(-2*c + Pi - 2*d*x)/4, 2]*Sec[(c + d*x)/2] + 2*Csc[(c + d*x)/2]*Sqrt[Sin
[c + d*x]])*Sin[c + d*x]^(3/2))/(d*(e*Sin[c + d*x])^(3/2))
```

3.112.3 Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 27, 264, 266, 827, 216, 219, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\sin(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sec(c+dx)(a(-\cos(c+dx))-a)}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 25$$

$$-\int -\frac{(\cos(c+dx)a+a)\sec(c+dx)}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 25$$

$$\int \frac{\sec(c+dx)(a\cos(c+dx)+a)}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{3/2}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx - a\int -\frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 25$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 3042$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx + a\int \frac{1}{\cos(c+dx)(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2}{(e\sin(c+dx))^{3/2}(e^2-e^2\sin^2(c+dx))}d(e\sin(c+dx))}{de} + a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{1}{(e\sin(c+dx))^{3/2}(e^2-e^2\sin^2(c+dx))}d(e\sin(c+dx))}{d} + a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 264$$

$$\frac{ae\left(\frac{\int \frac{\sqrt{e\sin(c+dx)}}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{e^2} - \frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d} + a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 266$$

$$\frac{ae\left(\frac{2\int \frac{e^2\sin^2(c+dx)}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{e^2} - \frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d} + a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 827$$

$$\frac{ae\left(\frac{2\left(\frac{1}{2}\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)} - \frac{1}{2}\int \frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}\right)}{e^2} - \frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d} +$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 216$$

$$\frac{ae\left(\frac{2\left(\frac{1}{2}\int\frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}+a\int\frac{1}{(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 219$$

$$a\int\frac{1}{(e\sin(c+dx))^{3/2}}dx+\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}$$

$$\downarrow 3116$$

$$a\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)+\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)+\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}$$

$$\downarrow 3121$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)+\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)+\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}-\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}$$

$$\downarrow 3119$$

$$\frac{ae\left(\frac{2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sin(c+dx)}{2\sqrt{e}}\right)}{2\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt{e}\sin(c+dx)}{2\sqrt{e}}\right)}{2\sqrt{e}}\right)}{e^2}-\frac{2}{e^2\sqrt{e\sin(c+dx)}}\right)}{d}+a\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(a*e*((2*(-1/2*ArcTan[Sqrt[e]*Sin[c + d*x]]/Sqrt[e] + ArcTanh[Sqrt[e]*Sin[
c + d*x]]/(2*Sqrt[e])))/e^2 - 2/(e^2*Sqrt[e*Sin[c + d*x]])))/d + a*((-2*Co
s[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]))
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.112.4 Maple [A] (verified)

Time = 10.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30

method	result
default	$\frac{-\frac{a \arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}-\frac{2a}{e\sqrt{e\sin(dx+c)}}+\frac{a \operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{a\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{-\sin(}\right.\right.}{d}$
parts	$\frac{a\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx}$

input
```
int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-a/e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-2*a/e/(e*sin(d*x+c))^(1/2
)+a/e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+a/e*(2*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/cos(d*x+c
)/(e*sin(d*x+c))^(1/2))/d
```

3.112.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.32

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \left[\frac{-8i\sqrt{2}a\sqrt{-ie}\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(d\right.$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(-8*I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 8*I*sqrt(2)*a*sqrt(I*
e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c))) - 2*a*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*
x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x +
c) - e))*sin(d*x + c) - a*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x +
c)^2 + 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e
*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)
/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8))*sin(d*x + c) - 16*(a*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)))/(d*e^2*
sin(d*x + c)), 1/8*(-8*I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 8*I*sqr
t(2)*a*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)
^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 +
e*sin(d*x + c) - e))*sin(d*x + c) + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e
*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c)
- 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin
(d*x + c) + 8))*sin(d*x + c) - 16*(a*cos(d*x + c) + a)*sqrt(e*sin(d*x +...
```

3.112.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*sin(c
+ d*x))**(3/2), x))
```

3.112.7 Maxima [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.112.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{3/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\sin(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{3/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+dx)}}{(e\,\sin(c + dx))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

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3.113.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$

output

```
a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(5/2)-2/3*a/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d
/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^
(1/2)/d/e^2/(e*sin(d*x+c))^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int \frac{a + a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\arctan\left(\sqrt{\sin(c+dx)}\right) - 3\text{arctanh}\left(\sqrt{\sin(c+dx)}\right) + 2\,\text{EllipticF}\right.}{6de^2\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/6*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-3*ArcTan[Sqrt[Sin[c + d*x]
]] - 3*ArcTanh[Sqrt[Sin[c + d*x]]] + 2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]
 + Csc[(c + d*x)/2]^2*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d*x]])/(d*e^2*Sqrt[
e*Sin[c + d*x]])
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 27, 264, 266, 756, 216, 219, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\sin(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a - a\csc\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sec(c+dx)(a(-\cos(c+dx))-a)}{(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 25$$
$$-\int -\frac{(\cos(c+dx)a+a)\sec(c+dx)}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 25$$

$$\int \frac{\sec(c+dx)(a\cos(c+dx)+a)}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 3042$$

$$\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{5/2}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx - a\int -\frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 25$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 3042$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx + a\int \frac{1}{\cos(c+dx)(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2}{(e\sin(c+dx))^{5/2}\left(e^2-e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{de} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{1}{(e\sin(c+dx))^{5/2}\left(e^2-e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 264$$

$$\frac{ae\left(\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}\left(e^2-e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{e^2} - \frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 266$$

$$\frac{ae\left(\frac{2\int \frac{1}{e^2-e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{e^2} - \frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 756$$

$$\frac{ae\left(\frac{2\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e}+\frac{\int \frac{1}{e^2\sin^2(c+dx)+e}d\sqrt{e\sin(c+dx)}}{2e}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}+a\int\frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 216$$

$$\frac{ae\left(\frac{2\left(\frac{\int \frac{1}{e-e^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e}+\frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}+a\int\frac{1}{(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 219$$

$$a\int\frac{1}{(e\sin(c+dx))^{5/2}}dx+\frac{ae\left(\frac{2\left(\frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}}+\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}$$

$$\downarrow 3116$$

$$a\left(\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)+\frac{ae\left(\frac{2\left(\frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}}+\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)+\frac{ae\left(\frac{2\left(\frac{\arctan(\sqrt{e}\sin(c+dx))}{2e^{3/2}}+\frac{\operatorname{arctanh}(\sqrt{e}\sin(c+dx))}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}$$

$$\downarrow 3121$$

$$\frac{a\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)+ae\left(\frac{2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}$$

$$\downarrow 3042$$

$$\frac{a\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)+ae\left(\frac{2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}$$

$$\downarrow 3120$$

$$\frac{ae\left(\frac{2\left(\frac{\arctan\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{e}\sin(c+dx)\right)}{2e^{3/2}}\right)}{e^2}-\frac{2}{3e^2(e\sin(c+dx))^{3/2}}\right)}{d}+a\left(\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(a*e*((2*(ArcTan[Sqrt[e]*Sin[c + d*x]]/(2*e^(3/2)) + ArcTanh[Sqrt[e]*Sin[c
 + d*x]]/(2*e^(3/2))))/e^2 - 2/(3*e^2*(e*Sin[c + d*x])^(3/2))))/d + a*((-2
*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c - Pi/2 + d
*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]))
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

3.113. $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.113.4 Maple [A] (verified)

Time = 11.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02

method	result
default	$-\frac{2a}{3e(e\sin(dx+c))^{\frac{3}{2}}}+\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}-\frac{a\left(\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sin(dx+c)^{\frac{5}{2}}\operatorname{EllipticF}\left(\sqrt{-\sin}\right.\right.}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a\left(\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sin(dx+c)^{\frac{5}{2}}\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a\left(-\frac{}{3e(e\,s}\right.$

input
```
int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*a/e/(e*sin(d*x+c))^(3/2)+a/e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/
2))+a/e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-1/3*a/e^2*((-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)
/(e*sin(d*x+c))^(1/2))/d
```

3.113.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 705, normalized size of antiderivative = 4.41

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \left[-\frac{6\,(a\cos(dx+c)-a)\sqrt{-e}\arctan\left(\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{e\sin(dx+c)}\sqrt{-e}}{4\left(e\cos(dx+c)^2-e\sin(dx+c)-e\right)}\right)+3\right.$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(6*(a*cos(d*x + c) - a)*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin
(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x
 + c) - e)) + 3*(a*cos(d*x + c) - a)*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e
*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c)
 - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
 c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*si
n(d*x + c) + 8)) - 8*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-I*e)*weier
strassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*(sqrt(2)*a*cos(d*x
 + c) - sqrt(2)*a)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 16*sqrt(e*sin(d*x + c))*a)/(d*e^3*cos(d*x + c) - d*e^3), 1/2
4*(6*(a*cos(d*x + c) - a)*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x +
 c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) -
 e)) + 3*(a*cos(d*x + c) - a)*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x
 + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqr
t(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*
e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) + 8*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-I*e)*weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 8*(sqrt(2)*a*cos(d*x + c) - s
qrt(2)*a)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c
)) + 16*sqrt(e*sin(d*x + c))*a)/(d*e^3*cos(d*x + c) - d*e^3)]
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\sin(dx + c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

3.113.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\sin(dx + c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{(e\,\sin(c + d\,x))^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.114 $\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^{5/2}\,dx$

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3.114.1 Optimal result

Integrand size = 25, antiderivative size = 194

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{2a^2e^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}$$
$$-\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}$$
$$-\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

output

```
-2*a^2*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(5/2)*arctan
h((e*sin(d*x+c))^(1/2)/e^(1/2))/d-4/3*a^2*e*(e*sin(d*x+c))^(3/2)/d-2/5*a^2
*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(3/2)
/d+9/5*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin
(d*x+c)^(1/2)
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \frac{2a^2 \cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)(e\sin(c+dx))^{5/2}\left(-15\arctan\left(\sqrt{}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(e*Si
n[c + d*x])^(5/2)*(-15*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 1
5*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 9*Sin[c + d*x]^(3/2)
- 10*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 9*Sqrt[Cos[c + d*x]^2]*Hype
rgeometric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^(3/2) + 3*Sin[c
+ d*x]^(7/2)))/(15*d*Sin[c + d*x]^(5/2))
```

3.114.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2 (e\sin(c+dx))^{5/2}\, dx$$

$$\downarrow 3042$$

$$\int \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2} dx$$

$$\downarrow 4360$$

$$\int \sec^2(c+dx)(a(-\cos(c+dx))-a)^2 (e\sin(c+dx))^{5/2} dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$\int \left(a^2(e\sin(c+dx))^{5/2}+a^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}+2a^2\sec(c+dx)(e\sin(c+dx))^{5/2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2e^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}-\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*A
rcTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*S
in[c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/
(5*d) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.114.4 Maple [A] (verified)

Time = 19.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
default	$\frac{a^2\left(-60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\cos(dx+c)+60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\cos(dx+c)+54\sqrt{-\sin(dx-}$
parts	$-\frac{a^2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{}}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-60*(e*sin(d*x+c))^(1/2)*e^(5/2)
*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)+60*(e*sin(d*x+c))^(1/2)*e
^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)+54*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))*e^3-27*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3+12*e^3*cos(d
*x+c)^4+40*e^3*cos(d*x+c)^3-42*e^3*cos(d*x+c)^2-40*cos(d*x+c)*e^3+30*e^3)/
d
```

3.114.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.96

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/60*(-54*I*sqrt(2)*a^2*sqrt(-I*e)*e^2*cos(d*x + c)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 54*I*sqrt(2)*
a^2*sqrt(I*e)*e^2*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c))) - 30*a^2*sqrt(-e)*e^2*arctan(1/4*(cos
(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x
 + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c) + 15*a^2*sqrt(-e)*e^2*cos(d*x
+ c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (
cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(
e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 4*(6*a^2*e^2*cos(d*x +
 c)^2 + 20*a^2*e^2*cos(d*x + c) - 15*a^2*e^2)*sqrt(e*sin(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)), 1/60*(-54*I*sqrt(2)*a^2*sqrt(-I*e)*e^2*cos(d*x +
c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 54*I*sqrt(2)*a^2*sqrt(I*e)*e^2*cos(d*x + c)*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 30*a^2*e^(5
/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*
sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c) + 15*a^2*e^(
5/2)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d
*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sq
rt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^...
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

3.114.8 Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (e\,\sin(c + d\,x))^{5/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.115 $\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^{3/2}\,dx$

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3.115.1 Optimal result

Integrand size = 25, antiderivative size = 192

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^{3/2}\,dx = \frac{2a^2e^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{a^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}$$

output

```
2*a^2*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(3/2)*arctanh
((e*sin(d*x+c))^(1/2)/e^(1/2))/d+1/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-4*a^2*e*(e*sin(d*x+c))^(1/2)/d
-2/3*a^2*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c
))^(1/2)/d
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 27.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06

$$\int (a + a\sec(c + dx))^2 (e\sin(c$$

$$+dx))^{3/2}\,dx = \frac{16a^2 e\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(6\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(16*a^2*e*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(6*ArcTan[S
qrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 6*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] - 12*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]] - Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c +
 d*x]^2]*Sqrt[Sin[c + d*x]] + 2*Sin[c + d*x]^(5/2))*Sin[ArcSin[Sin[c + d*x
]]/2]^4)/(3*d*Sin[c + d*x]^(9/2))
```

3.115.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2 (e\sin(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2} dx$$

$$\downarrow 4360$$

$$\int \sec^2(c+dx)(a(-\cos(c+dx))-a)^2 (e\sin(c+dx))^{3/2} dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$\int \left(a^2(e\sin(c+dx))^{3/2}+a^2\sec^2(c+dx)(e\sin(c+dx))^{3/2}+2a^2\sec(c+dx)(e\sin(c+dx))^{3/2}\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^2e^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*
Sin[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^
2*e*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.115.4 Maple [A] (verified)

Time = 13.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.05

method	result
default	$\frac{a^2\left(12\cos(dx+c)\sqrt{e\sin(dx+c)}\,e^{\frac{3}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+12\cos(dx+c)\sqrt{e\sin(dx+c)}\,e^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+\sqrt{-\sin(dx+c)+}\right.}{6\cos(dx-}$
parts	$-\frac{a^2e^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a^2e^2\sqrt{}}{}$

input
```
int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(12*cos(d*x+c)*(e*sin(d*x+c))^(1/2
)*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+12*cos(d*x+c)*(e*sin(d*x+c)
)^(1/2)*e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*e^2-4*e^2*cos(d*x+c)^2*sin(d*x+c)-24*e^2*sin(d*x+c)*cos(d*x+c
)+6*e^2*sin(d*x+c))/d
```

3.115.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.76

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \left[-\frac{2\sqrt{2}a^2\sqrt{-ie}e\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+2\sqrt{2}a^2\sqrt{ie}e\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))}{}\right.$$

$$\left.-\frac{2\sqrt{2}a^2\sqrt{-ie}e\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+2\sqrt{2}a^2\sqrt{ie}e\cos(dx+c)}{}\right]$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(2*sqrt(2)*a^2*sqrt(-I*e)*e*cos(d*x + c)*weierstrassPInverse(4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 2*sqrt(2)*a^2*sqrt(I*e)*e*cos(d*x + c)*we
ierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 6*a^2*sqrt(-e)*e*
arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt
(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c) - 3*a^2*sqrt(-e
)*e*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(7*cos(d*
x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqr
t(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 -
 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*(2*a^2*e
*cos(d*x + c)^2 + 12*a^2*e*cos(d*x + c) - 3*a^2*e)*sqrt(e*sin(d*x + c)))/(
d*cos(d*x + c)), -1/12*(2*sqrt(2)*a^2*sqrt(-I*e)*e*cos(d*x + c)*weierstras
sPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*sqrt(2)*a^2*sqrt(I*e)*e
*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 6
*a^2*e^(3/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d
*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c) - 3
*a^2*e^(3/2)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*
(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x
+ c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x
+ c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*
(2*a^2*e*cos(d*x + c)^2 + 12*a^2*e*cos(d*x + c) - 3*a^2*e)*sqrt(e*sin(d...
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

3.115.8 Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\, dx = \int (e\,\sin(c + d\,x))^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.116 $\int (a + a\sec(c+dx))^2 \sqrt{e\sin(c+dx)}\,dx$

3.116.1 Optimal result . . . 900
3.116.2 Mathematica [C] (warning: unable to verify) . . . 901
3.116.3 Rubi [A] (verified) . . . 901
3.116.4 Maple [A] (verified) . . . 903
3.116.5 Fricas [C] (verification not implemented) . . . 903
3.116.6 Sympy [F] . . . 904
3.116.7 Maxima [F] . . . 905
3.116.8 Giac [F] . . . 905
3.116.9 Mupad [F(-1)] . . . 905

3.116.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \sqrt{e\sin(c+dx)}\,dx = &-\frac{2a^2\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} \\ &+ \frac{2a^2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} \\ &+ \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} \\ &+ \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}\end{aligned}$$

output

```
a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e-2*a^2*arctan((e*sin(d*x+c))^(1/2)/
e^(1/2))*e^(1/2)/d+2*a^2*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d-a
^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2
)
```

3.116.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.75 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.22

$$\int (a + a\sec(c+dx))^2 \sqrt{e\sin(c+dx)}\,dx =$$

$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\sqrt{e\sin(c+dx)}\left(3\arctan\left(\sqrt{\sin(c+dx)}\right)\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt
[e*Sin[c + d*x]]*(3*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*Ar
cTanh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*Sin[c + d*x]^(3/2) + Sq
rt[Cos[c + d*x]^2]*Hypergeometric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c
+ d*x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2\sqrt{e\sin(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int \sec^2(c+dx)(a(-\cos(c+dx))-a)^2\sqrt{e\sin(c+dx)}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$\int \left(a^2\sqrt{e\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{e\sin(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*A
rcTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/
2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*
Sin[c + d*x])^(3/2))/(d*e)
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.116.4 Maple [A] (verified)

Time = 14.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.59

method	result
default	$-\frac{a^2\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{s}\right.}{}$
parts	$-\frac{a^2 e\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\left(2\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+$

input
```
int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))*e-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e+4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c)
)^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-4*cos(d*x+c)*e^(1/2)*(e*sin(d
*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+2*e*cos(d*x+c)^2-2*e)/d
```

3.116.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 677, normalized size of antiderivative = 4.91

$$\int (a + a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$$

$$= \left[\frac{2i\sqrt{2}a^2\sqrt{-ie}\cos(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*I*sqrt(2)*a^2*sqrt
(I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*a^2*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*s
in(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d
*x + c) - e))*cos(d*x + c) + a^2*sqrt(-e)*cos(d*x + c)*log((e*cos(d*x + c)
^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(
d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*
sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 - 2)*sin(d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))*a^2*sin(d*x + c))/(d*co
s(d*x + c)), 1/4*(2*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassZeta(
4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*I*sqrt
(2)*a^2*sqrt(I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a^2*sqrt(e)*arctan(1/4*(cos(d*x +
 c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2
 + e*sin(d*x + c) - e))*cos(d*x + c) + a^2*sqrt(e)*cos(d*x + c)*log((e*cos
(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2
- 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2
 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d
*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))*a^2*sin(d*x ...
```

3.116.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sqrt{e\sin(c+dx)}\,dx = a^2\left(\int \sqrt{e\sin(c+dx)}\,dx + \int 2\sqrt{e\sin(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\sin(c+dx)}\sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(2*sqrt(e*sin(c + d*x))*
sec(c + d*x), x) + Integral(sqrt(e*sin(c + d*x))*sec(c + d*x)**2, x))
```

3.116.7 Maxima [F]

$$\int (a + a\sec(c + dx))^2 \sqrt{e \sin(c + dx)}\, dx = \int (a \sec(dx + c) + a)^2 \sqrt{e \sin(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

3.116.8 Giac [F]

$$\int (a + a\sec(c + dx))^2 \sqrt{e \sin(c + dx)}\, dx = \int (a \sec(dx + c) + a)^2 \sqrt{e \sin(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 \sqrt{e \sin(c + dx)}\, dx = \int \sqrt{e \sin(c + dx)} \left(a + \frac{a}{\cos(c + dx)}\right)^2 dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

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3.117.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{3a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}$$

output

```
2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+2*a^2*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)-3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)+a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/
d/e
```

3.117.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 19.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\left(2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+2\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(2*ArcT
an[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 2*ArcTanh[Sqrt[Sin[c + d*x]]
]*Sqrt[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] + 3*Sqrt[Cos[c + d*x]^2]*Hyper
geometric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2]*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c
 + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

3.117.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}\,dx$$
$$\downarrow 4360$$
$$\int \frac{\sec^2(c+dx)(a(-\cos(c+dx))-a)^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$\downarrow 3042$$

$$\int \frac{\left(a \sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2 \sqrt{e \cos\left(c+dx-\frac{\pi}{2}\right)}} dx$$

$$\downarrow 3352$$

$$\int \left(\frac{a^2}{\sqrt{e \sin(c+dx)}} + \frac{a^2 \sec^2(c+dx)}{\sqrt{e \sin(c+dx)}} + \frac{2a^2 \sec(c+dx)}{\sqrt{e \sin(c+dx)}}\right) dx$$

$$\downarrow 2009$$

$$\frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a^2 \sec(c+dx)\sqrt{e \sin(c+dx)}}{de} + \frac{3a^2 \sqrt{\sin(c+dx)}\, \text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right), 2\right)}{d\sqrt{e \sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x
]*Sqrt[e*Sin[c + d*x]])/(d*e)
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.117.4 Maple [A] (verified)

Time = 11.95 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.17

method	result
default	$\frac{a^2\left(-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\sqrt{e}+4\cos(dx+c)\sqrt{e\sin(dx+c)}\,\arctan\left(\frac{\sqrt{}}{2\sqrt{e}\,\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}\right.\right.$
parts	$-\frac{a^2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{a^2\sqrt{\cos(dx+c)^2 e\sin(dx+c)}\left(\sqrt{-\sin(d}{2\sqrt{-e\sin(dx}$

input
```
int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-3*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/
2*2^(1/2))*e^(1/2)+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))
^(1/2)/e^(1/2))+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))+2*e^(1/2)*sin(d*x+c))/d
```

3.117.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.75

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$

$$= \left[\frac{6\sqrt{2}a^2\sqrt{-i\,e}\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+6\sqrt{2}a^2\sqrt{i\,e}\cos(dx+\right.$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(6*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 6*sqrt(2)*a^2*sqrt(I*e)*cos(d*x + c)*weierstr
assPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*a^2*sqrt(-e)*arctan(1
/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*
cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c) - a^2*sqrt(-e)*cos(d*x
+ c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (
cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(
e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))
*a^2)/(d*e*cos(d*x + c)), 1/4*(6*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weier
strassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 6*sqrt(2)*a^2*sqrt(I
*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*si
n(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c)
+ a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8
*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x
 + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x
 + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4
*sqrt(e*sin(d*x + c))*a^2)/(d*e*cos(d*x + c))]
```

3.117.6 Sympy [F]

$$\int \frac{(a + a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e
*sin(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*sin(c + d*x)), x))
```

3.117.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

3.117.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

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3.118.1 Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}$$

output

```
-2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+2*a^2*arctanh((e*sin
(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/
e^3-4*a^2/d/e/(e*sin(d*x+c))^(1/2)-2*a^2*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/
2)-2*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+5*a^2*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

3.118.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 19.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.60

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\cot(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\sqrt{e\sin(c+dx)}\left(6\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Cos[(c + d*x)/2]^4*Cot[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt
[e*Sin[c + d*x]]*(6*Hypergeometric2F1[-1/4, 1, 3/4, Sin[c + d*x]^2] + 6*Hy
pergeometric2F1[-1/4, 3/2, 3/4, Sin[c + d*x]^2] + Hypergeometric2F1[3/4, 3
/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^2))/(3*d*e^2*Sqrt[Cos[c + d*x]^2])
```

3.118.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\sin(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4360$$
$$\int \frac{\sec^2(c+dx)(a(-\cos(c+dx))-a)^2}{(e\sin(c+dx))^{3/2}}dx$$
$$\downarrow 3042$$

$$\int \frac{\left(a \sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2 \left(e \cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}} dx$$

$$\downarrow 3352$$

$$\int \left(\frac{a^2}{(e \sin(c+dx))^{3/2}} + \frac{a^2 \sec^2(c+dx)}{(e \sin(c+dx))^{3/2}} + \frac{2a^2 \sec(c+dx)}{(e \sin(c+dx))^{3/2}}\right) dx$$

$$\downarrow 2009$$

$$-\frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2 \text{arctanh}\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{3a^2 \sec(c+dx)(e \sin(c+dx))^{3/2}}{de^3} - \frac{5a^2 E\left(\frac{1}{2}(c+dx-\frac{\pi}{2}) \middle| 2\right) \sqrt{e \sin(c+dx)}}{de^2 \sqrt{\sin(c+dx)}} - \frac{4a^2}{de\sqrt{e \sin(c+dx)}} - \frac{2a^2 \cos(c+dx)}{de\sqrt{e \sin(c+dx)}} - \frac{2a^2 \sec(c+dx)}{de\sqrt{e \sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh
[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c +
d*x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c +
d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]
*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e
*Sin[c + d*x])^(3/2))/(d*e^3)
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.118.4 Maple [A] (verified)

Time = 17.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

method	result
default	$\frac{a^2\left(10e^{\frac{3}{2}}\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-5e^{\frac{3}{2}}\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+}\right.$
parts	$\frac{a^2\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(d}\right.$ $\frac{}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^(5/2)/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*a^2*(10*e^(3/2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-5*e^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-10*e^(3/2)*c
os(d*x+c)^2-8*e^(3/2)*cos(d*x+c)-4*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e
*sin(d*x+c))^(1/2)*cos(d*x+c)*e+4*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*(e
*sin(d*x+c))^(1/2)*cos(d*x+c)*e+2*e^(3/2))/d
```

3.118.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.53

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(-10*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*sin(d*x + c)*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 10*I*
sqrt(2)*a^2*sqrt(I*e)*cos(d*x + c)*sin(d*x + c)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a^2*sqrt(-e)*arc
tan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e
)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c)*sin(d*x + c) - a^2
*sqrt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(7*
cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c
))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x +
c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin(d*
x + c) - 4*(5*a^2*cos(d*x + c)^2 + 4*a^2*cos(d*x + c) - a^2)*sqrt(e*sin(d*
x + c)))/(d*e^2*cos(d*x + c)*sin(d*x + c)), 1/4*(-10*I*sqrt(2)*a^2*sqrt(-I
*e)*cos(d*x + c)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 10*I*sqrt(2)*a^2*sqrt(I*e)*cos(d*x +
 c)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(
d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x +
 c) - e))*cos(d*x + c)*sin(d*x + c) + a^2*sqrt(e)*cos(d*x + c)*log((e*cos(
d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)...
```

3.118.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{(e\sin(c + dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\sin(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c + dx)}{(e\sin(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c + dx)}{(e\sin(c + dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*s
in(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*sin(c + d*x))**(3/2)
, x))
```

3.118.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.118.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{(e\sin(c+dx))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.119 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

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3.119.1 Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{7a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} + \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}$$

output

```
2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+2*a^2*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)-4/3*a^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*
cos(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^
(3/2)-7/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x
)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*s
in(d*x+c))^(1/2)+5/3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e^3
```

3.119.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 18.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\left(3+4\sqrt{\cos^2(c+dx)}\csc^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},\sin^2(c+dx)\right)+4\sqrt{c}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/3*(a^2*Cos[(c + d*x)/2]^4*(3 + 4*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^2*Hy
pergeometric2F1[-3/4, 1, 1/4, Sin[c + d*x]^2] + 4*Sqrt[Cos[c + d*x]^2]*Csc
[c + d*x]^2*Hypergeometric2F1[-3/4, 3/2, 1/4, Sin[c + d*x]^2] + 3*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2])*Sec[c + d*x
]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[e*Sin[c + d*x]])/(d*e^3)
```

3.119.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\sin(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4360$$
$$\int \frac{\sec^2(c+dx)(a(-\cos(c+dx))-a)^2}{(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 3042$$

$$\int \frac{\left(a \sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2 \left(e \cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}} dx$$

$$\downarrow 3352$$

$$\int \left(\frac{a^2}{(e \sin(c+dx))^{5/2}} + \frac{a^2 \sec^2(c+dx)}{(e \sin(c+dx))^{5/2}} + \frac{2a^2 \sec(c+dx)}{(e \sin(c+dx))^{5/2}}\right) dx$$

$$\downarrow 2009$$

$$\frac{2a^2 \arctan\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2 \operatorname{arctanh}\left(\frac{\sqrt{e \sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{5a^2 \sec(c+dx)\sqrt{e \sin(c+dx)}}{3de^3} + \frac{7a^2 \sqrt{\sin(c+dx)}\, \text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}), 2\right)}{3de^2 \sqrt{e \sin(c+dx)}} - \frac{4a^2}{3de(e \sin(c+dx))^{3/2}} - \frac{2a^2 \cos(c+dx)}{3de(e \sin(c+dx))^{3/2}} - \frac{2a^2 \sec(c+dx)}{3de(e \sin(c+dx))^{3/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x
])^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*S
ec[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Se
c[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.119.4 Maple [A] (verified)

Time = 17.50 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.29

method	result
default	$\frac{a^2\left(7\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e^{\frac{7}{2}}-14e^{\frac{7}{2}}\cos(dx+c)^4-8e^{\frac{7}{2}}\cos(dx+c)^3+12($
parts	$-\frac{a^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{a^2\sqrt{\cos(}$

input
```
int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/e^(9/2)/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*a^2*(7*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))*e^(7/2)-14*e^(7/2)*cos(d*x+c)^4-8*e^(7/2)*cos(d
*x+c)^3+12*(e*sin(d*x+c))^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d
*x+c)^3*e^2+12*(e*sin(d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*
cos(d*x+c)^3*e^2+20*e^(7/2)*cos(d*x+c)^2+8*e^(7/2)*cos(d*x+c)-12*(e*sin(d*
x+c))^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-12*(e*sin(
d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-6*e^(7/
2))/d
```

3.119.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.63

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
[-1/12*(6*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*sqrt(-e)*arctan(1/4*(cos
(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x
 + c)^2 - e*sin(d*x + c) - e)) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))
*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^
2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) +
 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 14*(sqrt(2)*a^2*c
os(d*x + c)^2 - sqrt(2)*a^2*cos(d*x + c))*sqrt(-I*e)*weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 14*(sqrt(2)*a^2*cos(d*x + c)^2 - sqr
t(2)*a^2*cos(d*x + c))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) -
I*sin(d*x + c)) - 4*(7*a^2*cos(d*x + c) - 3*a^2)*sqrt(e*sin(d*x + c)))/(d*
e^3*cos(d*x + c)^2 - d*e^3*cos(d*x + c)), 1/12*(6*(a^2*cos(d*x + c)^2 - a^
2*cos(d*x + c))*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*s
qrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 3*(
a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*sqrt(e)*log((e*cos(d*x + c)^4 - 72*
e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c)
 - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
 c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*si
n(d*x + c) + 8)) + 14*(sqrt(2)*a^2*cos(d*x + c)^2 - sqrt(2)*a^2*cos(d*x +
c))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))...
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.119.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.119.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{(e\sin(c+dx))^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.120 $\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

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3.120.2 Mathematica [A] (verified) . . . 924
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3.120.5 Fricas [C] (verification not implemented) . . . 929
3.120.6 Sympy [F(-1)] . . . 930
3.120.7 Maxima [F] . . . 930
3.120.8 Giac [F] . . . 930
3.120.9 Mupad [F(-1)] . . . 931

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad}$$

output

```
2/5*e*(e*sin(d*x+c))^(5/2)/a/d+4/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*s
in(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-2/21*e^3*cos(d*x+c)*(e*sin(d*x+c)
)^(1/2)/a/d+2/7*e^3*cos(d*x+c)^3*(e*sin(d*x+c))^(1/2)/a/d
```

3.120.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(42+25\,\text{co}\right.}{105ad(1+\sec(c+$$

input

```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(40*EllipticF[(-2*c + Pi - 2*d*x)/4,
2] + (42 + 25*Cos[c + d*x] - 42*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sq
rt[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(105*a*d*(1 + Sec[c + d*x])*Sqrt[S
in[c + d*x]])
```

3.120.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3048, 3042, 3049, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{e^2\int\cos(c+dx)(e\sin(c+dx))^{3/2}dx}{a}-\frac{e^2\int\cos^2(c+dx)(e\sin(c+dx))^{3/2}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{3/2}dx}{a} - \frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{3/2}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int (e\sin(c+dx))^{3/2}d(e\sin(c+dx))}{ad} - \frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{3/2}dx}{a}$$

$$\downarrow 15$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{3/2}dx}{a}$$

$$\downarrow 3048$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\left(\frac{1}{7}e^2\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}dx - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\left(\frac{1}{7}e^2\int \frac{\cos(c+dx)^2}{\sqrt{e\sin(c+dx)}}dx - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3049$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\left(\frac{1}{7}e^2\left(\frac{2}{3}\int \frac{1}{\sqrt{e\sin(c+dx)}}dx + \frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right) - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\left(\frac{1}{7}e^2\left(\frac{2}{3}\int \frac{1}{\sqrt{e\sin(c+dx)}}dx + \frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right) - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3121$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{e^2\left(\frac{1}{7}e^2\left(\frac{2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right) - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - e^2\left(\frac{1}{7}e^2\left(\frac{2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right) - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

$$\downarrow 3120$$

$$\frac{\frac{2e(e\sin(c+dx))^{5/2}}{5ad} - e^2\left(\frac{1}{7}e^2\left(\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de} + \frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{3d\sqrt{e\sin(c+dx)}}\right) - \frac{2e\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7d}\right)}{a}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(2*e*(e*Sin[c + d*x])^(5/2))/(5*a*d) - (e^2*((-2*e*Cos[c + d*x]^3*Sqrt[e*S
in[c + d*x]])/(7*d) + (e^2*((4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]
])/(3*d*e)))/7))/a
```

3.120.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.120.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
default	$\frac{2e(e\sin(dx+c))^{\frac{5}{2}}}{5a}+\frac{2e^4\left(3\sin(dx+c)^5+\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-5\sin(dx+c)^3+2\sin(dx+c)\right.}{21a\cos(dx+c)\sqrt{e\sin(dx+c)}}$ $\frac{}{d}$

input
```
int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(2/5/a*e*(e*sin(d*x+c))^(5/2)+2/21*e^4*(3*sin(d*x+c)^5+(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/a/cos(d*x+c)/(e*sin(d*x+c))^(1/
2))/d
```

3.120.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(5\sqrt{2}\sqrt{-ie}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\sqrt{ie}e^3\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*sqrt(2)*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c)) - (15*e^3*cos(d*x + c)^3 - 21*e^3*cos(d*x + c)^2
- 5*e^3*cos(d*x + c) + 21*e^3)*sqrt(e*sin(d*x + c)))/(a*d)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin{(dx+c)})^{\frac{7}{2}}}{a\sec{(dx+c)}+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.120.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin{(dx+c)})^{\frac{7}{2}}}{a\sec{(dx+c)}+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{7/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

3.121.1 Optimal result . . . 932
3.121.2 Mathematica [C] (verified) . . . 932
3.121.3 Rubi [A] (verified) . . . 933
3.121.4 Maple [A] (verified) . . . 936
3.121.5 Fricas [C] (verification not implemented) . . . 936
3.121.6 Sympy [F(-1)] . . . 937
3.121.7 Maxima [F] . . . 937
3.121.8 Giac [F] . . . 937
3.121.9 Mupad [F(-1)] . . . 938

3.121.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

output
```
2/3*e*(e*sin(d*x+c))^(3/2)/a/d-2/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a/d+4
/5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c
)^(1/2)
```

3.121.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{2e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(10\sqrt{\csc^2(c)}\sin^2(c+dx)+6\csc(c)\csc(dx-\text{arc}\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(2*e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(10*Sqrt[Csc[c]^2]*Sin[c + d*x]^2 +
 6*Csc[c]*Csc[d*x - ArcTan[Cot[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Sin[d*x - ArcTan[Cot[c]]]^2] + 3*Csc[c]
*Sec[c]*(Sin[c + d*x - ArcTan[Cot[c]]] + 3*Sin[c - d*x + ArcTan[Cot[c]]])
 - 3*Sqrt[Csc[c]^2]*Sin[c + d*x]*(Sin[2*(c + d*x)] + 4*Tan[c])))/(15*a*d*Sq
rt[Csc[c]^2]*(1 + Sec[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

3.121.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3049, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{e^2\int \cos(c+dx)\sqrt{e\sin(c+dx)}dx}{a}-\frac{e^2\int \cos^2(c+dx)\sqrt{e\sin(c+dx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int \cos(c+dx)\sqrt{e\sin(c+dx)}dx}{a}-\frac{e^2\int \cos(c+dx)^2\sqrt{e\sin(c+dx)}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int \sqrt{e\sin(c+dx)}d(e\sin(c+dx))}{ad}-\frac{e^2\int \cos(c+dx)^2\sqrt{e\sin(c+dx)}dx}{a}$$

$$\downarrow 15$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\int \cos(c+dx)^2\sqrt{e\sin(c+dx)}dx}{a}$$

$$\downarrow 3049$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\left(\frac{2}{5}\int \sqrt{e\sin(c+dx)}dx+\frac{2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\left(\frac{2}{5}\int \sqrt{e\sin(c+dx)}dx+\frac{2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de}\right)}{a}$$

$$\downarrow 3121$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\left(\frac{2\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}+\frac{2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\left(\frac{2\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}+\frac{2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de}\right)}{a}$$

$$\downarrow 3119$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{e^2\left(\frac{2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}\right)}{a}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (e^2*((4*EllipticE[(c - Pi/2 + d*x)
/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) + (2*Cos[c + d*x]*(e
*Sin[c + d*x])^(3/2))/(5*d*e)))/a
```

3.121. $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}dx$

3.121.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.121.4 Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.66

method	result
default	$\frac{2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin}\right.}{15a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/15/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1
/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-5*cos(d*x+c)^3-3*co
s(d*x+c)^2+5*cos(d*x+c))/d
```

3.121.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(3i\sqrt{2}\sqrt{-ie}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{ie}\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/15*(3*I*sqrt(2)*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*sqrt(I*e)*e^2*weiers
trassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))
+ (3*e^2*cos(d*x + c) - 5*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/(a*d)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.121.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{5/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

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3.122.1 Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

output
```
4/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c
))^(1/2)+2*e*(e*sin(d*x+c))^(1/2)/a/d-2/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2
)/a/d
```

3.122.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{2\left(-2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(-3+\cos(c+dx))\sqrt{\sin(c+dx)}\right)(e\sin(c+dx))^{3/2}}{3ad\sin^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(-2*(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-3 + Cos[c + d*x])*Sqrt[Sin
[c + d*x]])*(e*Sin[c + d*x])^(3/2))/(3*a*d*Sin[c + d*x]^(3/2))
```

3.122.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3049, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a(-\cos(c+dx))-a}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{\cos(c+dx)a+a}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a\cos(c+dx)+a}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{e^2\int\frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx}{a}-\frac{e^2\int\frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int\frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx}{a}-\frac{e^2\int\frac{\cos(c+dx)^2}{\sqrt{e\sin(c+dx)}}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int\frac{1}{\sqrt{e\sin(c+dx)}}d(e\sin(c+dx))}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{\sqrt{e\sin(c+dx)}}dx}{a}$$

$$\downarrow 15$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{\sqrt{e\sin(c+dx)}}dx}{a}$$

$$\downarrow 3049$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\left(\frac{2}{3}\int\frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\left(\frac{2}{3}\int\frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right)}{a}$$

$$\downarrow 3121$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\left(\frac{2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right)}{a}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\left(\frac{2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}\right)}{a}$$

$$\downarrow 3120$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{e^2\left(\frac{2\cos(c+dx)\sqrt{e\sin(c+dx)}}{3de}+\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}\right)}{a}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (e^2*((4*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[e
*Sin[c + d*x]])/(3*d*e)))/a
```

3.122. $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

3.122.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.122.4 Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

method	result
default	$\frac{2e^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\cos(dx+c)^2\sin(dx+c)+3\cos(dx+c)\sin(dx+c)\right)}{3a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input

```
int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
2/3/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)-cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c))/d
```

3.122.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{2}\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}\sqrt{i\,e}e\text{weierstrassPInverse}(4,0,c\right.}{3\,ad}$$

input

```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output

```
-2/3*(sqrt(2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(
d*x + c)) + sqrt(2)*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) - I
*sin(d*x + c)) + (e*cos(d*x + c) - 3*e)*sqrt(e*sin(d*x + c)))/(a*d)
```

3.122.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.122.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.122.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{3/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx$

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3.123.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

output
```
-2*e/a/d/(e*sin(d*x+c))^(1/2)+2*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(1/2)-4*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.75 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(3\sin\left(\frac{c}{2}\right)+\sin\left(\frac{3c}{2}\right)\right)-2\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(2*Cos[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(Sec[c/2]*Sec[c]*(
3*Sin[c/2] + Sin[(3*c)/2]) - 2*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2*
Sqrt[Csc[c]^2]*Csc[c + d*x]*Csc[d*x - ArcTan[Cot[c]]]*HypergeometricPFQ[{-
1/2, -1/4}, {3/4}, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[c]*Sqrt[Sin[d*x - ArcT
an[Cot[c]]]^2] - (Csc[c]*Csc[c + d*x]*Sec[c]*(Sin[c + d*x - ArcTan[Cot[c]]
] + 3*Sin[c - d*x + ArcTan[Cot[c]]]))/Sqrt[Csc[c]^2]))/(a*d*(1 + Sec[c + d
*x]))
```

3.123.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3047, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a(-\cos(c+dx))-a}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{\cos(c+dx)a+a}dx$$
$$\downarrow 25$$
$$\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a\cos(c+dx)+a}dx$$
$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a}-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{3/2}}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int\frac{1}{(e\sin(c+dx))^{3/2}}d(e\sin(c+dx))}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{3/2}}dx}{a}$$

$$\downarrow 15$$

$$-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{3/2}}dx}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

$$\downarrow 3047$$

$$-\frac{e^2\left(-\frac{2\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

$$\downarrow 3121$$

$$-\frac{e^2\left(-\frac{2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

$$\downarrow 3119$$

$$-\frac{e^2\left(-\frac{4E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{a}-\frac{2e}{ad\sqrt{e\sin(c+dx)}}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) - (e^2*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Si
n[c + d*x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(
d*e^2*Sqrt[Sin[c + d*x]])))/a
```

3.123.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.123.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

method	result
default	$-\frac{2e\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin}\right.}{a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^2+cos(d*x+c))/d
```

3.123.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\sqrt{-ie}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(-I*e)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (I*sqrt(2)*co
s(d*x + c) + I*sqrt(2))*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(d*x + c) - I*sin(d*x + c))) + sqrt(e*sin(d*x + c))*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

3.123.6 Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.123.7 Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.123.8 Giac [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e \sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,\sqrt{e\,\sin(c+dx)}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

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3.124.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}}$$

output
```
-2/3*e/a/d/(e*sin(d*x+c))^(3/2)+2/3*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(3/2)-
4/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(
1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{2\cot\left(\frac{1}{2}(c+dx)\right)\left(-1+\cos(c+dx)-2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ad(1+\cos(c+dx))\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(2*Cot[(c + d*x)/2]*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d*x)/4
, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*(1 + Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

3.124.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)\sqrt{e\sin(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{(a(-\cos(c+dx))-a)\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}dx}{a}-\frac{e^2\int-\frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}dx}{a}$$

$$\downarrow 25$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}dx}{a}-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}dx}{a}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{5/2}}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int\frac{1}{(e\sin(c+dx))^{5/2}}d(e\sin(c+dx))}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{5/2}}dx}{a}$$

$$\downarrow 15$$

$$-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{5/2}}dx}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

$$\downarrow 3047$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

$$\downarrow 3121$$

$$-\frac{e^2\left(-\frac{2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(-\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{a}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}$$

input
```
Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) - (e^2*((-2*Cos[c + d*x])/(3*d*e*(e*
Sin[c + d*x])^(3/2)) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])))/a
```

3.124.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.124.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{2e}{3a(e\sin(dx+c))^{\frac{3}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+\sin(dx+c)^3-\sin(dx+c)\right)}{3a\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$	121

input
```
int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3/a*e/(e*sin(d*x+c))^(3/2)-2/3*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+sin(d
*x+c)^3-sin(d*x+c))/a/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.124.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2\left(\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\cos(dx+\right.\right.}{3\,(ade\cos(dx+c)+}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(I*
e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(e*sin(d
*x + c)))/(a*d*e*cos(d*x + c) + a*d*e)
```

3.124.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}\sec(c+dx)+\sqrt{e\sin(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/
a
```

3.124.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.124.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)}{a\sqrt{e\,\sin(c+d\,x)}\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

3.125.1 Optimal result . . . 960
3.125.2 Mathematica [C] (verified) . . . 960
3.125.3 Rubi [A] (verified) . . . 961
3.125.4 Maple [A] (verified) . . . 965
3.125.5 Fricas [C] (verification not implemented) . . . 965
3.125.6 Sympy [F] . . . 966
3.125.7 Maxima [F(-1)] . . . 966
3.125.8 Giac [F] . . . 966
3.125.9 Mupad [F(-1)] . . . 967

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}$$

output

```
-2/5*e/a/d/(e*sin(d*x+c))^(5/2)+2/5*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(5/2)-
4/5*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*(e*sin(d*x+c))^(1/2)/a/d/e^2/sin(d*x+c)^(1/2)
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-6-9\cos(c+$$

input

```
Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(Sec[(c + d*x)/2]^2*(Cos[c + d*x] + I*Sin[c + d*x])*(-6 - 9*Cos[c + d*x] +
 2*Sqrt[1 - E^((2*I)*(c + d*x))]*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2,
 3/4, 7/4, E^((2*I)*(c + d*x))] + (3*I)*Sin[c + d*x]))/(15*a*d*e*Sqrt[e*Si
n[c + d*x]])
```

3.125.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\sin(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{(a(-\cos(c+dx))-a)(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)(e\sin(c+dx))^{3/2}}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{3/2}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}dx}{a}-\frac{e^2\int-\frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}dx}{a}$$

$$\downarrow 25$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}dx}{a}-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}dx}{a}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{7/2}}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int\frac{1}{(e\sin(c+dx))^{7/2}}d(e\sin(c+dx))}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{7/2}}dx}{a}$$

$$\downarrow 15$$

$$-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{7/2}}dx}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3047$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3116$$

$$-\frac{e^2\left(-\frac{2\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\left(-\frac{\int\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3121$$

$$-\frac{e^2\left(-\frac{2\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

$$\downarrow 3119$$

$$-\frac{e^2\left(-\frac{2\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)}{a}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}$$

input
```
Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) - (e^2*((-2*Cos[c + d*x])/(5*d*e*(e*
Sin[c + d*x])^(5/2)) - (2*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) -
(2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[
c + d*x]])))/(5*e^2)))/a
```

3.125.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.125.4 Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2e}{5a(e\sin(dx+c))^{\frac{5}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{5}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(d}{ea\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

input
```
int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5/a*e/(e*sin(d*x+c))^(5/2)+2/5/e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*sin(d*x+c)^5-3*sin(d*x+c)^3+sin(d*x+c))/
a/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.125.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\sqrt{-ie}\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)\right.}{}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*sqrt(-I*e)*sin(d*x + c)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(I*e)*sin(d*x + c)*weierstrassZ
eta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (2*c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(e*sin(d*x + c)))/((a*d*e^2*cos(d*
x + c) + a*d*e^2)*sin(d*x + c))
```

3.125.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\sin(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(3/2)*sec(c + d*x) + (e*sin(c + d*x))**(3/2)
), x)/a
```

3.125.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.125.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{a\,(e\,\sin(c+dx))^{3/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

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3.126.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ade^2\sqrt{e\sin(c+dx)}}$$

output

```
-2/7*e/a/d/(e*sin(d*x+c))^(7/2)+2/7*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(7/2)-
4/21*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(3/2)-4/21*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*sin(d*x+c)^(1/2)/a/d/e^2/(e*sin(d*x+c))^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(4+2\cos(c+dx)+\cos(2(c+dx))+\csc^2\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{21ade(1+\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(-2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)] + Csc[(c + d*x)/2]^2*EllipticF[
(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a*d*e*(1 + Cos[c + d*x]
)*(e*Sin[c + d*x])^(3/2))
```

3.126.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\sin(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4360$$
$$\int -\frac{\cos(c+dx)}{(a(-\cos(c+dx))-a)(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 25$$
$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+a)(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 3042$$
$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{5/2}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}dx}{a}-\frac{e^2\int-\frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}dx}{a}$$

$$\downarrow 25$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}dx}{a}-\frac{e^2\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int\frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}dx}{a}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{9/2}}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int\frac{1}{(e\sin(c+dx))^{9/2}}d(e\sin(c+dx))}{ad}-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{9/2}}dx}{a}$$

$$\downarrow 15$$

$$-\frac{e^2\int\frac{\cos(c+dx)^2}{(e\sin(c+dx))^{9/2}}dx}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3047$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\int\frac{1}{(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3116$$

$$-\frac{e^2\left(-\frac{2\left(\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\left(\frac{\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3121$$

$$-\frac{e^2\left(-\frac{2\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{2\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(-\frac{2\left(\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)}{7e^2}-\frac{2\cos(c+dx)}{7de(e\sin(c+dx))^{7/2}}\right)}{a}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}$$

input
```
Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) - (e^2*((-2*Cos[c + d*x])/(7*d*e*(e*
Sin[c + d*x])^(7/2)) - (2*((-2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)
) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[
e*Sin[c + d*x]])))/(7*e^2)))/a
```

3.126.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.126.4 Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result
default	$-\frac{2e}{7a(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{9}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^5+5\sin(dx+c)^3-3\sin(dx+c)\right.}{21e^2a\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}$ / d

input
```
int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/7/a*e/(e*sin(d*x+c))^(7/2)-2/21/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
2*sin(d*x+c)^5+5*sin(d*x+c)^3-3*sin(d*x+c))/a/sin(d*x+c)^4/cos(d*x+c)/(e*s
in(d*x+c))^(1/2))/d
```

3.126.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}\cos(dx+c)^3+\sqrt{2}\cos(dx+c)^2-\sqrt{2}\cos(dx+c)-\sqrt{\right.\right.}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*((sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*cos(d*x +
 c) - sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d
*x + c)) + (sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*cos(
d*x + c) - sqrt(2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*s
in(d*x + c)) + (2*cos(d*x + c)^2 + 2*cos(d*x + c) + 3)*sqrt(e*sin(d*x + c)
))/(a*d*e^3*cos(d*x + c)^3 + a*d*e^3*cos(d*x + c)^2 - a*d*e^3*cos(d*x + c)
 - a*d*e^3)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.126.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.126.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\,\sin(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

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3.127.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{52e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} + \frac{4e(e\sin(c+dx))^{5/2}}{5a^2d}$$

output

```
4/5*e*(e*sin(d*x+c))^(5/2)/a^2/d-52/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-4*e^3*(e*sin(d*x+c))^(1/2)/a
^2/d+26/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+2/7*e^3*cos(d*x+c)^3*
(e*sin(d*x+c))^(1/2)/a^2/d
```

3.127.2 Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^3\left(520\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(756-305\cos(c+dx)+84\cos(2(c+dx))-15\cos(3(c+dx))\right.}{210a^2d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/210*(e^3*(520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (756 - 305*Cos[c +
d*x] + 84*Cos[2*(c + d*x)] - 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]])*Sqrt
[e*Sin[c + d*x]])/(a^2*d*Sqrt[Sin[c + d*x]])
```

3.127.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^4\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4 \int \left(\frac{a^2 \cos^4(c+dx)}{\sqrt{e \sin(c+dx)}} - \frac{2a^2 \cos^3(c+dx)}{\sqrt{e \sin(c+dx)}} + \frac{a^2 \cos^2(c+dx)}{\sqrt{e \sin(c+dx)}} \right) dx}{a^4}$$

$$\downarrow \text{2009}$$

$$\frac{e^4 \left(\frac{4a^2 (e \sin(c+dx))^{5/2}}{5de^3} - \frac{4a^2 \sqrt{e \sin(c+dx)}}{de} + \frac{2a^2 \cos^3(c+dx) \sqrt{e \sin(c+dx)}}{7de} + \frac{26a^2 \cos(c+dx) \sqrt{e \sin(c+dx)}}{21de} + \frac{52a^2 \sqrt{\sin(c+dx)}\, \text{EllipticF}}{21d\sqrt{e \sin(c+}} \right.}{a^4}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((52*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*d*S
qrt[e*Sin[c + d*x]]) - (4*a^2*Sqrt[e*Sin[c + d*x]])/(d*e) + (26*a^2*Cos[c
+ d*x]*Sqrt[e*Sin[c + d*x]])/(21*d*e) + (2*a^2*Cos[c + d*x]^3*Sqrt[e*Sin[c
 + d*x]])/(7*d*e) + (4*a^2*(e*Sin[c + d*x])^(5/2))/(5*d*e^3)))/a^4
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.127.4 Maple [A] (verified)

Time = 6.87 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2e^4\left(-15\cos(dx+c)^4\sin(dx+c)+65\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+42\cos(dx\right.}{105a^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^4*(-15*cos(d*x+c)^4*sin(d*x+c
)+65*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+42*cos(d*x+c)^3*sin(d*x+c)-65*cos(d*
x+c)^2*sin(d*x+c)+168*cos(d*x+c)*sin(d*x+c))/d
```

3.127.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(65\sqrt{2}\sqrt{-ie}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+65\sqrt{}\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/105*(65*sqrt(2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*sqrt(2)*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, cos(d
*x + c) - I*sin(d*x + c)) + (15*e^3*cos(d*x + c)^3 - 42*e^3*cos(d*x + c)^2
 + 65*e^3*cos(d*x + c) - 168*e^3)*sqrt(e*sin(d*x + c)))/(a^2*d)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.127.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\sin(c+dx))^{7/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.128 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

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3.128.8 Giac [F] . . . 987
3.128.9 Mupad [F(-1)] . . . 987

3.128.1 Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} + \frac{4e(e\sin(c+dx))^{3/2}}{3a^2d} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}$$

output

```
4/3*e*(e*sin(d*x+c))^(3/2)/a^2/d-12/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^
2/d+4*e^3/a^2/d/(e*sin(d*x+c))^(1/2)-2*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))
^(1/2)-2*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(1/2)+44/5*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.39

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^2(c+dx)\sec^2(c+dx)\left(\frac{16\cos(dx)\sin(c)}{3d} - \frac{16\sec\left(\frac{c}{2}\right)\sec(c)\left(8\sin\left(\frac{c}{2}\right)+3\sin\right.\right.}{5d}$$

$$-\frac{88\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(-\frac{\cot(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx-\arctan(\cot(c))\right.\right.}{\sqrt{1-\cos(dx-\arctan(\cot(c)))}\sqrt{1+\cos(dx-\arctan(\cot(c)))}\sqrt{1+\cot}}}{5d(a+a\sec(c+dx))^2\sin}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*Sec[c + d*x]^2*((16*Cos[d*x]*Sin[c])/
(3*d) - (16*Sec[c/2]*Sec[c]*(8*Sin[c/2] + 3*Sin[(3*c)/2]))/(5*d) - (4*Cos[
2*d*x]*Sin[2*c])/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d +
 (16*Cos[c]*Sin[d*x])/(3*d) - (4*Cos[2*c]*Sin[2*d*x])/(5*d))*(e*Sin[c + d*
x])^(5/2))/(a + a*Sec[c + d*x])^2 - (88*Cos[c/2 + (d*x)/2]^4*Sec[c]*Sec[c
+ d*x]^2*(e*Sin[c + d*x])^(5/2)*(-((Cot[c]*HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[d*x - ArcTan[Cot[c]]])/(Sqrt[1 -
Cos[d*x - ArcTan[Cot[c]]]]*Sqrt[1 + Cos[d*x - ArcTan[Cot[c]]]]*Sqrt[1 + Co
t[c]^2]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]])) - ((2*
Cos[d*x - ArcTan[Cot[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]^2)/(Cos[c]^2 + Sin[c]^
2) - (Cot[c]*Sin[d*x - ArcTan[Cot[c]]])/Sqrt[1 + Cot[c]^2])/Sqrt[Cos[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]]))/(5*d*(a + a*Sec[c + d*x])^2*
Sin[c + d*x]^(5/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^4\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4\int \left(\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}+\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(\frac{4a^2(e\sin(c+dx))^{3/2}}{3de^3}-\frac{12a^2\cos(c+dx)(e\sin(c+dx))^{3/2}}{5de^3}-\frac{44a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)\sqrt{e\sin(c+dx)}}{5de^2\sqrt{\sin(c+dx)}}+\frac{4a^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{de\sqrt{e\sin(c+dx)}}\right.}{a^4}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((4*a^2)/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[
e*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]^3)/(d*e*Sqrt[e*Sin[c + d*x]]) - (44
*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^2*Sqrt[
Sin[c + d*x]]) + (4*a^2*(e*Sin[c + d*x])^(3/2))/(3*d*e^3) - (12*a^2*Cos[c
+ d*x]*(e*Sin[c + d*x])^(3/2))/(5*d*e^3)))/a^4
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.128.4 Maple [A] (verified)

Time = 7.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

method	result
default	$\frac{2e^3\left(66\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-33\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{}\right.}{15a^2\cos(dx+c)\sqrt{e\sin(dx+}}$

input
```
int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(66*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2
^(1/2))-33*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-10*cos(d*x+c)^3
-33*cos(d*x+c)^2+40*cos(d*x+c))/d
```

3.128.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(\left(3\,e^2\cos(dx+c)^2-7\,e^2\cos(dx+c)-40\,e^2\right)\sqrt{e\sin(dx+c)}\sin(dx+c)+33\left(i\sqrt{2}e^2\cos(dx+c)+i\right.\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*((3*e^2*cos(d*x + c)^2 - 7*e^2*cos(d*x + c) - 40*e^2)*sqrt(e*sin(d*x
 + c))*sin(d*x + c) + 33*(I*sqrt(2)*e^2*cos(d*x + c) + I*sqrt(2)*e^2)*sqrt
(-I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 33*(-I*sqrt(2)*e^2*cos(d*x + c) - I*sqrt(2)*e^2)*sqrt(I*e)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/(a^2*d*cos(d*x + c) + a^2*d)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.128.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{5/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.129 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

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3.129.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}(c-\frac{\pi}{2}+dx),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{4e\sqrt{e\sin(c+dx)}}{a^2d} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}$$

output

```
4/3*e^3/a^2/d/(e*sin(d*x+c))^(3/2)-2/3*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))
^(3/2)-2/3*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(3/2)+4*e^2*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+4*e*(e*sin
(d*x+c))^(1/2)/a^2/d-4/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d
```

3.129.2 Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left((15+10\cos(c+dx)-\cos(2(c+dx)))\csc(c+\right.}{3a^2d(1+\sec(c+}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*((15 + 10*Cos[c + d*x] - Cos[2*(c + d
*x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2 + (24*EllipticF[(-2*c + Pi - 2*d*x)/
4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(3*a^2*d*(1 + Sec[c + d
*x])^2)
```

3.129.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^4\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4 \int \left(\frac{a^2 \cos^4(c+dx)}{(e \sin(c+dx))^{5/2}} - \frac{2a^2 \cos^3(c+dx)}{(e \sin(c+dx))^{5/2}} + \frac{a^2 \cos^2(c+dx)}{(e \sin(c+dx))^{5/2}} \right) dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4 \left(\frac{4a^2 \sqrt{e \sin(c+dx)}}{de^3} - \frac{4a^2 \cos(c+dx) \sqrt{e \sin(c+dx)}}{3de^3} - \frac{4a^2 \sqrt{\sin(c+dx)}\, \text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{de^2 \sqrt{e \sin(c+dx)}} + \frac{4a^2}{3de(e \sin(c+dx))^{3/2}} - \frac{2a^2 \cos^3(c+}{3de(e \sin(c+dx}$$

$$a^4$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((4*a^2)/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e
*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Cos[c + d*x]^3)/(3*d*e*(e*Sin[c + d*x])^
(3/2)) - (4*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*e^
2*Sqrt[e*Sin[c + d*x]]) + (4*a^2*Sqrt[e*Sin[c + d*x]])/(d*e^3) - (4*a^2*Co
s[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)))/a^4
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.129.4 Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

method	result
default	$-\frac{2e^3\left(3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\cos(dx+c)^6+6\cos(dx+c)^5+4\cos(dx+c)\right.}{3a^2(e\sin(dx+c))^{\frac{3}{2}}\cos(dx+c)\left(\cos(dx+c)^2-1\right)d}$

input
```
int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^2/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*e^3*(3*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^6+6*cos(d*x+c)^5+4*cos(d*x+c)^4-14*cos(
d*x+c)^3-3*cos(d*x+c)^2+8*cos(d*x+c))/d
```

3.129.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(3\left(\sqrt{2}e\cos\left(dx+c\right)+\sqrt{2}e\right)\sqrt{-i\,e}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+3\left(\sqrt{2}e\cos(\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*(sqrt(2)*e*cos(d*x + c) + sqrt(2)*e)*sqrt(-I*e)*weierstrassPInvers
e(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 3*(sqrt(2)*e*cos(d*x + c) + sqrt(
2)*e)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) +
 (e*cos(d*x + c)^2 - 5*e*cos(d*x + c) - 8*e)*sqrt(e*sin(d*x + c)))/(a^2*d*
cos(d*x + c) + a^2*d)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\sin(c+dx))^{3/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

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3.130.7 Maxima [F] . . . 999
3.130.8 Giac [F] . . . 999
3.130.9 Mupad [F(-1)] . . . 999

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = &\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}\\ &- \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}\\ &- \frac{4e}{a^2d\sqrt{e\sin(c+dx)}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}\\ &+ \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}\end{aligned}$$

output

```
4/5*e^3/a^2/d/(e*sin(d*x+c))^(5/2)-2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))
^(5/2)-2/5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(5/2)-4*e/a^2/d/(e*sin(d*
x+c))^(1/2)+16/5*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)-28/5*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

3.130.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{e\sin(c+dx)}\Big(\sec(c)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(49\sin\left(\frac{1}{2}(c-dx)\right)+35\sin\left(\frac{1}{2}(3c+\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(Sec[c]*Sec[(c + d
*x)/2]^3*(49*Sin[(c - d*x)/2] + 35*Sin[(3*c + d*x)/2] - 23*Sin[(c + 3*d*x)
/2] + 5*Sin[(5*c + 3*d*x)/2]) - 56*Sqrt[Csc[c]^2]*Csc[c + d*x]*Csc[d*x - A
rcTan[Cot[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x - ArcTan[Cot
[c]]]^2]*Sin[c]*Sqrt[Sin[d*x - ArcTan[Cot[c]]]^2] - 28*Sqrt[Csc[c]^2]*Csc[
c + d*x]*(Sin[c + d*x - ArcTan[Cot[c]]] + 3*Sin[c - d*x + ArcTan[Cot[c]]])
*Tan[c]))/(5*a^2*d*(1 + Sec[c + d*x])^2)
```

3.130.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4360$$
$$\int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^4\int\frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int\frac{\sin(c+dx-\frac{\pi}{2})^2(\sin(c+dx-\frac{\pi}{2})a+a)^2}{(e\cos(c+dx-\frac{\pi}{2}))^{7/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4\int\left(\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}+\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(\frac{28a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}-\frac{4a^2}{de^3\sqrt{e\sin(c+dx)}}+\frac{16a^2\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}+\frac{4a^2}{5de(e\sin(c+dx))^{5/2}}-\frac{2a^2\cos^3(c+dx)}{5de(e\sin(c+dx))^{5/2}}-\frac{2a}{5de(}\right.}{a^4}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((4*a^2)/(5*d*e*(e*Sin[c + d*x])^(5/2)) - (2*a^2*Cos[c + d*x])/(5*d*e
*(e*Sin[c + d*x])^(5/2)) - (2*a^2*Cos[c + d*x]^3)/(5*d*e*(e*Sin[c + d*x])^
(5/2)) - (4*a^2)/(d*e^3*Sqrt[e*Sin[c + d*x]]) + (16*a^2*Cos[c + d*x])/(5*d
*e^3*Sqrt[e*Sin[c + d*x]]) + (28*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c + d*x]])))/a^4
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.130.4 Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09

method	result
default	$-\frac{2e\left(-\frac{2e^2}{5(e\sin(dx+c))^{\frac{5}{2}}}+\frac{2}{\sqrt{e\sin(dx+c)}}\right)}{a^2}-\frac{2e\left(14\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-7\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+9\sin(dx+c)^5-11\sin(dx+c)^3+2\sin(dx+c)\right)}{5a^2\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

input
```
int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*e/a^2*(-2/5*e^2/(e*sin(d*x+c))^(5/2)+2/(e*sin(d*x+c))^(1/2))-2/5*e*(14
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-7*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+9*sin
(d*x+c)^5-11*sin(d*x+c)^3+2*sin(d*x+c))/a^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin
(d*x+c))^(1/2))/d
```

3.130.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{e\sin(dx+c)}(9\cos(dx+c)+8)\sin(dx+c)+7\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{}\right.\right.}{}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/5*(sqrt(e*sin(d*x + c))*(9*cos(d*x + c) + 8)*sin(d*x + c) + 7*(-I*sqrt(
2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(-I*e)*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 7*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*sqr
t(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.130.6 Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a
**2
```

3.130.7 Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.130.8 Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\sqrt{e\,\sin(c+dx)}}{a^2(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

3.131.1 Optimal result 1000
3.131.2 Mathematica [A] (verified) 1001
3.131.3 Rubi [A] (verified) 1001
3.131.4 Maple [A] (verified) 1003
3.131.5 Fricas [C] (verification not implemented) 1003
3.131.6 Sympy [F] 1004
3.131.7 Maxima [F(-1)] 1004
3.131.8 Giac [F] 1005
3.131.9 Mupad [F(-1)] 1005

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 190

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{4e}{3a^2d(e\sin(c+dx))^{3/2}} + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} + \frac{20\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}$$

output

```
4/7*e^3/a^2/d/(e*sin(d*x+c))^(7/2)-2/7*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))
^(7/2)-2/7*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(7/2)-4/3*e/a^2/d/(e*sin(
d*x+c))^(3/2)+16/21*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(3/2)-20/21*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx =$$
$$-\frac{\csc^3(c+dx)\left(16(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)+40\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{7}{2}}(c+dx\right.}{42a^2d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-1/42*(Csc[c + d*x]^3*(16*(8 + 11*Cos[c + d*x])*Sin[(c + d*x)/2]^4 + 40*El
lipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(a^2*d*Sqrt[e*Sin[c
 + d*x]])
```

3.131.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2\sqrt{e\sin(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4360$$
$$\int \frac{\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2\sqrt{e\sin(c+dx)}}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 3354$$

$$\frac{e^4 \int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{9/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4 \int \left(\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}} + \frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(\frac{20a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{21de^4\sqrt{e\sin(c+dx)}} - \frac{4a^2}{3de^3(e\sin(c+dx))^{3/2}} + \frac{16a^2\cos(c+dx)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{4a^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2a^2\cos^3(c+d}{7de(e\sin(c+dx)}\right.}{a^4}$$

input
```
Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(e^4*((4*a^2)/(7*d*e*(e*Sin[c + d*x])^(7/2)) - (2*a^2*Cos[c + d*x])/(7*d*e
*(e*Sin[c + d*x])^(7/2)) - (2*a^2*Cos[c + d*x]^3)/(7*d*e*(e*Sin[c + d*x])^
(7/2)) - (4*a^2)/(3*d*e^3*(e*Sin[c + d*x])^(3/2)) + (16*a^2*Cos[c + d*x])/
(21*d*e^3*(e*Sin[c + d*x])^(3/2)) + (20*a^2*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(21*d*e^4*Sqrt[e*Sin[c + d*x]])))/a^4
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.131.4 Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{4e^3\left(7\cos(dx+c)^2-4\right)}{21a^2(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(5\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{9}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+11\sin(dx+c)^5-17\sin(dx+c)^3+6\sin(\right.}{21a^2\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

input
```
int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4/21/a^2*e^3/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2-4)-2/21*(5*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))+11*sin(d*x+c)^5-17*sin(d*x+c)^3+6*sin(d*x+c))/a^2/sin
(d*x+c)^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.131.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2\left(5\left(\sqrt{2}\cos(dx+c)^2+2\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-i\,e}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+\right.}{2}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*(sqrt(2)*cos(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-
I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)
*cos(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(I*e)*weierstrassP
Inverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(e*sin(d*x + c))*(11*co
s(d*x + c) + 8))/(a^2*d*e*cos(d*x + c)^2 + 2*a^2*d*e*cos(d*x + c) + a^2*d*
e)
```

3.131.6 Sympy [F]

$$\int \frac{1}{(a + a\sec(c + dx))^2 \sqrt{e \sin(c + dx)}}\, dx = \frac{\int \frac{1}{\sqrt{e \sin(c+dx)} \sec^2(c+dx) + 2\sqrt{e \sin(c+dx)} \sec(c+dx) + \sqrt{e \sin(c+dx)}}\, dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*sin(c + d*x))*
sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a**2
```

3.131.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))^2 \sqrt{e \sin(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.131.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos(c+dx)^2}{a^2\sqrt{e\,\sin(c+dx)}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

3.132.1 Optimal result 1006
3.132.2 Mathematica [C] (verified) 1007
3.132.3 Rubi [A] (verified) 1007
3.132.4 Maple [A] (verified) 1009
3.132.5 Fricas [C] (verification not implemented) 1010
3.132.6 Sympy [F(-1)] 1010
3.132.7 Maxima [F(-1)] 1011
3.132.8 Giac [F] 1011
3.132.9 Mupad [F(-1)] 1011

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}$$

output

```
4/9*e^3/a^2/d/(e*sin(d*x+c))^(9/2)-2/9*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))
^(9/2)-2/9*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(9/2)-4/5*e/a^2/d/(e*sin(
d*x+c))^(5/2)+16/45*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(5/2)-4/15*cos(d*x+c
)/a^2/d/e/(e*sin(d*x+c))^(1/2)+4/15*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*si
n(d*x+c))^(1/2)/a^2/d/e^2/sin(d*x+c)^(1/2)
```

3.132.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-31-40\cos(\right.}{}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(Sec[(c + d*x)/2]^4*(Cos[c + d*x] + I*Sin[c + d*x])*(-31 - 40*Cos[c + d*x]
 - 19*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^4*Sqrt[1 - E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/E^((2*I)*(c +
 d*x)) + (16*I)*Sin[c + d*x] + (13*I)*Sin[2*(c + d*x)]))/(180*a^2*d*e*Sqrt
[e*Sin[c + d*x]])
```

3.132.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\sin(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4360$$
$$\int \frac{\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2(e\sin(c+dx))^{3/2}}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 3354$$

$$\frac{e^4 \int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{11/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4 \int \left(\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}} + \frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(-\frac{4a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)\sqrt{e\sin(c+dx)}}{15de^6\sqrt{\sin(c+dx)}} - \frac{4a^2\cos(c+dx)}{15de^5\sqrt{e\sin(c+dx)}} - \frac{4a^2}{5de^3(e\sin(c+dx))^{5/2}} + \frac{16a^2\cos(c+dx)}{45de^3(e\sin(c+dx))^{5/2}} + \frac{4a^2}{9de(e\sin(c+dx))^{9/2}}\right.}{a^4}$$

input
```
Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(e^4*((4*a^2)/(9*d*e*(e*Sin[c + d*x])^(9/2)) - (2*a^2*Cos[c + d*x])/(9*d*e
*(e*Sin[c + d*x])^(9/2)) - (2*a^2*Cos[c + d*x]^3)/(9*d*e*(e*Sin[c + d*x])^
(9/2)) - (4*a^2)/(5*d*e^3*(e*Sin[c + d*x])^(5/2)) + (16*a^2*Cos[c + d*x])/
(45*d*e^3*(e*Sin[c + d*x])^(5/2)) - (4*a^2*Cos[c + d*x])/(15*d*e^5*Sqrt[e*
Sin[c + d*x]]) - (4*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(15*d*e^6*Sqrt[Sin[c + d*x]])))/a^4
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.132.4 Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.95

method	result
default	$\frac{4e^3\left(9\cos(dx+c)^2-4\right)}{45a^2(e\sin(dx+c))^{\frac{9}{2}}}+\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{11}{2}}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{15}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,s}{ea^2\sin(dx+c)^5\cos(dx+c)\sqrt{}}$ … d

input
```
int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(4/45*e^3/a^2/(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2-4)+2/45/e*(6*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(11/2)*EllipticE((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(11/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^7
-19*sin(d*x+c)^5+23*sin(d*x+c)^3-10*sin(d*x+c))/a^2/sin(d*x+c)^5/cos(d*x+c
)/(e*sin(d*x+c))^(1/2))/d
```

3.132.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(3\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\sqrt{-i\,e}\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassP}\right.}{}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2
)*cos(d*x + c) - I*sqrt(2))*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (6*cos(d*x + c)
^3 + 12*cos(d*x + c)^2 + 19*cos(d*x + c) + 8)*sqrt(e*sin(d*x + c)))/((a^2*
d*e^2*cos(d*x + c)^2 + 2*a^2*d*e^2*cos(d*x + c) + a^2*d*e^2)*sin(d*x + c))
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.132.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)^2}{a^2\,(e\,\sin(c+dx))^{3/2}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

3.133.1 Optimal result 1012
3.133.2 Mathematica [A] (verified) 1013
3.133.3 Rubi [A] (verified) 1013
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3.133.1 Optimal result

Integrand size = 25, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx &= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}\\ &- \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}\\ &- \frac{4e}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}}\\ &- \frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}} + \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231a^2de^2\sqrt{e\sin(c+dx)}}\end{aligned}$$

output

```
4/11*e^3/a^2/d/(e*sin(d*x+c))^(11/2)-2/11*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+
c))^(11/2)-2/11*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(11/2)-4/7*e/a^2/d/(
e*sin(d*x+c))^(7/2)+16/77*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(7/2)-4/231*co
s(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)-4/231*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c+d\right.\right.}{1848a^2de^2\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
-1/1848*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^5*(52 + 97*Cos[c + d*x] + 4*Cos
[2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi
 - 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

3.133.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\sin(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4360$$
$$\int \frac{\cos^2(c+dx)}{(a(-\cos(c+dx))-a)^2(e\sin(c+dx))^{5/2}}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 3354$$

$$\frac{e^4 \int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{(e\sin(c+dx))^{13/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{13/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4 \int \left(\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}} + \frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(\frac{4a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{231de^6\sqrt{e\sin(c+dx)}} - \frac{4a^2\cos(c+dx)}{231de^5(e\sin(c+dx))^{3/2}} - \frac{4a^2}{7de^3(e\sin(c+dx))^{7/2}} + \frac{16a^2\cos(c+dx)}{77de^3(e\sin(c+dx))^{7/2}} + \frac{4a^2}{11de(e\sin(c+}$$
$$a^4$$

input
```
Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(e^4*((4*a^2)/(11*d*e*(e*Sin[c + d*x])^(11/2)) - (2*a^2*Cos[c + d*x])/(11*
d*e*(e*Sin[c + d*x])^(11/2)) - (2*a^2*Cos[c + d*x]^3)/(11*d*e*(e*Sin[c + d
*x])^(11/2)) - (4*a^2)/(7*d*e^3*(e*Sin[c + d*x])^(7/2)) + (16*a^2*Cos[c +
d*x])/(77*d*e^3*(e*Sin[c + d*x])^(7/2)) - (4*a^2*Cos[c + d*x])/(231*d*e^5*
(e*Sin[c + d*x])^(3/2)) + (4*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(231*d*e^6*Sqrt[e*Sin[c + d*x]]))/a^4
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.133.4 Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

method	result
default	$\frac{4e^3\left(11\cos(dx+c)^2-4\right)}{77a^2(e\sin(dx+c))^{\frac{11}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{13}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^7+47\sin(dx+c)^5-87\sin$ $\frac{231e^2a^2\sin(dx+c)^6\cos(dx+c)\sqrt{e\sin(dx+c)}}{d}$

input
```
int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4/77*e^3/a^2/(e*sin(d*x+c))^(11/2)*(11*cos(d*x+c)^2-4)-2/231/e^2*((-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(13/2)*EllipticF((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^7+47*sin(d*x+c)^5-87*sin(d*x+c)^3+
42*sin(d*x+c))/a^2/sin(d*x+c)^6/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.133.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}\cos(dx+c)^4+2\sqrt{2}\cos(dx+c)^3-2\sqrt{2}\cos(dx+c)\right.\right.}{}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/231*((sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*sqrt(2)*cos(
d*x + c) - sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*
sin(d*x + c)) + (sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*sqr
t(2)*cos(d*x + c) - sqrt(2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + (2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 47*cos(d*x
+ c) + 24)*sqrt(e*sin(d*x + c)))/(a^2*d*e^3*cos(d*x + c)^4 + 2*a^2*d*e^3*c
os(d*x + c)^3 - 2*a^2*d*e^3*cos(d*x + c) - a^2*d*e^3)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))^2(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))^2(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.133.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\,\sin(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx$

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3.134.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx$$
$$= \frac{a^3 \cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3a^3\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a^3\,\text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{3a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

output

```
3*a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1
+m)/d/e/(1+m)+a^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*si
n(d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1
/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*
a^3*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin
(d*x+c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

3.134.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.65

$$\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx$$
$$= \frac{a^3 (e\sin(c+dx))^m \left(3\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)\sin(c+dx) + \text{Hypergeometric2F1}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

output
```
(a^3*(e*Sin[c + d*x])^m*(3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2]*Sin[c + d*x] + Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + 3*Hypergeometric2F1[3/2, (1 + m)/2,
(3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

3.134.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4360, 25, 25, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx) + a)^3 (e\sin(c+dx))^m \, dx$$
$$\downarrow 3042$$
$$\int \left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^3 \left(e\cos\left(c + dx - \frac{\pi}{2}\right)\right)^m dx$$
$$\downarrow 4360$$
$$\int \sec^3(c+dx)\left(-(a(-\cos(c+dx)) - a)^3\right)(e\sin(c+dx))^m dx$$
$$\downarrow 25$$
$$-\int -(\cos(c+dx)a + a)^3 \sec^3(c+dx)(e\sin(c+dx))^m dx$$
$$\downarrow 25$$

$$\int \sec^3(c+dx)(a\cos(c+dx)+a)^3(e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3352$$

$$\int (a^3(e\sin(c+dx))^m + a^3\sec^3(c+dx)(e\sin(c+dx))^m + 3a^3\sec^2(c+dx)(e\sin(c+dx))^m + 3a^3\sec(c+dx)(e\si$$

$$\downarrow 2009$$

$$\frac{3a^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} +$$
$$\frac{a^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} +$$
$$\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} +$$
$$\frac{3a^3\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

output
```
(a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x
])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2
, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Co
s[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]
*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.134.4 Maple [F]

$$\int (a + a \sec(dx + c))^3 (e \sin(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

3.134.5 Fricas [F]

$$\int (a + a \sec(c + dx))^3 (e \sin(c + dx))^m \, dx = \int (a \sec(dx + c) + a)^3 (e \sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
 a^3)*(e*sin(d*x + c))^m, x)
```

3.134.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx = a^3 \left(\int (e\sin(c+dx))^m \, dx + \int 3(e\sin(c+dx))^m \sec(c+dx) \, dx + \int 3(e\sin(c+dx))^m \sec^2(c+dx) \, dx + \int (e\sin(c+dx))^m \sec^3(c+dx) \, dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

output
```
a**3*(Integral((e*sin(c + d*x))**m, x) + Integral(3*(e*sin(c + d*x))**m*se
c(c + d*x), x) + Integral(3*(e*sin(c + d*x))**m*sec(c + d*x)**2, x) + Inte
gral((e*sin(c + d*x))**m*sec(c + d*x)**3, x))
```

3.134.7 Maxima [F]

$$\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx = \int (a\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.134.8 Giac [F]

$$\int (a + a\sec(c+dx))^3 (e\sin(c+dx))^m \, dx = \int (a\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 (e\sin(c + dx))^m \, dx = \int (e \sin(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^3 dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.135 $\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^m \, dx$

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3.135.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^m \, dx$$
$$= \frac{a^2 \cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{2a^2\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

output

```
2*a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1
+m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*
x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+a^2*hypergeom(
[3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+c))^(1+m)
*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

3.135.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^m \, dx$$

$$= \frac{a^2 (e\sin(c+dx))^m \left(2\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)\sin(c+dx) + \sqrt{\cos^2(c+dx)}\,(\text{Hype}\right.}{d(1}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

output
```
(a^2*(e*Sin[c + d*x])^m*(2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + Hypergeometric2F1[3/2, (1 + m)/2, (3
 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

3.135.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2 (e\sin(c+dx))^m \, dx$$

$$\downarrow 3042$$

$$\int \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4360$$

$$\int \sec^2(c+dx)(a(-\cos(c+dx))-a)^2 (e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2 \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sin\left(c+dx+\frac{\pi}{2}\right)^2} dx$$

$$\downarrow 3352$$

$$\int \left(a^2(e\sin(c+dx))^m + a^2\sec^2(c+dx)(e\sin(c+dx))^m + 2a^2\sec(c+dx)(e\sin(c+dx))^m\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2a^2(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} + \frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} + \frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

output
```
(a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x
])^(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/
2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.135.4 Maple [F]

$$\int (a + a \sec(dx + c))^2 (e \sin(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

3.135.5 Fricas [F]

$$\int (a + a \sec(c + dx))^2 (e \sin(c + dx))^m \, dx = \int (a \sec(dx + c) + a)^2 (e \sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*sin(d*x + c))^
m, x)
```

3.135.6 Sympy [F]

$$\int (a + a \sec(c + dx))^2 (e \sin(c + dx))^m \, dx = a^2 \left(\int (e \sin(c + dx))^m \, dx + \int 2 (e \sin(c + dx))^m \sec(c + dx) \, dx + \int (e \sin(c + dx))^m \sec^2(c + dx) \, dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

output
```
a**2*(Integral((e*sin(c + d*x))**m, x) + Integral(2*(e*sin(c + d*x))**m*se
c(c + d*x), x) + Integral((e*sin(c + d*x))**m*sec(c + d*x)**2, x))
```

3.135.7 Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.135.8 Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (e\,\sin(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.136 $\int (a + a\sec(c+dx))(e\sin(c+dx))^m\,dx$

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3.136.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^m\,dx$$
$$= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

output
```
a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/
d/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2
)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^m\,dx$$
$$= \frac{a(e\sin(c+dx))^m\left(\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx) + \sqrt{\cos^2(c+dx)}\,\text{Hyperge}\right.}{d(1+m)}$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

output
```
(a*(e*Sin[c + d*x])^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c +
d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + m))
```

3.136.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 27, 278, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4360$$

$$\int -(\sec(c+dx)(a(-\cos(c+dx))-a)(e\sin(c+dx))^m)\,dx$$

$$\downarrow 25$$

$$-\int -((\cos(c+dx)a+a)\sec(c+dx)(e\sin(c+dx))^m)\,dx$$

$$\downarrow 25$$

$$\int \sec(c+dx)(a\cos(c+dx)+a)(e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int (e\sin(c+dx))^m dx + a\int \sec(c+dx)(e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$a\int (e\sin(c+dx))^m dx + a\int \frac{(e\sin(c+dx))^m}{\cos(c+dx)}dx$$

$$\downarrow 3044$$

$$\frac{a\int \frac{e^2(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{de}+a\int(e\sin(c+dx))^m dx$$

$$\downarrow 27$$

$$\frac{ae\int \frac{(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}+a\int(e\sin(c+dx))^m dx$$

$$\downarrow 278$$

$$a\int(e\sin(c+dx))^m dx+\frac{a(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 3122$$

$$\frac{a(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

output
```
(a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^
2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.136.4 Maple [F]

$$\int (a + a \sec(dx + c)) (e \sin(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

3.136.5 Fricas [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.136.6 Sympy [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^m \, dx = a\left(\int (e\sin(c + dx))^m \, dx + \int (e\sin(c + dx))^m \sec(c + dx) \, dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

output
```
a*(Integral((e*sin(c + d*x))**m, x) + Integral((e*sin(c + d*x))**m*sec(c +
 d*x), x))
```

3.136.7 Maxima [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.136.8 Giac [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^m\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^m\,dx = \int (e\,\sin(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right) dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.137 $\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$

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3.137.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}}$$

output
```
-e*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)+e*cos(d*x+c)*hypergeom([-1/2, -1/2+1/2*
m],[1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)/(cos(d*x+c)^2
)^(1/2)
```

3.137.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.38 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.83

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{i2^{1-m}\left(1-e^{2i(c+dx)}\right)^{-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2e^{i(c+dx)}(-2+m)m\,\text{Hypergeometri}\right.$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

output
```
(I*2^(1 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[c/2
 + (d*x)/2]^2*(2*E^(I*(c + d*x))*(-2 + m)*m*Hypergeometric2F1[(1 - m)/2, 1
 - m, (3 - m)/2, E^((2*I)*(c + d*x))] - 4*E^(I*(c + d*x))*(-2 + m)*m*Hyper
geometric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] - (-1 + m)*
(2*(-2 + m)*Hypergeometric2F1[1 - m, -1/2*m, 1 - m/2, E^((2*I)*(c + d*x))]
 - 2*E^((2*I)*(c + d*x))*m*Hypergeometric2F1[2 - m, 1 - m/2, 2 - m/2, E^((
2*I)*(c + d*x))] - (-2 + m)*(2*Hypergeometric2F1[2 - m, -1/2*m, 1 - m/2, E
^((2*I)*(c + d*x))] + Hypergeometric2F1[-m, -1/2*m, 1 - m/2, E^((2*I)*(c +
 d*x))])))*Sec[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 - E^((2*I)*(c + d*x)))^m
*(-2 + m)*(-1 + m)*m*(a + a*Sec[c + d*x])*Sin[c + d*x]^m)
```

3.137.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^m}{a(-\cos(c+dx))-a}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^m}{\cos(c+dx)a+a}dx$$
$$\downarrow 25$$
$$\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{a\cos(c+dx)+a}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 3318$$

$$\frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{m-2}dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{m-2}dx}{a}$$

$$\downarrow 3042$$

$$\frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{m-2}dx}{a} - \frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{m-2}dx}{a}$$

$$\downarrow 3044$$

$$\frac{e\int (e\sin(c+dx))^{m-2}d(e\sin(c+dx))}{ad} - \frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{m-2}dx}{a}$$

$$\downarrow 15$$

$$-\frac{e^2\int \cos(c+dx)^2(e\sin(c+dx))^{m-2}dx}{a} - \frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

$$\downarrow 3057$$

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}} - \frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

output
```
-((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeom
etric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1
 + m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

3.137.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.137.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{a + a \sec(dx+c)} dx$$

input
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

output
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

3.137.5 Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.137.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\sin(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

3.137.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.137.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\,\sin(c+dx))^m}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.138 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

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3.138.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)} - \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{2e(e\sin(c+dx))^{-1+m}}{a^2d(1-m)}$$

output

```
2*e^3*(e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)-2*e*(e*sin(d*x+c))^(-1+m)/a^2/d/(1
-m)-e^3*cos(d*x+c)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)
*(e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)/(cos(d*x+c)^2)^(1/2)-e^3*cos(d*x+c)*hyp
ergeom([-1/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-3+m)
/a^2/d/(3-m)/(cos(d*x+c)^2)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.96

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{i2^{4-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(-1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1-\frac{m}{2},e^{2i(c+dx)}\right)}{4m}+\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

output
```
((-I)*2^(4 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[
(c + d*x)/2]^4*(((-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, (2 + m)/2
, 1 - m/2, E^((2*I)*(c + d*x))])/(4*m) + (E^(I*(c + d*x))*((6 - 5*m + m^2)
*Hypergeometric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] + E^(
I*(c + d*x))*(-1 + m)*(E^(I*(c + d*x))*(-2 + m)*Hypergeometric2F1[2 - m, (
3 - m)/2, (5 - m)/2, E^((2*I)*(c + d*x))] - 2*(-3 + m)*Hypergeometric2F1[2
 - m, 1 - m/2, 2 - m/2, E^((2*I)*(c + d*x))]))/((1 - E^((2*I)*(c + d*x)))
^m*(-3 + m)*(-2 + m)*(-1 + m)) + (E^((2*I)*(c + d*x))*((4*E^(I*(c + d*x))*
Hypergeometric2F1[(3 - m)/2, 4 - m, (5 - m)/2, E^((2*I)*(c + d*x))])/(-3 +
 m) + (4*E^((3*I)*(c + d*x))*Hypergeometric2F1[4 - m, (5 - m)/2, (7 - m)/2
, E^((2*I)*(c + d*x))])/(-5 + m) - Hypergeometric2F1[4 - m, 1 - m/2, 2 - m
/2, E^((2*I)*(c + d*x))]/(-2 + m) - (6*E^((2*I)*(c + d*x))*Hypergeometric2
F1[4 - m, 2 - m/2, 3 - m/2, E^((2*I)*(c + d*x))])/(-4 + m) - (E^((4*I)*(c
+ d*x))*Hypergeometric2F1[4 - m, 3 - m/2, 4 - m/2, E^((2*I)*(c + d*x))])/(
-6 + m)))/(1 - E^((2*I)*(c + d*x)))^m)*Sec[c + d*x]^2*(e*Sin[c + d*x])^m)/
(a^2*d*(1 + Sec[c + d*x])^2*Sin[c + d*x]^m)
```

3.138.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(a(-\cos(c+dx))-a)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^4\int \cos^2(c+dx)(a-a\cos(c+dx))^2(e\sin(c+dx))^{m-4}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{m-4}\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^4\int \left(a^2\cos^4(c+dx)(e\sin(c+dx))^{m-4}-2a^2\cos^3(c+dx)(e\sin(c+dx))^{m-4}+a^2\cos^2(c+dx)(e\sin(c+dx))^{m-4}\right.}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(-\frac{2a^2(e\sin(c+dx))^{m-1}}{de^3(1-m)}-\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(c+dx)\right)}{de(3-m)\sqrt{\cos^2(c+dx)}}-\frac{a^2\cos(c+dx)(e\sin(c+dx))^r}{}\right.}{a^4}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((2*a^2*(e*Sin[c + d*x])^(-3 + m))/(d*e*(3 - m)) - (a^2*Cos[c + d*x]*
Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c +
 d*x])^(-3 + m))/(d*e*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (a^2*Cos[c + d*x]*Hy
pergeometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(-3 + m))/(d*e*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*a^2*(e*Sin[c + d*x]
)^(-1 + m))/(d*e^3*(1 - m))))/a^4
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.138.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{(a+a\sec(dx+c))^2} dx$$

input
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

output
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

3.138.5 Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2
), x)
```

3.138.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\sin(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral((e*sin(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.138.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.138.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^m}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.139 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

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3.139.1 Optimal result

Integrand size = 23, antiderivative size = 236

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)}$$
$$+\frac{e^5\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{3e^5\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{7e^3(e\sin(c+dx))^{-3+m}}{a^3d(3-m)}-\frac{3e(e\sin(c+dx))^{-1+m}}{a^3d(1-m)}$$

output

```
-4*e^5*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)+7*e^3*(e*sin(d*x+c))^(-3+m)/a^3/d
/(3-m)-3*e*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+e^5*cos(d*x+c)*hypergeom([-5/
2, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m
)/(cos(d*x+c)^2)^(1/2)+3*e^5*cos(d*x+c)*hypergeom([-3/2, -5/2+1/2*m],[-3/2
+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(cos(d*x+c)^2)^(1/
2)
```

3.139.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.09 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.22

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{i2^{4-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(-1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1-\frac{m}{2},e^{2i(c+dx)}\right)}{2m}+\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

output
```
((-I)*2^(4 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[
(c + d*x)/2]^6*(((-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, (2 + m)/2
, 1 - m/2, E^((2*I)*(c + d*x))])/(2*m) + (3*E^(I*(c + d*x))*((6 - 5*m + m^
2)*Hypergeometric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + m)*(E^(I*(c + d*x))*(-2 + m)*Hypergeometric2F1[2 - m,
 (3 - m)/2, (5 - m)/2, E^((2*I)*(c + d*x))] - 2*(-3 + m)*Hypergeometric2F1
[2 - m, 1 - m/2, 2 - m/2, E^((2*I)*(c + d*x))]))/((1 - E^((2*I)*(c + d*x)
))^m*(-3 + m)*(-2 + m)*(-1 + m)) + (6*E^((2*I)*(c + d*x))*((4*E^(I*(c + d*
x))*Hypergeometric2F1[(3 - m)/2, 4 - m, (5 - m)/2, E^((2*I)*(c + d*x))])/(
-3 + m) + (4*E^((3*I)*(c + d*x))*Hypergeometric2F1[4 - m, (5 - m)/2, (7 -
m)/2, E^((2*I)*(c + d*x))])/(-5 + m) - Hypergeometric2F1[4 - m, 1 - m/2, 2
 - m/2, E^((2*I)*(c + d*x))]/(-2 + m) - (6*E^((2*I)*(c + d*x))*Hypergeomet
ric2F1[4 - m, 2 - m/2, 3 - m/2, E^((2*I)*(c + d*x))])/(-4 + m) - (E^((4*I)
*(c + d*x))*Hypergeometric2F1[4 - m, 3 - m/2, 4 - m/2, E^((2*I)*(c + d*x))
])/(-6 + m)))/(1 - E^((2*I)*(c + d*x)))^m - (4*E^((3*I)*(c + d*x))*(-(Hype
rgeometric2F1[(3 - m)/2, 6 - m, (5 - m)/2, E^((2*I)*(c + d*x))]/(-3 + m))
 - (15*E^((2*I)*(c + d*x))*Hypergeometric2F1[(5 - m)/2, 6 - m, (7 - m)/2, E
^((2*I)*(c + d*x))])/(-5 + m) - (15*E^((4*I)*(c + d*x))*Hypergeometric2F1[
6 - m, (7 - m)/2, (9 - m)/2, E^((2*I)*(c + d*x))])/(-7 + m) - (E^((6*I)*(c
 + d*x))*Hypergeometric2F1[6 - m, (9 - m)/2, (11 - m)/2, E^((2*I)*(c + ...
```

3.139.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4360, 25, 25, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(a(-\cos(c+dx))-a)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(\cos(c+dx)a+a)^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(a\cos(c+dx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 3354$$

$$\frac{e^6\int \cos^3(c+dx)(a-a\cos(c+dx))^3(e\sin(c+dx))^{m-6}dx}{a^6}$$

$$\downarrow 3042$$

$$\frac{e^6\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{m-6}\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a-a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$

$$\downarrow 3352$$

$$\frac{e^6\int \left(-a^3\cos^6(c+dx)(e\sin(c+dx))^{m-6}+3a^3\cos^5(c+dx)(e\sin(c+dx))^{m-6}-3a^3\cos^4(c+dx)(e\sin(c+dx))^{m}\right.}{a^6}$$

$$\downarrow 2009$$

$$\frac{e^6\left(-\frac{3a^3(e\sin(c+dx))^{m-1}}{de^5(1-m)}+\frac{7a^3(e\sin(c+dx))^{m-3}}{de^3(3-m)}+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{de(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3}{}\right.}{a^6}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

output
```
(e^6*((-4*a^3*(e*Sin[c + d*x])^(-5 + m))/(d*e*(5 - m)) + (a^3*Cos[c + d*x]
*Hypergeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c
+ d*x])^(-5 + m))/(d*e*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c
+ d*x])^(-5 + m))/(d*e*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*a^3*(e*Sin[c + d
*x])^(-3 + m))/(d*e^3*(3 - m)) - (3*a^3*(e*Sin[c + d*x])^(-1 + m))/(d*e^5*
(1 - m))))/a^6
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.139.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

input
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

output
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

3.139.5 Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e \sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3
*a^3*sec(d*x + c) + a^3), x)
```

3.139.6 Sympy [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{(e \sin(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral((e*sin(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

3.139.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.139.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(c+dx)^3\,(e\,\sin(c+dx))^m}{a^3\,(\cos(c+dx)+1)^3}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

3.140 $\int (a + a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

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3.140.2 Mathematica [B] (warning: unable to verify) . . . 1053
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3.140.7 Maxima [F] . . . 1057
3.140.8 Giac [F] . . . 1058
3.140.9 Mupad [F(-1)] . . . 1058

3.140.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int (a + a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \frac{2ae\,\text{AppellF1}\left(-\frac{1}{2}, \frac{1-m}{2}, \frac{1}{2}(-2-m), \frac{1}{2}, \cos(c+dx), -\cos(c+dx)\right)(1-\cos(c$$

output
```
2*a*e*AppellF1(-1/2,-1-1/2*m,-1/2*m+1/2,1/2,-cos(d*x+c),cos(d*x+c))*(1-cos
(d*x+c))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((1+c
os(d*x+c))^(1/2*m))
```

3.140.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1243 vs. 2(106) = 212.

Time = 9.56 (sec) , antiderivative size = 1243, normalized size of antiderivative = 11.73

$$\int (a + a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

output

```
(4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c
 + d*x]))^(3/2)*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*(6*AppellF
1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*m*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] - 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 1/2, 1 +
m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 +
 m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + A
ppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - 4*m*AppellF1[(3 + m)/2, 3/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 6*AppellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] + 6*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(1 +
m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c +
d*x] + 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m,...
```

3.140.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{3/2}(e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4364$$

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\int \frac{(-\cos(c+dx)a-a)^{3/2}(e\sin(c+dx))^m}{(-\cos(c+dx))^{3/2}}dx}{\sqrt{a(-\cos(c+dx))-a}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\int\frac{(-e\cos(c+dx+\frac{\pi}{2}))^m(-\sin(c+dx+\frac{\pi}{2})a-a)^{3/2}}{(-\sin(c+dx+\frac{\pi}{2}))^{3/2}}dx}{\sqrt{a(-\cos(c+dx))-a}}$$

$$\downarrow 3365$$

$$-\frac{e\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}-\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int(-$$

$$\downarrow 152$$

$$\frac{ae\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1}{-}}}{d}$$

$$\downarrow 152$$

$$\frac{ae\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-\frac{}{}}}{d}$$

$$\downarrow 150$$

$$\frac{2ae\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-\frac{1}{2}}(a\cos(c+dx)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

output
```
(2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d
*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*(-a - a*Cos[c + d*x])^(-1/2 + (1 - m)/
2 + m/2)*(-a + a*Cos[c + d*x])^((1 - m)/2 + (-1 + m)/2)*Sqrt[a + a*Sec[c +
 d*x]]*(e*Sin[c + d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

3.140.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
 - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.140.4 Maple [F]

$$\int (a + a \sec(dx + c))^{\frac{3}{2}} (e \sin(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

3.140.5 Fricas [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}(e\sin(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
```

output
```
Timed out
```

3.140.7 Maxima [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}(e\sin(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.140.8 Giac [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}(e\sin(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
sage0*x
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int (e\,\sin(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$

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3.141.6 Sympy [F] 1063
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3.141.8 Giac [F] 1063
3.141.9 Mupad [F(-1)] 1064

3.141.1 Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx =$$
$$-\frac{2e\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-\frac{m}{2},\frac{3}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+$$
$$d$$

output
```
-2*e*AppellF1(1/2,-1/2*m,-1/2*m+1/2,3/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x
+c))^(-1/2*m+1/2)*cos(d*x+c)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/
d/((1+cos(d*x+c))^(1/2*m))
```

3.141.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 433 vs. 2(107) = 214.

Time = 2.37 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.05

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$$
$$=\frac{}{d(1+m)\left(\left(2(1+m)\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},2+m,\frac{5+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1+2m)\right.\right.}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

output
```
(4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Sec[c + d*x
])]*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(
3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + (1 + 2*m)*AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1
+ m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.141.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4364$$

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\int \frac{\sqrt{-\cos(c+dx)a-a}(e\sin(c+dx))^m}{\sqrt{-\cos(c+dx)}}dx}{\sqrt{a(-\cos(c+dx))-a}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)a-a}}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a(-\cos(c+dx))-a}}$$

$$\downarrow 3365$$

$$-\frac{e\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}-\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int \frac{(}{}}{d}$$

$$\downarrow 152$$

$$-\frac{e\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1}{2}}}{d}$$

$$\downarrow 152$$

$$-\frac{e\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-}}{d}$$

$$\downarrow 150$$

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}-\frac{1}{2}}(}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

output
```
(-2*e*AppellF1[1/2, (1 - m)/2, -1/2*m, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(
1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(-a - a*Cos[c + d*x])^(-1/2 + (
1 - m)/2 + m/2)*(-a + a*Cos[c + d*x])^((1 - m)/2 + (-1 + m)/2)*Sqrt[a + a*
Sec[c + d*x]]*(e*Sin[c + d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

3.141.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.141.4 Maple [F]

$$\int \sqrt{a + a \sec(dx + c)}\,(e \sin(dx + c))^m\,dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

3.141.5 Fricas [F]

$$\int \sqrt{a + a \sec(c + dx)}(e \sin(c + dx))^m\,dx = \int \sqrt{a \sec(dx + c) + a}(e \sin(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.141.6 Sympy [F]

$$\int \sqrt{a + a\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}(e\sin\left(c+dx\right))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*(e*sin(d*x+c))**m,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sin(c + d*x))**m, x)
```

3.141.7 Maxima [F]

$$\int \sqrt{a + a\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}(e\sin\left(dx+c\right))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.141.8 Giac [F]

$$\int \sqrt{a + a\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}(e\sin\left(dx+c\right))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
sage0*x
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}(e\sin(c + dx))^m \, dx = \int (e \sin(c + dx))^m \sqrt{a + \frac{a}{\cos(c + dx)}} \, dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.142.1 Optimal result . . . 1065
3.142.2 Mathematica [B] (warning: unable to verify) . . . 1065
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3.142.5 Fricas [F] . . . 1068
3.142.6 Sympy [F] . . . 1069
3.142.7 Maxima [F] . . . 1069
3.142.8 Giac [F] . . . 1069
3.142.9 Mupad [F(-1)] . . . 1070

3.142.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2e\,\text{AppellF1}\left(\frac{3}{2},\frac{1-m}{2},\frac{2-m}{2},\frac{5}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2/3*e*AppellF1(3/2,1-1/2*m,-1/2*m+1/2,5/2,-cos(d*x+c),cos(d*x+c))*(1-cos(
d*x+c))^(-1/2*m+1/2)*cos(d*x+c)*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-
1+m)/d/(a+a*sec(d*x+c))^(1/2)
```

3.142.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(115) = 230.

Time = 2.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.41

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{4(3+m)\,\text{Appel}}{d(1+m)\left((2(1+m)\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},2+m,\frac{5+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+\text{AppellF1}\right.}$$

input
```
Integrate[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin[c + d*x])
^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, 1 + m, (5
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (
3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*(1 + Cos[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])
```

3.142.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\sqrt{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4364$$

$$\frac{\sqrt{a(-\cos(c+dx))-a}\int \frac{\sqrt{-\cos(c+dx)}(e\sin(c+dx))^m}{\sqrt{-\cos(c+dx)a-a}}dx}{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a(-\cos(c+dx))-a}\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)a-a}}dx}{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 3365$$

$$-\frac{e(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int \sqrt{-\cos(c+dx)}(-\cos(c+dx)a-a)}{d\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 152$$

$$\frac{e(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int\sqrt{-\cos(c+d}}{ad\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 152$$

$$\frac{e(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}+\frac{m-1}{2}}(e\sin(c}{ad\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx}}$$

$$\downarrow 150$$

$$\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}}{3ad\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(-a - a*Cos[c + d*x])^(1/2 +
(1 - m)/2 + m/2)*(-a + a*Cos[c + d*x])^((1 - m)/2 + (-1 + m)/2)*(e*Sin[c +
 d*x])^(-1 + m))/(3*a*d*(1 + Cos[c + d*x])^(m/2)*Sqrt[a + a*Sec[c + d*x]])
```

3.142.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.142.4 Maple [F]

$$\int \frac{(e \sin (dx+c))^m}{\sqrt{a+a \sec (dx+c)}} dx$$

input
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

3.142.5 Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{\sqrt{a+a \sec(c+dx)}} dx = \int \frac{(e \sin (dx+c))^m}{\sqrt{a \sec (dx+c)+a}} dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.142.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sin(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.142.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.142.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\,\sin(c+d\,x))^m}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.143.1 Optimal result 1071
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3.143.3 Rubi [A] (verified) 1072
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3.143.6 Sympy [F] 1075
3.143.7 Maxima [F] 1075
3.143.8 Giac [F(-2)] 1075
3.143.9 Mupad [F(-1)] 1076

3.143.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2e\,\text{AppellF1}\left(\frac{5}{2},\frac{1-m}{2},\frac{4-m}{2},\frac{7}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+dx)(1+\cos(c+d$$
$$5ad\sqrt{a+a\sec(c+dx)}$$

output
```
-2/5*e*AppellF1(5/2,2-1/2*m,-1/2*m+1/2,7/2,-cos(d*x+c),cos(d*x+c))*(1-cos(
d*x+c))^(-1/2*m+1/2)*cos(d*x+c)^2*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^
(-1+m)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.143.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 484 vs. 2(120) = 240.

Time = 2.32 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.03

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{}{d(1+m)\left(-4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},1+m,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-t\right.}$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - 2*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*
Sin[c + d*x])^m)/(d*(1 + m)*(-4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (
3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2 + (2
*m*AppellF1[(3 + m)/2, -1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - 4*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, m, (5 + m)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[(3 + m)/2, 1/2,
1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c +
d*x]) + (3 + m)*AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]))*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.143.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4364$$

$$\frac{\sqrt{a(-\cos(c+dx))-a}\int \frac{(-\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-\cos(c+dx)a-a)^{3/2}}dx}{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a(-\cos(c+dx))-a}\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^{3/2}}dx}{\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 3365$$

$$-\frac{e(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int(-\cos(c+dx))^{3/2}(-\cos(c+dx)a-a}{d\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 152$$

$$-\frac{e(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}}(e\sin(c+dx))^{m-1}\int(-\cos(c+dx)}{a^2d\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\downarrow 152$$

$$-\frac{e(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}+\frac{m-1}{2}}(e\sin}{a^2d\sqrt{-\cos(c+dx)}\sqrt{a\sec(c+}$$

$$\downarrow 150$$

$$\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}+\frac{m}{2}+\frac{1}{2}}(a\cos(c+dx)-a)^{\frac{1-}{2}}}{5a^2d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(-a - a*Cos[c + d*x])^(1/2
+ (1 - m)/2 + m/2)*(-a + a*Cos[c + d*x])^((1 - m)/2 + (-1 + m)/2)*(e*Sin[c
 + d*x])^(-1 + m))/(5*a^2*d*(1 + Cos[c + d*x])^(m/2)*Sqrt[a + a*Sec[c + d*
x]])
```

3.143.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))  Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])  Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.143.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}} dx$$

input
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

3.143.5 Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}} dx = \int \frac{(e \sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}} dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 +
 2*a^2*sec(d*x + c) + a^2), x)
```

3.143.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.143.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

3.143.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%%{
%%{poly1[1,0]:[1,0,-2]%%},[5,12]%%%}+%%%{%%{[6,0]:[1,0,-2]%%},[5,10]%%%}+%
%%{%%{[15,0]
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\,\sin(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.144 $\int (a + a\sec(c+dx))^n (e\sin(c+dx))^m \, dx$

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3.144.8 Giac [F] 1081
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3.144.1 Optimal result

Integrand size = 23, antiderivative size = 130

$$\int (a + a\sec(c+dx))^n (e\sin(c+dx))^m \, dx =$$

$$-\frac{e\,\text{AppellF1}\left(1-n, \frac{1-m}{2}, \frac{1}{2}(1-m-2n), 2-n, \cos(c+dx), -\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+$$
$$d(1-n)$$

output
```
-e*AppellF1(1-n,1/2-1/2*m-n,-1/2*m+1/2,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(
d*x+c))^(-1/2*m+1/2)*cos(d*x+c)*(1+cos(d*x+c))^(1/2-1/2*m-n)*(a+a*sec(d*x+
c))^n*(e*sin(d*x+c))^(-1+m)/d/(1-n)
```

3.144.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 276 vs. 2(130) = 260.

Time = 2.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.12

$$\int (a + a\sec(c+dx))^n (e\sin(c+dx))^m \, dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2}, n, 1+m, \frac{3}{}\right.}{d(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \tan^2\left(\frac{1}{2}(c+dx)\right), -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx)\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*
x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m
, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) -
 4*((1 + m)*AppellF1[(3 + m)/2, n, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m, (5 + m)/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[(c + d*x)/2]^2))
```

3.144.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^n (e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n \int (-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n (e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n \int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m \left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n} \left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n dx$$

$$\downarrow 3365$$

$$-\frac{e(-\cos(c+dx))^n (a\cos(c+dx)-a)^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n (e\sin(c+dx))^{m-1}(a(-\cos(c+dx))-a)^{\frac{1-m}{2}-n}\int(}{d}$$

$$\downarrow 152$$

$$-\frac{e(-\cos(c+dx))^n (a\cos(c+dx)-a)^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n (e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}(a($$

$$\downarrow 152$$

$$-\frac{e(1-\cos(c+dx))^{\frac{1-m}{2}}(-\cos(c+dx))^n(a\cos(c+dx)-a)^{\frac{1-m}{2}+\frac{m-1}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos($$

$$\downarrow 150$$

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\cos(c+dx)-a)^{\frac{1-m}{2}+\frac{m-1}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+d$$

input
```
Int[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

output
```
-((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos
[c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^
((1 - m - 2*n)/2)*(-a - a*Cos[c + d*x])^((1 - m)/2 - n + (-1 + m + 2*n)/2)
*(-a + a*Cos[c + d*x])^((1 - m)/2 + (-1 + m)/2)*(a + a*Sec[c + d*x])^n*(e*
Sin[c + d*x])^(-1 + m))/(d*(1 - n)))
```

3.144.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.144.4 Maple [F]

$$\int (a + a \sec(dx + c))^n (e \sin(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

3.144.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n (e \sin(c + dx))^m \, dx = \int (a \sec(dx + c) + a)^n (e \sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.144.6 Sympy [F]

$$\int (a + a\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \int (a(\sec(c + dx) + 1))^n (e\sin(c + dx))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*(e*sin(c + d*x))**m, x)
```

3.144.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^n (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.144.8 Giac [F]

$$\int (a + a\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^n (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \int (e \sin(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
```

output
```
int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.145 $\int (a + a\sec(c+dx))^n \sin^7(c+dx)\,dx$

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3.145.9 Mupad [F(-1)] 1088

3.145.1 Optimal result

Integrand size = 21, antiderivative size = 180

$$\int (a + a\sec(c+dx))^n \sin^7(c+dx)\,dx =$$
$$-\frac{(3-n)(8-n)(16-n)\,\text{Hypergeometric2F1}(6,4+n,5+n,1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{42a^4d(1-n)(4+n)}$$
$$-\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4d(1-n)}$$
$$+\frac{\cos^7(c+dx)(a+a\sec(c+dx))^{4+n}\,(6(8-n)-(108-25n+n^2)\sec(c+dx))}{42a^4d(1-n)}$$

output
```
-1/42*(3-n)*(8-n)*(16-n)*hypergeom([6, 4+n],[5+n],1+sec(d*x+c))*(a+a*sec(d
*x+c))^(4+n)/a^4/d/(-n^2-3*n+4)-cos(d*x+c)^7*(1-sec(d*x+c))^2*(a+a*sec(d*x
+c))^(4+n)/a^4/d/(1-n)+1/42*cos(d*x+c)^7*(a+a*sec(d*x+c))^(4+n)*(48-6*n-(n
^2-25*n+108)*sec(d*x+c))/a^4/d/(1-n)
```

3.145.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int (a + a\sec(c+dx))^n \sin^7(c+dx)\,dx$$
$$= \frac{((4+n)\cos^5(c+dx)\,((24-25n+n^2)\cos(c+dx)+3(13+n+(-1+n)\cos(2(c+dx))))-(-384+20}{42d(-1}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

output
```
(((4 + n)*Cos[c + d*x]^5*((24 - 25*n + n^2)*Cos[c + d*x] + 3*(13 + n + (-1
 + n)*Cos[2*(c + d*x)])) - (-384 + 200*n - 27*n^2 + n^3)*Hypergeometric2F1
[6, 4 + n, 5 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^4*(a*(1 + Sec[c +
d*x]))^n)/(42*d*(-1 + n)*(4 + n))
```

3.145.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4361, 25, 27, 111, 27, 162, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^7\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4361$$

$$-\frac{\int -a^3\cos^8(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d(-\sec(c+dx))}{a^6d}$$

$$\downarrow 25$$

$$\frac{\int a^3\cos^8(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d(-\sec(c+dx))}{a^6d}$$

$$\downarrow 27$$

$$\frac{\int \cos^8(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d(-\sec(c+dx))}{a^3d}$$

$$\downarrow 111$$

$$\frac{\frac{\int a\cos^8(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+3}(-n-(4-n)\sec(c+dx)+8)d(-\sec(c+dx))}{a(1-n)} - \frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{n+4}}{a(1-n)}}{a^3d}$$

$$\downarrow 27$$

$$\frac{\frac{\int \cos^8(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+3}(-n-(4-n)\sec(c+dx)+8)d(-\sec(c+dx))}{1-n} - \frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{n+4}}{a(1-n)}}{a^3d}$$

$$\downarrow 162$$

$$\frac{\frac{1}{42}(3-n)(8-n)(16-n)\int \cos^6(c+dx)(\sec(c+dx)a+a)^{n+3}d(-\sec(c+dx)) + \frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-25n+108\right)\sec(c+dx)\right)(a\sec(c+dx)+a)^{n+4}}{42a}}{1-n} - \frac{\cos^7}{}$$

$$a^3d$$

$$\downarrow 75$$

$$\frac{\frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-25n+108\right)\sec(c+dx)\right)(a\sec(c+dx)+a)^{n+4}}{42a} - \frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,\text{Hypergeometric2F1}(6,n+4,n+5,\sec(c+dx)+1)}{42a(n+4)}}{1-n} -$$

$$a^3d$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

output
```
(-((Cos[c + d*x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a*(
1 - n))) + (-1/42*((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5
+ n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a*(4 + n)) + (Cos[c
+ d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^2)*Sec[
c + d*x]))/(42*a))/(1 - n))/(a^3*d)
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.145.4 Maple [F]

$$\int (a + a \sec(dx + c))^n \sin(dx + c)^7 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

3.145.5 Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^7\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*(a*se
c(d*x + c) + a)^n*sin(d*x + c), x)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**7,x)
```

output
```
Timed out
```

3.145.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^7\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

3.145.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^7\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\,dx = \int \sin(c + d\,x)^7 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.146 $\int (a + a\sec(c+dx))^n \sin^5(c+dx)\,dx$

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3.146.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int (a + a\sec(c+dx))^n \sin^5(c+dx)\,dx$$
$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3d}$$
$$+ \frac{(32-13n+n^2)\,\text{Hypergeometric2F1}(4,3+n,4+n,1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{20a^3d(3+n)}$$

output
```
1/20*(12-n)*cos(d*x+c)^4*(a+a*sec(d*x+c))^(3+n)/a^3/d-1/5*cos(d*x+c)^5*(a+
a*sec(d*x+c))^(3+n)/a^3/d+1/20*(n^2-13*n+32)*hypergeom([4, 3+n],[4+n],1+se
c(d*x+c))*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)
```

3.146.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int (a + a\sec(c+dx))^n \sin^5(c+dx)\,dx =$$
$$-\frac{((3+n)\cos^4(c+dx)(-12+n+4\cos(c+dx)) - (32-13n+n^2)\,\text{Hypergeometric2F1}(4,3+n,4+n}{20d(3+n)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

output
```
-1/20*(((3 + n)*Cos[c + d*x]^4*(-12 + n + 4*Cos[c + d*x]) - (32 - 13*n + n
^2)*Hypergeometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x
])^3*(a*(1 + Sec[c + d*x]))^n)/(d*(3 + n))
```

3.146.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4361, 27, 100, 27, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4361$$

$$-\frac{\int a^2\cos^6(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{n+2}d(-\sec(c+dx))}{a^4d}$$

$$\downarrow 27$$

$$-\frac{\int \cos^6(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{n+2}d(-\sec(c+dx))}{a^2d}$$

$$\downarrow 100$$

$$-\frac{\frac{\int -a\cos^5(c+dx)(-n-5\sec(c+dx)+12)(\sec(c+dx)a+a)^{n+2}d(-\sec(c+dx))}{5a}+\frac{\cos^5(c+dx)(a\sec(c+dx)+a)^{n+3}}{5a}}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{5}\int -\cos^5(c+dx)(-n-5\sec(c+dx)+12)(\sec(c+dx)a+a)^{n+2}d(-\sec(c+dx))+\frac{\cos^5(c+dx)(a\sec(c+dx)+a)^{n+3}}{5a}}{a^2d}$$

$$\downarrow 87$$

$$-\frac{\frac{1}{5}\left(\frac{1}{4}(n^2-13n+32)\int\cos^4(c+dx)(\sec(c+dx)a+a)^{n+2}d(-\sec(c+dx))-\frac{(12-n)\cos^4(c+dx)(a\sec(c+dx)+a)^{n+3}}{4a}\right)+}{a^2d}$$

$$\downarrow 75$$

$$-\frac{\frac{1}{5}\left(-\frac{(n^2-13n+32)(a\sec(c+dx)+a)^{n+3}\,\text{Hypergeometric2F1}(4,n+3,n+4,\sec(c+dx)+1)}{4a(n+3)}-\frac{(12-n)\cos^4(c+dx)(a\sec(c+dx)+a)^{n+3}}{4a}\right)+\frac{\cos^5}{}}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

output
```
-(((Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a) + (-1/4*((12 - n)*C
os[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/a - ((32 - 13*n + n^2)*Hyperge
ometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n)
)/(4*a*(3 + n)))/5)/(a^2*d))
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)  Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.146.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sin (dx + c)^5 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

3.146.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sin^5(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \sin (dx + c)^5 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n*si
n(d*x + c), x)
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int (a + a \sec(c + dx))^n \sin^5(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.146.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\,dx = \int \sin(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.147 $\int (a + a\sec(c+dx))^n \sin^3(c+dx)\,dx$

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3.147.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a + a\sec(c+dx))^n \sin^3(c+dx)\,dx$$
$$= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2 d} - \frac{(4-n)\,\text{Hypergeometric2F1}(3,2+n,3+n,1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{3a^2 d(2+n)}$$

output
```
1/3*cos(d*x+c)^3*(a+a*sec(d*x+c))^(2+n)/a^2/d-1/3*(4-n)*hypergeom([3, 2+n]
,[3+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

3.147.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))^n \sin^3(c+dx)\,dx$$
$$= \frac{((2+n)\cos^3(c+dx) + (-4+n)\,\text{Hypergeometric2F1}(3,2+n,3+n,1+\sec(c+dx)))\,(1+\sec(c+dx))}{3d(2+n)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

output
```
(((2 + n)*Cos[c + d*x]^3 + (-4 + n)*Hypergeometric2F1[3, 2 + n, 3 + n, 1 +
 Sec[c + d*x]])*(1 + Sec[c + d*x])^2*(a*(1 + Sec[c + d*x]))^n)/(3*d*(2 + n
))
```

3.147.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4361, 25, 27, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4361$$

$$-\frac{\int -a\cos^4(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d(-\sec(c+dx))}{a^2d}$$

$$\downarrow 25$$

$$\frac{\int a\cos^4(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d(-\sec(c+dx))}{a^2d}$$

$$\downarrow 27$$

$$\frac{\int \cos^4(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d(-\sec(c+dx))}{ad}$$

$$\downarrow 87$$

$$\frac{\frac{1}{3}(4-n)\int -\cos^3(c+dx)(\sec(c+dx)a+a)^{n+1}d(-\sec(c+dx))+\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a}}{ad}$$

$$\downarrow 75$$

$$\frac{\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a}-\frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,\text{Hypergeometric2F1}(3,n+2,n+3,\sec(c+dx)+1)}{3a(n+2)}}{ad}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

output
```
((Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a) - ((4 - n)*Hypergeome
tric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(
3*a*(2 + n)))/(a*d)
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.147.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sin (dx + c)^3 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

3.147.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sin^3(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \sin (dx + c)^3 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int (a + a \sec(c + dx))^n \sin^3(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

output
```
Timed out
```

3.147.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^3(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^3\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.147.8 Giac [F(-2)]

Exception generated.

$$\int (a + a\sec(c + dx))^n \sin^3(c + dx)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^3(c + dx)\,dx = \int \sin(c + d\,x)^3 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.148 $\int (a + a\sec(c+dx))^n \sin(c+dx)\,dx$

3.148.1 Optimal result 1099
3.148.2 Mathematica [A] (verified) 1099
3.148.3 Rubi [A] (verified) 1100
3.148.4 Maple [F] 1101
3.148.5 Fricas [F] 1101
3.148.6 Sympy [F] 1101
3.148.7 Maxima [F] 1102
3.148.8 Giac [F] 1102
3.148.9 Mupad [B] (verification not implemented) 1102

3.148.1 Optimal result

Integrand size = 19, antiderivative size = 42

$$\int (a + a\sec(c+dx))^n \sin(c+dx)\,dx = \frac{\text{Hypergeometric2F1}(2, 1+n, 2+n, 1+\sec(c+dx))(a + a\sec(c+dx))^{1+n}}{ad(1+n)}$$

output
```
hypergeom([2, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

3.148.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c+dx))^n \sin(c+dx)\,dx = \frac{\text{Hypergeometric2F1}(2, 1+n, 2+n, 1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

output
```
(Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(1 + n))/(a*d*(1 + n))
```

3.148.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4361, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4361$$

$$-\frac{\int \cos^2(c+dx)(\sec(c+dx)a+a)^n d(-\sec(c+dx))}{d}$$

$$\downarrow 75$$

$$\frac{(a\sec(c+dx)+a)^{n+1}\,\text{Hypergeometric2F1}(2,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

output
```
(Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])
^(1 + n))/(a*d*(1 + n))
```

3.148.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.148.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sin (dx + c)\, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

3.148.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sin(c + dx)\, dx = \int (a \sec (dx + c) + a)^n \sin (dx + c)\ dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.148.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \sin(c + dx)\, dx = \int (a(\sec (c + dx) + 1))^n \sin (c + dx)\, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x), x)
```

3.148.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.148.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\,dx = \frac{\cos(c + dx)\left(a + \frac{a}{\cos(c+dx)}\right)^n {}_2\mathrm{F}_1(1 - n, -n;\ 2 - n;\ -\cos(c + dx))}{d\,(\cos(c + dx) + 1)^n\,(n - 1)}$$

input
```
int(sin(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

output
```
(cos(c + d*x)*(a + a/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -cos(c
+ d*x)))/(d*(cos(c + d*x) + 1)^n*(n - 1))
```

3.149 $\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$

3.149.1 Optimal result 1103
3.149.2 Mathematica [B] (verified) 1103
3.149.3 Rubi [A] (verified) 1104
3.149.4 Maple [F] 1105
3.149.5 Fricas [F] 1106
3.149.6 Sympy [F] 1106
3.149.7 Maxima [F] 1106
3.149.8 Giac [F] 1107
3.149.9 Mupad [F(-1)] 1107

3.149.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}$$

output
```
-1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

3.149.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(40) = 80.

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{2^{-1+n}\,\text{Hypergeometric2F1}\left(1,1-n,2-n,\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{-}}{d(-1+n)}$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

output
```
(2^(-1 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/
2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)
/(d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

3.149.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4361, 25, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4361$$

$$-\frac{a^2\int -\frac{(\sec(c+dx)a+a)^{n-1}}{a(1-\sec(c+dx))}d(-\sec(c+dx))}{d}$$

$$\downarrow 25$$

$$\frac{a^2\int \frac{(\sec(c+dx)a+a)^{n-1}}{a(1-\sec(c+dx))}d(-\sec(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a\int \frac{(\sec(c+dx)a+a)^{n-1}}{1-\sec(c+dx)}d(-\sec(c+dx))}{d}$$

$$\downarrow 78$$

$$-\frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

input
```
Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

output
```
-1/2*(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(d*n)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.149.4 Maple [F]

$$\int \csc(dx+c)\,(a+a\sec(dx+c))^n\,dx$$

input
```
int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

3.149.5 Fricas [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.149.6 Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x), x)
```

3.149.7 Maxima [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.149.8 Giac [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x),x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x), x)
```

3.150 $\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$

3.150.1 Optimal result 1108
3.150.2 Mathematica [A] (verified) 1108
3.150.3 Rubi [A] (verified) 1109
3.150.4 Maple [F] 1111
3.150.5 Fricas [F] 1112
3.150.6 Sympy [F] 1112
3.150.7 Maxima [F] 1112
3.150.8 Giac [F] 1113
3.150.9 Mupad [F(-1)] 1113

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}$$
$$- \frac{(2+n)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{8dn}$$

output
```
-1/4*a*(2-n)*(a+a*sec(d*x+c))^(-1+n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)
/d/(1-sec(d*x+c))-1/8*(2+n)*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+
a*sec(d*x+c))^n/d/n
```

3.150.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\left(2^{1+n}\,\text{Hypergeometric2F1}\left(1,1-n\right.\right.$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

output
```
(Cos[c + d*x]*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Hyper
geometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^n + 2^n*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c +
d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c
+ d*x])^n))/(8*d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

3.150.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4361, 27, 100, 25, 27, 88, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4361$$

$$-\frac{a^4\int \frac{\sec^2(c+dx)(\sec(c+dx)a+a)^{n-2}}{a^2(1-\sec(c+dx))^2}d(-\sec(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\int \frac{\sec^2(c+dx)(\sec(c+dx)a+a)^{n-2}}{(1-\sec(c+dx))^2}d(-\sec(c+dx))}{d}$$

$$\downarrow 100$$

$$-\frac{a^2\left(\frac{\int -\frac{a(n+2\sec(c+dx))(\sec(c+dx)a+a)^{n-2}}{1-\sec(c+dx)}d(-\sec(c+dx))}{2a}-\frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\left(-\frac{\int \frac{a(n+2\sec(c+dx))(\sec(c+dx)a+a)^{n-2}}{1-\sec(c+dx)}d(-\sec(c+dx))}{2a}-\frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\left(-\frac{1}{2}\int \frac{(n+2\sec(c+dx))(\sec(c+dx)a+a)^{n-2}}{1-\sec(c+dx)}d(-\sec(c+dx)) - \frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 88$$

$$-\frac{a^2\left(\frac{1}{2}\left(\frac{(2-n)(a\sec(c+dx)+a)^{n-1}}{2a(1-n)} - \frac{(n+2)\int \frac{(\sec(c+dx)a+a)^{n-1}}{1-\sec(c+dx)}d(-\sec(c+dx))}{2a}\right) - \frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 78$$

$$-\frac{a^2\left(\frac{1}{2}\left(\frac{(n+2)(a\sec(c+dx)+a)^n \text{ Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{4a^2n} + \frac{(2-n)(a\sec(c+dx)+a)^{n-1}}{2a(1-n)}\right) - \frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((a^2*(-1/2*(a + a*Sec[c + d*x])^(-1 + n)/(a*(1 - Sec[c + d*x])) + (((2 -
 n)*(a + a*Sec[c + d*x])^(-1 + n))/(2*a*(1 - n)) + ((2 + n)*Hypergeometric
2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(4*a^2*n))/
2))/d)
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))  Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))  Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)  Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.150.4 Maple [F]

$$\int \csc(dx+c)^3 (a+a\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

3.150.5 Fricas [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.150.6 Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**3, x)
```

3.150.7 Maxima [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.150.8 Giac [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^3}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^3,x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.151 $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

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3.151.1 Optimal result

Integrand size = 21, antiderivative size = 240

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{a^2(12+9n+n^2)\,\text{Hypergeometric2F1}\left(1,-2+n,-1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-2+n}}{16d(2-n)}$$
$$+\frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2}-\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2}$$
$$-\frac{a^2(a+a\sec(c+dx))^{-2+n}\left(12+4n-7n^2-n^3-2(1-n)(6+n)\sec(c+dx)\right)}{8d\left(2-3n+n^2\right)(1-\sec(c+dx))}$$

output

```
1/16*a^2*(n^2+9*n+12)*hypergeom([1, -2+n],[-1+n],1/2+1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(-2+n)/d/(2-n)+1/4*a^2*(3+n)*sec(d*x+c)^2*(a+a*sec(d*x+c))^(-2
+n)/d/(1-n)/(1-sec(d*x+c))^2-a^2*sec(d*x+c)^3*(a+a*sec(d*x+c))^(-2+n)/d/(1
-n)/(1-sec(d*x+c))^2-1/8*a^2*(a+a*sec(d*x+c))^(-2+n)*(12+4*n-7*n^2-n^3-2*(
1-n)*(6+n)*sec(d*x+c))/d/(n^2-3*n+2)/(1-sec(d*x+c))
```

3.151.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 492 vs. $2(240) = 480$.

Time = 3.89 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.05

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx =$$

$$-\frac{\cos(c+dx)(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\left(2^{1+n}\cot^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

output
```
-1/64*(Cos[c + d*x]*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Cot[(c + d*x)/2]^4
*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) - 3*2^n*n*Cot[(c
+ d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 2^n*
n^2*Cot[(c + d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1
+ n) - 3*2^(2 + n)*(-2 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x
]*Sec[(c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 +
 n) - 2^n*(-18 + 7*n + n^2)*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x
]*Sec[(c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 +
 n) + 32*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 12*n*Sec[c + d*x]*(1 + Sec[c
+ d*x])^n - 12*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n + 2*n*
Sec[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 2*Sec[(c + d*x)/2]^
4*Sec[c + d*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d*x)/2]^4*Sec[c + d*x]*
(1 + Sec[c + d*x])^n))/(d*(-2 + n)*(-1 + n)*(1 + Sec[c + d*x])^n)
```

3.151.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4361, 25, 27, 111, 25, 27, 166, 27, 163, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^n}{\cos\left(c + dx - \frac{\pi}{2}\right)^5} dx$$

$$\downarrow 4361$$

$$-\frac{a^6 \int -\frac{\sec^4(c+dx)(\sec(c+dx)a+a)^{n-3}}{a^3(1-\sec(c+dx))^3} d(-\sec(c+dx))}{d}$$

$$\downarrow 25$$

$$\frac{a^6 \int \frac{\sec^4(c+dx)(\sec(c+dx)a+a)^{n-3}}{a^3(1-\sec(c+dx))^3} d(-\sec(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^3 \int \frac{\sec^4(c+dx)(\sec(c+dx)a+a)^{n-3}}{(1-\sec(c+dx))^3} d(-\sec(c+dx))}{d}$$

$$\downarrow 111$$

$$\frac{a^3 \left(\frac{\int -\frac{a\sec^2(c+dx)(\sec(c+dx)a+a)^{n-3}(n\sec(c+dx)+3)}{(1-\sec(c+dx))^3} d(-\sec(c+dx))}{a(1-n)} - \frac{\sec^3(c+dx)(a\sec(c+dx)+a)^{n-2}}{a(1-n)(1-\sec(c+dx))^2} \right)}{d}$$

$$\downarrow 25$$

$$\frac{a^3 \left(-\frac{\int \frac{a\sec^2(c+dx)(\sec(c+dx)a+a)^{n-3}(n\sec(c+dx)+3)}{(1-\sec(c+dx))^3} d(-\sec(c+dx))}{a(1-n)} - \frac{\sec^3(c+dx)(a\sec(c+dx)+a)^{n-2}}{a(1-n)(1-\sec(c+dx))^2} \right)}{d}$$

$$\downarrow 27$$

$$\frac{a^3 \left(-\frac{\int \frac{\sec^2(c+dx)(\sec(c+dx)a+a)^{n-3}(n\sec(c+dx)+3)}{(1-\sec(c+dx))^3} d(-\sec(c+dx))}{1-n} - \frac{\sec^3(c+dx)(a\sec(c+dx)+a)^{n-2}}{a(1-n)(1-\sec(c+dx))^2} \right)}{d}$$

$$\downarrow 166$$

$$\frac{a^3 \left(-\frac{\int -\frac{a\sec(c+dx)(\sec(c+dx)a+a)^{n-3}(2(n+3)-(1-n)(n+6)\sec(c+dx))}{(1-\sec(c+dx))^2} d(-\sec(c+dx))}{4a} - \frac{(n+3)\sec^2(c+dx)(a\sec(c+dx)+a)^{n-2}}{4a(1-\sec(c+dx))^2} \right)}{1-n} - \frac{\sec^3(c+dx)(a\sec(c-}{a(1-n)(1-\sec(}$$

$$\downarrow 27$$

$$\frac{a^3 \left(-\frac{\frac{1}{4}\int -\frac{\sec(c+dx)(\sec(c+dx)a+a)^{n-3}(2(n+3)-(1-n)(n+6)\sec(c+dx))}{(1-\sec(c+dx))^2} d(-\sec(c+dx)) - \frac{(n+3)\sec^2(c+dx)(a\sec(c+dx)+a)^{n-2}}{4a(1-\sec(c+dx))^2} \right)}{1-n} - \frac{\sec^3(c+dx)(a\sec(}{a(1-n)(1-se}$$

3.151. $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

$$\downarrow 163$$

$$a^3\left(-\frac{\frac{1}{4}\left(\frac{\left(-2(1-n)(n+6)\sec(c+dx)-n^3-7n^2+4n+12\right)(a\sec(c+dx)+a)^{n-2}}{2a(2-n)(1-\sec(c+dx))}-\frac{1}{2}(1-n)\left(n^2+9n+12\right)\int\frac{(\sec(c+dx)a+a)^{n-3}}{1-\sec(c+dx)}d(-\sec(c+dx))\right)-\frac{(n+3)\sec^2}{}}{1-n}\right)$$

$$\downarrow 78$$

$$a^3\left(-\frac{\frac{1}{4}\left(\frac{\left(-2(1-n)(n+6)\sec(c+dx)-n^3-7n^2+4n+12\right)(a\sec(c+dx)+a)^{n-2}}{2a(2-n)(1-\sec(c+dx))}-\frac{(1-n)\left(n^2+9n+12\right)(a\sec(c+dx)+a)^{n-2}\,\text{Hypergeometric2F1}\left(1,n-2,n-1,\frac{1}{2}(\sec\right.}{4a(2-n)}}{1-n}\right)$$

$$d$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

output
```
(a^3*(-((Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(a*(1 - n)*(1 - Sec
[c + d*x])^2)) - (-1/4*((3 + n)*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(-2 +
n))/(a*(1 - Sec[c + d*x])^2) + (-1/4*((1 - n)*(12 + 9*n + n^2)*Hypergeomet
ric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^(-2 +
 n))/(a*(2 - n)) + ((a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3
 - 2*(1 - n)*(6 + n)*Sec[c + d*x]))/(2*a*(2 - n)*(1 - Sec[c + d*x])))/4)/(1
 - n)))/d
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.151.4 Maple [F]

$$\int \csc(dx+c)^5 (a+a\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

3.151.5 Fricas [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^5 \, dx$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n \, dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**n,x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^5\,dx$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

3.151.8 Giac [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^5\,dx$$

input
```
integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^5}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^5,x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^5, x)
```

3.152 $\int (a + a\sec(c+dx))^n \sin^4(c+dx)\,dx$

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3.152.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\int (a + a\sec(c+dx))^n \sin^4(c+dx)\,dx =$$

$$-\frac{\text{AppellF1}\left(1-n,-\frac{1}{2},\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+dx))^{\frac{1}{2}-n}(n-n\cos(c+dx)}{d(1-n)\sqrt{1-\cos(c+dx)}}$$

$$-\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d}$$

$$+\frac{2^{\frac{1}{2}+n}\,\text{AppellF1}\left(\frac{1}{2},-4+n,\frac{1}{2}-n,\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\cos^n(c+dx)(1+\cos(c+dx)}{d}$$

output

```
-cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d+2^(1/2+n)*AppellF1(1/2,-4+n,1/
2-n,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*cos(d*x+c)^n*(1+cos(d*x+c))^(-1/2
-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d-AppellF1(1-n,1/2-n,-1/2,2-n,-cos(d*x+c
),cos(d*x+c))*(1+cos(d*x+c))^(1/2-n)*(n-n*cos(d*x+c))*cot(d*x+c)*(a+a*sec(
d*x+c))^n/d/(1-n)/(1-cos(d*x+c))^(1/2)
```

3.152.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.75 (sec) , antiderivative size = 7069, normalized size of antiderivative = 30.73

$$\int (a + a\sec(c + dx))^n \sin^4(c + dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

output
```
Result too large to show
```

3.152.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.38, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4364, 3042, 3360, 3042, 3266, 3042, 3265, 3042, 3264, 150, 3525, 27, 3042, 3487, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c + dx)(a\sec(c + dx) + a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c + dx - \frac{\pi}{2}\right)^4 \left(a - a\csc\left(c + dx - \frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4364$$

$$(-\cos(c + dx))^n (a(-\cos(c + dx)) - a)^{-n} (a\sec(c + dx) + a)^n \int (-\cos(c + dx))^{-n} (-\cos(c + dx)a - a)^n \sin^4(c + dx)dx$$

$$\downarrow 3042$$

$$(-\cos(c + dx))^n (a(-\cos(c + dx)) - a)^{-n} (a\sec(c + dx) + a)^n \int \cos\left(c + dx + \frac{\pi}{2}\right)^4 \left(-\sin\left(c + dx + \frac{\pi}{2}\right)\right)^{-n} \left(-\sin\left(c + dx + \frac{\pi}{2}\right)a - a\right)^n dx$$

$$\downarrow 3360$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n\left(1-2\cos^2(c+dx)\right)dx+\int(-\cos(c+dx))^{4-n}(-\cos(c+dx)a-$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n dx+\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx\right.\right.\right.$$

$$\downarrow 3266$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+(\cos(c+dx)+1)\right.$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+(\cos(c+dx)+1)\right.$$

$$\downarrow 3265$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+(-\cos(c+dx))^-\right.$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+(-\cos(c+dx))^-\right.$$

$$\downarrow 3264$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{\sin(c+dx)(-\mathrm{co}}{\right.}$$

$$\downarrow 150$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\left(\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(1-2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{2^{n+\frac{1}{2}}\sin(c+dx)}{\right.}$$

$$\downarrow 3525$$

$$a)^n \left(\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \int 2(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n(an-an\cos(c+dx))dx}{2a} + \frac{2^{n+\frac{1}{2}}\sin(c+dx)(-\cos(c+dx))^{-n}\,c}{} \right.$$

$$\downarrow 27$$

$$a)^n \left(\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \int (-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n(an-an\cos(c+dx))dx}{a} + \frac{2^{n+\frac{1}{2}}\sin(c+dx)(-\cos(c+dx))^{-n}\cos}{} \right.$$

$$\downarrow 3042$$

$$a)^n \left(\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \int \left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n\left(an-an\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a} + \frac{2^{n+\frac{1}{2}}\sin(c+dx)(-}{} \right.$$

$$\downarrow 3487$$

$$a)^n \left(-\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \csc(c+dx)\sqrt{a(-\cos(c+dx))-a}\sqrt{an-an\cos(c+dx)}\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^{n-\frac{1}{2}}\sqrt{an}}{ad} \right.$$

$$\downarrow 152$$

$$a)^n \left(-\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \csc(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a(-\cos(c+dx))-a)^n\sqrt{an-an\cos(c+dx)}\int(-\cos(c+dx))^{-n}(\cos(c+}{ad} \right.$$

$$\downarrow 152$$

$$a)^n \left(-\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ \csc(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a(-\cos(c+dx))-a)^n(an-an\cos(c+dx))\int\sqrt{1-\cos(c+dx)}(-\cos(}{ad\sqrt{1-\cos(c+dx)}} \right.$$

$$\downarrow 150$$

$$a)^n \left(\frac{(-\cos(c+dx))^n (a(-\cos(c+dx))-a)^{-n} (a\sec(c+dx)+ 2^{n+\frac{1}{2}}\sin(c+dx)(-\cos(c+dx))^{-n}\cos^n(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a(-\cos(c+dx))-a)^n\,\text{AppellF1}\left(\frac{1}{2},}{d} \right.$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

output
```
((-Cos[c + d*x])^n*(a + a*Sec[c + d*x])^n*((AppellF1[1 - n, -1/2, 1/2 - n,
 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(-Cos[c + d*x])^(1 - n)*(1 + Cos[c +
d*x])^(1/2 - n)*(-a - a*Cos[c + d*x])^n*(a*n - a*n*Cos[c + d*x])*Csc[c + d
*x])/(a*d*(1 - n)*Sqrt[1 - Cos[c + d*x]]) + ((-Cos[c + d*x])^(1 - n)*(-a -
 a*Cos[c + d*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/
2 - n, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Co
s[c + d*x])^(-1/2 - n)*(-a - a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(-Cos[c +
d*x])^n)))/(-a - a*Cos[c + d*x])^n
```

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3360
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/d^4   Int[(d*Sin[e
 + f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3525
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Simp[1/(b*d*(m + n + 2))   Int[(a + b*Sin[e + f*
x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1
)) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.152.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sin (dx + c)^4 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

3.152.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sin^4(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \sin (dx + c)^4 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n, x
)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int (a + a \sec(c + dx))^n \sin^4(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**4,x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^4(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.152.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^4(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^4(c + dx)\,dx = \int \sin(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.153 $\int (a + a\sec(c+dx))^n \sin^2(c+dx)\,dx$

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3.153.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (a + a\sec(c+dx))^n \sin^2(c+dx)\,dx = -\frac{\text{AppellF1}\left(1-n,-\frac{1}{2},-\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{\frac{1}{2}-}}{d(1-n)}$$

output
```
-AppellF1(1-n,-1/2-n,-1/2,2-n,-cos(d*x+c),cos(d*x+c))*(1+cos(d*x+c))^(1/2-
n)*cot(d*x+c)*(a+a*sec(d*x+c))^n*(1-cos(d*x+c))^(1/2)/d/(1-n)
```

3.153.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.34 (sec) , antiderivative size = 4297, normalized size of antiderivative = 45.23

$$\int (a + a\sec(c+dx))^n \sin^2(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

output
```
(2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 +
Sec[c + d*x]))^n*Sin[(c + d*x)/2]*(Cos[2*(c + d*x)]*(-1/4*(1 + Sec[c + d*x
])^n - ((1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2 - ((1 + Sec[c + d*x])^n*Sin
[c + d*x]^4)/4) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] + (I/2)*(1 +
 Sec[c + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)] + (I/4)*(1 + Sec[c + d*x]
)^n*Sin[c + d*x]^4*Sin[2*(c + d*x)] + Cos[c + d*x]^4*(-1/4*(Cos[2*(c + d*x
)]*(1 + Sec[c + d*x])^n) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)]) +
Cos[c + d*x]^3*((-I)*Cos[2*(c + d*x)]*(1 + Sec[c + d*x])^n*Sin[c + d*x] -
(1 + Sec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)]) + Cos[c + d*x]^2*(Cos[
2*(c + d*x)]*((1 + Sec[c + d*x])^n/2 + (3*(1 + Sec[c + d*x])^n*Sin[c + d*x
]^2)/2) - (I/2)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] - ((3*I)/2)*(1 + Sec
[c + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)]) + Cos[c + d*x]*(Cos[2*(c + d
*x)]*(I*(1 + Sec[c + d*x])^n*Sin[c + d*x] + I*(1 + Sec[c + d*x])^n*Sin[c +
 d*x]^3) + (1 + Sec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)] + (1 + Sec[c
 + d*x])^n*Sin[c + d*x]^3*Sin[2*(c + d*x)]))*((3*AppellF1[1/2, n, 2, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1
/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3
/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2,
1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^
2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/...
```

3.153.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4364, 3042, 3353, 25, 3042, 3487, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n\sin^2(c+dx)dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+$$
$$a)^n\int\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n dx$$

$$\downarrow 3353$$

$$\frac{(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int-(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^{n+1}(a-}{a^2}$$

$$\downarrow 25$$

$$-\frac{(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^{n+1}(a-}{a^2}$$

$$\downarrow 3042$$

$$-\frac{(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-\right.}{a^2}$$

$$\downarrow 3487$$

$$\frac{\csc(c+dx)\sqrt{a-a\cos(c+dx)}(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-}}{a^2d}$$

$$\downarrow 152$$

$$\frac{\csc(c+dx)(a(-\cos(c+dx))-a)\sqrt{a-a\cos(c+dx)}(-\cos(c+dx))^n(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n}{a^2d}$$

$$\downarrow 152$$

$$\frac{\csc(c+dx)(a(-\cos(c+dx))-a)(a-a\cos(c+dx))(-\cos(c+dx))^n(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n}{a^2d\sqrt{1-\cos(c+dx)}}$$

$$\downarrow 150$$

$$\frac{\cot(c+dx)(a(-\cos(c+dx))-a)(a-a\cos(c+dx))(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,\right.}{a^2d(1-n)\sqrt{1-\cos(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

output
```
(AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(-1/2 - n)*(-a - a*Cos[c + d*x])*(a - a*Cos[c + d*x])*Cot[c
+ d*x]*(a + a*Sec[c + d*x])^n)/(a^2*d*(1 - n)*Sqrt[1 - Cos[c + d*x]])
```

3.153.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3353
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[1/b^2   Int[(d*Sin[e
 + f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[
n, 0])
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4364

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.153.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sin (dx + c)^2 \, dx$$

input

```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

output

```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

3.153.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \sin (dx + c)^2 \, dx$$

input

```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
```

output

```
integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n, x)
```

3.153.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \sin^2 (c + dx) \, dx$$

input

```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**2,x)
```

output

```
Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

3.153.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.153.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx)\,dx = \int \sin(c + d\,x)^2 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.154 $\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$

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3.154.9 Mupad [F(-1)] 1140

3.154.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n}n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))}{d}$$

output
```
-cot(d*x+c)*(a+a*sec(d*x+c))^n/d+2^(-1/2+n)*n*hypergeom([1/2, 3/2-n],[3/2]
,1/2-1/2*sec(d*x+c))*(1+sec(d*x+c))^(-1/2-n)*(a+a*sec(d*x+c))^n*tan(d*x+c)
/d
```

3.154.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}\left(\cot^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},n,\frac{1}{2},\tan^2\left(\frac{1}{2}(c+dx)\right)\right) - \text{Hypergeometric2F1}\left(\frac{1}{2},n,\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((2^(-1 + n)*(Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c +
 d*x)/2]^2] - Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c +
 d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Se
c[c + d*x]))^n*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^n))
```

3.154.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4363, 25, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4363$$

$$-an\int -\sec(c+dx)(\sec(c+dx)a+a)^{n-1}dx-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 25$$

$$an\int \sec(c+dx)(\sec(c+dx)a+a)^{n-1}dx-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 3042$$

$$an\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{n-1}dx-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 4315$$

$$n(\sec(c+dx)+1)^{-n}(a\sec(c+dx)+a)^n\int \sec(c+dx)(\sec(c+dx)+1)^{n-1}dx-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 3042$$

$$n(\sec(c+dx)+1)^{-n}(a\sec(c+dx)+a)^n\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{n-1}dx-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 4314$$

$$-\frac{n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\int\frac{(\sec(c+dx)+1)^{n-\frac{3}{2}}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}}-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

$$\downarrow 79$$

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometri
c2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n
)*(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

3.154.4 Maple [F]

$$\int \csc(dx+c)^2 (a+a\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

3.154.5 Fricas [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.154.6 Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

3.154.7 Maxima [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.154.8 Giac [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^2,x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$

3.155.1 Optimal result 1141
3.155.2 Mathematica [A] (verified) 1142
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3.155.7 Maxima [F] 1147
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3.155.9 Mupad [F(-1)] 1148

3.155.1 Optimal result

Integrand size = 21, antiderivative size = 349

$$\begin{aligned}
&\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx \\
&= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\,(1-4n^2)\,(1-\cos(c+dx))^2} \\
&\quad - \frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} \\
&\quad - \frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d\,(3-8n+4n^2)\,(a-a\cos(c+dx))^2(a+a\cos(c+dx))} \\
&\quad + \frac{n(7-3n-n^2)\cos(c+dx)\left(\frac{1+\cos(c+dx)}{1-\cos(c+dx)}\right)^{-\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2}-n,1-n,2-n,-\frac{2\cos(c+dx)}{1-\cos(c+dx)}\right)}{d(1-2n)(3-2n)(1-n)(1+2n)(1-\cos(c+dx))^2}
\end{aligned}$$

output

```
(n^2-n+2)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(-4*n^2+1)/(1
-cos(d*x+c))^2-a^4*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(a-a
*cos(d*x+c))^2/(a+a*cos(d*x+c))^2-a^3*(4-n)*cos(d*x+c)*(a+a*sec(d*x+c))^n*
sin(d*x+c)/d/(4*n^2-8*n+3)/(a-a*cos(d*x+c))^2/(a+a*cos(d*x+c))+n*(-n^2-3*n
+7)*cos(d*x+c)*((1+cos(d*x+c))/(1-cos(d*x+c)))^(-1/2-n)*hypergeom([1-n, -1
/2-n],[2-n],-2*cos(d*x+c)/(1-cos(d*x+c)))*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/
(-8*n^4+20*n^3-10*n^2-5*n+3)/(1-cos(d*x+c))^2
```

3.155.2 Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{(a(1+\sec(c+dx)))^n\left(-2\cot^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},n,\frac{1}{2},\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(\cos(c+dx)\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

output
```
((a*(1 + Sec[c + d*x]))^n*((-2*Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2,
n, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(3*2^n*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^n + 2*(1 + Sec[c + d*x])^n + n*(1 + Sec[c +
 d*x])^n))/(1 + Sec[c + d*x])^n + (-(Cos[c + d*x]*(4*n*Cos[c + d*x] + (-3
+ n)*(3 + Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2) + (24*
Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^n*(-3*2^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n - 2*(1 + Sec[c +
d*x])^n + n*(2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c +
d*x])^n)))/(1 + Sec[c + d*x])^n)/(4*(-3 + 2*n)))*Tan[(c + d*x)/2])/(24*d)
```

3.155.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4364, 3042, 3362, 144, 25, 27, 172, 27, 172, 27, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sec(c+dx)+a)^n\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$
$$\downarrow 4364$$
$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n\csc^4(c+dx)dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int\frac{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n}{\cos\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 3362$$

$$-\frac{a^6\sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^{n-}}{(a\cos(c+dx)-a)^{5/2}}}{d\sqrt{a\cos(c+dx)-a}}$$

$$\downarrow 144$$

$$-\frac{a^6\sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\left(\frac{\int-\frac{a^2(-n-2\cos(c+dx)+2)(-\cos(c+dx))^{-}}{(a\cos(c+dx)-a)}}{a^3(3-}\right.}{d\sqrt{a\cos(c+dx)-a}}$$

$$\downarrow 25$$

$$-\frac{a^6\sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\left(-\frac{\int\frac{a^2(-n-2\cos(c+dx)+2)(-\cos(c+dx))}{(a\cos(c+dx)-a}}{a^3(3-}\right.}{d\sqrt{a\cos(c+dx)-a}}$$

$$\downarrow 27$$

$$-\frac{a^6\sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\left(-\frac{\int\frac{(-n-2\cos(c+dx)+2)(-\cos(c+dx))^{-n}}{(a\cos(c+dx)-a)^{}}}{a(3-}\right.}{d\sqrt{a\cos(c+dx)-a}}$$

$$\downarrow 172$$

$$-\frac{a^6\sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\left(-\frac{\int\frac{a^2(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)}{(a\cos(c+dx)}}{a^3}\right.}{d\sqrt{a\cos(c+}}$$

$$\downarrow 27$$

$$a^6 \sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n \left(-\frac{\int \frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^{n-}}{(a\cos(c+dx)-}}{a(1} \right)$$

$$-\frac{}{d\sqrt{a\cos(c+}}$$

$$\downarrow 172$$

$$a^6 \sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n \left(-\frac{\int \frac{a^2n\left(-n^2-3n+7\right)(-\cos(c+dx))^{-n}}{(a\cos(c+dx)-a)}}{a^3(2n-} \right)$$

$$-\frac{}{}$$

$$\downarrow 27$$

$$a^6 \sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n \left(-\frac{n\left(-n^2-3n+7\right)\int \frac{(-\cos(c+dx))^{-n}(-}{(a\cos(c+}}{a(2n+} \right)$$

$$-\frac{}{}$$

$$\downarrow 142$$

$$a^6 \sin(c+dx)(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n \left(-\frac{n\left(-n^2-3n+7\right)(-\cos(c+dx))^{1-n}\left(\frac{co}{1-}\right.}{} \right)$$

$$-\frac{}{}$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((a^6*(-Cos[c + d*x])^n*(-a - a*Cos[c + d*x])^(-1/2 - n)*(-(((-Cos[c + d*
x])^(1 - n)*(-a - a*Cos[c + d*x])^(-3/2 + n))/(a^2*(3 - 2*n)*(-a + a*Cos[c
 + d*x])^(3/2))) - (-(((4 - n)*(-Cos[c + d*x])^(1 - n)*(-a - a*Cos[c + d*x
])^(-1/2 + n))/(a^2*(1 - 2*n)*(-a + a*Cos[c + d*x])^(3/2))) + (-(((2 - n +
 n^2)*(-Cos[c + d*x])^(1 - n)*(-a - a*Cos[c + d*x])^(1/2 + n))/(a^2*(1 + 2
*n)*(-a + a*Cos[c + d*x])^(3/2))) - (n*(7 - 3*n - n^2)*(-Cos[c + d*x])^(1
- n)*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*(-a - a*Cos[c + d*
x])^(1/2 + n)*Hypergeometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/
(1 - Cos[c + d*x])])/(a^2*(1 - n)*(1 + 2*n)*(-a + a*Cos[c + d*x])^(3/2)))/
(a*(1 - 2*n)))/(a*(3 - 2*n)))*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*Sqrt
[-a + a*Cos[c + d*x]]))
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]/(a^(
p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])   Subst[Int[(d*
x)^n*(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2), x], x, Sin[e + f*x]],
 x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2
] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.155.4 Maple [F]

$$\int \csc(dx+c)^4 (a+a\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

3.155.5 Fricas [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
```

output
```
Timed out
```

3.155.7 Maxima [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.155.8 Giac [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^4}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^4,x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.156 $\int (a + a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

3.156.1 Optimal result 1149
3.156.2 Mathematica [B] (warning: unable to verify) 1149
3.156.3 Rubi [A] (verified) 1150
3.156.4 Maple [F] 1152
3.156.5 Fricas [F] 1152
3.156.6 Sympy [F(-1)] 1153
3.156.7 Maxima [F] 1153
3.156.8 Giac [F] 1153
3.156.9 Mupad [F(-1)] 1154

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int (a + a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx =$$

$$-\frac{\text{AppellF1}\left(1-n,-\frac{1}{4},-\frac{1}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

output
```
-AppellF1(1-n,-1/4-n,-1/4,2-n,-cos(d*x+c),cos(d*x+c))*cos(d*x+c)*(1+cos(d*
x+c))^(-1/4-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2)/d/(1-n)/(1-cos(d*x+c))^
(1/4)
```

3.156.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 382 vs. 2(105) = 210.

Time = 16.60 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.64

$$\int (a + a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{}{d\left(2\left(3\,\text{AppellF1}\left(\frac{5}{4},n,\frac{5}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-5\,\text{AppellF1}\left(\frac{5}{4},n,\frac{7}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx\right)\right)\right)$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

output

```
(10*(AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
 AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(1 +
 Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(5/2))/(d*(2*(3*Appel
lF1[5/4, n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF
1[5/4, n, 7/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF
1[5/4, 1 + n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*App
ellF1[5/4, 1 + n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1
+ Cos[c + d*x]) + 5*AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*(1 + Cos[c + d*x]) - 5*AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.156.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n\sin^{\frac{3}{2}}(c+dx)dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n dx$$

$$\downarrow 3365$$

$$-\frac{\sqrt{\sin(c+dx)}(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)}{d\sqrt[4]{a\cos(c+dx)-a}}$$

$$\downarrow 152$$

$$-\frac{\sqrt{\sin(c+dx)}(-\cos(c+dx))^n(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(\cos(c+dx)+1)^n}{d\sqrt[4]{a\cos(c+dx)-a}}$$

$$\downarrow 152$$

$$-\frac{\sqrt{\sin(c+dx)}(-\cos(c+dx))^n(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n\int\sqrt[4]{1-\cos(c+dx)}(-\cos(c+dx))^{-n}}{d\sqrt[4]{1-\cos(c+dx)}}$$

$$\downarrow 150$$

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,-\frac{1}{4},-n-\frac{1}{4},2-n,\cos(c+\right.}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

output
```
-((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos
[c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c
+ d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```

3.156.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.156.4 Maple [F]

$$\int (a + a \sec(dx + c))^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

3.156.5 Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a \sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.156.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.156.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int \sin(c + d\,x)^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.157 $\int (a + a\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$

3.157.1 Optimal result 1155
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3.157.7 Maxima [F] 1159
3.157.8 Giac [F] 1159
3.157.9 Mupad [F(-1)] 1160

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int (a + a\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx =$$
$$-\frac{\text{AppellF1}\left(1-n, \frac{1}{4}, \frac{1}{4}-n, 2-n, \cos(c+dx), -\cos(c+dx)\right) \sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(1+\cos(c-}{d(1-n)\sqrt{\sin(c+dx)}}$$

output
```
-AppellF1(1-n,1/4-n,1/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/4)*c
os(d*x+c)*(1+cos(d*x+c))^(1/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(1/
2)
```

3.157.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(105) = 210.

Time = 3.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int (a + a\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$$
$$= \frac{14\,\text{AppellF1}\left(\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right),\right.}{d\left(6\left(3\,\text{AppellF1}\left(\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right), -\tan^2\left(\frac{1}{2}(c+dx)\right)\right) - 2n\,\text{AppellF1}\left(\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan^2\left(\frac{1}{2}\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

output

```
(14*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(3/2))/(d*(6*(3*App
ellF1[7/4, n, 5/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Ap
pellF1[7/4, 1 + n, 3/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-
1 + Cos[c + d*x]) + 21*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.157.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(c+dx)}(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n\sqrt{\sin(c+dx)}dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n dx$$

$$\downarrow 3365$$

$$-\frac{\sqrt[4]{a\cos(c+dx)-a}(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)a-}{\sqrt[4]{a\cos(c+dx)-a}}}{d\sqrt{\sin(c+dx)}}$$

$$\downarrow 152$$

$$-\frac{\sqrt[4]{a\cos(c+dx)-a}(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(\cos(c+dx)+1)^{n-\frac{1}{4}}}{\sqrt[4]{a\cos(c+dx)-a}}d}{d\sqrt{\sin(c+dx)}}$$

$$\downarrow 152$$

$$-\frac{\sqrt[4]{1-\cos(c+dx)}(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(\cos(c+dx)+1)^{n-\frac{1}{4}}}{\sqrt[4]{1-\cos(c+dx)}}d\cos}{d\sqrt{\sin(c+dx)}}$$

$$\downarrow 150$$

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,\frac{1}{4},\frac{1}{4}-n,2-n,\cos(c+\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

output
```
-((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 -
Cos[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c
 + d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

3.157.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.157.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sqrt{\sin (dx + c)} dx$$

input
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

output
```
int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

3.157.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx = \int (a \sec (dx + c) + a)^n \sqrt{\sin (dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.157.6 Sympy [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx = \int (a(\sec(c + dx) + 1))^n \sqrt{\sin(c + dx)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*sqrt(sin(c + d*x)), x)
```

3.157.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\sin(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.157.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\sin(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx = \int \sqrt{\sin(c + d\,x)} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.158 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

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3.158.2 Mathematica [B] (warning: unable to verify) 1161
3.158.3 Rubi [A] (verified) 1162
3.158.4 Maple [F] 1164
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3.158.6 Sympy [F] 1165
3.158.7 Maxima [F] 1165
3.158.8 Giac [F] 1165
3.158.9 Mupad [F(-1)] 1166

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = -\frac{\text{AppellF1}\left(1-n,\frac{3}{4},\frac{3}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos(c+dx)(1+\cos(c+dx))^{3/4-n}(a+a\sec(c+dx))^n}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

output
```
-AppellF1(1-n,3/4-n,3/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(3/4)*c
os(d*x+c)*(1+cos(d*x+c))^(3/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(3/
2)
```

3.158.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.

Time = 3.53 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \frac{10\,\text{AppellF1}\left(\frac{1}{4},n,\frac{1}{2},\frac{5}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-t\right.}{d\left(2\left(\text{AppellF1}\left(\frac{5}{4},n,\frac{3}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.-2n\,\text{AppellF1}\left(\frac{5}{4},1+n,\frac{1}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-t\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

output
```
(10*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Sin[c + d*x]])/(d*(2*(Appel
lF1[5/4, n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Appel
lF1[5/4, 1 + n, 1/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 +
Cos[c + d*x]) + 5*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.158.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4364, 3042, 3365, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^n}{\sqrt{\sin(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int \frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int \frac{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n}{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 3365$$

$$-\frac{(a\cos(c+dx)-a)^{3/4}(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n\int \frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)}{(a\cos(c+dx)-a)^{3/4}}}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$\downarrow 152$$

$$-\frac{(a\cos(c+dx)-a)^{3/4}(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(\cos(c+dx)+1)^{n-\frac{3}{4}}}{(a\cos(c+dx)-a)^{3/4}}}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$\downarrow 152$$

$$-\frac{(1-\cos(c+dx))^{3/4}(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))^{-n}(\cos(c+dx)+1)^{n-\frac{3}{4}}}{(1-\cos(c+dx))^{3/4}}d}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$\downarrow 150$$

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,\frac{3}{4},\frac{3}{4}-n,2-n,\cos(c\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

output
```
-((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 -
Cos[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c
 + d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

3.158.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.158.4 Maple [F]

$$\int \frac{(a + a \sec (dx + c))^n}{\sqrt{\sin (dx + c)}} dx$$

input
```
int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

output
```
int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

3.158.5 Fricas [F]

$$\int \frac{(a + a \sec(c + dx))^n}{\sqrt{\sin(c + dx)}} dx = \int \frac{(a \sec (dx + c) + a)^n}{\sqrt{\sin (dx + c)}} dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.158.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\sin(c + dx)}}\,dx = \int \frac{(a(\sec(c + dx) + 1))^n}{\sqrt{\sin(c + dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n/sqrt(sin(c + d*x)), x)
```

3.158.7 Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\sin(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\sin(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.158.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\sin(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\sin(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\sin(c + dx)}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin(c + d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.159 $\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

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3.159.2 Mathematica [B] (warning: unable to verify) 1167
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3.159.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\text{AppellF1}\left(1-n,\frac{5}{4},\frac{5}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c+dx)(1+\cos($$
$$d(1-n)\sin^{\frac{5}{2}}(c+dx)$$

output
```
-AppellF1(1-n,5/4-n,5/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(5/4)*c
os(d*x+c)*(1+cos(d*x+c))^(5/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(5/
2)
```

3.159.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.

Time = 4.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = -\frac{6\,\text{AppellF1}\left(-\frac{1}{4},n,-\frac{1}{2},\frac{3}{4},\right.}{d\left(-2\left(\text{AppellF1}\left(\frac{3}{4},n,\frac{1}{2},\frac{7}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2n\,\text{AppellF1}\left(\frac{3}{4},1+n,-\frac{1}{2},\frac{7}{4},\tan^2\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

output
```
(-6*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n)/(d*(-2*(AppellF1[3/4, n, 1/2,
 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[3/4, 1 + n,
-1/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) +
 3*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x]))*Sqrt[Sin[c + d*x]])
```

3.159.3 Rubi [A] (verified)

Time $= 0.52$ (sec) , antiderivative size $= 133$, normalized size of antiderivative $= 1.27$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used $= \{3042, 4364, 3042, 3365, 152, 152, 150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}dx$$

$$\downarrow 4364$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int \frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)a-a)^n}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{-n}(a\sec(c+dx)+a)^n\int \frac{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{-n}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^n}{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 3365$$

$$-\frac{(a\cos(c+dx)-a)^{5/4}(-\cos(c+dx))^n(a(-\cos(c+dx))-a)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n\int \frac{(-\cos(c+dx))^{-n}(-\cos(c+dx)}{(a\cos(c+dx)-a)^{5/4}}}{d\sin^{\frac{5}{2}}(c+dx)}$$

$$\downarrow 152$$

$$\frac{(a(-\cos(c+dx))-a)(a\cos(c+dx)-a)^{5/4}(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n\int\frac{(-\cos(c+dx))}{(a}}{ad\sin^{\frac{5}{2}}(c+dx)}$$

$$\downarrow 152$$

$$-\frac{\sqrt[4]{1-\cos(c+dx)}(a(-\cos(c+dx))-a)(a\cos(c+dx)-a)(-\cos(c+dx))^n(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)}{a^2d\sin^{\frac{5}{2}}(c+dx)}$$

$$\downarrow 150$$

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(a(-\cos(c+dx))-a)(a\cos(c+dx)-a)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)}{a^2d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

output
```
-((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 -
Cos[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(-a - a*Cos[
c + d*x])*(-a + a*Cos[c + d*x])*(a + a*Sec[c + d*x])^n)/(a^2*d*(1 - n)*Sin
[c + d*x]^(5/2)))
```

3.159.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2)))   Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p
- 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 4364
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m])   Int[(g*Cos[e + f*x])^p*((
b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p
}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

3.159.4 Maple [F]

$$\int \frac{(a + a \sec (dx + c))^n}{\sin (dx + c)^{\frac{3}{2}}} dx$$

input
```
int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

output
```
int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

3.159.5 Fricas [F]

$$\int \frac{(a + a \sec(c + dx))^n}{\sin^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \sec (dx + c) + a)^n}{\sin (dx + c)^{\frac{3}{2}}} dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1),
x)
```

3.159.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a(\sec(c + dx) + 1))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n/sin(c + d*x)**(3/2), x)
```

3.159.7 Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sin(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.159.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sin(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c + d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.160 $\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx$

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3.160.2 Mathematica [A] (verified) 1173
3.160.3 Rubi [A] (verified) 1174
3.160.4 Maple [A] (verified) 1176
3.160.5 Fricas [A] (verification not implemented) 1177
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3.160.7 Maxima [A] (verification not implemented) 1177
3.160.8 Giac [B] (verification not implemented) 1178
3.160.9 Mupad [B] (verification not implemented) 1178

3.160.1 Optimal result

Integrand size = 19, antiderivative size = 119

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx = -\frac{a\cos(c + dx)}{d} + \frac{3b\cos^2(c + dx)}{2d} + \frac{a\cos^3(c + dx)}{d} - \frac{3b\cos^4(c + dx)}{4d} - \frac{3a\cos^5(c + dx)}{5d} + \frac{b\cos^6(c + dx)}{6d} + \frac{a\cos^7(c + dx)}{7d} - \frac{b\log(\cos(c + dx))}{d}$$

output
```
-a*cos(d*x+c)/d+3/2*b*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*b*cos(d*x+c)^4/d
-3/5*a*cos(d*x+c)^5/d+1/6*b*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-b*ln(cos(d
*x+c))/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx = -\frac{35a\cos(c + dx)}{64d} + \frac{7a\cos(3(c + dx))}{64d} - \frac{7a\cos(5(c + dx))}{320d} + \frac{a\cos(7(c + dx))}{448d} - \frac{b\left(-\frac{3}{2}\cos^2(c + dx) + \frac{3}{4}\cos^4(c + dx) - \frac{1}{6}\cos^6(c + dx) + \log(\cos(c + dx))\right)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

output
```
(-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(
c + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (b*((-3*Cos[c + d*x]^2
)/2 + (3*Cos[c + d*x]^4)/4 - Cos[c + d*x]^6/6 + Log[Cos[c + d*x]]))/d
```

3.160.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^7\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -\left(\sin^6(c+dx)\tan(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \sin^6(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^7\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int (b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^3\sec(c+dx)d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^3\sec(c+dx)}{a}d(a\cos(c+dx))}{a^6d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6+3\cos^4(c+dx)a^6-3\cos^2(c+dx)a^6+a^6-b\cos^5(c+dx)a^5+3b\cos^3(c+dx)a^5-3b\cos\right.}{a^6d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{3}{5}a^7\cos^5(c+dx)-a^7\cos^3(c+dx)+a^7\cos(c+dx)-\frac{1}{6}a^6b\cos^6(c+dx)+\frac{3}{4}a^6b\cos^4(c+d}{a^6d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

output
```
-((a^7*Cos[c + d*x] - (3*a^6*b*Cos[c + d*x]^2)/2 - a^7*Cos[c + d*x]^3 + (3
*a^6*b*Cos[c + d*x]^4)/4 + (3*a^7*Cos[c + d*x]^5)/5 - (a^6*b*Cos[c + d*x]^
6)/6 - (a^7*Cos[c + d*x]^7)/7 + a^6*b*Log[a*Cos[c + d*x]])/(a^6*d))
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.160.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+$
parallelrisch	$\frac{-3675a\cos(dx+c)+15a\cos(7dx+7c)-147a\cos(5dx+5c)+735a\cos(3dx+3c)+35b\cos(6dx+6c)-420b\cos(4dx+4c)+30}{6720}$
risch	$ibx+\frac{29b\,e^{2i(dx+c)}}{128d}+\frac{29b\,e^{-2i(dx+c)}}{128d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{b\cos}{}$
norman	$\frac{-\frac{32a}{35d}-\frac{128b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{14b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{(96a+70b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{(96a+128b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{(32a}{}$ $\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+
c)+b*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx$$
$$= \frac{60\,a\cos(dx + c)^7 + 70\,b\cos(dx + c)^6 - 252\,a\cos(dx + c)^5 - 315\,b\cos(dx + c)^4 + 420\,a\cos(dx + c)^3 + 6}{420\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 -
315*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a
*cos(d*x + c) - 420*b*log(-cos(d*x + c)))/d
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**7,x)
```

output
```
Timed out
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx$$
$$= \frac{60\,a\cos(dx + c)^7 + 70\,b\cos(dx + c)^6 - 252\,a\cos(dx + c)^5 - 315\,b\cos(dx + c)^4 + 420\,a\cos(dx + c)^3 + 6}{420\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 -
315*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a
*cos(d*x + c) - 420*b*log(cos(d*x + c)))/d
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.66

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx$$

$$= \frac{420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{384\,a+1089\,b-\frac{2688\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8463\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

output
```
1/420*(420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*b*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (384*a + 1089*b - 2
688*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8463*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 8064*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2874
9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 - 56035*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3 + 56035*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*b*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*b*(cos(d*x + c) - 1)^6/(cos(d*x + c
) + 1)^6 - 1089*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

3.160.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int (a + b\sec(c + dx))\sin^7(c + dx)\,dx =$$

$$-\frac{a\cos(c+d\,x) - a\cos(c+d\,x)^3 + \frac{3\,a\cos(c+d\,x)^5}{5} - \frac{a\cos(c+d\,x)^7}{7} - \frac{3\,b\cos(c+d\,x)^2}{2} + \frac{3\,b\cos(c+d\,x)^4}{4} - \frac{b\cos(c+d\,x)^6}{6} -}{d}$$

input
```
int(sin(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - a*cos(c + d*x)^3 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c +
d*x)^7)/7 - (3*b*cos(c + d*x)^2)/2 + (3*b*cos(c + d*x)^4)/4 - (b*cos(c + d
*x)^6)/6 + b*log(cos(c + d*x)))/d
```

3.161 $\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx$

3.161.1 Optimal result 1179
3.161.2 Mathematica [A] (verified) 1179
3.161.3 Rubi [A] (verified) 1180
3.161.4 Maple [A] (verified) 1182
3.161.5 Fricas [A] (verification not implemented) 1183
3.161.6 Sympy [F] 1183
3.161.7 Maxima [A] (verification not implemented) 1183
3.161.8 Giac [B] (verification not implemented) 1184
3.161.9 Mupad [B] (verification not implemented) 1184

3.161.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx = -\frac{a\cos(c + dx)}{d} + \frac{b\cos^2(c + dx)}{d} + \frac{2a\cos^3(c + dx)}{3d} - \frac{b\cos^4(c + dx)}{4d} - \frac{a\cos^5(c + dx)}{5d} - \frac{b\log(\cos(c + dx))}{d}$$

output
```
-a*cos(d*x+c)/d+b*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*b*cos(d*x+c)^4/d
-1/5*a*cos(d*x+c)^5/d-b*ln(cos(d*x+c))/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx = -\frac{5a\cos(c + dx)}{8d} + \frac{5a\cos(3(c + dx))}{48d} - \frac{a\cos(5(c + dx))}{80d} - \frac{b\left(-\cos^2(c + dx) + \frac{1}{4}\cos^4(c + dx) + \log(\cos(c + dx))\right)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

output
```
(-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c +
d*x)])/(80*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]
))/d
```

3.161.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -\left(\sin^4(c+dx)\tan(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \sin^4(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int (b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^2\sec(c+dx)d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^2\sec(c+dx)}{a}d(a\cos(c+dx))}{a^4 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\cos^4(c+dx)a^4-2\cos^2(c+dx)a^4+a^4+b\cos^3(c+dx)a^3-2b\cos(c+dx)a^3+b\sec(c+dx)a^3\right)d(a\cos(c+$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)-\frac{2}{3}a^5\cos^3(c+dx)+a^5\cos(c+dx)+\frac{1}{4}a^4b\cos^4(c+dx)-a^4b\cos^2(c+dx)+a^4b\log(a\cos(c+$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

output
```
-((a^5*Cos[c + d*x] - a^4*b*Cos[c + d*x]^2 - (2*a^5*Cos[c + d*x]^3)/3 + (a
^4*b*Cos[c + d*x]^4)/4 + (a^5*Cos[c + d*x]^5)/5 + a^4*b*Log[a*Cos[c + d*x]
])/(a^4*d))
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.161.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$\frac{-300a\cos(dx+c)-6a\cos(5dx+5c)+50a\cos(3dx+3c)-15b\cos(4dx+4c)+180\cos(2dx+2c)b+480b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-4}{480d}$
risch	$ibx+\frac{3b\,e^{2i(dx+c)}}{16d}+\frac{3b\,e^{-2i(dx+c)}}{16d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{b\cos(4d}{32}$
norman	$\frac{-\frac{16a}{15d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{10b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{(16a+6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{2(16a+15b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+b*(-1/4*sin(d*x
+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx =$$
$$-\frac{12\,a\cos(dx+c)^5 + 15\,b\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 60\,b\cos(dx+c)^2 + 60\,a\cos(dx+c) + 60\,b}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 6
0*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(-cos(d*x + c)))/d
```

3.161.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx = \int (a + b\sec(c + dx))\sin^5(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**5,x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x)**5, x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx =$$
$$-\frac{12\,a\cos(dx+c)^5 + 15\,b\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 60\,b\cos(dx+c)^2 + 60\,a\cos(dx+c) + 60\,b}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 6
0*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(cos(d*x + c)))/d
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(81) = 162$.

Time $= 0.33$ (sec) , antiderivative size $= 248$, normalized size of antiderivative $= 2.85$

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx$$

$$= \frac{60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{64\,a+137\,b-\frac{320\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a(}{(co}}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*b*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (64*a + 137*b - 320*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 640*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*b*(cos(d*x + c) - 1)^3/(cos(d*
x + c) + 1)^3 + 805*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*b*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 1)^5)/d
```

3.161.9 Mupad [B] (verification not implemented)

Time $= 13.94$ (sec) , antiderivative size $= 67$, normalized size of antiderivative $= 0.77$

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx =$$

$$-\frac{a\cos(c+d\,x) - \frac{2\,a\cos(c+d\,x)^3}{3} + \frac{a\cos(c+d\,x)^5}{5} - b\cos(c+d\,x)^2 + \frac{b\cos(c+d\,x)^4}{4} + b\ln(\cos(c+d\,x))}{d}$$

input
```
int(sin(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d*x)^5)/5 - b*cos(c
 + d*x)^2 + (b*cos(c + d*x)^4)/4 + b*log(cos(c + d*x)))/d
```

3.162 $\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$

3.162.1 Optimal result 1185
3.162.2 Mathematica [A] (verified) 1185
3.162.3 Rubi [A] (verified) 1186
3.162.4 Maple [A] (verified) 1188
3.162.5 Fricas [A] (verification not implemented) 1188
3.162.6 Sympy [F] 1189
3.162.7 Maxima [A] (verification not implemented) 1189
3.162.8 Giac [A] (verification not implemented) 1189
3.162.9 Mupad [B] (verification not implemented) 1190

3.162.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx = -\frac{a\cos(c + dx)}{d} + \frac{b\cos^2(c + dx)}{2d} + \frac{a\cos^3(c + dx)}{3d} - \frac{b\log(\cos(c + dx))}{d}$$

output
```
-a*cos(d*x+c)/d+1/2*b*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-b*ln(cos(d*x+c))
/d
```

3.162.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx = -\frac{3a\cos(c + dx)}{4d} + \frac{a\cos(3(c + dx))}{12d} - \frac{b\left(-\frac{1}{2}\cos^2(c + dx) + \log(\cos(c + dx))\right)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

output
```
(-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*(-1/2*Cos[c +
 d*x]^2 + Log[Cos[c + d*x]]))/d
```

3.162.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^2(c+dx)\tan(c+dx)(-a\cos(c+dx)-b))\,dx$$

$$\downarrow 25$$

$$-\int -((b+a\cos(c+dx))\sin^2(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^2(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int (b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)\sec(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))\sec(c+dx)}{a}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+a^2-b\cos(c+dx)a+b\sec(c+dx)a\right)d(a\cos(c+dx))}{a^2d}$$

$$\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a^3\cos(c+dx)-\frac{1}{2}a^2b\cos^2(c+dx)+a^2b\log(a\cos(c+dx))}{a^2d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

output
```
-((a^3*Cos[c + d*x] - (a^2*b*Cos[c + d*x]^2)/2 - (a^3*Cos[c + d*x]^3)/3 +
a^2*b*Log[a*Cos[c + d*x]])/(a^2*d))
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.162.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$ibx+\frac{b\,e^{2i(dx+c)}}{8d}+\frac{b\,e^{-2i(dx+c)}}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$
parallelrisch	$\frac{12b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-12b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-9a\cos(dx+c)+3\cos(2dx+2c)b+a\cos(3dx+$
norman	$\frac{-\frac{4a}{3d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{(4a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)+b*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)
)))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx$$
$$=\frac{2\,a\cos(dx+c)^3+3\,b\cos(dx+c)^2-6\,a\cos(dx+c)-6\,b\log(-\cos(dx+c))}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(
-cos(d*x + c)))/d
```

3.162.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx = \int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**3,x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x)**3, x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$$
$$= \frac{2\,a\cos(dx + c)^3 + 3\,b\cos(dx + c)^2 - 6\,a\cos(dx + c) - 6\,b\log(\cos(dx + c))}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(
cos(d*x + c)))/d
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$$
$$= -\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{2\,ad^2\cos(dx + c)^3 + 3\,bd^2\cos(dx + c)^2 - 6\,ad^2\cos(dx + c)}{6\,d^3}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-b*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*b*d^2
*cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$$
$$= -\frac{a\cos(c + dx) - \frac{a\cos(c+dx)^3}{3} - \frac{b\cos(c+dx)^2}{2} + b\ln(\cos(c + dx))}{d}$$

input
```
int(sin(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) - (a*cos(c + d*x)^3)/3 - (b*cos(c + d*x)^2)/2 + b*log(cos
(c + d*x)))/d
```

3.163 $\int (a + b\sec(c + dx))\sin(c + dx)\,dx$

3.163.1 Optimal result 1191
3.163.2 Mathematica [A] (verified) 1191
3.163.3 Rubi [A] (verified) 1192
3.163.4 Maple [A] (verified) 1194
3.163.5 Fricas [A] (verification not implemented) 1194
3.163.6 Sympy [F] 1195
3.163.7 Maxima [A] (verification not implemented) 1195
3.163.8 Giac [A] (verification not implemented) 1195
3.163.9 Mupad [B] (verification not implemented) 1196

3.163.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int (a + b\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\cos(c + dx)}{d} - \frac{b\log(\cos(c + dx))}{d}$$

output
```
-a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

3.163.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c)\cos(dx)}{d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\sin(c)\sin(dx)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

output
```
-((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

3.163.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3200, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4359$$

$$\int -(\tan(c+dx)(-a\cos(c+dx)-b))dx$$

$$\downarrow 25$$

$$-\int -((b+a\cos(c+dx))\tan(c+dx))dx$$

$$\downarrow 25$$

$$\int \tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\tan\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3200$$

$$-\frac{\int \frac{(b+a\cos(c+dx))\sec(c+dx)}{a}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{b\sec(c+dx)}{a}+1\right)d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{b\log(a\cos(c+dx))+a\cos(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

output
```
-((a*Cos[c + d*x] + b*Log[a*Cos[c + d*x]])/d)
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.163.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
default	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
parts	$-\frac{a\cos(dx+c)}{d}+\frac{b\ln(\sec(dx+c))}{d}$	26
risch	$ibx+\frac{2ibc}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}-\frac{a\cos(dx+c)}{d}$	45
parallelrisch	$\frac{-a\cos(dx+c)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+a}{d}$	60
norman	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	89

input
```
int((a+b*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/sec(d*x+c)+b*ln(sec(d*x+c)))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

3.163.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin(c + dx)\,dx = \int (a + b\sec(c + dx))\sin(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c),x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x), x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a + b\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\cos(dx + c) + b\log(\cos(dx + c))}{d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c) + b*log(cos(d*x + c)))/d
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (a + b\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\cos(dx + c)}{d} - \frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

output
```
-a*cos(d*x + c)/d - b*log(abs(cos(d*x + c))/abs(d))/d
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a + b\sec(c + dx))\sin(c + dx)\,dx = -\frac{a\,\cos(c + d\,x) + b\,\ln(\cos(c + d\,x))}{d}$$

input
```
int(sin(c + d*x)*(a + b/cos(c + d*x)),x)
```

output
```
-(a*cos(c + d*x) + b*log(cos(c + d*x)))/d
```

3.164 $\int \csc(c+dx)(a+b\sec(c+dx))\,dx$

3.164.1 Optimal result 1197
3.164.2 Mathematica [B] (verified) 1197
3.164.3 Rubi [A] (verified) 1198
3.164.4 Maple [A] (verified) 1200
3.164.5 Fricas [A] (verification not implemented) 1201
3.164.6 Sympy [F] 1201
3.164.7 Maxima [A] (verification not implemented) 1201
3.164.8 Giac [B] (verification not implemented) 1202
3.164.9 Mupad [B] (verification not implemented) 1202

3.164.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{b\log(\tan(c+dx))}{d}$$

output
```
-a*arctanh(cos(d*x+c))/d+b*ln(tan(d*x+c))/d
```

3.164.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\log(\sin(c+dx))}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*Log[Cos[c/2 + (d*x)/2]])/d) - (b*Log[Cos[c + d*x]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d + (b*Log[Sin[c + d*x]])/d
```

3.164.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3313, 25, 3042, 3100, 14, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -(\csc(c+dx)\sec(c+dx)(-a\cos(c+dx)-b))dx$$

$$\downarrow 25$$

$$-\int -((b+a\cos(c+dx))\csc(c+dx)\sec(c+dx))dx$$

$$\downarrow 25$$

$$\int \csc(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3313$$

$$a\int \csc(c+dx)dx - b\int -\csc(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc(c+dx)dx + b\int \csc(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)dx + b\int \csc(c+dx)\sec(c+dx)dx$$

$$\downarrow 3100$$

$$a\int \csc(c+dx)dx + \frac{b\int \cot(c+dx)d\tan(c+dx)}{d}$$

$$\downarrow 14$$

$$a\int \csc(c+dx)dx + \frac{b\log(\tan(c+dx))}{d}$$

$$\downarrow 4257$$

$$\frac{b\log(\tan(c+dx))}{d} - \frac{a\operatorname{arctanh}(\cos(c+dx))}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

3.164.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3313
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[Cos[e + f*x]^
p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[Cos[e + f*x]^p*(d*Sin[e + f*x
])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2
] && IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] |
| LtQ[p + 1, -n, 2*p + 1])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.164.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+b\ln(\tan(dx+c))}{d}$	33
default	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+b\ln(\tan(dx+c))}{d}$	33
parallelrisch	$\frac{-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{d}$	50
norman	$\frac{(a+b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	55
risch	$-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{\ln\left(e^{i(dx+c)}+1\right)b}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{\ln\left(e^{i(dx+c)}-1\right)b}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	89

input
```
int(csc(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(-cot(d*x+c)+csc(d*x+c))+b*ln(tan(d*x+c)))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2\,b\log(-\cos(dx+c)) + (a-b)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - (a+b)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b*log(-cos(d*x + c)) + (a - b)*log(1/2*cos(d*x + c) + 1/2) - (a +
b)*log(-1/2*cos(d*x + c) + 1/2))/d
```

3.164.6 Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x), x)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(a-b)\log(\cos(dx+c)+1) - (a+b)\log(\cos(dx+c)-1) + 2\,b\log(\cos(dx+c))}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*((a - b)*log(cos(d*x + c) + 1) - (a + b)*log(cos(d*x + c) - 1) + 2*b*
log(cos(d*x + c)))/d
```

3.164.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(26) = 52$.

Time $= 0.28$ (sec) , antiderivative size $= 61$, normalized size of antiderivative $= 2.35$

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*b*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.164.9 Mupad [B] (verification not implemented)

Time $= 0.11$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 2.42$

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \frac{\frac{a\ln(\cos(c+dx)-1)}{2} - b\ln(\cos(c+dx)) - \frac{a\ln(\cos(c+dx)+1)}{2} + \frac{b\ln(\cos(c+dx)-1)}{2} + \frac{b\ln(\cos(c+dx)+1)}{2}}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x),x)
```

output
```
((a*log(cos(c + d*x) - 1))/2 - b*log(cos(c + d*x)) - (a*log(cos(c + d*x) +
 1))/2 + (b*log(cos(c + d*x) - 1))/2 + (b*log(cos(c + d*x) + 1))/2)/d
```

3.165 $\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$

3.165.1 Optimal result 1203
3.165.2 Mathematica [A] (verified) 1203
3.165.3 Rubi [A] (verified) 1204
3.165.4 Maple [A] (verified) 1207
3.165.5 Fricas [B] (verification not implemented) 1207
3.165.6 Sympy [F] 1208
3.165.7 Maxima [A] (verification not implemented) 1208
3.165.8 Giac [B] (verification not implemented) 1208
3.165.9 Mupad [B] (verification not implemented) 1209

3.165.1 Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{b\log(\tan(c+dx))}{d}$$

output
```
-1/2*a*arctanh(cos(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-1/2*a*cot(d*x+c)*csc(d*x
+c)/d+b*ln(tan(d*x+c))/d
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.88

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\csc^2(c+dx)}{2d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log(\sin(c+dx))}{d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
-1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Log[Cos[(c +
 d*x)/2]])/(2*d) - (b*Log[Cos[c + d*x]])/d + (a*Log[Sin[(c + d*x)/2]])/(2*
d) + (b*Log[Sin[c + d*x]])/d + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

3.165.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3313, 25, 3042, 3100, 244, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^3(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^3(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3313$$

$$a\int \csc^3(c+dx)dx - b\int -\csc^3(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^3(c+dx)dx + b\int \csc^3(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^3dx + b\int \csc(c+dx)^3\sec(c+dx)dx$$

$$\downarrow 3100$$

$$a\int \csc(c+dx)^3dx + \frac{b\int \cot^3(c+dx)\left(\tan^2(c+dx)+1\right)d\tan(c+dx)}{d}$$

$$\downarrow 244$$

$$a\int \csc(c+dx)^3dx + \frac{b\int \left(\cot^3(c+dx)+\cot(c+dx)\right)d\tan(c+dx)}{d}$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^3dx + \frac{b\left(\log(\tan(c+dx)) - \frac{1}{2}\cot^2(c+dx)\right)}{d}$$

$$\downarrow 4255$$

$$a\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) + \frac{b\left(\log(\tan(c+dx)) - \frac{1}{2}\cot^2(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) + \frac{b\left(\log(\tan(c+dx)) - \frac{1}{2}\cot^2(c+dx)\right)}{d}$$

$$\downarrow 4257$$

$$a\left(-\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) + \frac{b\left(\log(\tan(c+dx)) - \frac{1}{2}\cot^2(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
a*(-1/2*ArcTanh[Cos[c + d*x]]/d - (Cot[c + d*x]*Csc[c + d*x])/(2*d)) + (b*
(-1/2*Cot[c + d*x]^2 + Log[Tan[c + d*x]]))/d
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3313
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[Cos[e + f*x]^
p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[Cos[e + f*x]^p*(d*Sin[e + f*x
])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2
] && IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] |
| LtQ[p + 1, -n, 2*p + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.165.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-8b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-8b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(4a+8b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a}{8d}$
norman	$\frac{-\frac{a+b}{8d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{(a+2b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$
risch	$\frac{a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}+e^{i(dx+c)}a}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+1\right)b}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}-1\right.}{d}$

input
```
int(csc(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+b*(-1/2
/sin(d*x+c)^2+ln(tan(d*x+c))))
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,a\cos(dx+c) - 4\left(b\cos(dx+c)^2 - b\right)\log\left(-\cos(dx+c)\right) - \left((a-2\,b)\cos(dx+c)^2 - a + 2\,b\right)\log\left(\frac{1}{2}\,c\right.}{4\left(d\cos(dx+c)^2 - d\right.}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*cos(d*x + c) - 4*(b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - ((a
- 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a + 2*b)*
cos(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c) + 1/2) + 2*b)/(d*cos(d*x +
 c)^2 - d)
```

3.165.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x)**3, x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{(a-2b)\log(\cos(dx+c)+1)-(a+2b)\log(\cos(dx+c)-1)+4b\log(\cos(dx+c))-\frac{2(a\cos(dx+c)+b}{\cos(dx+c)^2-1}}{4d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*((a - 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) +
4*b*log(cos(d*x + c)) - 2*(a*cos(d*x + c) + b)/(cos(d*x + c)^2 - 1))/d
```

3.165.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.64

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{2(a+2b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{8d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*b*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 2*a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
)*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\frac{\frac{b}{2}+\frac{a\cos(c+dx)}{2}}{\cos(c+dx)^2-1} + \ln\left(\cos\left(c+dx\right)-1\right)\left(\frac{a}{4}+\frac{b}{2}\right) - \ln\left(\cos\left(c+dx\right)+1\right)\left(\frac{a}{4}-\frac{b}{2}\right) - b\ln\left(\cos\left(c+dx\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^3,x)
```

output
```
((b/2 + (a*cos(c + d*x))/2)/(cos(c + d*x)^2 - 1) + log(cos(c + d*x) - 1)*(
a/4 + b/2) - log(cos(c + d*x) + 1)*(a/4 - b/2) - b*log(cos(c + d*x)))/d
```

3.166 $\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$

3.166.1 Optimal result 1210
3.166.2 Mathematica [A] (verified) 1210
3.166.3 Rubi [A] (warning: unable to verify) 1211
3.166.4 Maple [A] (verified) 1215
3.166.5 Fricas [B] (verification not implemented) 1215
3.166.6 Sympy [F] 1216
3.166.7 Maxima [A] (verification not implemented) 1216
3.166.8 Giac [B] (verification not implemented) 1217
3.166.9 Mupad [B] (verification not implemented) 1217

3.166.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{3a\operatorname{arctanh}(\cos(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{d} - \frac{b\cot^4(c+dx)}{4d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{b\log(\tan(c+dx))}{d}$$

output
```
-3/8*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)^2/d-1/4*b*cot(d*x+c)^4/d-3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d+b*ln(tan(d*x+c))/d
```

3.166.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.76

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\csc^2(c+dx)}{2d} - \frac{b\csc^4(c+dx)}{4d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{b\log(\cos(c+dx))}{d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{b\log(\sin(c+dx))}{d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
(-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[
c + d*x]^2)/(2*d) - (b*Csc[c + d*x]^4)/(4*d) - (3*a*Log[Cos[(c + d*x)/2]])
/(8*d) - (b*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (b*
Log[Sin[c + d*x]])/d + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/
2]^4)/(64*d)
```

3.166.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3313, 25, 3042, 3100, 243, 49, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^5(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3313$$

$$a\int \csc^5(c+dx)dx - b\int -\csc^5(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^5(c+dx)dx + b\int \csc^5(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^5dx + b\int \csc(c+dx)^5\sec(c+dx)dx$$

$$\downarrow 3100$$

$$a\int \csc(c+dx)^5dx + \frac{b\int \cot^5(c+dx)\left(\tan^2(c+dx)+1\right)^2 d\tan(c+dx)}{d}$$

$$\downarrow 243$$

$$a\int \csc(c+dx)^5dx + \frac{b\int \cot^3(c+dx)\left(\tan^2(c+dx)+1\right)^2 d\tan^2(c+dx)}{2d}$$

$$\downarrow 49$$

$$a\int \csc(c+dx)^5dx + \frac{b\int \left(\cot^3(c+dx)+2\cot^2(c+dx)+\cot(c+dx)\right) d\tan^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^5dx + \frac{b\left(-\frac{1}{2}\cot^2(c+dx)-2\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4255$$

$$a\left(\frac{3}{4}\int \csc^3(c+dx)dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) + \frac{b\left(-\frac{1}{2}\cot^2(c+dx)-2\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 3042$$

$$a\left(\frac{3}{4}\int \csc(c+dx)^3dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) + \frac{b\left(-\frac{1}{2}\cot^2(c+dx)-2\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4255$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) + \frac{b\left(-\frac{1}{2}\cot^2(c+dx) - 2\cot(c+dx) + \log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 3042$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) + \frac{b\left(-\frac{1}{2}\cot^2(c+dx) - 2\cot(c+dx) + \log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4257$$

$$a\left(\frac{3}{4}\left(-\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) + \frac{b\left(-\frac{1}{2}\cot^2(c+dx) - 2\cot(c+dx) + \log\left(\tan^2(c+dx)\right)\right)}{2d}$$

input
```
Int[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
a*(-1/4*(Cot[c + d*x]*Csc[c + d*x]^3)/d + (3*(-1/2*ArcTanh[Cos[c + d*x]]/d
 - (Cot[c + d*x]*Csc[c + d*x])/(2*d)))/4) + (b*(-2*Cot[c + d*x] - Cot[c +
d*x]^2/2 + Log[Tan[c + d*x]^2]))/(2*d)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3313
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[Cos[e + f*x]^
p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[Cos[e + f*x]^p*(d*Sin[e + f*x
])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2
] && IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] |
| LtQ[p + 1, -n, 2*p + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.166.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c$
default	$\frac{a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c$
parallelrisch	$\frac{-64b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-64b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(24a+64b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-8a-12b)}{64d}$
norman	$\frac{-\frac{a+b}{64d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}+\frac{(2a-3b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}-\frac{(2a+3b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}+8b\,e^{6i(dx+c)}-11a\,e^{5i(dx+c)}-32b\,e^{4i(dx+c)}-11a\,e^{3i(dx+c)}+8b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a}{4d\left(e^{2i(dx+c)}-1\right)^4}+\frac{3a\ln\left(e^{i(dx+c)}-1\right.}{8d}$

input
```
int(csc(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(-cot(d*x+c)+c
sc(d*x+c)))+b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c))))
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(92) = 184.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.01

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{6\,a\cos(dx+c)^3+8\,b\cos(dx+c)^2-10\,a\cos(dx+c)-16\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+b\right)\log(-}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 10*a*cos(d*x + c) - 16*(b*
cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-cos(d*x + c)) - ((3*a - 8*b)
*cos(d*x + c)^4 - 2*(3*a - 8*b)*cos(d*x + c)^2 + 3*a - 8*b)*log(1/2*cos(d*
x + c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2
 + 3*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*b)/(d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^2 + d)
```

3.166.6 Sympy [F]

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^5(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x)**5, x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{(3a-8b)\log(\cos(dx+c)+1)-(3a+8b)\log(\cos(dx+c)-1)+16b\log(\cos(dx+c))-\frac{2\left(3a\cos(d}{16d}$$

input
```
integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*((3*a - 8*b)*log(cos(d*x + c) + 1) - (3*a + 8*b)*log(cos(d*x + c) -
1) + 16*b*log(cos(d*x + c)) - 2*(3*a*cos(d*x + c)^3 + 4*b*cos(d*x + c)^2 -
 5*a*cos(d*x + c) - 6*b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

3.166.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(92) = 184$.

Time $= 0.32$ (sec) , antiderivative size $= 266$, normalized size of antiderivative $= 2.66$

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{4\,(3\,a+8\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 64\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{\left(a+b-\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a(\cos(d}{(\cos(dx-}}{(\cos(dx+c)}}{64\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/64*(4*(3*a + 8*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64
*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a + b - 8*a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 18*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 -
 8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.166.9 Mupad [B] (verification not implemented)

Time $= 13.91$ (sec) , antiderivative size $= 117$, normalized size of antiderivative $= 1.17$

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a}{16}+\frac{b}{2}\right)}{d} - \frac{-\frac{3\,a\cos(c+d\,x)^3}{8}-\frac{b\cos(c+d\,x)^2}{2}+\frac{5\,a\,\cos(c+d\,x)}{8}+\frac{3\,b}{4}}{d\,\left(\cos\left(c+d\,x\right)^4-2\cos\left(c+d\,x\right)^2+1\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a}{16}-\frac{b}{2}\right)}{d} - \frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^5,x)
```

output

```
(log(cos(c + d*x) - 1)*((3*a)/16 + b/2))/d - ((3*b)/4 + (5*a*cos(c + d*x))
/8 - (3*a*cos(c + d*x)^3)/8 - (b*cos(c + d*x)^2)/2)/(d*(cos(c + d*x)^4 - 2
*cos(c + d*x)^2 + 1)) - (log(cos(c + d*x) + 1)*((3*a)/16 - b/2))/d - (b*lo
g(cos(c + d*x)))/d
```

3.167 $\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$

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3.167.1 Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{5a\operatorname{arctanh}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b\cot^6(c+dx)}{6d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{b\log(\tan(c+dx))}{d}$$

output

```
-5/16*a*arctanh(cos(d*x+c))/d-3/2*b*cot(d*x+c)^2/d-3/4*b*cot(d*x+c)^4/d-1/
6*b*cot(d*x+c)^6/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d-5/24*a*cot(d*x+c)*csc(d*
x+c)^3/d-1/6*a*cot(d*x+c)*csc(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

3.167.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = &-\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} \\ &- \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{b\csc^2(c+dx)}{2d} \\ &- \frac{b\csc^4(c+dx)}{4d} - \frac{b\csc^6(c+dx)}{6d} \\ &- \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} \\ &- \frac{b\log(\cos(c+dx))}{d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} \\ &+ \frac{b\log(\sin(c+dx))}{d} + \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} \\ &+ \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

output
```
(-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[
(c + d*x)/2]^6)/(384*d) - (b*Csc[c + d*x]^2)/(2*d) - (b*Csc[c + d*x]^4)/(4
*d) - (b*Csc[c + d*x]^6)/(6*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (b*L
og[Cos[c + d*x]])/d + (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (b*Log[Sin[c +
d*x]])/d + (5*a*Sec[(c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
 + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

3.167.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.053$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3313, 25, 3042, 3100, 243, 49, 2009, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a - b\csc\left(c + dx - \frac{\pi}{2}\right)}{\cos\left(c + dx - \frac{\pi}{2}\right)^7} dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^7(c + dx)\sec(c + dx)(-a\cos(c + dx) - b)\right) dx$$

$$\downarrow 25$$

$$-\int -\left((b + a\cos(c + dx))\csc^7(c + dx)\sec(c + dx)\right) dx$$

$$\downarrow 25$$

$$\int \csc^7(c + dx)\sec(c + dx)(a\cos(c + dx) + b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b - a\sin\left(c + dx - \frac{\pi}{2}\right)}{\sin\left(c + dx - \frac{\pi}{2}\right)\cos\left(c + dx - \frac{\pi}{2}\right)^7} dx$$

$$\downarrow 25$$

$$-\int \frac{b - a\sin\left(\frac{1}{2}(2c - \pi) + dx\right)}{\cos\left(\frac{1}{2}(2c - \pi) + dx\right)^7 \sin\left(\frac{1}{2}(2c - \pi) + dx\right)} dx$$

$$\downarrow 3313$$

$$a\int \csc^7(c + dx)dx - b\int -\csc^7(c + dx)\sec(c + dx)dx$$

$$\downarrow 25$$

$$a\int \csc^7(c + dx)dx + b\int \csc^7(c + dx)\sec(c + dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c + dx)^7 dx + b\int \csc(c + dx)^7\sec(c + dx)dx$$

$$\downarrow 3100$$

$$a\int \csc(c + dx)^7 dx + \frac{b\int \cot^7(c + dx)\left(\tan^2(c + dx) + 1\right)^3 d\tan(c + dx)}{d}$$

$$\downarrow 243$$

$$a\int \csc(c + dx)^7 dx + \frac{b\int \cot^4(c + dx)\left(\tan^2(c + dx) + 1\right)^3 d\tan^2(c + dx)}{2d}$$

$$\downarrow 49$$

$$a\int \csc(c+dx)^7dx + \frac{b\int \left(\cot^4(c+dx)+3\cot^3(c+dx)+3\cot^2(c+dx)+\cot(c+dx)\right)d\tan^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^7dx + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4255$$

$$a\left(\frac{5}{6}\int \csc^5(c+dx)dx - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right) + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\int \csc(c+dx)^5dx - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right) + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4255$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \csc^3(c+dx)dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right) + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \csc(c+dx)^3dx - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right) + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 4255$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right) + \frac{b\left(-\frac{1}{3}\cot^3(c+dx)-\frac{3}{2}\cot^2(c+dx)-3\cot(c+dx)+\log\left(\tan^2(c+dx)\right)\right)}{2d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(c+dx)dx - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) - \frac{\cot(c+dx)\csc^5(c+dx)}{6d}\right.$$
$$\left.\frac{b\left(-\frac{1}{3}\cot^3(c+dx) - \frac{3}{2}\cot^2(c+dx) - 3\cot(c+dx) + \log\left(\tan^2(c+dx)\right)\right)}{2d}\right)$$

$$\downarrow 4257$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{\cot(c+dx)\csc(c+dx)}{2d}\right) - \frac{\cot(c+dx)\csc^3(c+dx)}{4d}\right) - \frac{\cot(c+dx)\csc^5(c+\right.$$
$$\left.\frac{b\left(-\frac{1}{3}\cot^3(c+dx) - \frac{3}{2}\cot^2(c+dx) - 3\cot(c+dx) + \log\left(\tan^2(c+dx)\right)\right)}{2d}\right)$$

input
```
Int[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

output
```
a*(-1/6*(Cot[c + d*x]*Csc[c + d*x]^5)/d + (5*(-1/4*(Cot[c + d*x]*Csc[c + d
*x]^3)/d + (3*(-1/2*ArcTanh[Cos[c + d*x]]/d - (Cot[c + d*x]*Csc[c + d*x])/
(2*d)))/4))/6) + (b*(-3*Cot[c + d*x] - (3*Cot[c + d*x]^2)/2 - Cot[c + d*x]
^3/3 + Log[Tan[c + d*x]^2]))/(2*d)
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3313
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[Cos[e + f*x]^
p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[Cos[e + f*x]^p*(d*Sin[e + f*x
])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2
] && IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] |
| LtQ[p + 1, -n, 2*p + 1])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.167.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c)+\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}\right.}{d}$
default	$\frac{a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c)+\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}\right.}{d}$
parallelrisch	$\frac{-384b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-384b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(120a+384b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-9a-}{3}$
norman	$\frac{-\frac{a+b}{384d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{384d}+\frac{(3a-4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{128d}-\frac{(3a+4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{128d}+\frac{(15a-29b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128d}-\frac{(15a+29b)\tan\left(\frac{dx}{2}+\right.}{128d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}$
risch	$\frac{15a\,e^{11i(dx+c)}+48b\,e^{10i(dx+c)}-85a\,e^{9i(dx+c)}-288b\,e^{8i(dx+c)}+198a\,e^{7i(dx+c)}+736b\,e^{6i(dx+c)}+198a\,e^{5i(dx+c)}-288b\,e^{4}}{24d\left(e^{2i(dx+c)}-1\right)^6}$

input
```
int(csc(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5
/16*ln(-cot(d*x+c)+csc(d*x+c)))+b*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-1/2/
sin(d*x+c)^2+ln(tan(d*x+c))))
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.

Time $= 0.27$ (sec) , antiderivative size $= 284$, normalized size of antiderivative $= 2.03$

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{30\,a\cos(dx+c)^5+48\,b\cos(dx+c)^4-80\,a\cos(dx+c)^3-120\,b\cos(dx+c)^2+66\,a\cos(dx+c)-96\,(b}{}$$

input
```
integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(30*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 80*a*cos(d*x + c)^3 - 12
0*b*cos(d*x + c)^2 + 66*a*cos(d*x + c) - 96*(b*cos(d*x + c)^6 - 3*b*cos(d*
x + c)^4 + 3*b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - 3*((5*a - 16*b)*co
s(d*x + c)^6 - 3*(5*a - 16*b)*cos(d*x + c)^4 + 3*(5*a - 16*b)*cos(d*x + c)
^2 - 5*a + 16*b)*log(1/2*cos(d*x + c) + 1/2) + 3*((5*a + 16*b)*cos(d*x + c
)^6 - 3*(5*a + 16*b)*cos(d*x + c)^4 + 3*(5*a + 16*b)*cos(d*x + c)^2 - 5*a
- 16*b)*log(-1/2*cos(d*x + c) + 1/2) + 88*b)/(d*cos(d*x + c)^6 - 3*d*cos(d
*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{3\,(5\,a-16\,b)\log\left(\cos\left(dx+c\right)+1\right)-3\,(5\,a+16\,b)\log\left(\cos\left(dx+c\right)-1\right)+96\,b\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(1\right.}{96\,d}$$

input
```
integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(3*(5*a - 16*b)*log(cos(d*x + c) + 1) - 3*(5*a + 16*b)*log(cos(d*x +
 c) - 1) + 96*b*log(cos(d*x + c)) - 2*(15*a*cos(d*x + c)^5 + 24*b*cos(d*x
+ c)^4 - 40*a*cos(d*x + c)^3 - 60*b*cos(d*x + c)^2 + 33*a*cos(d*x + c) + 4
4*b)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(126) = 252$.

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.55

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{12\,(5\,a+16\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 384\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{\left(a+b-\frac{9\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(}{(co}\right.}{}$$

input
```
integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/384*(12*(5*a + 16*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
 384*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 9*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 87*b*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 110*a*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 - 352*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c)
 + 1)^3/(cos(d*x + c) - 1)^3 - 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 87*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 - 12*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(c
os(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x
+ c) + 1)^3)/d
```

3.167.9 Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{\frac{5\,a\cos(c+d\,x)^5}{16}+\frac{b\cos(c+d\,x)^4}{2}-\frac{5\,a\cos(c+d\,x)^3}{6}-\frac{5\,b\cos(c+d\,x)^2}{4}+\frac{11\,a\,\cos(c+d\,x)}{16}+\frac{11\,b}{12}}{d\,\left(\cos\left(c+d\,x\right)^6-3\cos\left(c+d\,x\right)^4+3\cos\left(c+d\,x\right)^2-1\right)}$$
$$+\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{5\,a}{32}+\frac{b}{2}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{5\,a}{32}-\frac{b}{2}\right)}{d}-\frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^7,x)
```

output
```
((11*b)/12 + (11*a*cos(c + d*x))/16 - (5*a*cos(c + d*x)^3)/6 + (5*a*cos(c
+ d*x)^5)/16 - (5*b*cos(c + d*x)^2)/4 + (b*cos(c + d*x)^4)/2)/(d*(3*cos(c
+ d*x)^2 - 3*cos(c + d*x)^4 + cos(c + d*x)^6 - 1)) + (log(cos(c + d*x) - 1
)*((5*a)/32 + b/2))/d - (log(cos(c + d*x) + 1)*((5*a)/32 - b/2))/d - (b*lo
g(cos(c + d*x)))/d
```

3.168 $\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx$

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3.168.1 Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx = \frac{5ax}{16} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{5a\cos(c + dx)\sin(c + dx)}{16d} - \frac{b\sin^3(c + dx)}{3d} - \frac{5a\cos(c + dx)\sin^3(c + dx)}{24d} - \frac{b\sin^5(c + dx)}{5d} - \frac{a\cos(c + dx)\sin^5(c + dx)}{6d}$$

output
```
5/16*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+
c)/d-1/3*b*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*b*sin(d*x+c
)^5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

3.168.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx = \frac{5a(c + dx)}{16d} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{b\sin^3(c + dx)}{3d} - \frac{b\sin^5(c + dx)}{5d} - \frac{15a\sin(2(c + dx))}{64d} + \frac{3a\sin(4(c + dx))}{64d} - \frac{a\sin(6(c + dx))}{192d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

output
```
(5*a*(c + d*x))/(16*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d
- (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d) - (15*a*Sin[2*(c + d
*x)])/(64*d) + (3*a*Sin[4*(c + d*x)])/(64*d) - (a*Sin[6*(c + d*x)])/(192*d
)
```

3.168.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 254, 2009, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -\left(\sin^5(c+dx)\tan(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \sin^5(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^6(c+dx)dx + b\int \sin^5(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^6dx + b\int \sin(c+dx)^5\tan(c+dx)dx$$

$$\downarrow 3072$$

$$a\int \sin(c+dx)^6dx + \frac{b\int \frac{\sin^6(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\downarrow 254$$

$$a\int \sin(c+dx)^6dx + \frac{b\int \left(-\sin^4(c+dx) - \sin^2(c+dx) + \frac{1}{1-\sin^2(c+dx)} - 1\right)d\sin(c+dx)}{d}$$

$$\downarrow 2009$$

$$a\int \sin(c+dx)^6dx + \frac{b\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\int \sin^4(c+dx)dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{b\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\int \sin(c+dx)^4dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{b\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{b\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(c+dx)^2dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) + \frac{b\left(\operatorname{arctanh}(\sin(c+dx)) - \frac{1}{5}\sin^5(c+dx) - \frac{1}{3}\sin^3(c+dx) - \sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 24$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

output
```
(b*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3 - Sin[c + d*x]
^5/5))/d + a*(-1/6*(Cos[c + d*x]*Sin[c + d*x]^5)/d + (5*(-1/4*(Cos[c + d*x
]*Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.168.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.168.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(d}{d}$
default	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(d}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(sec}{d}$
parallelrisch	$\frac{300axd-1320\sin(dx+c)b-5a\sin(6dx+6c)+45a\sin(4dx+4c)-225a\sin(2dx+2c)-12\sin(5dx+5c)b+140\sin(3dx+3c)b-}{960d}$
risch	$\frac{5ax}{16}+\frac{11ib\,e^{i(dx+c)}}{16d}-\frac{11ib\,e^{-i(dx+c)}}{16d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{b\sin(5dx+5c}{80d}$
norman	$\frac{5ax}{16}+\frac{15ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{75ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16}+\frac{25ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{75ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16}+\frac{15ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8}+\frac{5ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/
16*d*x+5/16*c)+b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x
+c)+tan(d*x+c))))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$$
$$=\frac{75\,adx+120\,b\log\left(\sin\left(dx+c\right)+1\right)-120\,b\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^5+48\,b\cos\left(dx+\right.\right.}{240\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(75*a*d*x + 120*b*log(sin(d*x + c) + 1) - 120*b*log(-sin(d*x + c) +
1) - (40*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 1
76*b*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*b)*sin(d*x + c))/d
```

3.168.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx = \int (a + b\sec(c + dx))\sin^6(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**6,x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x)**6, x)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx$$
$$= \frac{5\left(4\sin(2dx + 2c)^3 + 60dx + 60c + 9\sin(4dx + 4c) - 48\sin(2dx + 2c)\right)a - 32\left(6\sin(dx + c)^5 + 10\right.}{960d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a - 32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx$$
$$= \frac{75(dx + c)a + 240b\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 240b\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(75a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{11}\right.}{}}{}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
1/240*(75*(d*x + c)*a + 240*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(75*a*tan(1/2*d*x + 1/2*c)^11 - 240*
b*tan(1/2*d*x + 1/2*c)^11 + 425*a*tan(1/2*d*x + 1/2*c)^9 - 1520*b*tan(1/2*
d*x + 1/2*c)^9 + 990*a*tan(1/2*d*x + 1/2*c)^7 - 4128*b*tan(1/2*d*x + 1/2*c
)^7 - 990*a*tan(1/2*d*x + 1/2*c)^5 - 4128*b*tan(1/2*d*x + 1/2*c)^5 - 425*a
*tan(1/2*d*x + 1/2*c)^3 - 1520*b*tan(1/2*d*x + 1/2*c)^3 - 75*a*tan(1/2*d*x
 + 1/2*c) - 240*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.168.9 Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.61

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5\,a\,\mathrm{atan}\left(\frac{125\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\left(\frac{125\,a^3}{64}+20\,a\,b^2\right)}+\frac{20\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{125\,a^3}{64}+20\,a\,b^2}\right)}{8\,d}$$
$$+\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{25\,a^2\,b}{4}+64\,b^3}+\frac{25\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\left(\frac{25\,a^2\,b}{4}+64\,b^3\right)}\right)}{d}$$
$$-\frac{\left(2\,b-\frac{5\,a}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{38\,b}{3}-\frac{85\,a}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{172\,b}{5}-\frac{33\,a}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{33\,a}{4}+\frac{172\,b}{5}\right)\tan}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\right.}$$

input
```
int(sin(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

output
```
(5*a*atan((125*a^3*tan(c/2 + (d*x)/2))/(64*(20*a*b^2 + (125*a^3)/64)) + (2
0*a*b^2*tan(c/2 + (d*x)/2))/(20*a*b^2 + (125*a^3)/64)))/(8*d) + (2*b*atanh
((64*b^3*tan(c/2 + (d*x)/2))/((25*a^2*b)/4 + 64*b^3) + (25*a^2*b*tan(c/2 +
 (d*x)/2))/(4*((25*a^2*b)/4 + 64*b^3))))/d - (tan(c/2 + (d*x)/2)*((5*a)/8
+ 2*b) - tan(c/2 + (d*x)/2)^11*((5*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^3*((85
*a)/24 + (38*b)/3) - tan(c/2 + (d*x)/2)^9*((85*a)/24 - (38*b)/3) + tan(c/2
 + (d*x)/2)^5*((33*a)/4 + (172*b)/5) - tan(c/2 + (d*x)/2)^7*((33*a)/4 - (1
72*b)/5))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/
2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c
/2 + (d*x)/2)^12 + 1))
```

3.169 $\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$

3.169.1 Optimal result 1237
3.169.2 Mathematica [A] (verified) 1237
3.169.3 Rubi [A] (verified) 1238
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3.169.7 Maxima [A] (verification not implemented) 1242
3.169.8 Giac [B] (verification not implemented) 1242
3.169.9 Mupad [B] (verification not implemented) 1243

3.169.1 Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx = \frac{3ax}{8} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{3a\cos(c + dx)\sin(c + dx)}{8d} - \frac{b\sin^3(c + dx)}{3d} - \frac{a\cos(c + dx)\sin^3(c + dx)}{4d}$$

output
```
3/8*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)
/d-1/3*b*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

3.169.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx = \frac{3a(c + dx)}{8d} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{b\sin^3(c + dx)}{3d} - \frac{a\sin(2(c + dx))}{4d} + \frac{a\sin(4(c + dx))}{32d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
 (b*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x
)])/(32*d)
```

3.169.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 254, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin^3(c+dx)\tan(c+dx)(-a\cos(c+dx)-b))\,dx$$

$$\downarrow 25$$

$$-\int -((b+a\cos(c+dx))\sin^3(c+dx)\tan(c+dx))\,dx$$

$$\downarrow 25$$

$$\int \sin^3(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^4(c+dx)dx + b\int \sin^3(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^4dx + b\int \sin(c+dx)^3\tan(c+dx)dx$$

$$\downarrow 3072$$

$$a\int \sin(c+dx)^4dx + \frac{b\int \frac{\sin^4(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\downarrow 254$$

$$a\int \sin(c+dx)^4dx+\frac{b\int\left(-\sin^2(c+dx)+\frac{1}{1-\sin^2(c+dx)}-1\right)d\sin(c+dx)}{d}$$

$$\downarrow 2009$$

$$a\int \sin(c+dx)^4dx+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{3}{4}\int \sin^2(c+dx)dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3042$$

$$a\left(\frac{3}{4}\int \sin(c+dx)^2dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 24$$

$$a\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)+\frac{b\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

output
```
(b*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3))/d + a*(-1/4*
(Cos[c + d*x]*Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2
*d)))/4)
```

3.169.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.169.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{36axd-120\sin(dx+c)b+3a\sin(4dx+4c)-24a\sin(2dx+2c)+96b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-96b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\sin(3$
risch	$\frac{3ax}{8}+\frac{5ib\,e^{i(dx+c)}}{8d}-\frac{5ib\,e^{-i(dx+c)}}{8d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{(3a-8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}-\frac{(3a+8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b*(-1
/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$
$$=\frac{9\,adx+12\,b\log\left(\sin\left(dx+c\right)+1\right)-12\,b\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2-\right.}{24\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + 12*b*log(sin(d*x + c) + 1) - 12*b*log(-sin(d*x + c) + 1) +
 (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*b)*sin(
d*x + c))/d
```

3.169.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx = \int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x)**4, x)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$$
$$= \frac{3\,(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c))a - 16\,\left(2\,\sin(dx + c)^3 - 3\,\log(\sin(dx + c) + 1) + 3\right.}{96\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a - 16*(2*
sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*b)/d
```

3.169.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(81) = 162.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$$
$$= \frac{9\,(dx + c)a + 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 - 24\,b\,\tan\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a + 24*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*b*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1
/2*d*x + 1/2*c)^7 + 33*a*tan(1/2*d*x + 1/2*c)^5 - 104*b*tan(1/2*d*x + 1/2*
c)^5 - 33*a*tan(1/2*d*x + 1/2*c)^3 - 104*b*tan(1/2*d*x + 1/2*c)^3 - 9*a*ta
n(1/2*d*x + 1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
1)^4)/d
```

3.169.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.00

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$
$$= \frac{3\,a\,\operatorname{atan}\left(\frac{27\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\left(\frac{27\,a^3}{8}+24\,a\,b^2\right)}+\frac{24\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{27\,a^3}{8}+24\,a\,b^2}\right)}{4\,d}+\frac{2\,b\,\operatorname{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\,a^2\,b+64\,b^3}+\frac{9\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\,a^2\,b+64\,b^3}\right)}{d}$$
$$-\frac{\left(2\,b-\frac{3\,a}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{26\,b}{3}-\frac{11\,a}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{11\,a}{4}+\frac{26\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{3\,a}{4}+2\,b\right)\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

output
```
(3*a*atan((27*a^3*tan(c/2 + (d*x)/2))/(8*(24*a*b^2 + (27*a^3)/8)) + (24*a*
b^2*tan(c/2 + (d*x)/2))/(24*a*b^2 + (27*a^3)/8)))/(4*d) + (2*b*atanh((64*b
^3*tan(c/2 + (d*x)/2))/(9*a^2*b + 64*b^3) + (9*a^2*b*tan(c/2 + (d*x)/2))/(
9*a^2*b + 64*b^3)))/d - (tan(c/2 + (d*x)/2)*((3*a)/4 + 2*b) - tan(c/2 + (d
*x)/2)^7*((3*a)/4 - 2*b) + tan(c/2 + (d*x)/2)^3*((11*a)/4 + (26*b)/3) - ta
n(c/2 + (d*x)/2)^5*((11*a)/4 - (26*b)/3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*t
an(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.170 $\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$

3.170.1 Optimal result 1244
3.170.2 Mathematica [A] (verified) 1244
3.170.3 Rubi [A] (verified) 1245
3.170.4 Maple [A] (verified) 1247
3.170.5 Fricas [A] (verification not implemented) 1248
3.170.6 Sympy [F] 1249
3.170.7 Maxima [A] (verification not implemented) 1249
3.170.8 Giac [B] (verification not implemented) 1249
3.170.9 Mupad [B] (verification not implemented) 1250

3.170.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx = \frac{ax}{2} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{a\cos(c + dx)\sin(c + dx)}{2d}$$

output
```
1/2*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)
/d
```

3.170.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx = \frac{a(c + dx)}{2d} + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b\sin(c + dx)}{d} - \frac{a\sin(2(c + dx))}{4d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

output
```
(a*(c + d*x))/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (
a*Sin[2*(c + d*x)])/(4*d)
```

3.170.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3072, 262, 219, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4360$$

$$\int -(\sin(c+dx)\tan(c+dx)(-a\cos(c+dx)-b))dx$$

$$\downarrow 25$$

$$-\int -((b+a\cos(c+dx))\sin(c+dx)\tan(c+dx))dx$$

$$\downarrow 25$$

$$\int \sin(c+dx)\tan(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 3317$$

$$a\int \sin^2(c+dx)dx + b\int \sin(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \sin(c+dx)^2dx + b\int \sin(c+dx)\tan(c+dx)dx$$

$$\downarrow 3072$$

$$a\int \sin(c+dx)^2dx + \frac{b\int \frac{\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\downarrow 262$$

$$a\int \sin(c+dx)^2dx+\frac{b\left(\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 219$$

$$a\int \sin(c+dx)^2dx+\frac{b(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}$$

$$\downarrow 3115$$

$$a\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{b(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}$$

$$\downarrow 24$$

$$a\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{b(\operatorname{arctanh}(\sin(c+dx))-\sin(c+dx))}{d}$$

input
```
Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

output
```
(b*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]))/d + a*(x/2 - (Cos[c + d*x]*Sin[
c + d*x])/(2*d))
```

3.170.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.170.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parts	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$\frac{2axd-4\sin(dx+c)b-a\sin(2dx+2c)+4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$
risch	$\frac{ax}{2}+\frac{ib\,e^{i(dx+c)}}{2d}-\frac{ib\,e^{-i(dx+c)}}{2d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{ax}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(ta\right.}{}$

input
```
int((a+b*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b*(-sin(d*x+c)+ln(sec(d*
x+c)+tan(d*x+c))))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$$
$$=\frac{adx+b\log(\sin(dx+c)+1)-b\log(-\sin(dx+c)+1)-(a\cos(dx+c)+2b)\sin(dx+c)}{2d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) - (a*cos(d
*x + c) + 2*b)*sin(d*x + c))/d
```

3.170.6 Sympy [F]

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx = \int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))*sin(c + d*x)**2, x)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$
$$= \frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a + 2\,b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1) - 2\sin(dx + c))}{4\,d}$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*b*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

3.170.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$
$$= \frac{(dx + c)a + 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,b\tan\left(\frac{1}{2}\,dx-\right.\right.}{\left(\tan(\right.}$$
$$2\,d$$

input
```
integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a + 2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x +
 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^2)/d
```

3.170.9 Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx = \frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} - \frac{a\,\sin(2c+2dx)}{4d} - \frac{b\,\sin(c+dx)}{d}$$

input
```
int(sin(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

output
```
(a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2*c + 2*d*x))/(4*d) - (b*sin(c + d
*x))/d
```

3.171 $\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$

3.171.1 Optimal result 1251
3.171.2 Mathematica [C] (verified) 1251
3.171.3 Rubi [A] (verified) 1252
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3.171.9 Mupad [B] (verification not implemented) 1257

3.171.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)}{d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-b*csc(d*x+c)/d
```

3.171.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*Cot[c + d*x])/d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Si
n[c + d*x]^2])/d
```

3.171.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 262, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^2(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^2(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^2(c+dx)dx - b\int -\csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^2(c+dx)dx + b\int \csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^2dx + b\int \csc(c+dx)^2\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^2dx - \frac{b\int -\frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$a\int \csc(c+dx)^2dx + \frac{b\int \frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 262$$

$$a\int \csc(c+dx)^2dx - \frac{b\left(\csc(c+dx) - \int \frac{1}{1-\csc^2(c+dx)}d\csc(c+dx)\right)}{d}$$

$$\downarrow 219$$

$$a\int \csc(c+dx)^2dx - \frac{b(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int 1d\cot(c+dx)}{d} - \frac{b(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 24$$

$$-\frac{a\cot(c+dx)}{d} - \frac{b(\csc(c+dx) - \operatorname{arctanh}(\csc(c+dx)))}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*Cot[c + d*x])/d) - (b*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x]))/d
```

3.171.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.171.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-a\cot(dx+c)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$	42
default	$\frac{-a\cot(dx+c)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$	42
parallelrisch	$\frac{-2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{2d}$	69
risch	$-\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{d\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}$	71
norman	$\frac{-\frac{a+b}{2d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	79

input
```
int(csc(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*cot(d*x+c)+b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{b\log(\sin(dx+c)+1)\sin(dx+c)-b\log(-\sin(dx+c)+1)\sin(dx+c)-2a\cos(dx+c)-2b}{2d\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b*log(sin(d*x + c) + 1)*sin(d*x + c) - b*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 2*a*cos(d*x + c) - 2*b)/(d*sin(d*x + c))
```

3.171.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x)**2, x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2a}{\tan(dx+c)}}{2d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
 2*a/tan(d*x + c))/d
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(37) = 74.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.08

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c) - (a + b)/tan
(1/2*d*x + 1/2*c))/d
```

3.171.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,b\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a}{2}-\frac{b}{2}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^2,x)
```

output
```
(2*b*atanh(tan(c/2 + (d*x)/2)))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) + (
tan(c/2 + (d*x)/2)*(a/2 - b/2))/d
```

3.172 $\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$

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3.172.2 Mathematica [C] (verified) . . . 1258
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3.172.9 Mupad [B] (verification not implemented) . . . 1264

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-b*csc(d*x+c)/d
-1/3*b*csc(d*x+c)^3/d
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{b\csc^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*Csc
[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

3.172.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^4(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^4(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^4(c+dx)dx - b\int -\csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^4(c+dx)dx + b\int \csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^4dx + b\int \csc(c+dx)^4\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^4dx - \frac{b\int -\frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$a\int \csc(c+dx)^4dx + \frac{b\int \frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 254$$

$$a\int \csc(c+dx)^4dx + \frac{b\int \left(-\csc^2(c+dx) + \frac{1}{1-\csc^2(c+dx)} - 1\right)d\csc(c+dx)}{d}$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^4dx - \frac{b\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int \left(\cot^2(c+dx)+1\right)d\cot(c+dx)}{d} - \frac{b\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(\frac{1}{3}\cot^3(c+dx) + \cot(c+dx)\right)}{d} - \frac{b\left(-\operatorname{arctanh}(\csc(c+dx)) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + Cot[c + d*x]^3/3))/d) - (b*(-ArcTanh[Csc[c + d*x]] +
Csc[c + d*x] + Csc[c + d*x]^3/3))/d
```

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.172.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-24b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+(-9a-15b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2$
risch	$-\frac{2i\left(3b\,e^{5i(dx+c)}-10b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+2a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-\frac{a+b}{24d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24d}+\frac{(3a-5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}-\frac{(3a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$

input
```
int(csc(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c
)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.81

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{4\,a\cos(dx+c)^3+6\,b\cos(dx+c)^2-3\left(b\cos(dx+c)^2-b\right)\log\left(\sin(dx+c)+1\right)\sin(dx+c)+3\left(b\cos\right.}{6\left(d\cos(dx+c)^2-d\right)\sin(dx+}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)^2 - 3*(b*cos(d*x + c)^2 - b)*l
og(sin(d*x + c) + 1)*sin(d*x + c) + 3*(b*cos(d*x + c)^2 - b)*log(-sin(d*x
+ c) + 1)*sin(d*x + c) - 6*a*cos(d*x + c) - 8*b)/((d*cos(d*x + c)^2 - d)*s
in(d*x + c))
```

3.172.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^4(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x)**4, x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + \frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(65) = 130.

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 24\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{24\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*b*log(abs(t
an(1/2*d*x + 1/2*c) + 1)) - 24*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a*
tan(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c) - (9*a*tan(1/2*d*x + 1/2*
c)^2 + 15*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.172.9 Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a}{24}-\frac{b}{24}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left((3a+5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8d} + \frac{2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3a}{8}-\frac{5b}{8}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 + b/3
+ tan(c/2 + (d*x)/2)^2*(3*a + 5*b)))/(8*d) + (2*b*atanh(tan(c/2 + (d*x)/2)
))/d + (tan(c/2 + (d*x)/2)*((3*a)/8 - (5*b)/8))/d
```

3.173 $\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$

3.173.1 Optimal result 1265
3.173.2 Mathematica [C] (verified) 1265
3.173.3 Rubi [A] (verified) 1266
3.173.4 Maple [A] (verified) 1269
3.173.5 Fricas [A] (verification not implemented) 1269
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3.173.7 Maxima [A] (verification not implemented) 1270
3.173.8 Giac [B] (verification not implemented) 1270
3.173.9 Mupad [B] (verification not implemented) 1271

3.173.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+dx)}{5d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+
c)^5/d-b*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^5/d
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{b\csc^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sin^2(c+dx)\right)}{5d}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

output
```
(-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a
*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*Csc[c + d*x]^5*Hypergeometric2F1[
-5/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

3.173.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3101, 25, 254, 2009, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int -\left(\csc^6(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)\right)dx$$

$$\downarrow 25$$

$$-\int -\left((b+a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 25$$

$$\int \csc^6(c+dx)\sec(c+dx)(a\cos(c+dx)+b)dx$$

$$\downarrow 3042$$

$$\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$a\int \csc^6(c+dx)dx - b\int -\csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$a\int \csc^6(c+dx)dx + b\int \csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \csc(c+dx)^6dx + b\int \csc(c+dx)^6\sec(c+dx)dx$$

$$\downarrow 3101$$

$$a\int \csc(c+dx)^6dx - \frac{b\int -\frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$a\int \csc(c+dx)^6dx + \frac{b\int \frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 254$$

$$a\int \csc(c+dx)^6dx + \frac{b\int \left(-\csc^4(c+dx) - \csc^2(c+dx) + \frac{1}{1-\csc^2(c+dx)} - 1\right)d\csc(c+dx)}{d}$$

$$\downarrow 2009$$

$$a\int \csc(c+dx)^6dx - \frac{b\left(-\text{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 4254$$

$$-\frac{a\int \left(\cot^4(c+dx) + 2\cot^2(c+dx) + 1\right)d\cot(c+dx)}{d} - \frac{b\left(-\text{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(\frac{1}{5}\cot^5(c+dx) + \frac{2}{3}\cot^3(c+dx) + \cot(c+dx)\right)}{d} - \frac{b\left(-\text{arctanh}(\csc(c+dx)) + \frac{1}{5}\csc^5(c+dx) + \frac{1}{3}\csc^3(c+dx) + \csc(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(Cot[c + d*x] + (2*Cot[c + d*x]^3)/3 + Cot[c + d*x]^5/5))/d) - (b*(-A
rcTanh[Csc[c + d*x]] + Csc[c + d*x] + Csc[c + d*x]^3/3 + Csc[c + d*x]^5/5)
)/d
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.173.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-960b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+960b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(a\cos(5dx+5c)+10a\cos(dx+c)-5a\cos(3dx+3c)\right.}{960d}$
risch	$-\frac{2i\left(15b\,e^{9i(dx+c)}-80b\,e^{7i(dx+c)}+178b\,e^{5i(dx+c)}+80a\,e^{4i(dx+c)}-80b\,e^{3i(dx+c)}-40a\,e^{2i(dx+c)}+15b\,e^{i(dx+c)}+8a\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}-b$
norman	$\frac{-\frac{a+b}{160d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{160d}+\frac{(5a-11b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}+\frac{(5a-7b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96d}-\frac{(5a+7b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}-\frac{(5a+11b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

input
```
int(csc(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/5/sin(d
*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{16\,a\cos(dx+c)^5+30\,b\cos(dx+c)^4-40\,a\cos(dx+c)^3-70\,b\cos(dx+c)^2-15\left(b\cos(dx+c)^4-2\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/30*(16*a*cos(d*x + c)^5 + 30*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 7
0*b*cos(d*x + c)^2 - 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(si
n(d*x + c) + 1)*sin(d*x + c) + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 b)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 30*a*cos(d*x + c) + 46*b)/((d*co
s(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

3.173.6 Sympy [F]

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^6(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*csc(c + d*x)**6, x)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{b\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + \frac{2\left(15\tan(dx+c)^4-}{\tan(}}{30\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/30*(b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15
*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4
+ 10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(93) = 186.

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.92

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3\,b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 25\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 35\,b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 480\,b\log\left(|\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 + 25*a*tan(
1/2*d*x + 1/2*c)^3 - 35*b*tan(1/2*d*x + 1/2*c)^3 + 480*b*log(abs(tan(1/2*d
*x + 1/2*c) + 1)) - 480*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a*tan(1
/2*d*x + 1/2*c) - 330*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)
^4 + 330*b*tan(1/2*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^2 + 35*b*tan
(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx \\
&= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{5a}{96}-\frac{7b}{96}\right)}{d} \\
&\quad - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left((10a+22b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{5a}{3}+\frac{7b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32d} \\
&\quad + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{a}{160}-\frac{b}{160}\right)}{d} + \frac{2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5a}{16}-\frac{11b}{16}\right)}{d}
\end{aligned}$$

input
```
int((a + b/cos(c + d*x))/sin(c + d*x)^6,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((5*a)/96 - (7*b)/96))/d - (cot(c/2 + (d*x)/2)^5*(a/
5 + b/5 + tan(c/2 + (d*x)/2)^2*((5*a)/3 + (7*b)/3) + tan(c/2 + (d*x)/2)^4*
(10*a + 22*b)))/(32*d) + (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d + (2*b*a
tanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)*((5*a)/16 - (11*b)/16))/
d
```

3.174 $\int (a+b\sec(c+dx))^2 \sin^5(c+dx)\,dx$

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3.174.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int (a+b\sec(c+dx))^2 \sin^5(c+dx)\,dx = -\frac{(a^2-2b^2)\cos(c+dx)}{d} + \frac{2ab\cos^2(c+dx)}{d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3d} - \frac{ab\cos^4(c+dx)}{2d} - \frac{a^2\cos^5(c+dx)}{5d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

output
```
-(a^2-2*b^2)*cos(d*x+c)/d+2*a*b*cos(d*x+c)^2/d+1/3*(2*a^2-b^2)*cos(d*x+c)^
3/d-1/2*a*b*cos(d*x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a*b*ln(cos(d*x+c))/d+b
^2*sec(d*x+c)/d
```

3.174.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int (a+b\sec(c+dx))^2 \sin^5(c+dx)\,dx = -\frac{30(5a^2-14b^2)\cos(c+dx) - 180ab\cos(2(c+dx)) - 25a^2\cos(3(c+dx)) + 20b^2\cos(3(c+dx)) + 15ab}{240d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

output
```
-1/240*(30*(5*a^2 - 14*b^2)*Cos[c + d*x] - 180*a*b*Cos[2*(c + d*x)] - 25*a
^2*Cos[3*(c + d*x)] + 20*b^2*Cos[3*(c + d*x)] + 15*a*b*Cos[4*(c + d*x)] +
3*a^2*Cos[5*(c + d*x)] + 480*a*b*Log[Cos[c + d*x]] - 240*b^2*Sec[c + d*x])
/d
```

3.174.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin^3(c+dx)\tan^2(c+dx)(-a\cos(c+dx)-b)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3316$$

$$-\frac{\int (b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)^2\sec^2(c+dx)d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)^2\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\cos^4(c+dx)a^4+\left(1-\frac{2b^2}{a^2}\right)a^4+2b\cos^3(c+dx)a^3-4b\cos(c+dx)a^3+2b\sec(c+dx)a^3-\left(2a^2-b^2\right)\cos^2(c\right.}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)+\frac{1}{2}a^4b\cos^4(c+dx)-2a^4b\cos^2(c+dx)+2a^4b\log(a\cos(c+dx))-a^3b^2\sec(c+dx)-\frac{1}{3}a^3\left(2a\right.}{a^3d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

output
```
-((a^3*(a^2 - 2*b^2)*Cos[c + d*x] - 2*a^4*b*Cos[c + d*x]^2 - (a^3*(2*a^2 -
 b^2)*Cos[c + d*x]^3)/3 + (a^4*b*Cos[c + d*x]^4)/2 + (a^5*Cos[c + d*x]^5)/
5 + 2*a^4*b*Log[a*Cos[c + d*x]] - a^3*b^2*Sec[c + d*x])/(a^3*d))
```

3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.174.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+2ab\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+2ab\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+$
parallelrisch	$\frac{-960ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+960ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-960ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2}{}$
norman	$\frac{\frac{16a^2-80b^2}{15d}-\frac{32\left(a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{16ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{4\left(16a^2+15ab-80b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}+\frac{\left(16a^2+\right.}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$2iabx+\frac{3ab\,e^{2i(dx+c)}}{8d}-\frac{5a^2e^{i(dx+c)}}{16d}+\frac{7\,e^{i(dx+c)}b^2}{8d}-\frac{5\,e^{-i(dx+c)}a^2}{16d}+\frac{7\,e^{-i(dx+c)}b^2}{8d}+\frac{3ab\,e^{-2i(dx+c)}}{8d}+$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/4*s
in(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^6/cos(d*x+c)+
(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx =$$

$$-\frac{48\,a^2\cos(dx+c)^6+120\,ab\cos(dx+c)^5-480\,ab\cos(dx+c)^3-80\,(2\,a^2-b^2)\cos(dx+c)^4+480\,ab\,c}{240\,d\cos(dx+}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/240*(48*a^2*cos(d*x + c)^6 + 120*a*b*cos(d*x + c)^5 - 480*a*b*cos(d*x +
 c)^3 - 80*(2*a^2 - b^2)*cos(d*x + c)^4 + 480*a*b*cos(d*x + c)*log(-cos(d*
x + c)) + 195*a*b*cos(d*x + c) + 240*(a^2 - 2*b^2)*cos(d*x + c)^2 - 240*b^
2)/(d*cos(d*x + c))
```

3.174.6 Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx = \int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**5, x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx =$$

$$-\frac{6\,a^2\cos(dx+c)^5+15\,ab\cos(dx+c)^4-60\,ab\cos(dx+c)^2-10\,(2\,a^2-b^2)\cos(dx+c)^3+60\,ab\log(c}{30\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/30*(6*a^2*cos(d*x + c)^5 + 15*a*b*cos(d*x + c)^4 - 60*a*b*cos(d*x + c)^
2 - 10*(2*a^2 - b^2)*cos(d*x + c)^3 + 60*a*b*log(cos(d*x + c)) + 30*(a^2 -
 2*b^2)*cos(d*x + c) - 30*b^2/cos(d*x + c))/d
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(118) = 236$.

Time $= 0.39$ (sec) , antiderivative size $= 418$, normalized size of antiderivative $= 3.37$

$$\int (a + b\sec(c + dx))^2 \sin^5(c + dx)\, dx$$

$$= \frac{60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{60\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{32\,a^2+137\,ab-100\,b^2}{}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/30*(60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*b
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(a*b + b^2 + a*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1) + (32*a^2 + 137*a*b - 100*b^2 - 160*a^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 805*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 440*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^2*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 + 1970*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 640*
b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a*b*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 360*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3 + 805*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 60*b^2*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*b*(cos(d*x + c) - 1)^5/(cos(d*x +
c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

3.174.9 Mupad [B] (verification not implemented)

Time $= 13.95$ (sec) , antiderivative size $= 104$, normalized size of antiderivative $= 0.84$

$$\int (a + b\sec(c + dx))^2 \sin^5(c + dx)\, dx =$$

$$-\frac{\cos(c + d\,x)\,(a^2 - 2\,b^2) - \cos(c + d\,x)^3\left(\frac{2\,a^2}{3} - \frac{b^2}{3}\right) + \frac{a^2\cos(c+d\,x)^5}{5} - \frac{b^2}{\cos(c+d\,x)} - 2\,a\,b\cos(c + d\,x)^2 + \frac{a\,b}{}}{d}$$

input
```
int(sin(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

output
```
-(cos(c + d*x)*(a^2 - 2*b^2) - cos(c + d*x)^3*((2*a^2)/3 - b^2/3) + (a^2*c
os(c + d*x)^5)/5 - b^2/cos(c + d*x) - 2*a*b*cos(c + d*x)^2 + (a*b*cos(c +
d*x)^4)/2 + 2*a*b*log(cos(c + d*x)))/d
```

3.175 $\int (a + b\sec(c + dx))^2 \sin^3(c + dx)\, dx$

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3.175.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int (a + b\sec(c + dx))^2 \sin^3(c + dx)\, dx = -\frac{(a^2 - b^2)\cos(c + dx)}{d} + \frac{ab\cos^2(c + dx)}{d} + \frac{a^2\cos^3(c + dx)}{3d} - \frac{2ab\log(\cos(c + dx))}{d} + \frac{b^2\sec(c + dx)}{d}$$

output
```
-(a^2-b^2)*cos(d*x+c)/d+a*b*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a*b*ln
(cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

3.175.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int (a + b\sec(c + dx))^2 \sin^3(c + dx)\, dx = \frac{(-9a^2 + 12b^2)\cos(c + dx) + 6ab\cos(2(c + dx)) + a^2\cos(3(c + dx)) - 24ab\log(\cos(c + dx)) + 12b^2\sec(c$$

$$= \frac{}{12d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

output
```
((-9*a^2 + 12*b^2)*Cos[c + d*x] + 6*a*b*Cos[2*(c + d*x)] + a^2*Cos[3*(c +
d*x)] - 24*a*b*Log[Cos[c + d*x]] + 12*b^2*Sec[c + d*x])/(12*d)
```

3.175.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \sin(c+dx)\tan^2(c+dx)(-a\cos(c+dx)-b)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3316$$

$$-\frac{\int (b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)\sec^2(c+dx)d(a\cos(c+dx))}{a^3 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{ad}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+\left(1-\frac{b^2}{a^2}\right)a^2-2b\cos(c+dx)a+2b\sec(c+dx)a+b^2\sec^2(c+dx)\right)d(a\cos(c+dx))}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a\left(a^2-b^2\right)\cos(c+dx)-a^2b\cos^2(c+dx)+2a^2b\log(a\cos(c+dx))-ab^2\sec(c+dx)}{ad}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

output
```
-((a*(a^2 - b^2)*Cos[c + d*x] - a^2*b*Cos[c + d*x]^2 - (a^3*Cos[c + d*x]^3
)/3 + 2*a^2*b*Log[a*Cos[c + d*x]] - a*b^2*Sec[c + d*x])/(a*d))
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.175.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}$
default	$\frac{-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}$
parts	$-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}+\frac{2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+$
parallelrisch	$\frac{48ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-48ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-48ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+(-8a^2$ $24d\cos(dx+c)$
norman	$\frac{\frac{4a^2-12b^2}{3d}-\frac{4\left(a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{2\left(4a^2+6ab-12b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-$
risch	$2iabx+\frac{ab\,e^{2i(dx+c)}}{4d}-\frac{3a^2e^{i(dx+c)}}{8d}+\frac{e^{i(dx+c)}b^2}{2d}-\frac{3\,e^{-i(dx+c)}a^2}{8d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{ab\,e^{-2i(dx+c)}}{4d}+\frac{4iabc}{d}$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/2*sin(d*x+c)^2-ln(cos(
d*x+c)))+b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$=\frac{2\,a^2\cos(dx+c)^4+6\,ab\cos(dx+c)^3-12\,ab\cos(dx+c)\log(-\cos(dx+c))-3\,ab\cos(dx+c)-6\,(a^2-$$
$$6\,d\cos(dx+c)$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a^2*cos(d*x + c)^4 + 6*a*b*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)*log
(-cos(d*x + c)) - 3*a*b*cos(d*x + c) - 6*(a^2 - b^2)*cos(d*x + c)^2 + 6*b^
2)/(d*cos(d*x + c))
```

3.175.6 Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**3, x)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$= \frac{a^2\cos(dx+c)^3 + 3\,ab\cos(dx+c)^2 - 6\,ab\log(\cos(dx+c)) - 3\,(a^2-b^2)\cos(dx+c) + \frac{3\,b^2}{\cos(dx+c)}}{3\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/3*(a^2*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^2 - 6*a*b*log(cos(d*x + c)) -
 3*(a^2 - b^2)*cos(d*x + c) + 3*b^2/cos(d*x + c))/d
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$= -\frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{b^2}{d\cos(dx+c)}$$
$$+ \frac{a^2d^5\cos(dx+c)^3 + 3\,abd^5\cos(dx+c)^2 - 3\,a^2d^5\cos(dx+c) + 3\,b^2d^5\cos(dx+c)}{3\,d^6}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*x + c)) + 1/3*(a^2*d
^5*cos(d*x + c)^3 + 3*a*b*d^5*cos(d*x + c)^2 - 3*a^2*d^5*cos(d*x + c) + 3*
b^2*d^5*cos(d*x + c))/d^6
```

3.175.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int (a+b\sec(c+dx))^2 \sin^3(c+dx)\,dx$$
$$= \frac{\frac{a^2\cos(c+d\,x)^3}{3} - \cos(c+d\,x)\,(a^2-b^2) + \frac{b^2}{\cos(c+d\,x)} + a\,b\cos(c+d\,x)^2 - 2\,a\,b\,\ln(\cos(c+d\,x))}{d}$$

input
```
int(sin(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

output
```
((a^2*cos(c + d*x)^3)/3 - cos(c + d*x)*(a^2 - b^2) + b^2/cos(c + d*x) + a*
b*cos(c + d*x)^2 - 2*a*b*log(cos(c + d*x)))/d
```

3.176 $\int (a+b\sec(c+dx))^2 \sin(c+dx)\,dx$

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3.176.1 Optimal result

Integrand size = 19, antiderivative size = 42

$$\int (a+b\sec(c+dx))^2 \sin(c+dx)\,dx = -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

output
```
-a^2*cos(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

3.176.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))^2 \sin(c+dx)\,dx = \frac{-a^2\cos(c+dx) + b(-2a\log(\cos(c+dx)) + b\sec(c+dx))}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

output
```
(-(a^2*Cos[c + d*x]) + b*(-2*a*Log[Cos[c + d*x]] + b*Sec[c + d*x]))/d
```

3.176.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4360, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4360$$

$$\int \tan(c+dx)\sec(c+dx)(-a\cos(c+dx)-b)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\downarrow 3312$$

$$-\frac{\int (b+a\cos(c+dx))^2\sec^2(c+dx)d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{a\int \left(\frac{b^2\sec^2(c+dx)}{a^2}+\frac{2b\sec(c+dx)}{a}+1\right)d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\frac{b^2\sec(c+dx)}{a}+2b\log(a\cos(c+dx))+a\cos(c+dx)\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

output
```
-((a*(a*Cos[c + d*x] + 2*b*Log[a*Cos[c + d*x]] - (b^2*Sec[c + d*x])/a))/d)
```

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.176.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\sec(dx+c)b^2-\frac{a^2}{\sec(dx+c)}+2ab\ln(\sec(dx+c))}{d}$
default	$\frac{\sec(dx+c)b^2-\frac{a^2}{\sec(dx+c)}+2ab\ln(\sec(dx+c))}{d}$
parts	$-\frac{a^2\cos(dx+c)}{d}+\frac{b^2\sec(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c))}{d}$
risch	$2iabx-\frac{a^2e^{i(dx+c)}}{2d}-\frac{e^{-i(dx+c)}a^2}{2d}+\frac{4iabc}{d}+\frac{2b^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2ab\ln\left(e^{2i(dx+c)}+1\right)}{d}$
parallelrisch	$\frac{4ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-4ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-4ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\cos(2dx+$ $2d\cos(dx+c)$
norman	$\frac{\frac{2a^2-2b^2}{d}-\frac{2\left(a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2ab\ln\left(1+\tan\right.}{d}$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sec(d*x+c)*b^2-a^2/sec(d*x+c)+2*a*b*ln(sec(d*x+c)))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx = -\frac{a^2\cos(dx+c)^2+2ab\cos(dx+c)\log(-\cos(dx+c))-b^2}{d\cos(dx+c)}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) - b^2)/(d*cos
(d*x + c))
```

3.176.6 Sympy [F]

$$\int (a + b\sec(c + dx))^2 \sin(c + dx)\, dx = \int (a + b\sec(c + dx))^2 \sin(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c),x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sin(c + d*x), x)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a + b\sec(c + dx))^2 \sin(c + dx)\, dx = -\frac{a^2 \cos(dx + c) + 2ab \log(\cos(dx + c)) - \frac{b^2}{\cos(dx+c)}}{d}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-(a^2*cos(d*x + c) + 2*a*b*log(cos(d*x + c)) - b^2/cos(d*x + c))/d
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^2 \sin(c+dx)\, dx = -\frac{a^2 \cos(dx + c)}{d} - \frac{2ab \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{b^2}{d\cos(dx + c)}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-a^2*cos(d*x + c)/d - 2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d
*x + c))
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx = -\frac{a^2\cos(c+dx) - \frac{b^2}{\cos(c+dx)} + 2ab\ln(\cos(c+dx))}{d}$$

input
```
int(sin(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

output
```
-(a^2*cos(c + d*x) - b^2/cos(c + d*x) + 2*a*b*log(cos(c + d*x)))/d
```

3.177 $\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$

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3.177.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(a+b)^2\log(1-\cos(c+dx))}{2d} - \frac{2ab\log(\cos(c+dx))}{d} - \frac{(a-b)^2\log(1+\cos(c+dx))}{2d} + \frac{b^2\sec(c+dx)}{d}$$

output
```
1/2*(a+b)^2*ln(1-cos(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/2*(a-b)^2*ln(1+cos
(d*x+c))/d+b^2*sec(d*x+c)/d
```

3.177.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{-(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2ab\log(\cos(c+dx)) + a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2ab\log\left(\sin\left(\frac{1}{2}(c+dx\right)\right.\right.}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(-((a - b)^2*Log[Cos[(c + d*x)/2]]) - 2*a*b*Log[Cos[c + d*x]] + a^2*Log[Si
n[(c + d*x)/2]] + 2*a*b*Log[Sin[(c + d*x)/2]] + b^2*Log[Sin[(c + d*x)/2]]
+ b^2*Sec[c + d*x])/d
```

3.177.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4360, 3042, 3316, 27, 526, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int \csc(c+dx)\sec^2(c+dx)(-a\cos(c+dx)-b)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 3316$$

$$\frac{a\int \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)}{a^2-a^2\cos^2(c+dx)}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^3\int \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)}{a^2(a^2-a^2\cos^2(c+dx))}d(-a\cos(c+dx))}{d}$$

$$\downarrow 526$$

$$\frac{a^3\left(\frac{\int \frac{\left(2ba^2+\left(a^2+b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{a\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{a^2}+\frac{b^2\sec(c+dx)}{a^3}\right)}{d}$$

$$\downarrow 25$$

$$\frac{a^3\left(\frac{b^2\sec(c+dx)}{a^3}-\frac{\int -\frac{\left(2ba^2+\left(a^2+b^2\right)\cos(c+dx)a\right)\sec(c+dx)}{a\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{a^2}\right)}{d}$$

$$\downarrow 523$$

$$\frac{a^3\left(\frac{b^2\sec(c+dx)}{a^3}-\frac{\int\left(-\frac{(a-b)^2}{2a(\cos(c+dx)a+a)}-\frac{2b\sec(c+dx)}{a}-\frac{(a+b)^2}{2a(a-a\cos(c+dx))}\right)d(-a\cos(c+dx))}{a^2}\right)}{d}$$

$$\downarrow 2009$$

$$\frac{a^3\left(\frac{b^2\sec(c+dx)}{a^3}-\frac{\frac{(a-b)^2\log(a\cos(c+dx)+a)}{2a}+2b\log(-a\cos(c+dx))-\frac{(a+b)^2\log(a-a\cos(c+dx))}{2a}}{a^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^3*(-((2*b*Log[-(a*Cos[c + d*x])] - ((a + b)^2*Log[a - a*Cos[c + d*x]])/
(2*a) + ((a - b)^2*Log[a + a*Cos[c + d*x]])/(2*a))/a^2) + (b^2*Sec[c + d*x
])/a^3))/d
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 526
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol] :
> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemainder
[(c + d*x)^n, x, x]}, Simp[R*(x^(m + 1)/(a*(m + 1))), x] + Simp[1/a   Int[x
^(m + 1)*(ExpandToSum[a*Qx - b*R*x, x]/(a + b*x^2)), x], x]] /; FreeQ[{a, b
, c, d}, x] && IGtQ[n, 1] && ILtQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.177.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^2\ln(-\cot(dx+c)+\csc(dx+c))+2ab\ln(\tan(dx+c))+b^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)}{d}$
default	$\frac{a^2\ln(-\cot(dx+c)+\csc(dx+c))+2ab\ln(\tan(dx+c))+b^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)}{d}$
norman	$-\frac{2b^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{d}$
parallelrisch	$\frac{-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos(dx+c)(a+b)^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\cos(d$
risch	$\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}+\frac{2\ln(e^{i(dx+c)}-1)ab}{d}+\frac{\ln(e^{i(dx+c)}-1)b^2}{d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{2\ln(e^{i($

input
```
int(csc(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(-cot(d*x+c)+csc(d*x+c))+2*a*b*ln(tan(d*x+c))+b^2*(1/cos(d*x+c)
+ln(-cot(d*x+c)+csc(d*x+c))))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{4\,ab\cos(dx+c)\log(-\cos(dx+c)) + (a^2-2\,ab+b^2)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - (a^2+2\,a$$
$$2\,d\cos(dx+c)$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + (a^2 - 2*a*b + b^2)*cos(d*x
+ c)*log(1/2*cos(d*x + c) + 1/2) - (a^2 + 2*a*b + b^2)*cos(d*x + c)*log(-1
/2*cos(d*x + c) + 1/2) - 2*b^2)/(d*cos(d*x + c))
```

3.177.6 Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*csc(c + d*x), x)
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{4\,ab\log(\cos(dx+c)) + (a^2-2\,ab+b^2)\log(\cos(dx+c)+1) - (a^2+2\,ab+b^2)\log(\cos(dx+c)-1)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(4*a*b*log(cos(d*x + c)) + (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1)
- (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1) - 2*b^2/cos(d*x + c))/d
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{4\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-(a^2+2\,ab+b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 +
2*a*b + b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*(a*b +
b^2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1))/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^2}{2}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)^2}{2}+\frac{b^2}{\cos(c+d\,x)}-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2/sin(c + d*x),x)
```

output
```
((log(cos(c + d*x) - 1)*(a + b)^2)/2 - (log(cos(c + d*x) + 1)*(a - b)^2)/2
 + b^2/cos(c + d*x) - 2*a*b*log(cos(c + d*x)))/d
```

3.178 $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

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3.178.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\begin{aligned}\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = &-\frac{(2ab+(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2d}\\ &+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}\\ &-\frac{2ab\log(\cos(c+dx))}{d}\\ &-\frac{(a-3b)(a-b)\log(1+\cos(c+dx))}{4d}\\ &+\frac{b^2\sec(c+dx)}{d}\end{aligned}$$

output
```
-1/2*(2*a*b+(a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b)*(a+3*b)*ln(1-co
s(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/4*(a-3*b)*(a-b)*ln(1+cos(d*x+c))/d+b^
2*sec(d*x+c)/d
```

3.178.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. $2(114) = 228$.

Time = 1.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.89

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{\csc^4(c+dx)\left(2a^2-2b^2+2(a^2+3b^2)\cos(2(c+dx))-a^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4ab\,\mathrm{co}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/2*(Csc[c + d*x]^4*(2*a^2 - 2*b^2 + 2*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - a
^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4*a*b*Cos[3*(c + d*x)]*Log[Cos
[(c + d*x)/2]] - 3*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*a*b*Cos[
3*(c + d*x)]*Log[Cos[c + d*x]] + a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]
] + 4*a*b*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3*b^2*Cos[3*(c + d*x)]*
Log[Sin[(c + d*x)/2]] + Cos[c + d*x]*(8*a*b + (a^2 - 4*a*b + 3*b^2)*Log[Co
s[(c + d*x)/2]] + 4*a*b*Log[Cos[c + d*x]] - a^2*Log[Sin[(c + d*x)/2]] - 4*
a*b*Log[Sin[(c + d*x)/2]] - 3*b^2*Log[Sin[(c + d*x)/2]])))/(d*(Csc[(c + d*
x)/2]^2 - Sec[(c + d*x)/2]^2))
```

3.178.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 3042, 3316, 27, 532, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int \csc^3(c+dx)\sec^2(c+dx)(-a\cos(c+dx)-b)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3316$$

$$\frac{a^3\int \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)}{(a^2-a^2\cos^2(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^5\int \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)}{a^2(a^2-a^2\cos^2(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 532$$

$$\frac{a^5\left(-\frac{\int -\frac{\left(2b^2+4a\cos(c+dx)b+\left(a^2+b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a^2\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{2a^2}-\frac{a\left(\frac{b^2}{a^2}+1\right)\cos(c+dx)+2b}{2a^2(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 25$$

$$\frac{a^5\left(\frac{\int \frac{\left(2b^2+4a\cos(c+dx)b+\left(a^2+b^2\right)\cos^2(c+dx)\right)\sec^2(c+dx)}{a^2\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{2a^2}-\frac{a\left(\frac{b^2}{a^2}+1\right)\cos(c+dx)+2b}{2a^2(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 2333$$

$$\frac{a^5\left(\frac{\int\left(\frac{2b^2\sec^2(c+dx)}{a^4}+\frac{4b\sec(c+dx)}{a^3}+\frac{(a+b)(a+3b)}{2a^3(a-a\cos(c+dx))}+\frac{(a-3b)(a-b)}{2a^3(\cos(c+dx)a+a)}\right)d(-a\cos(c+dx))}{2a^2}-\frac{a\left(\frac{b^2}{a^2}+1\right)\cos(c+dx)+2b}{2a^2(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 2009$$

$$\frac{a^5\left(\frac{2b^2\sec(c+dx)}{a^3}+\frac{(a+b)(a+3b)\log(a-a\cos(c+dx))}{2a^3}-\frac{(a-3b)(a-b)\log(a\cos(c+dx)+a)}{2a^3}-\frac{4b\log(-a\cos(c+dx))}{a^2}}{2a^2}-\frac{a\left(\frac{b^2}{a^2}+1\right)\cos(c+dx)+2b}{2a^2(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

3.178. $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

output
```
(a^5*(-1/2*(2*b + a*(1 + b^2/a^2)*Cos[c + d*x])/(a^2*(a^2 - a^2*Cos[c + d*
x]^2)) + ((-4*b*Log[-(a*Cos[c + d*x])])/a^2 + ((a + b)*(a + 3*b)*Log[a - a
*Cos[c + d*x]])/(2*a^3) - ((a - 3*b)*(a - b)*Log[a + a*Cos[c + d*x]])/(2*a
^3) + (2*b^2*Sec[c + d*x])/a^3)/(2*a^2)))/d
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.178.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c}\right.}{d}$
norman	$\frac{\frac{a^2+2ab+b^2}{8d}+\frac{\left(a^2-2ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d}-\frac{\left(a^2+9b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{\left(a^2+4ab+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2ab\ln\left(\tan\left(\right.\right.}{d}$
parallelrisch	$\frac{16ba\left(1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16ba\left(1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\right.}{8d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
risch	$\frac{a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+4ab\,e^{4i(dx+c)}+2a^2e^{3i(dx+c)}-2b^2e^{3i(dx+c)}+4ab\,e^{2i(dx+c)}+a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\,l}{}$

input
```
int(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+2*a*b
*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/
cos(d*x+c)+3/2*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.80

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{4\,ab\cos(dx+c)+2\left(a^2+3\,b^2\right)\cos(dx+c)^2-4\,b^2-8\left(ab\cos(dx+c)^3-ab\cos(dx+c)\right)\log(-\cos(dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(4*a*b*cos(d*x + c) + 2*(a^2 + 3*b^2)*cos(d*x + c)^2 - 4*b^2 - 8*(a*b*
cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-cos(d*x + c)) - ((a^2 - 4*a*b + 3*
b^2)*cos(d*x + c)^3 - (a^2 - 4*a*b + 3*b^2)*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + ((a^2 + 4*a*b + 3*b^2)*cos(d*x + c)^3 - (a^2 + 4*a*b + 3*b^2
)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*
x + c))
```

3.178.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2 \csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**3, x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{8\,ab\log(\cos(dx+c)) + (a^2-4\,ab+3\,b^2)\log(\cos(dx+c)+1) - (a^2+4\,ab+3\,b^2)\log(\cos(dx+c)-}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(8*a*b*log(cos(d*x + c)) + (a^2 - 4*a*b + 3*b^2)*log(cos(d*x + c) + 1
) - (a^2 + 4*a*b + 3*b^2)*log(cos(d*x + c) - 1) - 2*(2*a*b*cos(d*x + c) +
(a^2 + 3*b^2)*cos(d*x + c)^2 - 2*b^2)/(cos(d*x + c)^3 - cos(d*x + c)))/d
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(108) = 216$.

Time = 0.33 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.75

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{16\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\left(a^2+4\,ab+3\,b^2\right)\ln}{8}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/8*(16*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + a^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^2 + 4*a*b + 3*b
^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - (a^2 + 2*a*b + b^2
 + 6*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14*b^2*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*
b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = {}& \frac{\ln(\cos(c+dx)-1)\,(a+b)\,(a+3b)}{4d}\\ & -\frac{\ln(\cos(c+dx)+1)\,(a-b)\,(a-3b)}{4d}\\ & -\frac{2ab\ln(\cos(c+dx))}{d}\\ & -\frac{\cos(c+dx)^2\left(\frac{a^2}{2}+\frac{3b^2}{2}\right)-b^2+ab\cos(c+dx)}{d\left(\cos(c+dx)-\cos(c+dx)^3\right)}\end{aligned}$$

input
```
int((a + b/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

output
```
(log(cos(c + d*x) - 1)*(a + b)*(a + 3*b))/(4*d) - (log(cos(c + d*x) + 1)*(
a - b)*(a - 3*b))/(4*d) - (2*a*b*log(cos(c + d*x)))/d - (cos(c + d*x)^2*(a
^2/2 + (3*b^2)/2) - b^2 + a*b*cos(c + d*x))/(d*(cos(c + d*x) - cos(c + d*x
)^3))
```

3.179 $\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\, dx$

3.179.1 Optimal result 1305
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3.179.1 Optimal result

Integrand size = 21, antiderivative size = 175

$$\begin{aligned}\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\, dx = {} & \frac{5}{16}\left(a^2 - 6b^2\right) x + \frac{2ab\operatorname{arctanh}(\sin(c + dx))}{d} \\ & - \frac{2ab\sin(c + dx)}{d} \\ & - \frac{(11a^2 - 18b^2)\cos(c + dx)\sin(c + dx)}{16d} \\ & + \frac{(13a^2 - 6b^2)\cos^3(c + dx)\sin(c + dx)}{24d} \\ & - \frac{a^2\cos^5(c + dx)\sin(c + dx)}{6d} - \frac{2ab\sin^3(c + dx)}{3d} \\ & - \frac{2ab\sin^5(c + dx)}{5d} + \frac{b^2\tan(c + dx)}{d}\end{aligned}$$

output

```
5/16*(a^2-6*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/16*(11
*a^2-18*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(13*a^2-6*b^2)*cos(d*x+c)^3*sin(
d*x+c)/d-1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d-2/5*a*b*
sin(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

3.179.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^2 \sin^6(c+dx)\,dx$$
$$= \frac{60\left(5(a^2-6b^2)(c+dx)-32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

output
```
(60*(5*(a^2 - 6*b^2)*(c + d*x) - 32*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 32*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2128*a*b*Sin[c
 + d*x] + (-185*a^2 + 1410*b^2 - 5*(29*a^2 - 84*b^2)*Cos[2*(c + d*x)] + 23
2*a*b*Cos[3*(c + d*x)] + 35*a^2*Cos[4*(c + d*x)] - 30*b^2*Cos[4*(c + d*x)]
 - 24*a*b*Cos[5*(c + d*x)] - 5*a^2*Cos[6*(c + d*x)])*Tan[c + d*x])/(960*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4360, 3042, 3390, 3042, 3072, 254, 2009, 4889, 360, 25, 2345, 27, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$
$$\downarrow 4360$$
$$\int \sin^4(c+dx)\tan^2(c+dx)(-a\cos(c+dx)-b)^2 dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3390$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\sin^4(c+dx)\tan^2(c+dx)dx+2ab\int \sin^5(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$\int \left(b^2+a^2\cos(c+dx)^2\right)\sin(c+dx)^4\tan(c+dx)^2dx+2ab\int \sin(c+dx)^5\tan(c+dx)dx$$

$$\downarrow 3072$$

$$\int \left(b^2+a^2\cos(c+dx)^2\right)\sin(c+dx)^4\tan(c+dx)^2dx+\frac{2ab\int \frac{\sin^6(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\downarrow 254$$

$$\frac{\int \left(b^2+a^2\cos(c+dx)^2\right)\sin(c+dx)^4\tan(c+dx)^2dx+2ab\int \left(-\sin^4(c+dx)-\sin^2(c+dx)+\frac{1}{1-\sin^2(c+dx)}-1\right)d\sin(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\int \left(b^2+a^2\cos(c+dx)^2\right)\sin(c+dx)^4\tan(c+dx)^2dx+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 4889$$

$$\frac{\int \frac{\tan^6(c+dx)\left(a^2+b^2+b^2\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)^4}d\tan(c+dx)}{d}+\frac{2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 360$$

$$\frac{-\frac{1}{6}\int -\frac{6b^2\tan^6(c+dx)+6a^2\tan^4(c+dx)-6a^2\tan^2(c+dx)+a^2}{\left(\tan^2(c+dx)+1\right)^3}d\tan(c+dx)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}}{d}+\frac{2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\int \frac{6b^2\tan^6(c+dx)+6a^2\tan^4(c+dx)-6a^2\tan^2(c+dx)+a^2}{\left(\tan^2(c+dx)+1\right)^3}d\tan(c+dx)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}}{d}+\frac{2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 2345$$

$$\frac{\frac{1}{6}\left(\frac{\left(13a^2-6b^2\right)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}-\frac{1}{4}\int\frac{3\left(-8b^2\tan^4(c+dx)-8\left(a^2-b^2\right)\tan^2(c+dx)+3a^2-2b^2\right)}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)\right)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{\left(13a^2-6b^2\right)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}-\frac{3}{4}\int\frac{-8b^2\tan^4(c+dx)-8\left(a^2-b^2\right)\tan^2(c+dx)+3a^2-2b^2}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)\right)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 1471$$

$$\frac{\frac{1}{6}\left(\frac{\left(13a^2-6b^2\right)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}-\frac{3}{4}\left(\frac{\left(11a^2-18b^2\right)\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}-\frac{1}{2}\int\frac{5a^2-14b^2+16b^2\tan^2(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)\right)\right)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 299$$

$$\frac{\frac{1}{6}\left(\frac{\left(13a^2-6b^2\right)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5\left(a^2-6b^2\right)\int\frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)-16b^2\tan(c+dx)\right)+\frac{\left(11a^2-18b^2\right)\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)\right)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{\left(13a^2-6b^2\right)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5\left(a^2-6b^2\right)\arctan(\tan(c+dx))-16b^2\tan(c+dx)\right)+\frac{\left(11a^2-18b^2\right)\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)\right)-\frac{a^2\tan(c+dx)}{6\left(\tan^2(c+dx)+1\right)^3}+2ab\left(\operatorname{arctanh}(\sin(c+dx))-\frac{1}{5}\sin^5(c+dx)-\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

output
```
(2*a*b*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x] - Sin[c + d*x]^3/3 - Sin[c +
d*x]^5/5))/d + (-1/6*(a^2*Tan[c + d*x])/(1 + Tan[c + d*x]^2)^3 + (((13*a^2
 - 6*b^2)*Tan[c + d*x])/(4*(1 + Tan[c + d*x]^2)^2) - (3*((-5*(a^2 - 6*b^2)
*ArcTan[Tan[c + d*x]] - 16*b^2*Tan[c + d*x])/2 + ((11*a^2 - 18*b^2)*Tan[c
+ d*x])/(2*(1 + Tan[c + d*x]^2))))/4)/6)/d
```

3.179. $\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3390
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[2*a*(b/d)
Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*
x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.179.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+2ab\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(se}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+2ab\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(se}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\right.\right.}{d}$
parallelrisch	$\frac{-3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\left(-180a^2+450b^2\right)\sin(3dx+3c)+(40a}{}$
risch	$\frac{5a^2x}{16}-\frac{15b^2x}{8}+\frac{ie^{-2i(dx+c)}b^2}{4d}-\frac{15ia^2e^{-2i(dx+c)}}{128d}+\frac{11iabe^{i(dx+c)}}{8d}-\frac{ie^{2i(dx+c)}b^2}{4d}+\frac{15ia^2e^{2i(dx+c)}}{128d}-\frac{11ia}{}$
norman	$\frac{\left(-\frac{5a^2}{16}+\frac{15b^2}{8}\right)x+\left(-\frac{45a^2}{16}+\frac{135b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{25a^2}{16}+\frac{75b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{25a^2}{16}+\frac{75b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-}{}$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)+2*a*b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(
d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$$
$$=\frac{75\left(a^2-6\,b^2\right)dx\cos\left(dx+c\right)+240\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-240\,ab\cos\left(dx+c\right)\log\left(-\sin\left(d\right.\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(75*(a^2 - 6*b^2)*d*x*cos(d*x + c) + 240*a*b*cos(d*x + c)*log(sin(d*
x + c) + 1) - 240*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (40*a^2*cos(d*
x + c)^6 + 96*a*b*cos(d*x + c)^5 - 352*a*b*cos(d*x + c)^3 - 10*(13*a^2 - 6
*b^2)*cos(d*x + c)^4 + 736*a*b*cos(d*x + c) + 15*(11*a^2 - 18*b^2)*cos(d*x
 + c)^2 - 240*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

output
```
Timed out
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\,dx$$

$$= \frac{5\left(4\sin\left(2\,dx + 2\,c\right)^3 + 60\,dx + 60\,c + 9\sin\left(4\,dx + 4\,c\right) - 48\sin\left(2\,dx + 2\,c\right)\right)a^2 - 64\left(6\sin\left(dx + c\right)^5 + 1\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^2 - 64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(s
in(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a*b - 120*(
15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*ta
n(d*x + c)^2 + 1) - 8*tan(d*x + c))*b^2)/d
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(163) = 326$.

Time $= 0.38$ (sec) , antiderivative size $= 379$, normalized size of antiderivative $= 2.17$

$$\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\, dx$$

$$= \frac{480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 75\,(a^2 - 6\,b^2)(dx + c) - \frac{480\,b^2\,\tan}{\tan\left(\frac{1}{2}\,d\right.}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 480*a*b*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 75*(a^2 - 6*b^2)*(d*x + c) - 480*b^2*tan(1/2*d*x +
1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(75*a^2*tan(1/2*d*x + 1/2*c)^11 -
480*a*b*tan(1/2*d*x + 1/2*c)^11 - 210*b^2*tan(1/2*d*x + 1/2*c)^11 + 425*a^
2*tan(1/2*d*x + 1/2*c)^9 - 3040*a*b*tan(1/2*d*x + 1/2*c)^9 - 870*b^2*tan(1
/2*d*x + 1/2*c)^9 + 990*a^2*tan(1/2*d*x + 1/2*c)^7 - 8256*a*b*tan(1/2*d*x
+ 1/2*c)^7 - 660*b^2*tan(1/2*d*x + 1/2*c)^7 - 990*a^2*tan(1/2*d*x + 1/2*c)
^5 - 8256*a*b*tan(1/2*d*x + 1/2*c)^5 + 660*b^2*tan(1/2*d*x + 1/2*c)^5 - 42
5*a^2*tan(1/2*d*x + 1/2*c)^3 - 3040*a*b*tan(1/2*d*x + 1/2*c)^3 + 870*b^2*t
an(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c) - 480*a*b*tan(1/2*d*x
+ 1/2*c) + 210*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.179.9 Mupad [B] (verification not implemented)

Time $= 16.74$ (sec) , antiderivative size $= 231$, normalized size of antiderivative $= 1.32$

$$\int (a + b\sec(c + dx))^2 \sin^6(c + dx)\, dx$$

$$= \frac{\frac{5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)a^2}{8} + 4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,a\,b - \frac{15\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)b^2}{4}}{d}$$
$$- \frac{\frac{15\,a^2\,\sin(c+d\,x)}{128} - \frac{5\,b^2\,\sin(c+d\,x)}{4} + \frac{3\,a^2\,\sin(3\,c+3\,d\,x)}{32} - \frac{a^2\,\sin(5\,c+5\,d\,x)}{48} + \frac{a^2\,\sin(7\,c+7\,d\,x)}{384} - \frac{15\,b^2\,\sin(3\,c+3\,d\,x)}{64} + \frac{b^2\,\sin(5\,c-}{64}}{d\,\cos\left(c+d\,x\right)}$$

input
```
int(sin(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

output
```
((5*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (15*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + 4*a*b*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d - ((15*a^2*sin(c + d*x))/128 - (5*b^2*sin(c + d*x))/4
+ (3*a^2*sin(3*c + 3*d*x))/32 - (a^2*sin(5*c + 5*d*x))/48 + (a^2*sin(7*c +
 7*d*x))/384 - (15*b^2*sin(3*c + 3*d*x))/64 + (b^2*sin(5*c + 5*d*x))/64 +
(59*a*b*sin(2*c + 2*d*x))/48 - (2*a*b*sin(4*c + 4*d*x))/15 + (a*b*sin(6*c
+ 6*d*x))/80)/(d*cos(c + d*x))
```

3.180 $\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$

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3.180.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx = {} & \frac{3}{8}\left(a^2 - 4b^2\right) x + \frac{2ab\operatorname{arctanh}(\sin(c + dx))}{d} \\ & - \frac{b(28a^2 + b^2)\sin(c + dx)}{6ad} \\ & - \frac{(39a^2 + 2b^2)\cos(c + dx)\sin(c + dx)}{24d} \\ & - \frac{(12a^2 + b^2)(b + a\cos(c + dx))^2 \sin(c + dx)}{12abd} \\ & + \frac{(b + a\cos(c + dx))^3 \sin(c + dx)}{4ad} \\ & + \frac{(b + a\cos(c + dx))^3 \tan(c + dx)}{bd}\end{aligned}$$

output

```
3/8*(a^2-4*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-1/6*b*(28*a^2+b^2)*sin(d*x+c
)/a/d-1/24*(39*a^2+2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/12*(12*a^2+b^2)*(b+a*c
os(d*x+c))^2*sin(d*x+c)/a/b/d+1/4*(b+a*cos(d*x+c))^3*sin(d*x+c)/a/d+(b+a*c
os(d*x+c))^3*tan(d*x+c)/b/d
```

3.180.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))^2 \sin^4(c+dx)\,dx$$
$$= \frac{12\left(3(a^2-4b^2)(c+dx) - 16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

output
```
(12*(3*(a^2 - 4*b^2)*(c + d*x) - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 208*a*b*Sin[c
+ d*x] + (-6*(3*a^2 - 4*b^2)*Cos[2*(c + d*x)] + 16*a*b*Cos[3*(c + d*x)] +
3*(-7*a^2 + 40*b^2 + a^2*Cos[4*(c + d*x)]))*Tan[c + d*x])/(96*d)
```

3.180.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4360, 3042, 3373, 25, 3042, 3528, 3042, 3512, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4 \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$
$$\downarrow 4360$$
$$\int \sin^2(c+dx)\tan^2(c+dx)(-a\cos(c+dx)-b)^2 dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4 \left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2} dx$$
$$\downarrow 3373$$

$$\frac{-\frac{\int -(b+a\cos(c+dx))^2\left(8a^2-5b\cos(c+dx)a-\left(12a^2+b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 25$$

$$\frac{\frac{\int (b+a\cos(c+dx))^2\left(8a^2-5b\cos(c+dx)a-\left(12a^2+b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{(b+a\sin(c+dx+\frac{\pi}{2}))^2\left(8a^2-5b\sin(c+dx+\frac{\pi}{2})a+\left(-12a^2-b^2\right)\sin(c+dx+\frac{\pi}{2})^2\right)}{\sin(c+dx+\frac{\pi}{2})}dx}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3528$$

$$\frac{\frac{1}{3}\int (b+a\cos(c+dx))\left(24ba^2-17b^2\cos(c+dx)a-b\left(39a^2+2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\int \frac{(b+a\sin(c+dx+\frac{\pi}{2}))\left(24ba^2-17b^2\sin(c+dx+\frac{\pi}{2})a-b\left(39a^2+2b^2\right)\sin(c+dx+\frac{\pi}{2})^2\right)}{\sin(c+dx+\frac{\pi}{2})}dx-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3512$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\int \left(48a^2b^2-4\left(28a^2+b^2\right)\cos^2(c+dx)b^2+9a\left(a^2-4b^2\right)\cos(c+dx)b\right)\sec(c+dx)dx-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{48a^2b^2-4\left(28a^2+b^2\right)\sin(c+dx+\frac{\pi}{2})^2b^2+9a\left(a^2-4b^2\right)\sin(c+dx+\frac{\pi}{2})b}{\sin(c+dx+\frac{\pi}{2})}dx-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}}{4ab}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}{\sin(c+dx)(a\cos(c+dx)+b)^3}$$

$$\downarrow 3502$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(16a^2b^2+3a\left(a^2-4b^2\right)\cos(c+dx)b\right)\sec(c+dx)dx-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+d}{2d}\right.}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

$$\downarrow 27$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3\int\left(16a^2b^2+3a\left(a^2-4b^2\right)\cos(c+dx)b\right)\sec(c+dx)dx-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+}{2d}\right.}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{16a^2b^2+3a\left(a^2-4b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\left(12a^2+b^2\right)\sin(c+}{}}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

$$\downarrow 3214$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3\left(16a^2b^2\int\sec(c+dx)dx+3abx\left(a^2-4b^2\right)\right)-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{(12a}{}}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3\left(16a^2b^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+3abx\left(a^2-4b^2\right)\right)-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

$$\downarrow 4257$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3\left(\frac{16a^2b^2\operatorname{arctanh}(\sin(c+dx))}{d}+3abx\left(a^2-4b^2\right)\right)-\frac{4b^2\left(28a^2+b^2\right)\sin(c+dx)}{d}\right)-\frac{ab\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{(12}{}}{\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

output
```
((b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + (-1/3*((12*a^2 + b^2)*(b +
 a*Cos[c + d*x])^2*Sin[c + d*x])/d + (-1/2*(a*b*(39*a^2 + 2*b^2)*Cos[c + d
*x]*Sin[c + d*x])/d + (3*(3*a*b*(a^2 - 4*b^2)*x + (16*a^2*b^2*ArcTanh[Sin[
c + d*x]])/d) - (4*b^2*(28*a^2 + b^2)*Sin[c + d*x])/d)/2)/3)/(4*a*b) + ((b
 + a*Cos[c + d*x])^3*Tan[c + d*x])/(b*d)
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3373
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 2)/(b*d
^2*f*(m + n + 4))), x] + Simp[1/(a*b*d*(n + 1)*(m + n + 4))   Int[(a + b*Si
n[e + f*x])^m*(d*Sin[e + f*x])^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m +
n + 2)*(m + n + 4) + a*b*(m + 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*
(m + n + 3)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e,
f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !
m < -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.180.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2$
default	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\right.}{d}$
parallelrisch	$\frac{72a^2xd\cos(dx+c)-288b^2dx\cos(dx+c)+384ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-384ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-}{19}$
risch	$\frac{3a^2x}{8}-\frac{3b^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{5iabe^{i(dx+c)}}{4d}-\frac{5iabe^{-i(dx+c)}}{4d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{ie^{-2i(dx+c}}{8d}$
norman	$\frac{\left(-\frac{3a^2}{8}+\frac{3b^2}{2}\right)x+\left(-\frac{9a^2}{8}+\frac{9b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{3a^2}{4}+3b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{3a^2}{4}-3b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3a^2}{8}-\frac{3b^}{2}\right.}$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a
*b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c
)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$$
$$=\frac{9\left(a^2-4\,b^2\right)dx\cos\left(dx+c\right)+24\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-24\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+\right.\right.}{2}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/24*(9*(a^2 - 4*b^2)*d*x*cos(d*x + c) + 24*a*b*cos(d*x + c)*log(sin(d*x +
 c) + 1) - 24*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*a^2*cos(d*x + c
)^4 + 16*a*b*cos(d*x + c)^3 - 64*a*b*cos(d*x + c) - 3*(5*a^2 - 4*b^2)*cos(
d*x + c)^2 + 24*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.180.6 Sympy [F]

$$\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx = \int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**4, x)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$$

$$= \frac{3\,(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c))a^2 - 32\,\left(2\,\sin(dx + c)^3 - 3\,\log(\sin(dx + c) + 1) + \right.}{96\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^2 - 32*(
2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*s
in(d*x + c))*a*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2
*tan(d*x + c))*b^2)/d
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.60

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$$

$$= \frac{48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\,(a^2-4\,b^2)(dx+c)-\frac{48\,b^2\tan\left(\frac{1}{2}\,dx\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 9*(a^2 - 4*b^2)*(d*x + c) - 48*b^2*tan(1/2*d*x + 1/2*c
)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*
tan(1/2*d*x + 1/2*c)^7 - 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^2*tan(1/2*d*
x + 1/2*c)^5 - 208*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1/2*c
)^5 - 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 208*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*
b^2*tan(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d
*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)
/d
```

3.180.9 Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx = \frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d} - \frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$+ \frac{a^2\cos(c+d\,x)^3\,\sin(c+d\,x)}{4\,d} + \frac{b^2\,\sin(c+d\,x)}{d\,\cos(c+d\,x)}$$
$$- \frac{8\,a\,b\,\sin(c+d\,x)}{3\,d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$- \frac{5\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)}{8\,d}$$
$$+ \frac{b^2\,\cos(c+d\,x)\,\sin(c+d\,x)}{2\,d}$$
$$+ \frac{2\,a\,b\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d}$$

input
```
int(sin(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

output
```
(3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) - (3*b^2*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^2*cos(c + d*x)^3*sin(c + d*x)
)/(4*d) + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (8*a*b*sin(c + d*x))/(3*d)
 + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (5*a^2*cos(c +
 d*x)*sin(c + d*x))/(8*d) + (b^2*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*a*b
*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.181 $\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx$

3.181.1 Optimal result 1325
3.181.2 Mathematica [A] (verified) 1325
3.181.3 Rubi [A] (verified) 1326
3.181.4 Maple [A] (verified) 1327
3.181.5 Fricas [A] (verification not implemented) 1328
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3.181.7 Maxima [A] (verification not implemented) 1329
3.181.8 Giac [B] (verification not implemented) 1329
3.181.9 Mupad [B] (verification not implemented) 1330

3.181.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx = \frac{a^2 x}{2} - b^2 x + \frac{2ab\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{2ab\sin(c + dx)}{d} - \frac{a^2\cos(c + dx)\sin(c + dx)}{2d} + \frac{b^2\tan(c + dx)}{d}$$

output
```
1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

3.181.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx = -\frac{-2a^2c + 4b^2c - 2a^2dx + 4b^2dx + 8ab\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - 8ab\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right.}{4d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

output
```
-1/4*(-2*a^2*c + 4*b^2*c - 2*a^2*d*x + 4*b^2*d*x + 8*a*b*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)] - 4*b^2*Tan[c + d*x])/d
```

3.181.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4359, 3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4359$$

$$\int \tan^2(c+dx)(-a\cos(c+dx)-b)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\tan\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3201$$

$$\int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

input
```
Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

output
```
(a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])
/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.181.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
parts	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2(\tan(dx+c)-dx-c)}{d}+\frac{2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$\frac{-16ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+16ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-8ab\sin(2dx+2c)-a^2\sin(3dx+3c)+4dx(a$
risch	$\frac{a^2x}{2}-b^2x+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{iabe^{i(dx+c)}}{d}-\frac{iabe^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ib^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2ab\ln\left(e^{i(dx$
norman	$\frac{\left(-\frac{a^2}{2}+b^2\right)x+\left(-\frac{a^2}{2}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{2}-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{a^2}{2}-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{\left(a^2-4ab-2b^2\right)\tan\left(\frac{dx}{2}\right.}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c)))+b^2*(tan(d*x+c)-d*x-c))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx$$
$$= \frac{(a^2 - 2\,b^2)dx\cos(dx + c) + 2\,ab\cos(dx + c)\log(\sin(dx + c) + 1) - 2\,ab\cos(dx + c)\log(-\sin(dx + c) +}{2\,d\cos(dx + c)}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c)
+ 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4
*a*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.181.6 Sympy [F]

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx = \int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**2, x)
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx$$
$$= \frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a^2 - 4\,(dx + c - \tan(dx + c))b^2 + 4\,ab(\log(\sin(dx + c) + 1) - \log(\sin(dx +$$
$$4\,d$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*b^2
 + 4*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))
/d
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(73) = 146.

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int (a + b\sec(c + dx))^2 \sin^2(c + dx)\, dx$$
$$= \frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + (a^2 - 2\,b^2)(dx + c) - \frac{4\,b^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + (a^2 - 2*b^2)*(d*x + c) - 4*b^2*tan(1/2*d*x + 1/2*c)/(tan
(1/2*d*x + 1/2*c)^2 - 1) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d
*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.181.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = \frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} - \frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{b^2\,\sin(c+dx)}{d\,\cos(c+dx)} - \frac{2\,a\,b\,\sin(c+dx)}{d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} - \frac{a^2\,\cos(c+dx)\,\sin(c+dx)}{2\,d}$$

input
```
int(sin(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) -
(2*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.182 $\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$

3.182.1 Optimal result 1331
3.182.2 Mathematica [B] (verified) 1331
3.182.3 Rubi [A] (verified) 1332
3.182.4 Maple [A] (verified) 1335
3.182.5 Fricas [A] (verification not implemented) 1336
3.182.6 Sympy [F] 1336
3.182.7 Maxima [A] (verification not implemented) 1336
3.182.8 Giac [B] (verification not implemented) 1337
3.182.9 Mupad [B] (verification not implemented) 1337

3.182.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

output
```
2*a*b*arctanh(sin(d*x+c))/d-(a^2+b^2)*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d+b^2*
tan(d*x+c)/d
```

3.182.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(59) = 118.

Time = 1.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(4ab\cos(c+dx)+(a^2+2b^2)\cos(2(c+dx))+a\left(a+2b\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{4d\left(-1+\cot^2\left(\frac{1}{2}(c+\right.\right.}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/4*(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(4*a*b*Cos[c + d*x] + (a^2 + 2*b
^2)*Cos[2*(c + d*x)] + a*(a + 2*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(d*(-1 +
 Cot[(c + d*x)/2]^2))
```

3.182.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 3042, 3390, 25, 3042, 3101, 25, 262, 219, 4889, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4360$$

$$\int \csc^2(c+dx)\sec^2(c+dx)(-a\cos(c+dx)-b)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3390$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^2(c+dx)\sec^2(c+dx)dx - 2ab\int -\csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^2(c+dx)\sec^2(c+dx)dx + 2ab\int \csc^2(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^2}dx + 2ab\int \csc(c+dx)^2\sec(c+dx)dx$$

$$\downarrow 3101$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^2}dx - \frac{2ab\int -\frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^2}dx + \frac{2ab\int \frac{\csc^2(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 262$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^2}dx - \frac{2ab\left(\csc(c+dx)-\int \frac{1}{1-\csc^2(c+dx)}d\csc(c+dx)\right)}{d}$$

$$\downarrow 219$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^2}dx - \frac{2ab(\csc(c+dx)-\operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 4889$$

$$\frac{\int \cot^2(c+dx)\left(a^2+b^2+b^2\tan^2(c+dx)\right)d\tan(c+dx)}{d} - \frac{2ab(\csc(c+dx)-\operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 244$$

$$\frac{\int \left(b^2+\left(a^2+b^2\right)\cot^2(c+dx)\right)d\tan(c+dx)}{d} - \frac{2ab(\csc(c+dx)-\operatorname{arctanh}(\csc(c+dx)))}{d}$$

$$\downarrow 2009$$

$$\frac{b^2\tan(c+dx)-\left(a^2+b^2\right)\cot(c+dx)}{d} - \frac{2ab(\csc(c+dx)-\operatorname{arctanh}(\csc(c+dx)))}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*a*b*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x]))/d + (-((a^2 + b^2)*Cot[c
+ d*x]) + b^2*Tan[c + d*x])/d
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3390
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[2*a*(b/d)
Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*
x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.182.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-a^2\cot(dx+c)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{-a^2\cot(dx+c)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
parallelrisch	$\frac{-8ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+8ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\left(\left(a^2+2b^2\right)\cos(2dx+2c)+a(a-4b)\right)\sec\left(\frac{dx}{2}+\right.}{4d\cos(dx+c)}$
risch	$-\frac{2i\left(2ab\,e^{3i(dx+c)}+a^2e^{2i(dx+c)}+2ab\,e^{i(dx+c)}+a^2+2b^2\right)}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}-\frac{2ab\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2ab\ln\left(e^{i(dx+c)}+i\right)}{d}$
norman	$\frac{\frac{a^2+2ab+b^2}{2d}-\frac{\left(a^2+3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{\left(a^2-2ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{d}$

input
```
int(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*cot(d*x+c)+2*a*b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(
1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{ab\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c) - ab\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c) - 2}{d\cos(dx+c)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a*b*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a*b*cos(d*x + c)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*b*cos(d*x + c) - (a^2 + 2*b^2)*cos
(d*x + c)^2 + b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

3.182.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**2, x)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{ab\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + b^2\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + \frac{a^2}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + b
^2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

3.182.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(59) = 118$.

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.83

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\tan\left(\frac{1}{2}\,dx+\right.}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + a^2*tan(1/2*d*x + 1/2*c) - 2*a*b*tan(1/2*d*x + 1/2*c) + b
^2*tan(1/2*d*x + 1/2*c) - (a^2*tan(1/2*d*x + 1/2*c)^2 + 2*a*b*tan(1/2*d*x
+ 1/2*c)^2 + 5*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2 - 2*a*b - b^2)/(tan(1/2*d*
x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

3.182.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a-b)^2}{2\,d} - \frac{2\,a\,b+a^2+b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(a^2+2\,a\,b+5\,b^2)}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)} + \frac{4\,a\,b\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (2*a*b + a^2 + b^2 - tan(c/2 + (d*x
)/2)^2*(2*a*b + a^2 + 5*b^2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)
/2)^3)) + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.183 $\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$

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3.183.2 Mathematica [B] (verified) . . . 1338
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3.183.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} + \frac{b^2\tan(c+dx)}{d}$$

output
```
2*a*b*arctanh(sin(d*x+c))/d-(a^2+2*b^2)*cot(d*x+c)/d-1/3*(a^2+b^2)*cot(d*x
+c)^3/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d+b^2*tan(d*x+c)/d
```

3.183.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(100) = 200.

Time = 1.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.59

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-3a^2-14ab\cos(c+dx)-2(a^2+4b^2)\cos(2(c+dx))+6ab\cos(3(c+\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-3*a^2 - 14*a*b*Cos[c + d*x] - 2*(
a^2 + 4*b^2)*Cos[2*(c + d*x)] + 6*a*b*Cos[3*(c + d*x)] + a^2*Cos[4*(c + d*
x)] + 4*b^2*Cos[4*(c + d*x)] - 6*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[2*(c + d*x)] + 6*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
2*(c + d*x)] + 3*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*(c + d
*x)] - 3*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(
96*d*(-1 + Cot[(c + d*x)/2]^2))
```

3.183.3 Rubi [A] (verified)

Time $= 0.69$ (sec) , antiderivative size $= 87$, normalized size of antiderivative $= 0.87$, number of steps used $= 14$, number of rules used $= 13$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used $= \{3042, 4360, 3042, 3390, 25, 3042, 3101, 25, 254, 2009, 4889, 355, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4360$$

$$\int \csc^4(c+dx)\sec^2(c+dx)(-a\cos(c+dx)-b)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 3390$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^4(c+dx)\sec^2(c+dx)dx - 2ab\int -\csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^4(c+dx)\sec^2(c+dx)dx + 2ab\int \csc^4(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^4}dx + 2ab\int \csc(c+dx)^4\sec(c+dx)dx$$

$$\downarrow 3101$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^4}dx - \frac{2ab\int -\frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^4}dx + \frac{2ab\int \frac{\csc^4(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 254$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^4}dx + \frac{2ab\int \left(-\csc^2(c+dx)+\frac{1}{1-\csc^2(c+dx)}-1\right)d\csc(c+dx)}{d}$$

$$\downarrow 2009$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^4}dx - \frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 4889$$

$$\frac{\int \cot^4(c+dx)\left(\tan^2(c+dx)+1\right)\left(a^2+b^2+b^2\tan^2(c+dx)\right)d\tan(c+dx)}{d} - \frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 355$$

$$\frac{\int \left(\left(a^2+b^2\right)\cot^4(c+dx)+\left(a^2+2b^2\right)\cot^2(c+dx)+b^2\right)d\tan(c+dx)}{d} - \frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}\left(a^2+b^2\right)\cot^3(c+dx)-\left(a^2+2b^2\right)\cot(c+dx)+b^2\tan(c+dx)}{d} - \frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*a*b*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x] + Csc[c + d*x]^3/3))/d + (-
((a^2 + 2*b^2)*Cot[c + d*x]) - ((a^2 + b^2)*Cot[c + d*x]^3)/3 + b^2*Tan[c
+ d*x])/d
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3390
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[2*a*(b/d)
Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*
x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.183.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}$
parallelrisch	$\frac{-96ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+96ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\left(a^2+4b^2\right)\cos(2dx+2c)+\right.}{48d\cos(dx+c)}$
risch	$-\frac{4i\left(3ab\,e^{7i(dx+c)}-7ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}-7ab\,e^{3i(dx+c)}-2a^2e^{2i(dx+c)}-8b^2e^{2i(dx+c)}+3ab\,e^{i(dx+c)}+a^2+4b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{\frac{a^2+2ab+b^2}{24d}-\frac{3\left(a^2+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}+\frac{\left(a^2-2ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{24d}+\frac{\left(2a^2-7ab+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{6d}+\frac{\left(2a^2+7ab+5b^2\right)\tan\left(\frac{dx}{2}\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3-1/sin
(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/s
in(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c)))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{6\,ab\cos(dx+c)^3+2\left(a^2+4\,b^2\right)\cos(dx+c)^4-8\,ab\cos(dx+c)-3\left(a^2+4\,b^2\right)\cos(dx+c)^2-3\,(ab\,co}{3\,(}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(6*a*b*cos(d*x + c)^3 + 2*(a^2 + 4*b^2)*cos(d*x + c)^4 - 8*a*b*cos(d*
x + c) - 3*(a^2 + 4*b^2)*cos(d*x + c)^2 - 3*(a*b*cos(d*x + c)^3 - a*b*cos(
d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*(a*b*cos(d*x + c)^3 - a*b
*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 3*b^2)/((d*cos(d*x +
c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

3.183.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\csc^4(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**4, x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{ab\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+b^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\tan(d\right.}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(a*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) + b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(96) = 192$.

Time $= 0.34$ (sec) , antiderivative size $= 226$, normalized size of antiderivative $= 2.26$

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 48ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 4}{}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(
1/2*d*x + 1/2*c)^3 + 48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a^2*tan(1/2*d*x + 1/2*c) - 30*a*b*tan
(1/2*d*x + 1/2*c) + 21*b^2*tan(1/2*d*x + 1/2*c) - 48*b^2*tan(1/2*d*x + 1/2
*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (9*a^2*tan(1/2*d*x + 1/2*c)^2 + 30*a*b*
tan(1/2*d*x + 1/2*c)^2 + 21*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2 + 2*a*b + b^2
)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.183.9 Mupad [B] (verification not implemented)

Time $= 14.68$ (sec) , antiderivative size $= 182$, normalized size of antiderivative $= 1.82$

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^2}{24\,d}$$
$$- \frac{\frac{2ab}{3} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(3a^2+10ab+23b^2) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{8a^2}{3}+\frac{28ab}{3}+\frac{20b^2}{3}\right) + \frac{a^2}{3}+\frac{b^2}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2}{8}-\frac{3ab}{4}+\frac{5b^2}{8}+\frac{(a-b)^2}{4}\right)}{d} + \frac{4ab\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - ((2*a*b)/3 - tan(c/2 + (d*x)/2)^
4*(10*a*b + 3*a^2 + 23*b^2) + tan(c/2 + (d*x)/2)^2*((28*a*b)/3 + (8*a^2)/3
 + (20*b^2)/3) + a^2/3 + b^2/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (
d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*(a^2/8 - (3*a*b)/4 + (5*b^2)/8 + (a - b)
^2/4))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.184 $\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$

3.184.1 Optimal result 1346
3.184.2 Mathematica [B] (verified) 1346
3.184.3 Rubi [A] (verified) 1347
3.184.4 Maple [A] (verified) 1350
3.184.5 Fricas [A] (verification not implemented) 1351
3.184.6 Sympy [F(-1)] 1351
3.184.7 Maxima [A] (verification not implemented) 1352
3.184.8 Giac [B] (verification not implemented) 1352
3.184.9 Mupad [B] (verification not implemented) 1353

3.184.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+3b^2)\cot(c+dx)}{d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{(a^2+b^2)\cot^5(c+dx)}{5d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{2ab\csc^5(c+dx)}{5d} + \frac{b^2\tan(c+dx)}{d}$$

output
```
2*a*b*arctanh(sin(d*x+c))/d-(a^2+3*b^2)*cot(d*x+c)/d-1/3*(2*a^2+3*b^2)*cot
(d*x+c)^3/d-1/5*(a^2+b^2)*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*
x+c)^3/d-2/5*a*b*csc(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

3.184.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 368 vs. 2(143) = 286.

Time = 1.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.57

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40a^2+196ab\cos(c+dx)+20(a^2+6b^2)\cos(2(c+dx))-130ab\cos(\right.$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/7680*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^5*(40*a^2 + 196*a*b*Cos[c + d
*x] + 20*(a^2 + 6*b^2)*Cos[2*(c + d*x)] - 130*a*b*Cos[3*(c + d*x)] - 16*a^
2*Cos[4*(c + d*x)] - 96*b^2*Cos[4*(c + d*x)] + 30*a*b*Cos[5*(c + d*x)] + 4
*a^2*Cos[6*(c + d*x)] + 24*b^2*Cos[6*(c + d*x)] + 75*a*b*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sin[2*(c + d*x)] - 60*a*b*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]]*Sin[4*(c + d*x)] + 60*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]*Sin[4*(c + d*x)] + 15*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
6*(c + d*x)] - 15*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[6*(c +
d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2))
```

3.184.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 3042, 3390, 25, 3042, 3101, 25, 254, 2009, 4889, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int \csc^6(c+dx)\sec^2(c+dx)(-a\cos(c+dx)-b)^2dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 3390$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6(c+dx)\sec^2(c+dx)dx-2ab\int -\csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 25$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6(c+dx)\sec^2(c+dx)dx+2ab\int \csc^6(c+dx)\sec(c+dx)dx$$

$$\downarrow 3042$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^6}dx+2ab\int \csc(c+dx)^6\sec(c+dx)dx$$

$$\downarrow 3101$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^6}dx-\frac{2ab\int -\frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 25$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^6}dx+\frac{2ab\int \frac{\csc^6(c+dx)}{1-\csc^2(c+dx)}d\csc(c+dx)}{d}$$

$$\downarrow 254$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^6}dx+\frac{2ab\int \left(-\csc^4(c+dx)-\csc^2(c+dx)+\frac{1}{1-\csc^2(c+dx)}-1\right)d\csc(c+dx)}{d}$$

$$\downarrow 2009$$

$$\int \frac{b^2+a^2\cos(c+dx)^2}{\cos(c+dx)^2\sin(c+dx)^6}dx-\frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 4889$$

$$\frac{\int \cot^6(c+dx)\left(\tan^2(c+dx)+1\right)^2\left(a^2+b^2+b^2\tan^2(c+dx)\right)d\tan(c+dx)}{d}-\frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 355$$

$$\frac{\int \left(\left(a^2+b^2\right)\cot^6(c+dx)+\left(2a^2+3b^2\right)\cot^4(c+dx)+\left(a^2+3b^2\right)\cot^2(c+dx)+b^2\right)d\tan(c+dx)}{d}-\frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{5}\left(a^2+b^2\right)\cot^5(c+dx)-\frac{1}{3}\left(2a^2+3b^2\right)\cot^3(c+dx)-\left(a^2+3b^2\right)\cot(c+dx)+b^2\tan(c+dx)}{d}-$$
$$\frac{2ab\left(-\operatorname{arctanh}(\csc(c+dx))+\frac{1}{5}\csc^5(c+dx)+\frac{1}{3}\csc^3(c+dx)+\csc(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*a*b*(-ArcTanh[Csc[c + d*x]] + Csc[c + d*x] + Csc[c + d*x]^3/3 + Csc[c
+ d*x]^5/5))/d + (-((a^2 + 3*b^2)*Cot[c + d*x]) - ((2*a^2 + 3*b^2)*Cot[c +
 d*x]^3)/3 - ((a^2 + b^2)*Cot[c + d*x]^5)/5 + b^2*Tan[c + d*x])/d
```

3.184.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3390
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[2*a*(b/d)
Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*
x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.184.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+2ab\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+}{d}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+2ab\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+}{d}$
parallelrisch	$\frac{-3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^5\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(5(a^2+6}{1920}$
risch	$-\frac{4i\left(15ab\,e^{11i(dx+c)}-65ab\,e^{9i(dx+c)}+98ab\,e^{7i(dx+c)}+40a^2e^{6i(dx+c)}+98ab\,e^{5i(dx+c)}+20a^2e^{4i(dx+c)}+120b^2e^{4i(dx+c)}-\right.}{15d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{\frac{a^2+2ab+b^2}{160d}-\frac{5\left(a^2+7b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d}+\frac{\left(a^2-2ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{160d}+\frac{\left(11a^2-32ab+21b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{240d}+\frac{\left(11a^2+32ab+21b^2}{240}\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$

input
```
int(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/5
/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2
*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d*x+c)/
cos(d*x+c)-16/5*cot(d*x+c)))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{30\,ab\cos(dx+c)^5+8\,(a^2+6\,b^2)\cos(dx+c)^6-70\,ab\cos(dx+c)^3-20\,(a^2+6\,b^2)\cos(dx+c)^4+46}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(30*a*b*cos(d*x + c)^5 + 8*(a^2 + 6*b^2)*cos(d*x + c)^6 - 70*a*b*cos
(d*x + c)^3 - 20*(a^2 + 6*b^2)*cos(d*x + c)^4 + 46*a*b*cos(d*x + c) + 15*(
a^2 + 6*b^2)*cos(d*x + c)^2 - 15*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^
3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a*b*cos(d*x
 + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1)*
sin(d*x + c) - 15*b^2)/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x
 + c))*sin(d*x + c))
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{ab\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+3b^2\left(\frac{15\tan(dx}{15\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(a*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*b^2*((15*tan(d*x
+ c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(
d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(135) = 270.

Time = 0.36 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.28

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$=\frac{3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+25a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-70ab\tan}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2
*tan(1/2*d*x + 1/2*c)^5 + 25*a^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b*tan(1/2*d
*x + 1/2*c)^3 + 45*b^2*tan(1/2*d*x + 1/2*c)^3 + 960*a*b*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) - 960*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a^2*ta
n(1/2*d*x + 1/2*c) - 660*a*b*tan(1/2*d*x + 1/2*c) + 570*b^2*tan(1/2*d*x +
1/2*c) - 960*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*
a^2*tan(1/2*d*x + 1/2*c)^4 + 660*a*b*tan(1/2*d*x + 1/2*c)^4 + 570*b^2*tan(
1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*b*tan(1/2*d*x +
1/2*c)^2 + 45*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(1/2*
d*x + 1/2*c)^5)/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.73

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^2}{160\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^2}{32}-\frac{5\,a\,b}{48}+\frac{7\,b^2}{96}+\frac{(a-b)^2}{48}\right)}{d}$$
$$- \frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{22\,a^2}{15}+\frac{64\,a\,b}{15}+\frac{14\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(10\,a^2+44\,a\,b+102\,b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{25}{3}\right.}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{7\,a^2}{32}-\frac{19\,a\,b}{16}+\frac{35\,b^2}{32}+\frac{3\,(a-b)^2}{32}\right)}{d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) + (tan(c/2 + (d*x)/2)^3*(a^2/32 -
 (5*a*b)/48 + (7*b^2)/96 + (a - b)^2/48))/d - ((2*a*b)/5 + tan(c/2 + (d*x)
/2)^2*((64*a*b)/15 + (22*a^2)/15 + (14*b^2)/5) - tan(c/2 + (d*x)/2)^6*(44*
a*b + 10*a^2 + 102*b^2) + tan(c/2 + (d*x)/2)^4*((118*a*b)/3 + (25*a^2)/3 +
 35*b^2) + a^2/5 + b^2/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 32*tan(c/2 + (d*x)
/2)^7)) + (tan(c/2 + (d*x)/2)*((7*a^2)/32 - (19*a*b)/16 + (35*b^2)/32 + (3
*(a - b)^2)/32))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.185 $\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx$

3.185.1 Optimal result 1354
3.185.2 Mathematica [A] (verified) 1354
3.185.3 Rubi [A] (verified) 1355
3.185.4 Maple [A] (verified) 1357
3.185.5 Fricas [A] (verification not implemented) 1358
3.185.6 Sympy [F(-1)] 1358
3.185.7 Maxima [A] (verification not implemented) 1359
3.185.8 Giac [B] (verification not implemented) 1359
3.185.9 Mupad [B] (verification not implemented) 1360

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx = -\frac{a(a^2-6b^2)\cos(c+dx)}{d} + \frac{b(6a^2-b^2)\cos^2(c+dx)}{2d} + \frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d} - \frac{3a^2b\cos^4(c+dx)}{4d} - \frac{a^3\cos^5(c+dx)}{5d} - \frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

output
```
-a*(a^2-6*b^2)*cos(d*x+c)/d+1/2*b*(6*a^2-b^2)*cos(d*x+c)^2/d+1/3*a*(2*a^2-
3*b^2)*cos(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-b*(3
*a^2-2*b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

3.185.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx$$
$$= \frac{-60a(5a^2-42b^2)\cos(c+dx) + 60(9a^2b-2b^3)\cos(2(c+dx)) + 50a^3\cos(3(c+dx)) - 120ab^2\cos(3(c+$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

output
```
(-60*a*(5*a^2 - 42*b^2)*Cos[c + d*x] + 60*(9*a^2*b - 2*b^3)*Cos[2*(c + d*x
)] + 50*a^3*Cos[3*(c + d*x)] - 120*a*b^2*Cos[3*(c + d*x)] - 45*a^2*b*Cos[4
*(c + d*x)] - 6*a^3*Cos[5*(c + d*x)] - 1440*a^2*b*Log[Cos[c + d*x]] + 960*
b^3*Log[Cos[c + d*x]] + 1440*a*b^2*Sec[c + d*x] + 240*b^3*Sec[c + d*x]^2)/
(480*d)
```

3.185.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin^2(c+dx)\tan^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin^2(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3316$$

$$-\frac{\int(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)^2\sec^3(c+dx)d(a\cos(c+dx))}{a^5d}$$

$$\downarrow 27$$

$$-\frac{\int\frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)^2\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 522$$

$$-\frac{\int\left(\cos^4(c+dx)a^4+\left(1-\frac{6b^2}{a^2}\right)a^4+3b\cos^3(c+dx)a^3-\left(2a^2-3b^2\right)\cos^2(c+dx)a^2+3b^2\sec^2(c+dx)a^2+b^3\sec\right.}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)+\frac{3}{4}a^4b\cos^4(c+dx)-3a^3b^2\sec(c+dx)-\frac{1}{2}a^2b^3\sec^2(c+dx)-\frac{1}{2}a^2b\left(6a^2-b^2\right)\cos^2(c+dx)+}{a^2d}$$

input
```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

output
```
-((a^3*(a^2 - 6*b^2)*Cos[c + d*x] - (a^2*b*(6*a^2 - b^2)*Cos[c + d*x]^2)/2
 - (a^3*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/3 + (3*a^4*b*Cos[c + d*x]^4)/4 + (
a^5*Cos[c + d*x]^5)/5 + a^2*b*(3*a^2 - 2*b^2)*Log[a*Cos[c + d*x]] - 3*a^3*
b^2*Sec[c + d*x] - (a^2*b^3*Sec[c + d*x]^2)/2)/(a^2*d))
```

3.185.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.185.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+3a^2b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3ab^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{d}$
default	$\frac{-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+3a^2b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3ab^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{d}$
parts	$-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)}{d}+$
parallelrisch	$\frac{2880\left(a^2-\frac{2b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-2880\left(a^2-\frac{2b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2880\left(a$
norman	$\frac{\left(16a^3+24a^2b-48ab^2+16b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{16a^3-240ab^2}{15d}-\frac{\left(6a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{\left(18a^2b-12b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{(16a^3}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$
risch	$\frac{6ib\,a^2c}{d}-\frac{4ib^3c}{d}+\frac{5\,e^{3i(dx+c)}a^3}{96d}-\frac{e^{3i(dx+c)}ab^2}{8d}+\frac{9\,e^{2i(dx+c)}a^2b}{16d}-\frac{e^{2i(dx+c)}b^3}{8d}-\frac{5a^3e^{i(dx+c)}}{16d}+\frac{21\,e^{i(dx+c)}}{8d}$

input
```
int((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/4
*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(sin(d*x+c)^6/cos(d
*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)^
6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.03

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{96\,a^3\cos(dx+c)^7 + 360\,a^2b\cos(dx+c)^6 - 160\,(2\,a^3 - 3\,ab^2)\cos(dx+c)^5 - 240\,(6\,a^2b - b^3)\cos(dx+$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/480*(96*a^3*cos(d*x + c)^7 + 360*a^2*b*cos(d*x + c)^6 - 160*(2*a^3 - 3*
a*b^2)*cos(d*x + c)^5 - 240*(6*a^2*b - b^3)*cos(d*x + c)^4 - 1440*a*b^2*co
s(d*x + c) + 480*(a^3 - 6*a*b^2)*cos(d*x + c)^3 + 480*(3*a^2*b - 2*b^3)*co
s(d*x + c)^2*log(-cos(d*x + c)) - 240*b^3 + 15*(39*a^2*b - 8*b^3)*cos(d*x
+ c)^2)/(d*cos(d*x + c)^2)
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

output
```
Timed out
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-20\,(2\,a^3-3\,ab^2)\cos(dx+c)^3-30\,(6\,a^2b-b^3)\cos(dx+c)^2}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 20*(2*a^3 - 3*a*b
^2)*cos(d*x + c)^3 - 30*(6*a^2*b - b^3)*cos(d*x + c)^2 + 60*(a^3 - 6*a*b^2
)*cos(d*x + c) + 60*(3*a^2*b - 2*b^3)*log(cos(d*x + c)) - 30*(6*a*b^2*cos(
d*x + c) + b^3)/cos(d*x + c)^2)/d
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(160) = 320.

Time = 0.44 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.09

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 1)) - 60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 1)) + 30*(9*a^2*b + 12*a*b^2 - 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (64*a^3 + 411*a^2*b - 600*a*b^2 -
 274*b^3 - 320*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2415*a^2*b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 2640*a*b^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1490*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^3*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^2*b*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 - 3840*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
- 3100*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^2*b*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2160*a*b^2*(cos(d*x + c) - 1)^3/(cos(d
*x + c) + 1)^3 + 3100*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2415
*a^2*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 360*a*b^2*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 1490*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c)
 + 1)^4 - 411*a^2*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 274*b^3*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 1)^5)/d
```

3.185.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx =$$

$$-\frac{\cos(c+dx)^3\left(a\,b^2-\frac{2\,a^3}{3}\right)-\cos(c+dx)^2\left(3\,a^2\,b-\frac{b^3}{2}\right)+\ln(\cos(c+dx))\,(3\,a^2\,b-2\,b^3)-\frac{\frac{b^3}{2}+3\,a\,\cos(c}{\cos(c+d}}{d}$$

input
```
int(sin(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

output
```
-(cos(c + d*x)^3*(a*b^2 - (2*a^3)/3) - cos(c + d*x)^2*(3*a^2*b - b^3/2) +
log(cos(c + d*x))*(3*a^2*b - 2*b^3) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c
 + d*x)^2 - cos(c + d*x)*(6*a*b^2 - a^3) + (a^3*cos(c + d*x)^5)/5 + (3*a^2
*b*cos(c + d*x)^4)/4)/d
```

3.186 $\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx$

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3.186.2 Mathematica [A] (verified) 1361
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3.186.9 Mupad [B] (verification not implemented) 1366

3.186.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx = -\frac{a(a^2 - 3b^2)\cos(c + dx)}{d} + \frac{3a^2 b\cos^2(c + dx)}{2d} + \frac{a^3\cos^3(c + dx)}{3d} - \frac{b(3a^2 - b^2)\log(\cos(c + dx))}{d} + \frac{3ab^2\sec(c + dx)}{d} + \frac{b^3\sec^2(c + dx)}{2d}$$

output

```
-a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/
d-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2
/d
```

3.186.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

$$\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx = \frac{-9a(a^2 - 4b^2)\cos(c + dx) + 9a^2 b\cos(2(c + dx)) + a^3\cos(3(c + dx)) - 36a^2 b\log(\cos(c + dx)) + 12b^3\log(}{12d}$$

input

```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

output
```
(-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^
2*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

3.186.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4359$$

$$\int \tan^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan^3(c+dx)(a\cos(c+dx)+b)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\tan\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3200$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\frac{b^3\sec^3(c+dx)}{a}+3b^2\sec^2(c+dx)+\frac{\left(3a^2b-b^3\right)\sec(c+dx)}{a}-a^2\cos^2(c+dx)+a^2\left(1-\frac{3b^2}{a^2}\right)-3ab\cos(c+dx)\right)d(a\,c}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a\left(a^2-3b^2\right)\cos(c+dx)+b\left(3a^2-b^2\right)\log(a\cos(c+dx))-\frac{3}{2}a^2b\cos^2(c+dx)-3ab^2\sec(c+}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

output
```
-((a*(a^2 - 3*b^2)*Cos[c + d*x] - (3*a^2*b*Cos[c + d*x]^2)/2 - (a^3*Cos[c
+ d*x]^3)/3 + b*(3*a^2 - b^2)*Log[a*Cos[c + d*x]] - 3*a*b^2*Sec[c + d*x] -
 (b^3*Sec[c + d*x]^2)/2)/d)
```

3.186.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.186.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3ab^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+}{d}$
default	$\frac{-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3ab^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+}{d}$
parts	$-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}+\frac{3ab^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}$
parallelrisch	$\frac{72(1+\cos(2dx+2c))b\left(a^2-\frac{b^2}{3}\right)\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-72(1+\cos(2dx+2c))b\left(a^2-\frac{b^2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-72(1+\cos(2dx$
risch	$3ia^2bx+\frac{6ib\,a^2c}{d}+\frac{e^{3i(dx+c)}a^3}{24d}+\frac{3\,e^{2i(dx+c)}a^2b}{8d}-\frac{3a^3e^{i(dx+c)}}{8d}+\frac{3\,e^{i(dx+c)}ab^2}{2d}-\frac{3a^3e^{-i(dx+c)}}{8d}+\frac{3\,e^{-i(}$
norman	$\frac{-\frac{4a^3-36ab^2}{3d}-\frac{\left(6a^2b-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{2\left(2a^3-3a^2b+6ab^2-3b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{\left(4a^3+18a^2b-36ab^2-6b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}+}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/2*sin(d*x+c)^2-ln(co
s(d*x+c)))+3*a*b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b
^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{4\,a^3\cos(dx+c)^5+18\,a^2b\cos(dx+c)^4-9\,a^2b\cos(dx+c)^2+36\,ab^2\cos(dx+c)-12\,(a^3-3\,ab^2)\cos(d}{12\,d\cos(dx+c)^2}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c
)^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^
2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

3.186.6 Sympy [F]

$$\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx = \int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**3, x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx$$
$$= \frac{2\,a^3 \cos(dx + c)^3 + 9\,a^2 b \cos(dx + c)^2 - 6\,(a^3 - 3\,ab^2)\cos(dx + c) - 6\,(3\,a^2 b - b^3)\log(\cos(dx + c)) + \frac{3(}{}}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(2*a^3*cos(d*x + c)^3 + 9*a^2*b*cos(d*x + c)^2 - 6*(a^3 - 3*a*b^2)*cos
(d*x + c) - 6*(3*a^2*b - b^3)*log(cos(d*x + c)) + 3*(6*a*b^2*cos(d*x + c)
+ b^3)/cos(d*x + c)^2)/d
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^3 \sin^3(c+dx)\,dx$$
$$= -\frac{(3a^2b-b^3)\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6ab^2\cos(dx+c)+b^3}{2d\cos(dx+c)^2}$$
$$+\frac{2a^3d^8\cos(dx+c)^3+9a^2bd^8\cos(dx+c)^2-6a^3d^8\cos(dx+c)+18ab^2d^8\cos(dx+c)}{6d^9}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
-(3*a^2*b - b^3)*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^2*cos(d*x +
c) + b^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^2*b*d^8
*cos(d*x + c)^2 - 6*a^3*d^8*cos(d*x + c) + 18*a*b^2*d^8*cos(d*x + c))/d^9
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))^3 \sin^3(c+dx)\,dx$$
$$= \frac{\frac{b^3}{2}+3a\cos(c+dx)b^2}{\cos(c+dx)^2} - \ln(\cos(c+dx))\,(3a^2b-b^3) + \cos(c+dx)\,(3ab^2-a^3) + \frac{a^3\cos(c+dx)^3}{3} + \frac{3a^2b\cos(c+dx}{2}$$
$$d$$

input
```
int(sin(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

output
```
((b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*
b - b^3) + cos(c + d*x)*(3*a*b^2 - a^3) + (a^3*cos(c + d*x)^3)/3 + (3*a^2*
b*cos(c + d*x)^2)/2)/d
```

3.187 $\int (a + b\sec(c + dx))^3 \sin(c + dx)\, dx$

3.187.1 Optimal result 1367
3.187.2 Mathematica [A] (verified) 1367
3.187.3 Rubi [A] (verified) 1368
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3.187.5 Fricas [A] (verification not implemented) 1370
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3.187.7 Maxima [A] (verification not implemented) 1371
3.187.8 Giac [A] (verification not implemented) 1371
3.187.9 Mupad [B] (verification not implemented) 1372

3.187.1 Optimal result

Integrand size = 19, antiderivative size = 64

$$\int (a + b\sec(c + dx))^3 \sin(c + dx)\, dx = -\frac{a^3 \cos(c + dx)}{d} - \frac{3a^2 b \log(\cos(c + dx))}{d} + \frac{3ab^2 \sec(c + dx)}{d} + \frac{b^3 \sec^2(c + dx)}{2d}$$

output
```
-a^3*cos(d*x+c)/d-3*a^2*b*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*se
c(d*x+c)^2/d
```

3.187.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int (a + b\sec(c + dx))^3 \sin(c + dx)\, dx = \frac{-2a^3 \cos(c + dx) + b(-6a^2 \log(\cos(c + dx)) + 6ab\sec(c + dx) + b^2 \sec^2(c + dx))}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

output
```
(-2*a^3*Cos[c + d*x] + b*(-6*a^2*Log[Cos[c + d*x]] + 6*a*b*Sec[c + d*x] +
b^2*Sec[c + d*x]^2))/(2*d)
```

3.187.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \tan(c+dx)\sec^2(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 3312$$

$$-\frac{\int (b+a\cos(c+dx))^3\sec^3(c+dx)d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{a^2\int \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 49$$

$$-\frac{a^2 \int \left(\frac{b^3 \sec^3(c+dx)}{a^3}+\frac{3b^2 \sec^2(c+dx)}{a^2}+\frac{3b \sec(c+dx)}{a}+1\right) d(a\cos(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{b^3 \sec^2(c+dx)}{2a^2}-\frac{3b^2 \sec(c+dx)}{a}+3b\log(a\cos(c+dx))+a\cos(c+dx)\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

output
```
-((a^2*(a*Cos[c + d*x] + 3*b*Log[a*Cos[c + d*x]] - (3*b^2*Sec[c + d*x])/a
- (b^3*Sec[c + d*x]^2)/(2*a^2)))/d)
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.187.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{b^3\sec(dx+c)^2}{2}+3\sec(dx+c)ab^2+3a^2b\ln(\sec(dx+c))-\frac{a^3}{\sec(dx+c)}}{d}$
default	$\frac{\frac{b^3\sec(dx+c)^2}{2}+3\sec(dx+c)ab^2+3a^2b\ln(\sec(dx+c))-\frac{a^3}{\sec(dx+c)}}{d}$
parts	$-\frac{a^3\cos(dx+c)}{d}+\frac{b^3\sec(dx+c)^2}{2d}+\frac{3a^2b\ln(\sec(dx+c))}{d}+\frac{3ab^2\sec(dx+c)}{d}$
risch	$3ia^2bx-\frac{a^3e^{i(dx+c)}}{2d}-\frac{a^3e^{-i(dx+c)}}{2d}+\frac{6iba^2c}{d}+\frac{2b^2\left(3ae^{3i(dx+c)}+be^{2i(dx+c)}+3e^{i(dx+c)}a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(e^{2i(dx+c)}\right.}{d}$
norman	$\frac{\left(4a^3+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{2a^3-6ab^2}{d}-\frac{\left(2a^3+6ab^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{3a^2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3a^2b\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{d}$
parallelrisch	$\frac{6a^2b(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-6a^2b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-6a^2b(1+\cos(2dx+2c))\ln\left(\tan\left(\right.\right.}{2d(1+\cos(2dx+}$

input

```
int((a+b*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/2*b^3*sec(d*x+c)^2+3*sec(d*x+c)*a*b^2+3*a^2*b*ln(sec(d*x+c))-a^3/se
c(d*x+c))
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$$
$$=-\frac{2\,a^3\cos(dx+c)^3+6\,a^2b\cos(dx+c)^2\log(-\cos(dx+c))-6\,ab^2\cos(dx+c)-b^3}{2\,d\cos(dx+c)^2}$$

input

```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2*log(-cos(d*x + c)) - 6
*a*b^2*cos(d*x + c) - b^3)/(d*cos(d*x + c)^2)
```

3.187.6 Sympy [F]

$$\int (a + b\sec(c + dx))^3 \sin(c + dx)\,dx = \int (a + b\sec(c + dx))^3 \sin(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c),x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sin(c + d*x), x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int (a + b\sec(c + dx))^3 \sin(c + dx)\,dx = -\frac{2\,a^3\cos(dx + c) + 6\,a^2 b\log(\cos(dx + c)) - \frac{6\,ab^2}{\cos(dx+c)} - \frac{b^3}{\cos(dx+c)^2}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(2*a^3*cos(d*x + c) + 6*a^2*b*log(cos(d*x + c)) - 6*a*b^2/cos(d*x + c
) - b^3/cos(d*x + c)^2)/d
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int (a + b\sec(c + dx))^3 \sin(c + dx)\,dx = -\frac{a^3\cos(dx + c)}{d} - \frac{3\,a^2 b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6\,ab^2\cos(dx + c) + b^3}{2\,d\cos(dx + c)^2}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

output
```
-a^3*cos(d*x + c)/d - 3*a^2*b*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b
^2*cos(d*x + c) + b^3)/(d*cos(d*x + c)^2)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^3 \sin(c+dx)\,dx$$
$$= -\frac{a^3\cos(c+dx) - \frac{\frac{b^3}{2}+3a\cos(c+dx)b^2}{\cos(c+dx)^2} + 3a^2 b \ln(\cos(c+dx))}{d}$$

input
```
int(sin(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

output
```
-(a^3*cos(c + d*x) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 + 3*a^2
*b*log(cos(c + d*x)))/d
```

3.188 $\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$

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3.188.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\begin{aligned}\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = {}& \frac{(a+b)^3\log(1-\cos(c+dx))}{2d}\\ & -\frac{b(3a^2+b^2)\log(\cos(c+dx))}{d}\\ & -\frac{(a-b)^3\log(1+\cos(c+dx))}{2d}\\ & +\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}\end{aligned}$$

output
```
1/2*(a+b)^3*ln(1-cos(d*x+c))/d-b*(3*a^2+b^2)*ln(cos(d*x+c))/d-1/2*(a-b)^3*
ln(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

3.188.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{-2(a-b)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b(3a^2+b^2)\log(\cos(c+dx))+2(a+b)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6ab^2}{2d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(-2*(a - b)^3*Log[Cos[(c + d*x)/2]] - 2*b*(3*a^2 + b^2)*Log[Cos[c + d*x]]
+ 2*(a + b)^3*Log[Sin[(c + d*x)/2]] + 6*a*b^2*Sec[c + d*x] + b^3*Sec[c + d
*x]^2)/(2*d)
```

3.188.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 526, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int \csc(c+dx)\sec^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3316$$

$$\frac{a\int -\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a\int \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^4\int \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{a^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\downarrow 526$$

$$-\frac{a^4\left(\frac{\int \frac{\left(\cos^2(c+dx)a^4+3b^2a^2+b\left(3a^2+b^2\right)\cos(c+dx)a\right)\sec^2(c+dx)}{a^2\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{a^2}-\frac{b^3\sec^2(c+dx)}{2a^4}\right)}{d}$$

$$\downarrow 2333$$

$$-\frac{a^4\left(\frac{\int\left(\frac{(a-b)^3}{2a^2(\cos(c+dx)a+a)}+\frac{3b^2\sec^2(c+dx)}{a^2}+\frac{\left(b^3+3a^2b\right)\sec(c+dx)}{a^3}+\frac{(a+b)^3}{2a^2(a-a\cos(c+dx))}\right)d(a\cos(c+dx))}{a^2}-\frac{b^3\sec^2(c+dx)}{2a^4}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^4\left(\frac{\frac{b\left(3a^2+b^2\right)\log(a\cos(c+dx))}{a^2}+\frac{(a-b)^3\log(a\cos(c+dx)+a)}{2a^2}-\frac{(a+b)^3\log(a-a\cos(c+dx))}{2a^2}-\frac{3b^2\sec(c+dx)}{a}}{a^2}-\frac{b^3\sec^2(c+dx)}{2a^4}\right)}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
-((a^4*(-1/2*(b^3*Sec[c + d*x]^2)/a^4 + ((b*(3*a^2 + b^2)*Log[a*Cos[c + d*
x]])/a^2 - ((a + b)^3*Log[a - a*Cos[c + d*x]])/(2*a^2) + ((a - b)^3*Log[a
+ a*Cos[c + d*x]])/(2*a^2) - (3*b^2*Sec[c + d*x])/a)/a^2))/d)
```

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 526
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol] :
> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemainder
[(c + d*x)^n, x, x]}, Simp[R*(x^(m + 1)/(a*(m + 1))), x] + Simp[1/a   Int[x
^(m + 1)*(ExpandToSum[a*Qx - b*R*x, x]/(a + b*x^2)), x], x]] /; FreeQ[{a, b
, c, d}, x] && IGtQ[n, 1] && ILtQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.188.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3\ln(-\cot(dx+c)+\csc(dx+c))+3a^2b\ln(\tan(dx+c))+3ab^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+b^3\left(\frac{1}{2\cos(dx+c)^2}\right.}{d}$
default	$\frac{a^3\ln(-\cot(dx+c)+\csc(dx+c))+3a^2b\ln(\tan(dx+c))+3ab^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+b^3\left(\frac{1}{2\cos(dx+c)^2}\right.}{d}$
norman	$\frac{\frac{6ab^2}{d}-\frac{2\left(3ab^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\left(3a^2+b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b(}{}$
parallelrisch	$\frac{-3\left(a^2+\frac{b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\left(a^2+\frac{b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(a+b)^3(1+co}{d(1+\cos(2dx+2c))}$
risch	$\frac{2b^2\left(3a\,e^{3i(dx+c)}+b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{3\ln\left(e^{i(dx+c)}-1\right)a^2b}{d}+\frac{3\ln\left(e^{i(dx+c)}-1\right)ab^2}{d}+$

input
```
int(csc(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(-cot(d*x+c)+csc(d*x+c))+3*a^2*b*ln(tan(d*x+c))+3*a*b^2*(1/cos(
d*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+b^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c))))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{6\,ab^2\cos(dx+c)-2\left(3\,a^2b+b^3\right)\cos(dx+c)^2\log\left(-\cos(dx+c)\right)-\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos(dx+c}{2\,d\cos(dx+}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*cos(d*x + c) - 2*(3*a^2*b + b^3)*cos(d*x + c)^2*log(-cos(d*x
+ c)) - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(d*x + c)^2*log(1/2*cos(d*x + c
) + 1/2) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^2*log(-1/2*cos(d*x
 + c) + 1/2) + b^3)/(d*cos(d*x + c)^2)
```

3.188.6 Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3 \csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*csc(c + d*x), x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{(a^3-3\,a^2b+3\,ab^2-b^3)\log(\cos(dx+c)+1)-(a^3+3\,a^2b+3\,ab^2+b^3)\log(\cos(dx+c)-1)+2\,(3\,a$$
$$2\,d$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(cos(d*x + c) + 1) - (a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*log(cos(d*x + c) - 1) + 2*(3*a^2*b + b^3)*log(cos(d*x
+ c)) - (6*a*b^2*cos(d*x + c) + b^3)/cos(d*x + c)^2)/d
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(96) = 192.

Time = 0.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,(3\,a^2b+b^3)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9\,a^2b+12\,ab^2+3\,b^3+\frac{18\,a}{}}{}}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*
x + c) + 1)) - 2*(3*a^2*b + b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 1)) + (9*a^2*b + 12*a*b^2 + 3*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.188.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\frac{\ln(\cos(c+dx)-1)\,(a+b)^3}{2} - \frac{\ln(\cos(c+dx)+1)\,(a-b)^3}{2} + \frac{\frac{b^3}{2}+3\,a\,\cos(c+dx)\,b^2}{\cos(c+dx)^2} - \ln\left(\cos\left(c+dx\right)\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^3/sin(c + d*x),x)
```

output
```
((log(cos(c + d*x) - 1)*(a + b)^3)/2 - (log(cos(c + d*x) + 1)*(a - b)^3)/2
 + (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^
2*b + b^3))/d
```

3.189 $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

3.189.1 Optimal result 1380
3.189.2 Mathematica [B] (verified) 1380
3.189.3 Rubi [A] (verified) 1382
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3.189.9 Mupad [B] (verification not implemented) 1388

3.189.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} \\
&\quad + \frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d} - \frac{b(3a^2+2b^2)\log(\cos(c+dx))}{d} \\
&\quad - \frac{(a-4b)(a-b)^2\log(1+\cos(c+dx))}{4d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}$$

output
```
-1/2*a^2*(b*(3+b^2/a^2)+a*(1+3*b^2/a^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+
b)^2*(a+4*b)*ln(1-cos(d*x+c))/d-b*(3*a^2+2*b^2)*ln(cos(d*x+c))/d-1/4*(a-4*
b)*(a-b)^2*ln(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

3.189.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 669 vs. 2(162) = 324.

Time = 7.22 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.13

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3}{d(b+a\cos(c+dx))^3} \\
&\quad + \frac{(-a^3-3a^2b-3ab^2-b^3)\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3} \\
&\quad + \frac{(-a^3+6a^2b-9ab^2+4b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} \\
&\quad + \frac{(-3a^2b-2b^3)\cos^3(c+dx)\log(\cos(c+dx))(a+b\sec(c+dx))^3}{d(b+a\cos(c+dx))^3} \\
&\quad + \frac{(a^3+6a^2b+9ab^2+4b^3)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} \\
&\quad + \frac{(a^3-3a^2b+3ab^2-b^3)\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad + \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad - \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output

```
(3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3)
 + ((-a^3 - 3*a^2*b - 3*a*b^2 - b^3)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a
+ b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x])^3) + ((-a^3 + 6*a^2*b - 9*a
*b^2 + 4*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)
/(2*d*(b + a*Cos[c + d*x])^3) + ((-3*a^2*b - 2*b^3)*Cos[c + d*x]^3*Log[Cos
[c + d*x]]*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3) + ((a^3 + 6*
a^2*b + 9*a*b^2 + 4*b^3)*Cos[c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Sec[c
 + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^3)/(8*d*(b + a*Co
s[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*C
os[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c
 + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) - (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/
(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

3.189.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 532, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4360$$

$$\int \csc^3(c+dx)\sec^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^3(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3316$$

$$\frac{a^3\int -\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^3\int \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^6\int \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)}{a^3\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 532$$

$$-\frac{a^6\left(\frac{a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)}{2a^2\left(a^2-a^2\cos^2(c+dx)\right)}-\frac{\int -\frac{\left(2b^3+6a\cos(c+dx)b^2+2a^2\left(\frac{b^2}{a^2}+3\right)\cos^2(c+dx)b+a^3\left(\frac{3b^2}{a^2}+1\right)\cos^3(c+dx)\right)\sec^3(c+dx)}{a^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2a^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6\left(\frac{\int \frac{\left(2b^3+6a\cos(c+dx)b^2+2a^2\left(\frac{b^2}{a^2}+3\right)\cos^2(c+dx)b+a^3\left(\frac{3b^2}{a^2}+1\right)\cos^3(c+dx)\right)\sec^3(c+dx)}{a^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2a^2}+\frac{a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)}{2a^2\left(a^2-a^2\cos^2(c+dx)\right)}\right)}{d}$$

$$\downarrow 2333$$

3.189. $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

$$-\frac{a^6\left(\frac{\int\left(\frac{2b^3\sec^3(c+dx)}{a^5}+\frac{6b^2\sec^2(c+dx)}{a^4}+\frac{2\left(2b^3+3a^2b\right)\sec(c+dx)}{a^5}+\frac{(a+b)^2(a+4b)}{2a^4(a-a\cos(c+dx))}+\frac{(a-4b)(a-b)^2}{2a^4(\cos(c+dx)a+a)}\right)d(a\cos(c+dx))}{2a^2}+\frac{a\left(\frac{3b^2}{a^2}+1\right)\cos(c+}{2a^2(a^2-a^2\,\mathrm{co}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^6\left(\frac{a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)}{2a^2(a^2-a^2\cos^2(c+dx))}+\frac{-\frac{b^3\sec^2(c+dx)}{a^4}-\frac{(a+b)^2(a+4b)\log(a-a\cos(c+dx))}{2a^4}+\frac{(a-4b)(a-b)^2\log(a\cos(c+dx)+a)}{2a^4}-\frac{6b^2\sec(c+dx)}{a^3}+\frac{2b(}{2a^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
-((a^6*((b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])/(2*a^2*(a^2 -
 a^2*Cos[c + d*x]^2)) + ((2*b*(3*a^2 + 2*b^2)*Log[a*Cos[c + d*x]])/a^4 - (
(a + b)^2*(a + 4*b)*Log[a - a*Cos[c + d*x]])/(2*a^4) + ((a - 4*b)*(a - b)^
2*Log[a + a*Cos[c + d*x]])/(2*a^4) - (6*b^2*Sec[c + d*x])/a^3 - (b^3*Sec[c
 + d*x]^2)/a^4)/(2*a^2)))/d)
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.189.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3ab^2\left(-\frac{1}{2\sin(dx+c)^2\cos(d}}{d}$
default	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3ab^2\left(-\frac{1}{2\sin(dx+c)^2\cos(d}}{d}$
norman	$\frac{-\frac{a^3+3a^2b+3ab^2+b^3}{8d}+\frac{\left(a^3+15ab^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}+\frac{\left(a^3-3a^2b+3ab^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d}-\frac{\left(2a^3-3a^2b+30ab^2-9b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
parallelrisch	$\frac{-192(1+\cos(2dx+2c))\left(a^2+\frac{2b^2}{3}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192(1+\cos(2dx+2c))\left(a^2+\frac{2b^2}{3}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+32(4b-}{}$
risch	$\frac{a^3e^{7i(dx+c)}+9ab^2e^{7i(dx+c)}+6a^2be^{6i(dx+c)}+4b^3e^{6i(dx+c)}+3a^3e^{5i(dx+c)}+3ab^2e^{5i(dx+c)}+12a^2be^{4i(dx+c)}+3a^3e^{3i(dx+}}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}-1\right)^2}$

input
```
int(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+3*a^2
*b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+3*a*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c
)+3/2/cos(d*x+c)+3/2*ln(-cot(d*x+c)+csc(d*x+c)))+b^3*(1/2/sin(d*x+c)^2/cos
(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c))))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.79

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{12\,ab^2\cos(dx+c) - 2\,(a^3+9\,ab^2)\cos(dx+c)^3 + 2\,b^3 - 2\,(3\,a^2b+2\,b^3)\cos(dx+c)^2 + 4\,((3\,a^2b+2\,b^3$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 9*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 2
*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 + 4*((3*a^2*b + 2*b^3)*cos(d*x + c)^4 -
(3*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)) + ((a^3 - 6*a^2*b + 9
*a*b^2 - 4*b^3)*cos(d*x + c)^4 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(d*x
 + c)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*
cos(d*x + c)^4 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d*x + c)^2)*log(-1/
2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2)
```

3.189.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**3, x)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{(a^3-6\,a^2b+9\,ab^2-4\,b^3)\log\left(\cos\left(dx+c\right)+1\right)-(a^3+6\,a^2b+9\,ab^2+4\,b^3)\log\left(\cos\left(dx+c\right)-1\right)+4}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*((a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*log(cos(d*x + c) + 1) - (a^3 + 6*a
^2*b + 9*a*b^2 + 4*b^3)*log(cos(d*x + c) - 1) + 4*(3*a^2*b + 2*b^3)*log(co
s(d*x + c)) + 2*(6*a*b^2*cos(d*x + c) - (a^3 + 9*a*b^2)*cos(d*x + c)^3 + b
^3 - (3*a^2*b + 2*b^3)*cos(d*x + c)^2)/(cos(d*x + c)^4 - cos(d*x + c)^2))/
d
```

3.189.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(154) = 308.

Time = 0.38 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.98

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,(a^3+6\,a^2b+9\,ab^2+4\,b^3)\log\left(\frac{|-}{|}\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^2*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 3*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b
^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 8*(3*a^2*b + 2*b^3
)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 + 3*a^2*b +
3*a*b^2 + b^3 - 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 18*a*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1
)/(cos(d*x + c) - 1) - 4*(9*a^2*b + 12*a*b^2 + 6*b^3 + 18*a^2*b*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.189.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= \frac{\ln(\cos(c+dx)-1)\,(a+b)^2\,(a+4b)}{4d} - \frac{\ln(\cos(c+dx))\,(3a^2b+2b^3)}{d} \\
&\quad - \frac{\cos(c+dx)^3\left(\frac{a^3}{2}+\frac{9ab^2}{2}\right) - \frac{b^3}{2} + \cos(c+dx)^2\left(\frac{3a^2b}{2}+b^3\right) - 3ab^2\cos(c+dx)}{d\left(\cos(c+dx)^2-\cos(c+dx)^4\right)} \\
&\quad - \frac{\ln(\cos(c+dx)+1)\,(a-b)^2\,(a-4b)}{4d}
\end{aligned}$$

input
```
int((a + b/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

output
```
(log(cos(c + d*x) - 1)*(a + b)^2*(a + 4*b))/(4*d) - (log(cos(c + d*x))*(3*
a^2*b + 2*b^3))/d - (cos(c + d*x)^3*((9*a*b^2)/2 + a^3/2) - b^3/2 + cos(c
+ d*x)^2*((3*a^2*b)/2 + b^3) - 3*a*b^2*cos(c + d*x))/(d*(cos(c + d*x)^2 -
cos(c + d*x)^4)) - (log(cos(c + d*x) + 1)*(a - b)^2*(a - 4*b))/(4*d)
```

3.190 $\int (a+b\sec(c+dx))^3 \sin^6(c+dx)\,dx$

3.190.1 Optimal result 1389
3.190.2 Mathematica [B] (verified) 1390
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3.190.8 Giac [B] (verification not implemented) 1396
3.190.9 Mupad [B] (verification not implemented) 1397

3.190.1 Optimal result

Integrand size = 21, antiderivative size = 299

$$\begin{aligned}\int (a+b\sec(c+dx))^3 \sin^6(c+dx)\,dx = {} & \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} \\ & - \frac{5b^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{3a^2b\sin(c+dx)}{d} \\ & + \frac{5b^3\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} \\ & - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} \\ & - \frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{3a^2b\sin^5(c+dx)}{5d} \\ & - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{45ab^2\tan(c+dx)}{8d} \\ & - \frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d} \\ & - \frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d} \\ & + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}\end{aligned}$$

output

```
5/16*a^3*x-45/8*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(
d*x+c))/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-5/16*a^3*cos(d*x+c)*si
n(d*x+c)/d-a^2*b*sin(d*x+c)^3/d+5/6*b^3*sin(d*x+c)^3/d-5/24*a^3*cos(d*x+c)
*sin(d*x+c)^3/d-3/5*a^2*b*sin(d*x+c)^5/d-1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d
+45/8*a*b^2*tan(d*x+c)/d-15/8*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d-3/4*a*b^2*si
n(d*x+c)^4*tan(d*x+c)/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

3.190.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 818 vs. $2(299) = 598$.

Time = 7.09 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.74

$$
\begin{aligned}
&\int (a + b\sec(c + dx))^3 \sin^6(c + dx)\, dx \\
&= \frac{5a(a^2 - 18b^2)\,(c + dx)\cos^3(c + dx)(a + b\sec(c + dx))^3}{16d(b + a\cos(c + dx))^3} \\
&\quad + \frac{(-6a^2b + 5b^3)\cos^3(c + dx)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)(a + b\sec(c + dx))^3}{2d(b + a\cos(c + dx))^3} \\
&\quad + \frac{(6a^2b - 5b^3)\cos^3(c + dx)\log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)(a + b\sec(c + dx))^3}{2d(b + a\cos(c + dx))^3} \\
&\quad + \frac{b^3\cos^3(c + dx)(a + b\sec(c + dx))^3}{4d(b + a\cos(c + dx))^3\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} \\
&\quad + \frac{3ab^2\cos^3(c + dx)(a + b\sec(c + dx))^3\sin\left(\frac{1}{2}(c + dx)\right)}{d(b + a\cos(c + dx))^3\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} \\
&\quad - \frac{b^3\cos^3(c + dx)(a + b\sec(c + dx))^3}{4d(b + a\cos(c + dx))^3\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)^2} \\
&\quad + \frac{3ab^2\cos^3(c + dx)(a + b\sec(c + dx))^3\sin\left(\frac{1}{2}(c + dx)\right)}{d(b + a\cos(c + dx))^3\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)} \\
&\quad + \frac{3b(-11a^2 + 6b^2)\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(c + dx)}{8d(b + a\cos(c + dx))^3} \\
&\quad - \frac{3a(5a^2 - 32b^2)\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(2(c + dx))}{64d(b + a\cos(c + dx))^3} \\
&\quad - \frac{b(-21a^2 + 4b^2)\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(3(c + dx))}{48d(b + a\cos(c + dx))^3} \\
&\quad + \frac{3a(a^2 - 2b^2)\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(4(c + dx))}{64d(b + a\cos(c + dx))^3} \\
&\quad - \frac{3a^2b\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(5(c + dx))}{80d(b + a\cos(c + dx))^3} \\
&\quad - \frac{a^3\cos^3(c + dx)(a + b\sec(c + dx))^3\sin(6(c + dx))}{192d(b + a\cos(c + dx))^3}
\end{aligned}
$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

output

```
(5*a*(a^2 - 18*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(16*d
*(b + a*Cos[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d
*x])^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos
[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*C
os[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (3*b*(-11*a^2 + 6*b^2)*Cos[c + d
*x]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(8*d*(b + a*Cos[c + d*x])^3) -
(3*a*(5*a^2 - 32*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x
)])/(64*d*(b + a*Cos[c + d*x])^3) - (b*(-21*a^2 + 4*b^2)*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[3*(c + d*x)])/(48*d*(b + a*Cos[c + d*x])^3) + (3*
a*(a^2 - 2*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(6
4*d*(b + a*Cos[c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^3*Sin[5*(c + d*x)])/(80*d*(b + a*Cos[c + d*x])^3) - (a^3*Cos[c + d*x]^3*(
a + b*Sec[c + d*x])^3*Sin[6*(c + d*x)])/(192*d*(b + a*Cos[c + d*x])^3)
```

3.190.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4360, 25, 25, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin^3(c+dx)\tan^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin^3(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3391$$

$$\int (a^3\sin^6(c+dx)+3a^2b\sin^5(c+dx)\tan(c+dx)+3ab^2\sin^4(c+dx)\tan^2(c+dx)+b^3\sin^3(c+dx)\tan^3(c+dx$$

$$\downarrow 2009$$

$$-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a^3\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\sin^5(c+dx)}{5d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{45ab^2\tan(c+dx)}{8d}-\frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d}-\frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d}-\frac{45}{8}ab^2x-\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5b^3\sin^3(c+dx)}{6d}+\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}$$

input

```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

output

```
(5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3
*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c +
d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*
x]^3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^
3)/(24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x
]^5)/(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[
c + d*x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c
 + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.190.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+3a^2b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(s}{}$
default	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+3a^2b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(s}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5}{}$
parallelrisch	$\frac{-5760(1+\cos(2dx+2c))\left(a^2-\frac{5b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5760(1+\cos(2dx+2c))\left(a^2-\frac{5b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+600}{}$
risch	$\frac{5a^3x}{16}-\frac{45ab^2x}{8}-\frac{15ie^{-2i(dx+c)}a^3}{128d}-\frac{9ie^{i(dx+c)}b^3}{8d}+\frac{7ie^{-3i(dx+c)}a^2b}{32d}+\frac{15ie^{2i(dx+c)}a^3}{128d}-\frac{7ie^{3i(dx+c)}a^2b}{32d}+$
norman	$\frac{\left(\frac{5}{16}a^3-\frac{45}{8}ab^2\right)x+\left(-\frac{25}{8}a^3+\frac{225}{4}ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-\frac{5}{4}a^3+\frac{45}{2}ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(-\frac{5}{4}a^3+\frac{45}{2}ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$

input
```
int((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)+3*a^2*b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/
4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+b^3*(1/2*sin(d
*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*
ln(sec(d*x+c)+tan(d*x+c))))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$$
$$=\frac{75\,(a^3-18\,ab^2)dx\cos(dx+c)^2+60\,(6\,a^2b-5\,b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-60\,(6\,a^2b-5\,b^3)c}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/240*(75*(a^3 - 18*a*b^2)*d*x*cos(d*x + c)^2 + 60*(6*a^2*b - 5*b^3)*cos(d
*x + c)^2*log(sin(d*x + c) + 1) - 60*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) - (40*a^3*cos(d*x + c)^7 + 144*a^2*b*cos(d*x + c)^6 - 1
0*(13*a^3 - 18*a*b^2)*cos(d*x + c)^5 - 16*(33*a^2*b - 5*b^3)*cos(d*x + c)^
4 - 720*a*b^2*cos(d*x + c) + 15*(11*a^3 - 54*a*b^2)*cos(d*x + c)^3 - 120*b
^3 + 16*(69*a^2*b - 35*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
2)
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^3 \sin^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

output
```
Timed out
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int (a + b \sec(c + dx))^3 \sin^6(c + dx)\, dx$$

$$= \frac{5\left(4 \sin(2\,dx + 2\,c)^3 + 60\,dx + 60\,c + 9 \sin(4\,dx + 4\,c) - 48 \sin(2\,dx + 2\,c)\right)a^3 - 96\left(6 \sin(dx + c)^5 + 10\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^3 - 96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(s
in(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2*b - 360
*(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*
tan(d*x + c)^2 + 1) - 8*tan(d*x + c))*a*b^2 + 80*(4*sin(d*x + c)^3 - 6*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x
 + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(273) = 546$.

Time $= 0.44$ (sec) , antiderivative size $= 563$, normalized size of antiderivative $= 1.88$

$$\int (a + b\sec(c + dx))^3 \sin^6(c + dx)\, dx$$

$$= \frac{75\,(a^3 - 18\,ab^2)(dx + c) + 120\,(6\,a^2b - 5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 120\,(6\,a^2b - 5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,a\right.\right.\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

output
```
1/240*(75*(a^3 - 18*a*b^2)*(d*x + c) + 120*(6*a^2*b - 5*b^3)*log(abs(tan(1
/2*d*x + 1/2*c) + 1)) - 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 240*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3
- 6*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2 + 2*(75*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*a^2*b*tan(1/2*d*
x + 1/2*c)^11 - 630*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 480*b^3*tan(1/2*d*x +
1/2*c)^11 + 425*a^3*tan(1/2*d*x + 1/2*c)^9 - 4560*a^2*b*tan(1/2*d*x + 1/2*
c)^9 - 2610*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 2720*b^3*tan(1/2*d*x + 1/2*c)^9
 + 990*a^3*tan(1/2*d*x + 1/2*c)^7 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 1
980*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 5760*b^3*tan(1/2*d*x + 1/2*c)^7 - 990*a
^3*tan(1/2*d*x + 1/2*c)^5 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a*b^
2*tan(1/2*d*x + 1/2*c)^5 + 5760*b^3*tan(1/2*d*x + 1/2*c)^5 - 425*a^3*tan(1
/2*d*x + 1/2*c)^3 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 2720*b^3*tan(1/2*d*x + 1/2*c)^3 - 75*a^3*tan(1/2*d*x + 1
/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) + 630*a*b^2*tan(1/2*d*x + 1/2*c) +
480*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.190.9 Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.25

$$\int (a+b\sec(c+dx))^3 \sin^6(c+dx)\,dx = \frac{7b^3\sin(c+dx)}{3d} + \frac{5a^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{8d} - \frac{5b^3\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{13a^3\cos(c+dx)^3\sin(c+dx)}{24d} - \frac{a^3\cos(c+dx)^5\sin(c+dx)}{6d} + \frac{b^3\sin(c+dx)}{2d\cos(c+dx)^2} - \frac{b^3\cos(c+dx)^2\sin(c+dx)}{3d} - \frac{11a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{23a^2b\sin(c+dx)}{5d} - \frac{45ab^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4d} + \frac{6a^2b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{27ab^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{3ab^2\sin(c+dx)}{d\cos(c+dx)} + \frac{11a^2b\cos(c+dx)^2\sin(c+dx)}{5d} - \frac{3ab^2\cos(c+dx)^3\sin(c+dx)}{4d} - \frac{3a^2b\cos(c+dx)^4\sin(c+dx)}{5d}$$

input
```
int(sin(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

output

```
(7*b^3*sin(c + d*x))/(3*d) + (5*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/(8*d) - (5*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (1
3*a^3*cos(c + d*x)^3*sin(c + d*x))/(24*d) - (a^3*cos(c + d*x)^5*sin(c + d*
x))/(6*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (b^3*cos(c + d*x)^2*
sin(c + d*x))/(3*d) - (11*a^3*cos(c + d*x)*sin(c + d*x))/(16*d) - (23*a^2*
b*sin(c + d*x))/(5*d) - (45*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(4*d) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2
7*a*b^2*cos(c + d*x)*sin(c + d*x))/(8*d) + (3*a*b^2*sin(c + d*x))/(d*cos(c
 + d*x)) + (11*a^2*b*cos(c + d*x)^2*sin(c + d*x))/(5*d) - (3*a*b^2*cos(c +
 d*x)^3*sin(c + d*x))/(4*d) - (3*a^2*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.191 $\int (a+b\sec(c+dx))^3 \sin^4(c+dx)\,dx$

3.191.1 Optimal result 1399
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3.191.1 Optimal result

Integrand size = 21, antiderivative size = 236

$$\begin{aligned}\int (a+b\sec(c+dx))^3 \sin^4(c+dx)\,dx = {}& \frac{3}{8}a\left(a^2-12b^2\right)x+\frac{3b(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}\\ &-\frac{b(17a^2-b^2)\sin(c+dx)}{2d}\\ &-\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d}\\ &-\frac{(6a^2-b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{4bd}\\ &-\frac{(4a^2-b^2)\,(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d}\\ &+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d}\\ &+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd}\end{aligned}$$

output

```
3/8*a*(a^2-12*b^2)*x+3/2*b*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-1/2*b*(17*a^2
-b^2)*sin(d*x+c)/d-1/8*a*(21*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(6*a^2
-b^2)*(b+a*cos(d*x+c))^2*sin(d*x+c)/b/d-1/4*(4*a^2-b^2)*(b+a*cos(d*x+c))^3
*sin(d*x+c)/b^2/d+a*(b+a*cos(d*x+c))^4*tan(d*x+c)/b^2/d+1/2*(b+a*cos(d*x+c
))^4*sec(d*x+c)*tan(d*x+c)/b/d
```

3.191.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(236) = 472$.

Time = 6.75 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.95

$$
\begin{aligned}
&\int (a + b\sec(c+dx))^3 \sin^4(c+dx)\,dx \\
&= \frac{3a(a^2-12b^2)\,(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3} \\
&\quad + \frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} \\
&\quad - \frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad + \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&\quad - \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&\quad + \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&\quad + \frac{b(-15a^2+4b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{4d(b+a\cos(c+dx))^3} \\
&\quad - \frac{a(a^2-3b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(2(c+dx))}{4d(b+a\cos(c+dx))^3} \\
&\quad + \frac{a^2b\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(3(c+dx))}{4d(b+a\cos(c+dx))^3} \\
&\quad + \frac{a^3\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(4(c+dx))}{32d(b+a\cos(c+dx))^3}
\end{aligned}
$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

output
```
(3*a*(a^2 - 12*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(8*d*
(b + a*Cos[c + d*x])^3) + (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*
x])^3) - (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos
[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*C
os[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b*(-15*a^2 + 4*b^2)*Cos[c + d*x
]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(4*d*(b + a*Cos[c + d*x])^3) - (a
*(a^2 - 3*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(4*
d*(b + a*Cos[c + d*x])^3) + (a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[3*(c + d*x)])/(4*d*(b + a*Cos[c + d*x])^3) + (a^3*Cos[c + d*x]^3*(a + b
*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(32*d*(b + a*Cos[c + d*x])^3)
```

3.191.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4360, 25, 25, 3042, 3372, 25, 3042, 3528, 27, 3042, 3528, 3042, 3512, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \sin(c+dx)\tan^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \sin(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3372$$

$$-\frac{\int -(b+a\cos(c+dx))^3\left(-2\left(4a^2-b^2\right)\cos^2(c+dx)-3ab\cos(c+dx)+3\left(2a^2-b^2\right)\right)\sec(c+dx)dx}{2b^2}+$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 25$$

$$\frac{\int (b+a\cos(c+dx))^3\left(-2\left(4a^2-b^2\right)\cos^2(c+dx)-3ab\cos(c+dx)+3\left(2a^2-b^2\right)\right)\sec(c+dx)dx}{2b^2}+$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(b+a\sin(c+dx+\frac{\pi}{2}))^3\left(-2(4a^2-b^2)\sin(c+dx+\frac{\pi}{2})^2-3ab\sin(c+dx+\frac{\pi}{2})+3(2a^2-b^2)\right)}{\sin(c+dx+\frac{\pi}{2})}dx}{2b^2}+$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3528$$

$$\frac{\frac{1}{4}\int 6(b+a\cos(c+dx))^2\left(-3a\cos(c+dx)b^2-\left(6a^2-b^2\right)\cos^2(c+dx)b+2\left(2a^2-b^2\right)b\right)\sec(c+dx)dx-\frac{(4a^2-b^2)}{2b^2}}{\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}}$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\int (b+a\cos(c+dx))^2\left(-3a\cos(c+dx)b^2-\left(6a^2-b^2\right)\cos^2(c+dx)b+2\left(2a^2-b^2\right)b\right)\sec(c+dx)dx-\frac{(4a^2-b^2)\,si}{2b^2}}{\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{2}\int \frac{(b+a\sin(c+dx+\frac{\pi}{2}))^2\left(-3a\sin(c+dx+\frac{\pi}{2})b^2-(6a^2-b^2)\sin(c+dx+\frac{\pi}{2})^2b+2(2a^2-b^2)b\right)}{\sin(c+dx+\frac{\pi}{2})}dx-\frac{(4a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b)^3}{2d}}{2b^2}+$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3528$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\int(b+a\cos(c+dx))\left(-13a\cos(c+dx)b^3-\left(21a^2-2b^2\right)\cos^2(c+dx)b^2+6\left(2a^2-b^2\right)b^2\right)\sec(c+dx)dx-\frac{b(}{}\right.}{2b^2}$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\int\frac{(b+a\sin(c+dx+\frac{\pi}{2}))\left(-13a\sin(c+dx+\frac{\pi}{2})b^3-(21a^2-2b^2)\sin(c+dx+\frac{\pi}{2})^2b^2+6(2a^2-b^2)b^2\right)}{\sin(c+dx+\frac{\pi}{2})}dx-\frac{b(6a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b}{3d}\right.}{2b^2}$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3512$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(-4\left(17a^2-b^2\right)\cos^2(c+dx)b^3+12\left(2a^2-b^2\right)b^3+3a\left(a^2-12b^2\right)\cos(c+dx)b^2\right)\sec(c+dx)dx-\frac{ab^2(21a}{}\right.\right.}{2b^2}$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{-4(17a^2-b^2)\sin(c+dx+\frac{\pi}{2})^2b^3+12(2a^2-b^2)b^3+3a(a^2-12b^2)\sin(c+dx+\frac{\pi}{2})b^2}{\sin(c+dx+\frac{\pi}{2})}dx-\frac{ab^2(21a^2-2b^2)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b(}{}\right.}{2b^2}$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3502$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(\int 3\left(4\left(2a^2-b^2\right)b^3+a\left(a^2-12b^2\right)\cos(c+dx)b^2\right)\sec(c+dx)dx-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin}{2}\right.\right.}{2b^2}$$
$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 27$$

3.191. $\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\left(4\left(2a^2-b^2\right)b^3+a\left(a^2-12b^2\right)\cos(c+dx)b^2\right)\sec(c+dx)dx-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin}{2}\right.\right.}{2b^2}$$

$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\int\frac{4\left(2a^2-b^2\right)b^3+a\left(a^2-12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b(6a^2}{2b^2}}$$

$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3214$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(4b^3\left(2a^2-b^2\right)\int\sec(c+dx)dx+ab^2x\left(a^2-12b^2\right)\right)-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin(c+dx)\cos(c+}{2d}\right.\right.}{2b^2}$$

$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(4b^3\left(2a^2-b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+ab^2x\left(a^2-12b^2\right)\right)-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin(c+dx)}{2d}\right.\right.}{2b^2}$$

$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

$$\downarrow 4257$$

$$\frac{\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(\frac{4b^3\left(2a^2-b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+ab^2x\left(a^2-12b^2\right)\right)-\frac{4b^3\left(17a^2-b^2\right)\sin(c+dx)}{d}\right)-\frac{ab^2\left(21a^2-2b^2\right)\sin(c+dx)\cos(c-}{2d}\right.\right.}{2b^2}$$

$$\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

input
```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

output
```
(-1/2*((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/d + (3*(-1/3*(b*
(6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/d + (-1/2*(a*b^2*(21*a^
2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/d + (3*(a*b^2*(a^2 - 12*b^2)*x + (4*
b^3*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/d) - (4*b^3*(17*a^2 - b^2)*Sin[c
+ d*x])/d)/2)/3))/2)/(2*b^2) + (a*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^
2*d) + ((b + a*Cos[c + d*x])^4*Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

3.191. $\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3372
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x]
)^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[1/(a^2*d^2*(n + 1)*(n + 2
))   Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^(n + 2)*Simp[a^2*n*(n + 2)
 - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f*x] - (a^2*(n + 1)*(n + 2)
- b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b,
d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
 &&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.191.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3$
default	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
parallelrisch	$\frac{-192(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+192(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+24adx(a$
risch	$\frac{3a^3x}{8}-\frac{9ab^2x}{2}-\frac{ie^{i(dx+c)}b^3}{2d}+\frac{ie^{-3i(dx+c)}a^2b}{8d}+\frac{ie^{2i(dx+c)}a^3}{8d}-\frac{ib^2\left(be^{3i(dx+c)}-6ae^{2i(dx+c)}-be^{i(dx+c)}-6a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}$
norman	$\frac{\left(\frac{3}{8}a^3-\frac{9}{2}ab^2\right)x+\left(-\frac{3}{2}a^3+18ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(-\frac{3}{8}a^3+\frac{9}{2}ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{3}{8}a^3+\frac{9}{2}ab^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(\frac{3}{4}a\right.}{}$

input
```
int((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+3*a
^2*b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*
c)+b^3*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*
ln(sec(d*x+c)+tan(d*x+c))))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$
$$=\frac{3\left(a^3-12ab^2\right)dx\cos(dx+c)^2+6\left(2a^2b-b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-6\left(2a^2b-b^3\right)\cos(dx$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/8*(3*(a^3 - 12*a*b^2)*d*x*cos(d*x + c)^2 + 6*(2*a^2*b - b^3)*cos(d*x + c
)^2*log(sin(d*x + c) + 1) - 6*(2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x
+ c) + 1) + (2*a^3*cos(d*x + c)^5 + 8*a^2*b*cos(d*x + c)^4 + 24*a*b^2*cos(
d*x + c) - (5*a^3 - 12*a*b^2)*cos(d*x + c)^3 + 4*b^3 - 8*(4*a^2*b - b^3)*c
os(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.191.6 Sympy [F]

$$\int (a + b \sec(c + dx))^3 \sin^4(c + dx)\, dx = \int (a + b \sec(c + dx))^3 \sin^4(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**4, x)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78

$$\int (a + b \sec(c + dx))^3 \sin^4(c + dx)\, dx$$

$$= \frac{(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c))a^3 - 16\left(2\,\sin(dx + c)^3 - 3\,\log(\sin(dx + c) + 1) + 3\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^3 - 16*(2*
sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*a^2*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2
*tan(d*x + c))*a*b^2 - 8*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(
sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.83

$$\int (a+b\sec(c+dx))^3 \sin^4(c+dx)\,dx$$

$$= \frac{3\,(a^3-12\,ab^2)(dx+c)+12\,(2\,a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,(2\,a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
1/8*(3*(a^3 - 12*a*b^2)*(d*x + c) + 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
8*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*t
an(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
1)^2 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 -
 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*t
an(1/2*d*x + 1/2*c)^5 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(1/
2*d*x + 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/
2*c)^3 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
3 + 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) - 24*a^2*b*
tan(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) + 8*b^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.191.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx = \frac{b^3\sin(c+dx)}{d} + \frac{3a^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4d} - \frac{3b^3\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{a^3\cos(c+dx)^3\sin(c+dx)}{4d} + \frac{b^3\sin(c+dx)}{2d\cos(c+dx)^2} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{4a^2b\sin(c+dx)}{d} - \frac{9ab^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{6a^2b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{3ab^2\sin(c+dx)}{d\cos(c+dx)} + \frac{a^2b\cos(c+dx)^2\sin(c+dx)}{d}$$

input
```
int(sin(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

output
```
(b^3*sin(c + d*x))/d + (3*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/(4*d) - (3*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos
(c + d*x)^3*sin(c + d*x))/(4*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
- (5*a^3*cos(c + d*x)*sin(c + d*x))/(8*d) - (4*a^2*b*sin(c + d*x))/d - (9*
a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*cos(c + d*x)*sin(c + d*x)
)/(2*d) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x)) + (a^2*b*cos(c + d*x)^2*
sin(c + d*x))/d
```

3.192 $\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$

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3.192.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx = \frac{1}{2}a\left(a^2-6b^2\right)x + \frac{b(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*(a^2-6*b^2)*x+1/2*b*(6*a^2-b^2)*arctanh(sin(d*x+c))/d-15/2*a^2*b*sin
(d*x+c)/d-5/2*a^3*cos(d*x+c)*sin(d*x+c)/d+3/2*a*(b+a*cos(d*x+c))^2*tan(d*x
+c)/d+1/2*(b+a*cos(d*x+c))^3*sec(d*x+c)*tan(d*x+c)/d
```

3.192.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(138) = 276.

Time = 1.20 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.37

$$\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$$
$$= \frac{\sec^2(c+dx)\left(a^3c-6ab^2c+a^3dx-6ab^2dx-6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^3\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

output
```
(Sec[c + d*x]^2*(a^3*c - 6*a*b^2*c + a^3*d*x - 6*a*b^2*d*x - 6*a^2*b*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + b^3*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - b^3*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(a*(a^2 - 6*b^2)*(c +
d*x) + (-6*a^2*b + b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*(-6*a
^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-3*a^2*b + 2*b^3)*S
in[c + d*x] - (a^3*Sin[2*(c + d*x)])/2 + 6*a*b^2*Sin[2*(c + d*x)] - 3*a^2*
b*Sin[3*(c + d*x)] - (a^3*Sin[4*(c + d*x)])/4))/(4*d)
```

3.192.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 4360, 25, 25, 3042, 3368, 3042, 3527, 3042, 3526, 3042, 3512, 27, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3 dx$$

$$\downarrow 4360$$

$$\int \tan^2(c+dx)\sec(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)dx$$

$$\downarrow 25$$

$$\int \tan^2(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3 dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3368$$

$$\int (1-\cos^2(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int \frac{\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3527$$

$$\frac{1}{2}\int (b+a\cos(c+dx))^2\left(-4a\cos^2(c+dx)-b\cos(c+dx)+3a\right)\sec^2(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-4a\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\sin\left(c+dx+\frac{\pi}{2}\right)+3a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}$$

$$\downarrow 3526$$

$$\frac{1}{2}\left(\int (b+a\cos(c+dx))\left(-10\cos^2(c+dx)a^2+6a^2-5b\cos(c+dx)a-b^2\right)\sec(c+dx)dx+\frac{3a\tan(c+dx)(a\cos}{d}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-10\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+6a^2-5b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a\tan(c+dx)}{}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3512$$

$$\frac{1}{2}\left(\frac{1}{2}\int 2\left(-15a^2b\cos^2(c+dx)+a\left(a^2-6b^2\right)\cos(c+dx)+b\left(6a^2-b^2\right)\right)\sec(c+dx)dx-\frac{5a^3\sin(c+dx)\cos(c+}{d}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\int \left(-15a^2b\cos^2(c+dx)+a\left(a^2-6b^2\right)\cos(c+dx)+b\left(6a^2-b^2\right)\right)\sec(c+dx)dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\int \frac{-15a^2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\left(a^2-6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+b\left(6a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}+\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3502$$

$$\frac{1}{2}\left(\int \left(b\left(6a^2-b^2\right)+a\left(a^2-6b^2\right)\cos(c+dx)\right)\sec(c+dx)dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}-\frac{15a^2b\sin(c+dx)}{d}+\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\int \frac{b\left(6a^2-b^2\right)+a\left(a^2-6b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}-\frac{15a^2b\sin(c+dx)}{d}+\frac{3a\tan(}{}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3214$$

$$\frac{1}{2}\left(b\left(6a^2-b^2\right)\int\sec(c+dx)dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}+ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{d}+\frac{3a\tan(c+}{}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(b\left(6a^2-b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}+ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{d}+\frac{3a\ta}{}\right.$$
$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right)$$

$$\downarrow 4257$$

$$\frac{1}{2}\left(-\frac{5a^3\sin(c+dx)\cos(c+dx)}{d}+\frac{b\left(6a^2-b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{d}+\frac{3a\ta\right.$$

$$\left.\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*(a^2 - 6*b^2
)*x + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/d - (15*a^2*b*Sin[c + d*x])/
d - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/d + (3*a*(b + a*Cos[c + d*x])^2*Tan[
c + d*x])/d)/2
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.192.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin}{2\cos}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin}{2\cos}\right.}{d}$
parts	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a\,b^2(\tan(dx+c)-dx}{d}$
parallelrisch	$\frac{-24(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4adx(a^2-}{8d(1-}$
risch	$\frac{a^3x}{2}-3a\,b^2x+\frac{ie^{2i(dx+c)}a^3}{8d}+\frac{3ie^{i(dx+c)}a^2b}{2d}-\frac{3ie^{-i(dx+c)}a^2b}{2d}-\frac{ie^{-2i(dx+c)}a^3}{8d}-\frac{ib^2\left(b\,e^{3i(dx+c)}-6a\,e^{2i(d}\right.}{d(e^{2i(dx+}$
norman	$\frac{\left(\frac{1}{2}a^3-3a\,b^2\right)x+\left(\frac{1}{2}a^3-3a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-a^3+6a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{\left(a^3-6a^2b-6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{3\left(a^3-}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\right.}$

input
```
int((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(-sin(d*x+c)+l
n(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(1/2*sin(d*x+c)^3
/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$$
$$= \frac{2\,(a^3-6\,ab^2)dx\cos(dx+c)^2+(6\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-(6\,a^2b-b^3)\cos(dx+c)}{4\,d\cos(}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(a^3 - 6*a*b^2)*d*x*cos(d*x + c)^2 + (6*a^2*b - b^3)*cos(d*x + c)^2
*log(sin(d*x + c) + 1) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c)
+ 1) - 2*(a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2 - 6*a*b^2*cos(d*x +
c) - b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.192.6 Sympy [F]

$$\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**2, x)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$$
$$= \frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^3-12\,(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)\right.}{4\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a*
b^2 - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) + 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1) - 2*sin(d*x + c)))/d
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(126) = 252$.

Time $= 0.36$ (sec) , antiderivative size $= 346$, normalized size of antiderivative $= 2.51$

$$\int (a + b\sec(c + dx))^3 \sin^2(c + dx)\, dx$$

$$= \frac{(a^3 - 6\,ab^2)(dx + c) + (6\,a^2b - b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (6\,a^2b - b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{}$$

input
```
integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*((a^3 - 6*a*b^2)*(d*x + c) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*ta
n(1/2*d*x + 1/2*c)^7 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 6*a*b^2*tan(1/2*d*
x + 1/2*c)^7 + b^3*tan(1/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 +
 6*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*t
an(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x
 + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^
3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(
1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^
2)/d
```

3.192.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{a^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} - \frac{b^3\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{b^3\sin(c+dx)}{2d\cos(c+dx)^2} - \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{3a^2b\sin(c+dx)}{d} - \frac{6ab^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{6a^2b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{3ab^2\sin(c+dx)}{d\cos(c+dx)}$$

input
```
int(sin(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

output
```
(a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^3*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
 - (a^3*cos(c + d*x)*sin(c + d*x))/(2*d) - (3*a^2*b*sin(c + d*x))/d - (6*a
*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(c + d*x))/(d*cos(c + d
*x))
```

3.193 $\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$

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3.193.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\begin{aligned}\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = {}& \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} \\ &+ \frac{3b^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ &- \frac{a^3\cot(c+dx)}{d} - \frac{3ab^2\cot(c+dx)}{d} \\ &- \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} \\ &+ \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}\end{aligned}$$

output

```
3*a^2*b*arctanh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)
/d-3*a*b^2*cot(d*x+c)/d-3*a^2*b*csc(d*x+c)/d-3/2*b^3*csc(d*x+c)/d+1/2*b^3*
csc(d*x+c)*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

3.193.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 406 vs. $2(133) = 266$.

Time = 2.44 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.05

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12a^2b+2b^3+6a(a^2+2b^2)\cos(c+dx)+6(2a^2b+b^3)\cos(2(c+dx))\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/16*(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]*(12*a^2*b + 2*b^3 + 6*a*(a^2 +
2*b^2)*Cos[c + d*x] + 6*(2*a^2*b + b^3)*Cos[2*(c + d*x)] + 2*a^3*Cos[3*(c
+ d*x)] + 12*a*b^2*Cos[3*(c + d*x)] + 6*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]]*Sin[c + d*x] + 3*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[c + d*x] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d
*x] - 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 6*a^2*
b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3*b^3*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 6*a^2*b*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 3*b^3*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$
$$\downarrow 4360$$

$$\int \csc^2(c+dx)\sec^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3391$$

$$-\int \left(-\sec^2\left(\frac{1}{2}(2c-\pi)+dx\right)a^3-3b\sec(c+dx)\sec^2\left(\frac{1}{2}(2c-\pi)+dx\right)a^2-3b^2\sec^2(c+dx)\sec^2\left(\frac{1}{2}(2c-\pi)\right.\right.$$

$$\downarrow 2009$$

$$-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{6ab^2\cot(2c+2dx)}{d}+\frac{3b^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{3b^3\csc(c+dx)}{2d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) -
(a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[2*c + 2*d*x])/d - (3*a^2*b*Csc[c + d*x
])/d - (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d
)
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.193.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-a^3\cot(dx+c)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3ab^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3\left(\frac{}{2\sin(dx}\right.}{d}$
default	$\frac{-a^3\cot(dx+c)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3ab^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3\left(\frac{}{2\sin(dx}\right.}{d}$
parallelrisch	$\frac{-3(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{3\sec\left(\frac{dx}{2}+\frac{c}{2}\right)}{}}{d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3+3a^2b+3ab^2+b^3}{2d}+\frac{\left(a^3-3a^2b+3ab^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2d}-\frac{3\left(a^3-a^2b+7ab^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2d}+\frac{3\left(a^3+a^2b+7ab^2+b^3\right)\tan\left(\frac{}{}\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
risch	$-\frac{i\left(6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+2a^3e^{4i(dx+c)}+12a^2b\,e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4a^3e^{2i(dx+c)}+12ab^2e^{2i(dx+c)}+6a^2b\,e\right.}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}$

input
```
int(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*cot(d*x+c)+3*a^2*b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a
*b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b^3*(1/2/sin(d*x+c)/cos(d*x+c)
^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\left(2a^2b+b^3\right)\cos(dx+c)^2\log\left(\sin(dx+c)+1\right)\sin(dx+c) - 3\left(2a^2b+b^3\right)\cos(dx+c)^2\log\left(-\sin(dx+c)\right.}{4d\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) -
 3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) + 12
*a*b^2*cos(d*x + c) - 4*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 6*(2*a^2*
b + b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^2*sin(d*x + c))
```

3.193.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**2, x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{b^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\right.\right.}{4d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^2*b*(2/sin(d*x + c) -
 log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a*b^2*(1/tan(d*x + c)
 - tan(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.69

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$=\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\left(2a^2b+b^3\right)\mathrm{l}}{}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(a^3*tan(1/2*d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 3*(2*a^2*b + b^3)*log(abs(t
an(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c) - 2*(6*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x
 + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.193.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.36

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6\,a^2\,b+3\,b^3\right)}{d}$$
$$-\frac{3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2\,a^3+6\,a^2\,b+18\,a\,b^2+4\,b^3\right)+3\,a^2\,b+a^3+b^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^3+3\,a^2\,b+15\right.}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a-b\right)^3}{2\,d}$$

input
```
int((a + b/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(6*a^2*b + 3*b^3))/d - (3*a*b^2 - tan(c/2 + (d*
x)/2)^2*(18*a*b^2 + 6*a^2*b + 2*a^3 + 4*b^3) + 3*a^2*b + a^3 + b^3 + tan(c
/2 + (d*x)/2)^4*(15*a*b^2 + 3*a^2*b + a^3 - b^3))/(d*(2*tan(c/2 + (d*x)/2)
 - 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)
*(a - b)^3)/(2*d)
```

3.194 $\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$

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3.194.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = {}& \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}\\ &+\frac{5b^3\operatorname{arctanh}(\sin(c+dx))}{2d}\\ &-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}\\ &-\frac{a^3\cot^3(c+dx)}{3d}-\frac{ab^2\cot^3(c+dx)}{d}\\ &-\frac{3a^2b\csc(c+dx)}{d}-\frac{5b^3\csc(c+dx)}{2d}\\ &-\frac{a^2b\csc^3(c+dx)}{d}-\frac{5b^3\csc^3(c+dx)}{6d}\\ &+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d}+\frac{3ab^2\tan(c+dx)}{d}\end{aligned}$$

output

```
3*a^2*b*arctanh(sin(d*x+c))/d+5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)
/d-6*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-a*b^2*cot(d*x+c)^3/d-3*a^2*
b*csc(d*x+c)/d-5/2*b^3*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-5/6*b^3*csc(d*x+c
)^3/d+1/2*b^3*csc(d*x+c)^3*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

3.194.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 610 vs. $2(205) = 410$.

Time = 1.44 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.98

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(84a^2b+22b^3+32a(a^2+3b^2)\cos(c+dx)+8(6a^2b+5b^3)\cos(2(c+\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/768*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^3*(84*a^2*b + 22*b^3 + 32*a*(a
^2 + 3*b^2)*Cos[c + d*x] + 8*(6*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 4*a^3*Co
s[3*(c + d*x)] + 48*a*b^2*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 3
0*b^3*Cos[4*(c + d*x)] - 4*a^3*Cos[5*(c + d*x)] - 48*a*b^2*Cos[5*(c + d*x)
] + 36*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 30*b^
3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 36*a^2*b*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 30*b^3*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]]*Sin[3*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]]*Sin[3*(c + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]*Sin[3*(c + d*x)] - 15*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3
*(c + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c +
 d*x)] - 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]
+ 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*
b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(-1 + C
ot[(c + d*x)/2]^2)^2)
```

3.194.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a - b\csc\left(c + dx - \frac{\pi}{2}\right)\right)^3}{\cos\left(c + dx - \frac{\pi}{2}\right)^4} dx$$

$$\downarrow 4360$$

$$\int \csc^4(c + dx)\sec^3(c + dx)\left(-(-a\cos(c + dx) - b)^3\right) dx$$

$$\downarrow 25$$

$$-\int -(b + a\cos(c + dx))^3 \csc^4(c + dx)\sec^3(c + dx) dx$$

$$\downarrow 25$$

$$\int \csc^4(c + dx)\sec^3(c + dx)(a\cos(c + dx) + b)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b - a\sin\left(c + dx - \frac{\pi}{2}\right)\right)^3}{\sin\left(c + dx - \frac{\pi}{2}\right)^3 \cos\left(c + dx - \frac{\pi}{2}\right)^4} dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b - a\sin\left(\frac{1}{2}(2c - \pi) + dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c - \pi) + dx\right)^4 \sin\left(\frac{1}{2}(2c - \pi) + dx\right)^3} dx$$

$$\downarrow 3391$$

$$-\int \left(-a^3\sec^4\left(\frac{1}{2}(2c - \pi) + dx\right) - b^3\sec^3(c + dx)\sec^4\left(\frac{1}{2}(2c - \pi) + dx\right) - 3ab^2\sec^2(c + dx)\sec^4\left(\frac{1}{2}(2c - \pi) -$$

$$\downarrow 2009$$

$$-\frac{a^3\cot^3(c + dx)}{3d} - \frac{a^3\cot(c + dx)}{d} + \frac{3a^2b\operatorname{arctanh}(\sin(c + dx))}{d} - \frac{a^2b\csc^3(c + dx)}{d} - \frac{3a^2b\csc(c + dx)}{d} + \frac{3ab^2\tan(c + dx)}{d} - \frac{ab^2\cot^3(c + dx)}{d} - \frac{6ab^2\cot(c + dx)}{d} + \frac{5b^3\operatorname{arctanh}(\sin(c + dx))}{2d} - \frac{5b^3\csc^3(c + dx)}{6d} - \frac{5b^3\csc(c + dx)}{2d} + \frac{b^3\csc^3(c + dx)\sec^2(c + dx)}{2d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) -
(a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*
d) - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c +
d*x])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b
^3*Csc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.194.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3}\right.}{}$
default	$\frac{a^3\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3}\right.}{}$
parallelrisch	$\frac{-576(1+\cos(2dx+2c))b\left(a^2+\frac{5b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+576(1+\cos(2dx+2c))b\left(a^2+\frac{5b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\csc\left(\right.}{}$
norman	$\frac{-\frac{a^3+3a^2b+3a\,b^2+b^3}{24d}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{24d}+\frac{\left(7a^3-39a^2b+57a\,b^2-25b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{24d}-\frac{\left(7a^3+39a^2b+57a\,b^2+\right.}{24}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\right.\right.}$
risch	$-\frac{i\left(18a^2b\,e^{9i(dx+c)}+15b^3e^{9i(dx+c)}-24a^2b\,e^{7i(dx+c)}-20b^3e^{7i(dx+c)}-12a^3e^{6i(dx+c)}-84a^2b\,e^{5i(dx+c)}-22b^3e^{5i(dx+c)}-\right.}{3d(e}$

input
```
int(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1/3/sin(d*x+c)^3-1/s
in(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)
+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+b^3*(-1/3/sin(d*x+c)^3/cos(d*x+
c)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/sin(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.27

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{8\left(a^3+12\,ab^2\right)\cos(dx+c)^5+6\left(6\,a^2b+5\,b^3\right)\cos(dx+c)^4+36\,ab^2\cos(dx+c)-12\left(a^3+12\,ab^2\right)\cos($$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/12*(8*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 6*(6*a^2*b + 5*b^3)*cos(d*x + c
)^4 + 36*a*b^2*cos(d*x + c) - 12*(a^3 + 12*a*b^2)*cos(d*x + c)^3 + 6*b^3 -
 8*(6*a^2*b + 5*b^3)*cos(d*x + c)^2 - 3*((6*a^2*b + 5*b^3)*cos(d*x + c)^4
- (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 3
*((6*a^2*b + 5*b^3)*cos(d*x + c)^4 - (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log
(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*s
in(d*x + c))
```

3.194.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3 \csc^4(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**4, x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{b^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log\left(\sin(dx+c)+1\right)+15\log\left(\sin(dx+c)-1\right)\right)+6a^2b\left(\frac{2\left(3\sin\right.}{\sin}\right.$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(b^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5
- sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) +
 6*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + 12*a*b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x +
c)^3 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.76

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 9a^3\tan\left(\frac{1}{2}\right.}{}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^
2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x
 + 1/2*c) - 45*a^2*b*tan(1/2*d*x + 1/2*c) + 63*a*b^2*tan(1/2*d*x + 1/2*c) -
 27*b^3*tan(1/2*d*x + 1/2*c) + 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
4*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*t
an(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
1)^2 - (9*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 6
3*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 27*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3 + 3*a
^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.194.9 Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.27

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^3}{24\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\,b\,(a-b)^2}{4} - \frac{3\,(a-b)^3}{8}\right)}{d}$$

$$- \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7a^3}{3}+13a^2b+19ab^2+\frac{25b^3}{3}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{17a^3}{3}+29a^2b+89ab^2+\frac{77b^3}{3}\right) + ab^2 -}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - 16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 8\tan\left(\frac{c}{2}\right.\right.}$$

$$- \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(a^2b\,6\mathrm{i}+b^3\,5\mathrm{i}\right)\,1\mathrm{i}}{d}$$

input
```
int((a + b/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

output

```
(tan(c/2 + (d*x)/2)^3*(a - b)^3)/(24*d) - (tan(c/2 + (d*x)/2)*((3*b*(a - b
)^2)/4 - (3*(a - b)^3)/8))/d - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*
5i)*1i)/d - (tan(c/2 + (d*x)/2)^2*(19*a*b^2 + 13*a^2*b + (7*a^3)/3 + (25*b
^3)/3) - tan(c/2 + (d*x)/2)^4*(89*a*b^2 + 29*a^2*b + (17*a^3)/3 + (77*b^3)
/3) + a*b^2 + a^2*b + a^3/3 + b^3/3 + tan(c/2 + (d*x)/2)^6*(69*a*b^2 + 15*
a^2*b + 3*a^3 + b^3))/(d*(8*tan(c/2 + (d*x)/2)^3 - 16*tan(c/2 + (d*x)/2)^5
 + 8*tan(c/2 + (d*x)/2)^7))
```

3.195 $\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$

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3.195.1 Optimal result

Integrand size = 21, antiderivative size = 279

$$\begin{aligned}\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = {}& \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} \\ &+ \frac{7b^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\ &- \frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} \\ &- \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot^3(c+dx)}{d} \\ &- \frac{a^3\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^5(c+dx)}{5d} \\ &- \frac{3a^2b\csc(c+dx)}{d} - \frac{7b^3\csc(c+dx)}{2d} \\ &- \frac{a^2b\csc^3(c+dx)}{d} - \frac{7b^3\csc^3(c+dx)}{6d} \\ &- \frac{3a^2b\csc^5(c+dx)}{5d} - \frac{7b^3\csc^5(c+dx)}{10d} \\ &+ \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}\end{aligned}$$

output

```
3*a^2*b*arctanh(sin(d*x+c))/d+7/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)
/d-9*a*b^2*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-3*a*b^2*cot(d*x+c)^3/d-1/5*
a^3*cot(d*x+c)^5/d-3/5*a*b^2*cot(d*x+c)^5/d-3*a^2*b*csc(d*x+c)/d-7/2*b^3*c
sc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-7/6*b^3*csc(d*x+c)^3/d-3/5*a^2*b*csc(d*x+
c)^5/d-7/10*b^3*csc(d*x+c)^5/d+1/2*b^3*csc(d*x+c)^5*sec(d*x+c)^2/d+3*a*b^2
*tan(d*x+c)/d
```

3.195.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 812 vs. $2(279) = 558$.

Time = 2.16 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.91

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(1176a^2b+412b^3+80a(5a^2+18b^2)\cos(c+dx)+66(6a^2b+7b^3)\cos\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/61440*(Csc[(c + d*x)/2]^9*Sec[(c + d*x)/2]^5*(1176*a^2*b + 412*b^3 + 80
*a*(5*a^2 + 18*b^2)*Cos[c + d*x] + 66*(6*a^2*b + 7*b^3)*Cos[2*(c + d*x)] +
 16*a^3*Cos[3*(c + d*x)] + 288*a*b^2*Cos[3*(c + d*x)] - 600*a^2*b*Cos[4*(c
 + d*x)] - 700*b^3*Cos[4*(c + d*x)] - 48*a^3*Cos[5*(c + d*x)] - 864*a*b^2*
Cos[5*(c + d*x)] + 180*a^2*b*Cos[6*(c + d*x)] + 210*b^3*Cos[6*(c + d*x)] +
 16*a^3*Cos[7*(c + d*x)] + 288*a*b^2*Cos[7*(c + d*x)] + 450*a^2*b*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 525*b^3*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 450*a^2*b*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]*Sin[c + d*x] - 525*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]*Sin[c + d*x] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
3*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c +
 d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)
] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 27
0*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 315*b^
3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 270*a^2*b*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 315*b^3*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 90*a^2*b*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[...
```

3.195.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4360$$

$$\int \csc^6(c+dx)\sec^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)dx$$

$$\downarrow 25$$

$$-\int -(b+a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)dx$$

$$\downarrow 25$$

$$\int \csc^6(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx-\frac{\pi}{2}\right)^3\cos\left(c+dx-\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\downarrow 3391$$

$$-\int \left(-a^3\sec^6\left(\frac{1}{2}(2c-\pi)+dx\right)-b^3\sec^3(c+dx)\sec^6\left(\frac{1}{2}(2c-\pi)+dx\right)-3ab^2\sec^2(c+dx)\sec^6\left(\frac{1}{2}(2c-\pi)+\right.\right.$$

$$\downarrow 2009$$

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot^5(c+dx)}{5d}-\frac{3ab^2\cot^3(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}+\frac{7b^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{7b^3\csc^5(c+dx)}{10d}-\frac{7b^3\csc^3(c+dx)}{6d}-\frac{7b^3\csc(c+dx)}{2d}+\frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) -
(a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(
3*d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*
Cot[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2
*d) - (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*C
sc[c + d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5
*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.195.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{a^3\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+$
default	$\frac{a^3\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+$
parallelrisch	$\frac{-46080(1+\cos(2dx+2c))b\left(a^2+\frac{7b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+46080(1+\cos(2dx+2c))b\left(a^2+\frac{7b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2$
norman	$\frac{-\frac{a^3+3a^2b+3ab^2+b^3}{160d}+\frac{\left(a^3-3a^2b+3ab^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{160d}+\frac{\left(19a^3-87a^2b+117ab^2-49b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{480d}-\frac{\left(19a^3+87a^2b+117$
risch	$-\frac{i\left(90a^2b\,e^{13i(dx+c)}+105b^3e^{13i(dx+c)}-300a^2b\,e^{11i(dx+c)}-350b^3e^{11i(dx+c)}+198a^2b\,e^{9i(dx+c)}+231b^3e^{9i(dx+c)}+160a^3\right.}{}$

input
```
int(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1
/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3
*a*b^2*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d
*x+c)/cos(d*x+c)-16/5*cot(d*x+c))+b^3*(-1/5/sin(d*x+c)^5/cos(d*x+c)^2-7/15
/sin(d*x+c)^3/cos(d*x+c)^2+7/6/sin(d*x+c)/cos(d*x+c)^2-7/2/sin(d*x+c)+7/2*
ln(sec(d*x+c)+tan(d*x+c))))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.27

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{32\left(a^3+18ab^2\right)\cos(dx+c)^7+30\left(6a^2b+7b^3\right)\cos(dx+c)^6-80\left(a^3+18ab^2\right)\cos(dx+c)^5-70\left(6a^2\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(32*(a^3 + 18*a*b^2)*cos(d*x + c)^7 + 30*(6*a^2*b + 7*b^3)*cos(d*x +
 c)^6 - 80*(a^3 + 18*a*b^2)*cos(d*x + c)^5 - 70*(6*a^2*b + 7*b^3)*cos(d*x
+ c)^4 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*
b^3 + 46*(6*a^2*b + 7*b^3)*cos(d*x + c)^2 - 15*((6*a^2*b + 7*b^3)*cos(d*x
+ c)^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x +
c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*((6*a^2*b + 7*b^3)*cos(d*x +
 c)^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c
)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x
 + c)^4 + d*cos(d*x + c)^2)*sin(d*x + c))
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{b^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log\left(\sin(dx+c)+1\right)+105\log\left(\sin(dx+c)-1\right)\right)}{\cdots}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(b^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2
- 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*l
og(sin(d*x + c) - 1)) + 6*a^2*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 +
 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1))
+ 36*a*b^2*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*
tan(d*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x
+ c)^5)/d
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.78

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 9a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 25a^3\ta}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/480*(3*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*a
*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a^3*tan(1/
2*d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 135*a*b^2*tan(1/2*d*
x + 1/2*c)^3 - 55*b^3*tan(1/2*d*x + 1/2*c)^3 + 150*a^3*tan(1/2*d*x + 1/2*c
) - 990*a^2*b*tan(1/2*d*x + 1/2*c) + 1710*a*b^2*tan(1/2*d*x + 1/2*c) - 870
*b^3*tan(1/2*d*x + 1/2*c) + 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 48
0*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*t
an(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
1)^2 - (150*a^3*tan(1/2*d*x + 1/2*c)^4 + 990*a^2*b*tan(1/2*d*x + 1/2*c)^4
+ 1710*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 870*b^3*tan(1/2*d*x + 1/2*c)^4 + 25*
a^3*tan(1/2*d*x + 1/2*c)^2 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 135*a*b^2*
tan(1/2*d*x + 1/2*c)^2 + 55*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3 + 9*a^2*b +
 9*a*b^2 + 3*b^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.195.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.30

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^3}{160\,d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{19a^3}{15}+\frac{29a^2b}{5}+\frac{39ab^2}{5}+\frac{49b^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(10a^3+66a^2b+306ab^2+26b^3)-\tan\left(}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\right.}$$

$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21ab^2}{16}-\frac{3a^2b}{8}-\frac{a^3}{16}-\frac{7b^3}{8}+\frac{3(a-b)^2(a-4b)}{16}+\frac{3(a-b)^3}{16}\right)}{d}$$

$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{(a-b)^2(a-4b)}{48}+\frac{(a-b)^3}{32}\right)}{d}-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(a^2b\,6\mathrm{i}+b^3\,7\mathrm{i})\,1\mathrm{i}}{d}$$

input
```
int((a + b/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a - b)^3)/(160*d) - (tan(c/2 + (d*x)/2)^2*((39*a*b^
2)/5 + (29*a^2*b)/5 + (19*a^3)/15 + (49*b^3)/15) + tan(c/2 + (d*x)/2)^8*(3
06*a*b^2 + 66*a^2*b + 10*a^3 + 26*b^3) - tan(c/2 + (d*x)/2)^6*(411*a*b^2 +
 125*a^2*b + (55*a^3)/3 + (433*b^3)/3) + tan(c/2 + (d*x)/2)^4*((483*a*b^2)
/5 + (263*a^2*b)/5 + (103*a^3)/15 + (763*b^3)/15) + (3*a*b^2)/5 + (3*a^2*b
)/5 + a^3/5 + b^3/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 64*tan(c/2 + (d*x)/2)^7
 + 32*tan(c/2 + (d*x)/2)^9)) - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*
7i)*1i)/d + (tan(c/2 + (d*x)/2)*((21*a*b^2)/16 - (3*a^2*b)/8 - a^3/16 - (7
*b^3)/8 + (3*(a - b)^2*(a - 4*b))/16 + (3*(a - b)^3)/16))/d + (tan(c/2 + (
d*x)/2)^3*(((a - b)^2*(a - 4*b))/48 + (a - b)^3/32))/d
```

3.196 $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.196.2 Mathematica [A] (verified) 1445
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3.196.1 Optimal result

Integrand size = 21, antiderivative size = 223

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = &-\frac{(a^2-b^2)^3\cos(c+dx)}{a^7 d} - \frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6 d} \\ &+ \frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5 d} + \frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4 d} \\ &- \frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3 d} - \frac{b\cos^6(c+dx)}{6a^2 d} \\ &+ \frac{\cos^7(c+dx)}{7ad} + \frac{b(a^2-b^2)^3\log(b+a\cos(c+dx))}{a^8 d}\end{aligned}$$

output

```
-(a^2-b^2)^3*cos(d*x+c)/a^7/d-1/2*b*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^2/a^6
/d+1/3*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^3/a^5/d+1/4*b*(3*a^2-b^2)*cos(d*x+
c)^4/a^4/d-1/5*(3*a^2-b^2)*cos(d*x+c)^5/a^3/d-1/6*b*cos(d*x+c)^6/a^2/d+1/7
*cos(d*x+c)^7/a/d+b*(a^2-b^2)^3*ln(b+a*cos(d*x+c))/a^8/d
```

3.196.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-105a(35a^6-152a^4b^2+176a^2b^4-64b^6)\cos(c+dx)-105(29a^6b-40a^4b^3+16a^2b^5)\cos(2(c+dx))+7$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

output
```
(-105*a*(35*a^6 - 152*a^4*b^2 + 176*a^2*b^4 - 64*b^6)*Cos[c + d*x] - 105*(
29*a^6*b - 40*a^4*b^3 + 16*a^2*b^5)*Cos[2*(c + d*x)] + 735*a^7*Cos[3*(c +
d*x)] - 1260*a^5*b^2*Cos[3*(c + d*x)] + 560*a^3*b^4*Cos[3*(c + d*x)] + 420
*a^6*b*Cos[4*(c + d*x)] - 210*a^4*b^3*Cos[4*(c + d*x)] - 147*a^7*Cos[5*(c
+ d*x)] + 84*a^5*b^2*Cos[5*(c + d*x)] - 35*a^6*b*Cos[6*(c + d*x)] + 15*a^7
*Cos[7*(c + d*x)] + 6720*a^6*b*Log[b + a*Cos[c + d*x]] - 20160*a^4*b^3*Log
[b + a*Cos[c + d*x]] + 20160*a^2*b^5*Log[b + a*Cos[c + d*x]] - 6720*b^7*Lo
g[b + a*Cos[c + d*x]])/(6720*a^8*d)
```

3.196.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^7(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$
$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^7(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^7(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^7}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a\cos(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^8d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-a^6\cos^6(c+dx)+a^5b\cos^5(c+dx)+a^4\left(3a^2-b^2\right)\cos^4(c+dx)+a^3b\left(b^2-3a^2\right)\cos^3(c+dx)-a^2\left(3a^4-3\right.\right.}{a^8d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{1}{6}a^6b\cos^6(c+dx)+a\left(a^2-b^2\right)^3\cos(c+dx)-b\left(a^2-b^2\right)^3\log(a\cos(c+dx)+b)+\frac{1}{5}a^5(3a$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(a^2 - b^2)^3*Cos[c + d*x] + (a^2*b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[c +
 d*x]^2)/2 - (a^3*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/3 - (a^4*b*(3*
a^2 - b^2)*Cos[c + d*x]^4)/4 + (a^5*(3*a^2 - b^2)*Cos[c + d*x]^5)/5 + (a^6
*b*Cos[c + d*x]^6)/6 - (a^7*Cos[c + d*x]^7)/7 - b*(a^2 - b^2)^3*Log[b + a*
Cos[c + d*x]])/(a^8*d))
```

3.196. $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}dx$

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.196.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\cos(dx+c)^7 a^6}{7} - \frac{b\cos(dx+c)^6 a^5}{6} - \frac{3a^6\cos(dx+c)^5}{5} + \frac{a^4b^2\cos(dx+c)^5}{5} + \frac{3\cos(dx+c)^4 a^5 b}{4} - \frac{\cos(dx+c)^4 a^3 b^3}{4} + \cos(dx+c)^3 a^6 - \cos(dx+$
default	$\frac{\cos(dx+c)^7 a^6}{7} - \frac{b\cos(dx+c)^6 a^5}{6} - \frac{3a^6\cos(dx+c)^5}{5} + \frac{a^4b^2\cos(dx+c)^5}{5} + \frac{3\cos(dx+c)^4 a^5 b}{4} - \frac{\cos(dx+c)^4 a^3 b^3}{4} + \cos(dx+c)^3 a^6 - \cos(dx+$
parallelrisch	$\frac{320b(a-b)^3(a+b)^3\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right) - 320b(a-b)^3(a+b)^3\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right) - 7\left(\frac{5\left(29a^5b-40a^3b^3+16ab^5\right)}{7}\right.}{}$
norman	$\frac{\left(2a^5b+2a^4b^2-4a^3b^3-4a^2b^4+2ab^5+2b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{a^7d} + \frac{\left(14a^5b+16a^4b^2-24a^3b^3-28a^2b^4+10ab^5+12b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{a^7d} + \frac{-96}{}$
risch	$-\frac{7\cos(5dx+5c)}{320da} - \frac{b\cos(6dx+6c)}{192da^2} + \frac{7\cos(3dx+3c)}{64ad} + \frac{\cos(7dx+7c)}{448ad} + \frac{\cos(5dx+5c)b^2}{80da^3} + \frac{b\cos(4dx+4c)}{16da^2} - \frac{b^3c}{}$

input
```
int(sin(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^7*(1/7*cos(d*x+c)^7*a^6-1/6*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)
^5+1/5*a^4*b^2*cos(d*x+c)^5+3/4*cos(d*x+c)^4*a^5*b-1/4*cos(d*x+c)^4*a^3*b^
3+cos(d*x+c)^3*a^6-cos(d*x+c)^3*a^4*b^2+1/3*cos(d*x+c)^3*a^2*b^4-3/2*cos(d
*x+c)^2*a^5*b+3/2*cos(d*x+c)^2*a^3*b^3-1/2*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a
^6+3*cos(d*x+c)*a^4*b^2-3*cos(d*x+c)*a^2*b^4+b^6*cos(d*x+c))+b*(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/a^8*ln(b+a*cos(d*x+c)))
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{60\,a^7\cos(dx+c)^7 - 70\,a^6b\cos(dx+c)^6 - 84\,(3\,a^7 - a^5b^2)\cos(dx+c)^5 + 105\,(3\,a^6b - a^4b^3)\cos(dx+c)}{}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/420*(60*a^7*cos(d*x + c)^7 - 70*a^6*b*cos(d*x + c)^6 - 84*(3*a^7 - a^5*b
^2)*cos(d*x + c)^5 + 105*(3*a^6*b - a^4*b^3)*cos(d*x + c)^4 + 140*(3*a^7 -
 3*a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 - 210*(3*a^6*b - 3*a^4*b^3 + a^2*b^5)
*cos(d*x + c)^2 - 420*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c) +
 420*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b))/(a^8*d
)
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{60\,a^6\cos(dx+c)^7 - 70\,a^5 b\cos(dx+c)^6 - 84\,(3\,a^6 - a^4b^2)\cos(dx+c)^5 + 105\,(3\,a^5 b - a^3b^3)\cos(dx+c)^4 + 140\,(3\,a^6 - 3\,a^4b^2 + a^2b^4)\cos(dx+c)^3 - 210\,(3\,a$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/420*((60*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 84*(3*a^6 - a^4*
b^2)*cos(d*x + c)^5 + 105*(3*a^5*b - a^3*b^3)*cos(d*x + c)^4 + 140*(3*a^6
 - 3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(d*x + c)^2 - 420*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c))/a^7
 + 420*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b)/a^8)/
d
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. 2(211) = 422.

Time = 0.36 (sec) , antiderivative size = 1559, normalized size of antiderivative = 6.99

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/420*(420*(a^7*b - a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 - 3*a^2*b^
6 - a*b^7 + b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 420*(a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 1))/a^8 + (384*a^7 - 1089*a^6*b - 1848*a^5*b^2 + 3267*a^4*b^3 + 2240*a^3
*b^4 - 3267*a^2*b^5 - 840*a*b^6 + 1089*b^7 - 2688*a^7*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120
96*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24549*a^4*b^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 14000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 23709*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5040*a*b^
6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7623*b^7*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 8064*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2874
9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 32088*a^5*b^2*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 78687*a^4*b^3*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 35280*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 - 72807*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a*b^6*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 22869*b^7*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 13440*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 40320*a^5*b^2*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 136185*a^4*b^3*(cos(d*x + c...
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.12

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)^3\left(\frac{1}{a}-\frac{b^2\left(\frac{1}{a}-\frac{b^2}{3a^3}\right)}{a^2}\right)-\cos(c+dx)^5\left(\frac{3}{5a}-\frac{b^2}{5a^3}\right)-\cos(c+dx)\left(\frac{1}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)}{a^2}\right)+\frac{9}{}}{}$$

input
```
int(sin(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

output
```
(cos(c + d*x)^3*(1/a - (b^2*(1/a - b^2/(3*a^3)))/a^2) - cos(c + d*x)^5*(3/
(5*a) - b^2/(5*a^3)) - cos(c + d*x)*(1/a - (b^2*(3/a - (b^2*(3/a - b^2/a^3
))/a^2))/a^2) + cos(c + d*x)^7/(7*a) + (log(b + a*cos(c + d*x))*(a^6*b - b
^7 + 3*a^2*b^5 - 3*a^4*b^3))/a^8 - (b*cos(c + d*x)^6)/(6*a^2) - (b*cos(c +
 d*x)^2*(3/a - (b^2*(3/a - b^2/a^3))/a^2))/(2*a) + (b*cos(c + d*x)^4*(3/a
- b^2/a^3))/(4*a))/d
```

3.197 $\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$

3.197.1 Optimal result 1452
3.197.2 Mathematica [A] (verified) 1452
3.197.3 Rubi [A] (verified) 1453
3.197.4 Maple [A] (verified) 1455
3.197.5 Fricas [A] (verification not implemented) 1456
3.197.6 Sympy [F(-1)] 1456
3.197.7 Maxima [A] (verification not implemented) 1457
3.197.8 Giac [B] (verification not implemented) 1457
3.197.9 Mupad [B] (verification not implemented) 1458

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^2\cos(c+dx)}{a^5 d} - \frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4 d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3 d} + \frac{b\cos^4(c+dx)}{4a^2 d} - \frac{\cos^5(c+dx)}{5ad} + \frac{b(a^2-b^2)^2\log(b+a\cos(c+dx))}{a^6 d}$$

output
```
-(a^2-b^2)^2*cos(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*cos(d*x+c)^2/a^4/d+1/3*(2*
a^2-b^2)*cos(d*x+c)^3/a^3/d+1/4*b*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a/d+
b*(a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^6/d
```

3.197.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-60a(5a^4-14a^2b^2+8b^4)\cos(c+dx) - 60(3a^4b-2a^2b^3)\cos(2(c+dx)) + 50a^5\cos(3(c+dx)) - 40a^3b^2$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
(-60*a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x] - 60*(3*a^4*b - 2*a^2*b^3
)*Cos[2*(c + d*x)] + 50*a^5*Cos[3*(c + d*x)] - 40*a^3*b^2*Cos[3*(c + d*x)]
 + 15*a^4*b*Cos[4*(c + d*x)] - 6*a^5*Cos[5*(c + d*x)] + 480*a^4*b*Log[b +
a*Cos[c + d*x]] - 960*a^2*b^3*Log[b + a*Cos[c + d*x]] + 480*b^5*Log[b + a*
Cos[c + d*x]])/(480*a^6*d)
```

3.197.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^5(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos(c+dx)\sin^5(c+dx)}{b+a\cos(c+dx)}dx$$
$$\downarrow 25$$
$$\int \frac{\sin^5(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$
$$\downarrow 3042$$
$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^5}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$
$$\downarrow 25$$
$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a\cos(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^6 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(a^4\cos^4(c+dx)-a^3 b\cos^3(c+dx)-a^2\left(2a^2-b^2\right)\cos^2(c+dx)-ab\left(b^2-2a^2\right)\cos(c+dx)+\left(a^2-b^2\right)^2-\frac{}{b-}\right.}{a^6 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)-\frac{1}{4}a^4 b\cos^4(c+dx)+\frac{1}{2}a^2 b\left(2a^2-b^2\right)\cos^2(c+dx)+a\left(a^2-b^2\right)^2\cos(c+dx)-b\left(a^2-b^2\right)^2\log}{a^6 d}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(a^2 - b^2)^2*Cos[c + d*x] + (a^2*b*(2*a^2 - b^2)*Cos[c + d*x]^2)/2 -
 (a^3*(2*a^2 - b^2)*Cos[c + d*x]^3)/3 - (a^4*b*Cos[c + d*x]^4)/4 + (a^5*Co
s[c + d*x]^5)/5 - b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^6*d))
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.197.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+\frac{\cos(dx+c)^3a^2b^2}{3}+\cos(dx+c)^2a^3b-\frac{\cos(dx+c)^2ab^3}{2}+\cos(dx+c)a^4-2\cos(dx+c$
default	$\frac{-\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+\frac{\cos(dx+c)^3a^2b^2}{3}+\cos(dx+c)^2a^3b-\frac{\cos(dx+c)^2ab^3}{2}+\cos(dx+c)a^4-2\cos(dx+c$
parallelrisch	$480b(a-b)^2(a+b)^2\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-480b(a-b)^2(a+b)^2\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-6\Big(\left(30a^3b-20ab^3\right)\cos(2dx$
norman	$\frac{\left(2a^3b+2a^2b^2-2ab^3-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da^5}+\frac{-16a^4+50a^2b^2-30b^4}{15da^5}+\frac{2\left(5a^3b+6a^2b^2-3ab^3-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da^5}+\frac{\left(-16a^4+6a^3b+44\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$-\frac{\cos(5dx+5c)}{80da}-\frac{5e^{-i(dx+c)}}{16ad}-\frac{2ib^5c}{a^6d}+\frac{4ib^3c}{a^4d}-\frac{ixb^5}{a^6}+\frac{5\cos(3dx+3c)}{48ad}-\frac{2ibc}{a^2d}-\frac{ixb}{a^2}-\frac{5e^{i(dx+c)}}{16da}+\frac{b\ln}{}$

input
```
int(sin(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^5*(1/5*cos(d*x+c)^5*a^4-1/4*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*
a^4+1/3*cos(d*x+c)^3*a^2*b^2+cos(d*x+c)^2*a^3*b-1/2*cos(d*x+c)^2*a*b^3+cos
(d*x+c)*a^4-2*cos(d*x+c)*a^2*b^2+cos(d*x+c)*b^4)+b*(a^4-2*a^2*b^2+b^4)/a^6
*ln(b+a*cos(d*x+c)))
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{12\,a^5\cos(dx+c)^5 - 15\,a^4b\cos(dx+c)^4 - 20\,(2\,a^5 - a^3b^2)\cos(dx+c)^3 + 30\,(2\,a^4b - a^2b^3)\cos(dx+c)}{60\,a^6d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/60*(12*a^5*cos(d*x + c)^5 - 15*a^4*b*cos(d*x + c)^4 - 20*(2*a^5 - a^3*b
^2)*cos(d*x + c)^3 + 30*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2 + 60*(a^5 - 2*a
^3*b^2 + a*b^4)*cos(d*x + c) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x
+ c) + b))/(a^6*d)
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{12a^4\cos(dx+c)^5-15a^3b\cos(dx+c)^4-20\left(2a^4-a^2b^2\right)\cos(dx+c)^3+30\left(2a^3b-ab^3\right)\cos(dx+c)^2+60\left(a^4-2a^2b^2+b^4\right)\cos(dx+c)}{a^5} - \frac{60\left(a^4b-2}{}}{60\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/60*((12*a^4*cos(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 20*(2*a^4 - a^2*
b^2)*cos(d*x + c)^3 + 30*(2*a^3*b - a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 2*a^
2*b^2 + b^4)*cos(d*x + c))/a^5 - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*
x + c) + b)/a^6)/d
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(144) = 288$.

Time = 0.33 (sec) , antiderivative size = 867, normalized size of antiderivative = 5.70

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output

```
1/60*(60*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*log(abs(a
 + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1)))/(a^7 - a^6*b) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^6 + (64*a^5 - 137*a^4*b - 200*a
^3*b^2 + 274*a^2*b^3 + 120*a*b^4 - 137*b^5 - 320*a^5*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 880*a
^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1490*a^2*b^3*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 480*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 685*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^5*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 1280*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3100*a^2
*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 720*a*b^4*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 1370*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1970*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^3*b^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 3100*a^2*b^3*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 - 480*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 1370*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 120*a^3*b^2*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 1490*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1
)^4 + 120*a*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 685*b^5*(co...
```

3.197.9 Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(c+dx)\left(\frac{1}{a}-\frac{b^2\left(\frac{2}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)-\cos(c+dx)^3\left(\frac{2}{3a}-\frac{b^2}{3a^3}\right)+\frac{\cos(c+dx)^5}{5a}-\frac{b\cos(c+dx)^4}{4a^2}-\frac{\ln(b+a\cos(c+dx))\,(a}{a^6}}{d}$$

input

```
int(sin(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

output

```
-(cos(c + d*x)*(1/a - (b^2*(2/a - b^2/a^3))/a^2) - cos(c + d*x)^3*(2/(3*a)
 - b^2/(3*a^3)) + cos(c + d*x)^5/(5*a) - (b*cos(c + d*x)^4)/(4*a^2) - (log
(b + a*cos(c + d*x))*(a^4*b + b^5 - 2*a^2*b^3))/a^6 + (b*cos(c + d*x)^2*(2
/a - b^2/a^3))/(2*a))/d
```

3.198 $\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.198.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)\cos(c+dx)}{a^3 d} - \frac{b\cos^2(c+dx)}{2a^2 d} + \frac{\cos^3(c+dx)}{3ad} + \frac{b(a^2-b^2)\log(b+a\cos(c+dx))}{a^4 d}$$

output
```
-(a^2-b^2)*cos(d*x+c)/a^3/d-1/2*b*cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a/d+
b*(a^2-b^2)*ln(b+a*cos(d*x+c))/a^4/d
```

3.198.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(-9a^3+12ab^2)\cos(c+dx) - 3a^2 b\cos(2(c+dx)) + a^3\cos(3(c+dx)) + 12a^2 b\log(b+a\cos(c+dx)) - 12}{12a^4 d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
((-9*a^3 + 12*a*b^2)*Cos[c + d*x] - 3*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(
c + d*x)] + 12*a^2*b*Log[b + a*Cos[c + d*x]] - 12*b^3*Log[b + a*Cos[c + d*
x]])/(12*a^4*d)
```

3.198.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^3(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^3(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^3(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^3}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos(c+dx)(a^2-a^2\cos^2(c+dx))}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a\cos(c+dx)(a^2-a^2\cos^2(c+dx))}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+\left(1-\frac{b^2}{a^2}\right)a^2+b\cos(c+dx)a+\frac{b^3-a^2b}{b+a\cos(c+dx)}\right)d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a\left(a^2-b^2\right)\cos(c+dx)-b\left(a^2-b^2\right)\log(a\cos(c+dx)+b)+\frac{1}{2}a^2b\cos^2(c+dx)}{a^4d}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(a^2 - b^2)*Cos[c + d*x] + (a^2*b*Cos[c + d*x]^2)/2 - (a^3*Cos[c + d*
x]^3)/3 - b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d))
```

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.198.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+\cos(dx+c)b^2}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
default	$\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+\cos(dx+c)b^2}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
parallelrisch	$\frac{-9a^3\cos(dx+c)+12\cos(dx+c)ab^2+a^3\cos(3dx+3c)-3a^2b\cos(2dx+2c)-12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2b+12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$
norman	$\frac{\left(4a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a^3d}+\frac{-2ab+2b^2}{3da^3}+\frac{\left(4a^2-2ab-4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3da^3}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{(a+b)b(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b\right.}{a^4d}$
risch	$-\frac{ixb}{a^2}+\frac{ixb^3}{a^4}-\frac{b\mathrm{e}^{2i(dx+c)}}{8a^2d}-\frac{3\mathrm{e}^{i(dx+c)}}{8da}+\frac{\mathrm{e}^{i(dx+c)}b^2}{2a^3d}-\frac{3\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{\mathrm{e}^{-i(dx+c)}b^2}{2a^3d}-\frac{b\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{2ibc}{a^2d}$

input
```
int(sin(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*(1/3*cos(d*x+c)^3*a^2-1/2*a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+cos(d
*x+c)*b^2)+b*(a^2-b^2)/a^4*ln(b+a*cos(d*x+c)))
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\,a^3\cos(dx+c)^3 - 3\,a^2b\cos(dx+c)^2 - 6\,(a^3-ab^2)\cos(dx+c) + 6\,(a^2b-b^3)\log(a\cos(dx+c)+b)}{6\,a^4d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*cos(d
*x + c) + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b))/(a^4*d)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,a^2\cos(dx+c)^3 - 3\,ab\cos(dx+c)^2 - 6\,(a^2-b^2)\cos(dx+c)}{a^3} + \frac{6\,(a^2b-b^3)\log(a\cos(dx+c)+b)}{a^4}}{6\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*((2*a^2*cos(d*x + c)^3 - 3*a*b*cos(d*x + c)^2 - 6*(a^2 - b^2)*cos(d*x
+ c))/a^3 + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b)/a^4)/d
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= \frac{(a^2b-b^3)\log\left(\left|-a\cos(dx+c)-b\right|\right)}{a^4d} \\
&\quad + \frac{2\,a^2d^2\cos(dx+c)^3 - 3\,abd^2\cos(dx+c)^2 - 6\,a^2d^2\cos(dx+c) + 6\,b^2d^2\cos(dx+c)}{6\,a^3d^3}
\end{aligned}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a^2*b - b^3)*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/6*(2*a^2*d^2*cos(d
*x + c)^3 - 3*a*b*d^2*cos(d*x + c)^2 - 6*a^2*d^2*cos(d*x + c) + 6*b^2*d^2*
cos(d*x + c))/(a^3*d^3)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(c+d\,x)\left(\frac{1}{a}-\frac{b^2}{a^3}\right)-\frac{\cos(c+d\,x)^3}{3\,a}+\frac{b\cos(c+d\,x)^2}{2\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\,(a^2\,b-b^3)}{a^4}}{d}$$

input
```
int(sin(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

output
```
-(cos(c + d*x)*(1/a - b^2/a^3) - cos(c + d*x)^3/(3*a) + (b*cos(c + d*x)^2)
/(2*a^2) - (log(b + a*cos(c + d*x))*(a^2*b - b^3))/a^4)/d
```

3.199 $\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.199.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(c+dx)}{ad} + \frac{b\log(b+a\cos(c+dx))}{a^2 d}$$

output
```
-cos(d*x+c)/a/d+b*ln(b+a*cos(d*x+c))/a^2/d
```

3.199.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-a\cos(c+dx) + b\log(b+a\cos(c+dx))}{a^2 d}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-(a*Cos[c + d*x]) + b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

3.199.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\sin(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}{b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a\cos(c+dx)}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 49$$

$$-\frac{\int \left(1-\frac{b}{b+a\cos(c+dx)}\right) d(a\cos(c+dx))}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{a\cos(c+dx)-b\log(a\cos(c+dx)+b)}{a^2 d}$$

input
```
Int[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x] - b*Log[b + a*Cos[c + d*x]])/(a^2*d))
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.199.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
default	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
parallelrisch	$\frac{b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-a(\cos(dx+c)+1)}{a^2d}$	59
risch	$-\frac{ixb}{a^2}-\frac{e^{i(dx+c)}}{2da}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2ibc}{a^2d}+\frac{b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^2d}$	90
norman	$-\frac{2}{da\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{a^2d}-\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^2d}$	91

input
```
int(sin(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/sec(d*x+c)-1/a^2*b*ln(sec(d*x+c))+1/a^2*b*ln(a+b*sec(d*x+c)))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a\cos(dx+c)-b\log(a\cos(dx+c)+b)}{a^2d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c) - b*log(a*cos(d*x + c) + b))/(a^2*d)
```

3.199.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{\cos(dx+c)}{a} - \frac{b\log(a\cos(dx+c)+b)}{a^2}}{d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(cos(d*x + c)/a - b*log(a*cos(d*x + c) + b)/a^2)/d
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(dx+c)}{ad} + \frac{b\log\left(|-a\cos(dx+c)-b|\right)}{a^2 d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a*d) + b*log(abs(-a*cos(d*x + c) - b))/(a^2*d)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)-a\,\cos\left(c+d\,x\right)}{a^2\,d}$$

input
```
int(sin(c + d*x)/(a + b/cos(c + d*x)),x)
```

output
```
(b*log(b + a*cos(c + d*x)) - a*cos(c + d*x))/(a^2*d)
```

3.200 $\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.200.9 Mupad [B] (verification not implemented) 1476

3.200.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x
+c))/(a^2-b^2)/d
```

3.200.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right) + b\log(b+a\cos(c+dx)) + (a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)(a+b)d}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Lo
g[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

3.200.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4359$$

$$\int -\frac{\cot(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)}{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3200$$

$$-\frac{\int \frac{a\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\downarrow 587$$

$$-\frac{\frac{\int \frac{a^2-ab\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2}-\frac{b\int \frac{1}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^2-b^2}}{d}$$

$$\downarrow 16$$

$$-\frac{\frac{\int \frac{a^2-ab\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2} - \frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\downarrow 452$$

$$-\frac{\frac{a^2\int \frac{1}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))-b\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2} - \frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{a\operatorname{arctanh}(\cos(c+dx))-b\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2} - \frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\downarrow 240$$

$$-\frac{\frac{\frac{1}{2}b\log(a^2-a^2\cos^2(c+dx))+a\operatorname{arctanh}(\cos(c+dx))}{a^2-b^2} - \frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

input
```
Int[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-((-((b*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)) + (a*ArcTanh[Cos[c + d*x]] +
 (b*Log[a^2 - a^2*Cos[c + d*x]^2])/2)/(a^2 - b^2))/d)
```

3.200.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.200.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a-b)}{d(a^2-b^2)}$
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2a-2b}+\frac{b\ln(b+a\cos(dx+c))}{(a-b)(a+b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2a-2b}+\frac{b\ln(b+a\cos(dx+c))}{(a-b)(a+b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a+b)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{d(a^2-b^2)}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ibx}{a^2-b^2}-\frac{2ibc}{d(a^2-b^2)}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{d(a-b)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d(a+b)}+\frac{b\ln\left(e^{2i(dx+$

input
```
int(csc(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(b*ln(-2*a+sec(1/2*d*x+1/2*c)^2*(a-b))+ln(tan(1/2*d*x+1/2*c))*(a-b))/d/(a^
2-b^2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (
a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

3.200.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,b\log(a\cos(dx+c)+b)}{a^2-b^2} - \frac{\log(\cos(dx+c)+1)}{a-b} + \frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*b*log(a*cos(d*x + c) + b)/(a^2 - b^2) - log(cos(d*x + c) + 1)/(a -
b) + log(cos(d*x + c) - 1)/(a + b))/d
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) +
1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.200.9 Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln(\cos(c+d\,x)-1)}{2\,d\,(a+b)} - \frac{\ln(\cos(c+d\,x)+1)}{2\,d\,(a-b)} + \frac{b\,\ln(b+a\,\cos(c+d\,x))}{d\,(a^2-b^2)}$$

input
```
int(1/(sin(c + d*x)*(a + b/cos(c + d*x))),x)
```

output
```
log(cos(c + d*x) - 1)/(2*d*(a + b)) - log(cos(c + d*x) + 1)/(2*d*(a - b))
+ (b*log(b + a*cos(c + d*x)))/(d*(a^2 - b^2))
```

3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.201.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{a\log(1-\cos(c+dx))}{4(a+b)^2 d} - \frac{a\log(1+\cos(c+dx))}{4(a-b)^2 d} + \frac{a^2 b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^2 d}$$

output
```
1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d+1/4*a*ln(1-cos(d*x+c))/(a+b)
^2/d-1/4*a*ln(1+cos(d*x+c))/(a-b)^2/d+a^2*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2
/d
```

3.201.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right) - 4a\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2ab\log(b+a\cos(c+dx)) - (a-\right.}{8(a-b)^2(a+b)^2 d}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(-((a - b)^2*(a + b)*Csc[(c + d*x)/2]^2) - 4*a*((a + b)^2*Log[Cos[(c + d*x
)/2]] - 2*a*b*Log[b + a*Cos[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]]) +
 (a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*(a - b)^2*(a + b)^2*d)
```

3.201.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^2(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^2(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3316$$

$$\frac{a^3 \int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^2}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^3 \int \frac{\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^2}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^2 \int \frac{a\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^2}\,d(a\cos(c+dx))}{d}$$

$$\downarrow 593$$

$$-\frac{a^2\left(\frac{\int -\frac{b-a\cos(c+dx)}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)}\,d(a\cos(c+dx))}{2(a^2-b^2)} - \frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\left(-\frac{\int \frac{b-a\cos(c+dx)}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)}\,d(a\cos(c+dx))}{2(a^2-b^2)} - \frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 657$$

$$-\frac{a^2\left(-\frac{\int \left(\frac{-a-b}{2a(a-b)(\cos(c+dx)a+a)}+\frac{b-a}{2a(a+b)(a-a\cos(c+dx))}+\frac{2b}{(a-b)(a+b)(b+a\cos(c+dx))}\right)d(a\cos(c+dx))}{2(a^2-b^2)} - \frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))} - \frac{\frac{2b\log(a\cos(c+dx)+b)}{a^2-b^2}+\frac{(a-b)\log(a-a\cos(c+dx))}{2a(a+b)}-\frac{(a+b)\log(a\cos(c+dx)+a)}{2a(a-b)}}{2(a^2-b^2)}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
-((a^2*(-1/2*(b - a*Cos[c + d*x])/((a^2 - b^2)*(a^2 - a^2*Cos[c + d*x]^2))
 - (((a - b)*Log[a - a*Cos[c + d*x]])/(2*a*(a + b)) - ((a + b)*Log[a + a*C
os[c + d*x]])/(2*a*(a - b)) + (2*b*Log[b + a*Cos[c + d*x]])/(a^2 - b^2))/(
2*(a^2 - b^2))))/d)
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.201.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{a\ln(\cos(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{a\ln(\cos(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
parallelrisch	$\frac{8\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)a^2b-(a-b)\left(-4a(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a+b)\left((a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2(\right.\right.}{8d(a-b)^2(a+b)^2}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d(a-b)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{a^2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{d(a^4-2a^2b^2+b^4)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^2+2ab+b^2)}$
risch	$-\frac{iax}{2(a^2+2ab+b^2)}-\frac{iac}{2d(a^2+2ab+b^2)}+\frac{iax}{2a^2-4ab+2b^2}+\frac{iac}{2d(a^2-2ab+b^2)}-\frac{2ia^2bx}{a^4-2a^2b^2+b^4}-\frac{2ia^2bc}{d(a^4-2a^2b^2+b^4)}$

input
```
int(csc(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(4*a+4*b)/(cos(d*x+c)-1)+1/4*a/(a+b)^2*ln(cos(d*x+c)-1)+1/(4*a-4*b)
/(cos(d*x+c)+1)-1/4*a/(a-b)^2*ln(cos(d*x+c)+1)+b*a^2/(a+b)^2/(a-b)^2*ln(b+
a*cos(d*x+c)))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.86

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos(dx+c)-4\left(a^2b\cos(dx+c)^2-a^2b\right)\log(a\cos(dx+c)+b)-\left(a^3+2\,a\right.}{4\left((a^4\right.}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(d*x + c) - 4*(a^2*b*cos(d*x +
c)^2 - a^2*b)*log(a*cos(d*x + c) + b) - (a^3 + 2*a^2*b + a*b^2 - (a^3 + 2*
a^2*b + a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 2*a^2*
b + a*b^2 - (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 - 2*a^2*b^2 + b^4
)*d)
```

3.201.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{4a^2b\log(a\cos(dx+c)+b)}{a^4-2a^2b^2+b^4} - \frac{a\log(\cos(dx+c)+1)}{a^2-2ab+b^2} + \frac{a\log(\cos(dx+c)-1)}{a^2+2ab+b^2} + \frac{2(a\cos(dx+c)-b)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(4*a^2*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) - a*log(cos(d
*x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(cos(d*x + c) - 1)/(a^2 + 2*a*b +
b^2) + 2*(a*cos(d*x + c) - b)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.74

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{8a^2b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2a^2b^2+b^4}+\frac{2a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2}+\frac{\left(a+b-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^2+2ab+b^2)(\cos(dx+c)-1)}-\frac{\cos(dx+c}{(a-b)(\cos(dx}}{8d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*a^2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + 2*a*log(
abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2
*a*b + b^2)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a - b)*(cos(d*x + c
) + 1)))/d
```

3.201.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{4\,d\,{\left(a+b\right)}^2} - \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{a}{4\,{\left(a+b\right)}^2}-\frac{a}{4\,{\left(a-b\right)}^2}\right)}{d} - \frac{\frac{b}{2\,\left(a^2-b^2\right)}-\frac{a\,\cos(c+d\,x)}{2\,\left(a^2-b^2\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2-1\right)} - \frac{a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{4\,d\,{\left(a-b\right)}^2}$$

input
```
int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

output
```
(a*log(cos(c + d*x) - 1))/(4*d*(a + b)^2) - (log(b + a*cos(c + d*x))*(a/(4
*(a + b)^2) - a/(4*(a - b)^2)))/d - (b/(2*(a^2 - b^2)) - (a*cos(c + d*x))/
(2*(a^2 - b^2)))/(d*(cos(c + d*x)^2 - 1)) - (a*log(cos(c + d*x) + 1))/(4*d
*(a - b)^2)
```

3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.202.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = &\frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d}\\ &+\frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d}\\ &+\frac{a(3a+b)\log(1-\cos(c+dx))}{16(a+b)^3 d}\\ &-\frac{a(3a-b)\log(1+\cos(c+dx))}{16(a-b)^3 d}+\frac{a^4 b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^3 d}\end{aligned}$$

output

```
1/8*(4*a^2*b-a*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^2/d+1/4*(b-a
*cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)/d+1/16*a*(3*a+b)*ln(1-cos(d*x+c))/(a+b
)^3/d-1/16*a*(3*a-b)*ln(1+cos(d*x+c))/(a-b)^3/d+a^4*b*ln(b+a*cos(d*x+c))/(
a^2-b^2)^3/d
```

3.202.2 Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-2(a-b)^3\left(3a^2+4ab+b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-(a-b)^3(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)+8a\left(-\left((3a-b)(a+b)^3\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*(a - b)^3*(3*a^2 + 4*a*b + b^2)*Csc[(c + d*x)/2]^2 - (a - b)^3*(a + b)
^2*Csc[(c + d*x)/2]^4 + 8*a*(-((3*a - b)*(a + b)^3*Log[Cos[(c + d*x)/2]])
+ 8*a^3*b*Log[b + a*Cos[c + d*x]] + (a - b)^3*(3*a + b)*Log[Sin[(c + d*x)/
2]]) + 2*(a + b)^3*(3*a^2 - 4*a*b + b^2)*Sec[(c + d*x)/2]^2 + (a - b)^2*(a
 + b)^3*Sec[(c + d*x)/2]^4)/(64*(a - b)^3*(a + b)^3*d)
```

3.202.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 593, 25, 663, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\cot(c+dx)\csc^4(c+dx)}{-a\cos(c+dx)-b}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cot(c+dx)\csc^4(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^4(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3316$$

$$\frac{a^5\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^5\int \frac{\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^4\int \frac{a\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 593$$

$$-\frac{a^4\left(\frac{\int -\frac{b-3a\cos(c+dx)}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{4(a^2-b^2)}-\frac{b-a\cos(c+dx)}{4(a^2-b^2)(a^2-a^2\cos^2(c+dx))^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a^4\left(-\frac{\int \frac{b-3a\cos(c+dx)}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{4(a^2-b^2)}-\frac{b-a\cos(c+dx)}{4(a^2-b^2)(a^2-a^2\cos^2(c+dx))^2}\right)}{d}$$

$$\downarrow 663$$

$$-\frac{a^4\left(-\frac{\int\left(-\frac{(a+b)(3a-b)}{4a^3(a-b)^2(\cos(c+dx)a+a)}-\frac{3a-b}{4a^2(a+b)(a-a\cos(c+dx))^2}-\frac{(a-b)(3a+b)}{4a^3(a+b)^2(a-a\cos(c+dx))}+\frac{4b}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))}-\frac{3a+b}{4a^2(a-b)(\cos(c+dx)a+a)}\right.}{4(a^2-b^2)}\right.}{d}$$

3.202. $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\downarrow 2009$$

$$-\frac{a^4\left(-\frac{b-a\cos(c+dx)}{4(a^2-b^2)(a^2-a^2\cos^2(c+dx))^2}-\frac{-\frac{(a+b)(3a-b)\log(a\cos(c+dx)+a)}{4a^3(a-b)^2}+\frac{(a-b)(3a+b)\log(a-a\cos(c+dx))}{4a^3(a+b)^2}+\frac{4b\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^2}-\frac{3a-}{4a^2(a+b)(a-}}{4(a^2-b^2)}\right)}{d}$$

input
```
Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
-((a^4*(-1/4*(b - a*Cos[c + d*x])/((a^2 - b^2)*(a^2 - a^2*Cos[c + d*x]^2)^
2) - (-1/4*(3*a - b)/(a^2*(a + b)*(a - a*Cos[c + d*x])) + (3*a + b)/(4*a^2
*(a - b)*(a + a*Cos[c + d*x])) + ((a - b)*(3*a + b)*Log[a - a*Cos[c + d*x]
])/(4*a^3*(a + b)^2) - ((3*a - b)*(a + b)*Log[a + a*Cos[c + d*x]])/(4*a^3*
(a - b)^2) + (4*b*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^2)/(4*(a^2 - b^2)))
)/d)
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 663
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[1/c^p   Int[ExpandI
ntegrand[(d + e*x)^m*(f + g*x)^n*(-q + c*x)^p*(q + c*x)^p, x], x], x] /;  !
FractionalPowerFactorQ[q]] /; FreeQ[{a, c, d, e, f, g}, x] && ILtQ[p, -1] &
& IntegersQ[m, n] && NiceSqrtQ[(-a)*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.202.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b\,a^4\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(\cos(dx+c)+1)^2}-\frac{-3a+b}{16(a-b)^2(\cos(dx+c)+1)}-\frac{(3a-b)a\ln(\cos(dx+c)+1)}{16(a-b)^3}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)}$ … d
default	$\frac{\frac{b\,a^4\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(\cos(dx+c)+1)^2}-\frac{-3a+b}{16(a-b)^2(\cos(dx+c)+1)}-\frac{(3a-b)a\ln(\cos(dx+c)+1)}{16(a-b)^3}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)}$ … d
parallelrisch	$\frac{64\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)a^4b-(a-b)\left(-24(a-b)^2a\left(a+\frac{b}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a+b)\left((a+b)(a-b)^2\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{64d(a-b)^3(a+b)^3}$
norman	$\frac{-\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d(a-b)}+\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d\left(a^2-2ab+b^2\right)}-\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d\left(a^2+2ab+b^2\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{a^4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right.}{d(a^6-3a^4b^2+3a^2b^4-b^6)}$
risch	$-\frac{iabx}{8(a^3-3a^2b+3ab^2-b^3)}-\frac{iabc}{8d(a^3-3a^2b+3ab^2-b^3)}+\frac{3ia^2c}{8d(a^3-3a^2b+3ab^2-b^3)}-\frac{3ia^2x}{8(a^3+3a^2b+3ab^2+b^3)}-\frac{}{d(a^6}$

input
```
int(csc(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*a^4/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+1/2/(8*a-8*b)/(cos(d*x+c)+1)
^2-1/16*(-3*a+b)/(a-b)^2/(cos(d*x+c)+1)-1/16*(3*a-b)*a/(a-b)^3*ln(cos(d*x+
c)+1)-1/2/(8*a+8*b)/(cos(d*x+c)-1)^2-1/16*(-3*a-b)/(a+b)^2/(cos(d*x+c)-1)+
1/16*(3*a+b)/(a+b)^3*a*ln(cos(d*x+c)-1))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(172) = 344$.

Time $= 0.37$ (sec) , antiderivative size $= 469$, normalized size of antiderivative $= 2.62$

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{12\,a^4b - 16\,a^2b^3 + 4\,b^5 + 2\,(3\,a^5 - 2\,a^3b^2 - ab^4)\cos(dx+c)^3 - 8\,(a^4b - a^2b^3)\cos(dx+c)^2 - 2\,(5\,a^5 - 6\,a$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(12*a^4*b - 16*a^2*b^3 + 4*b^5 + 2*(3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*
x + c)^3 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(5*a^5 - 6*a^3*b^2 + a*b
^4)*cos(d*x + c) + 16*(a^4*b*cos(d*x + c)^4 - 2*a^4*b*cos(d*x + c)^2 + a^4
*b)*log(a*cos(d*x + c) + b) - (3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4 + (3*a^
5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 + 8*a^4*b + 6*a
^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^5 - 8*a
^4*b + 6*a^3*b^2 - a*b^4 + (3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x +
 c)^4 - 2*(3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(-1/2*c
os(d*x + c) + 1/2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4
 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*d)
```

3.202.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.50

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\frac{16\,a^4b\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(3\,a^2-ab)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(3\,a^2+ab)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(4\,a^2b\cos(dx+c)^2-(3\,a^3+ab^2)\cos(dx+c)^3-\right.}{(a^4-2\,a^2b^2+b^4)\cos(dx+c)^4+a^4-2\,a^2b^2+b}}{16\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(16*a^4*b*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - (3*a^2 - a*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
3*a^2 + a*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(4*
a^2*b*cos(d*x + c)^2 - (3*a^3 + a*b^2)*cos(d*x + c)^3 - 6*a^2*b + 2*b^3 +
(5*a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^
4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

3.202.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(172) = 344.

Time = 0.35 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.34

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\frac{64\,a^4b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{4\,(3\,a^2+ab)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)+}{(\cos(dx+c)+}}{a^2-2\,ab+b^2}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/64*(64*a^4*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6) + 4*(3*a^2 + a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a
^3 + 3*a^2*b + 3*a*b^2 + b^3) - (8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2 - 2*a
*b + b^2) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 6*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2))/d
```

3.202.9 Mupad [B] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.66

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{3a^2b-b^3}{4\left(a^2-b^2\right)^2} + \frac{\cos(c+dx)^3\left(3a^3+ab^2\right)}{8\left(a^4-2a^2b^2+b^4\right)} - \frac{a^2b\cos(c+dx)^2}{2\left(a^2-b^2\right)^2} - \frac{a\cos(c+dx)\left(5a^2-b^2\right)}{8\left(a^4-2a^2b^2+b^4\right)}}{d\left(\cos(c+dx)^4 - 2\cos(c+dx)^2 + 1\right)} + \frac{\ln\left(\cos(c+dx)-1\right)\left(\frac{3}{16(a+b)} - \frac{5b}{16(a+b)^2} + \frac{b^2}{8(a+b)^3}\right)}{d} - \frac{\ln\left(\cos(c+dx)+1\right)\left(\frac{b^2}{8(a-b)^3} + \frac{5b}{16(a-b)^2} + \frac{3}{16(a-b)}\right)}{d} + \frac{a^4b\ln\left(b+a\cos(c+dx)\right)}{d\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}$$

input
```
int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

output
```
((3*a^2*b - b^3)/(4*(a^2 - b^2)^2) + (cos(c + d*x)^3*(a*b^2 + 3*a^3))/(8*(
a^4 + b^4 - 2*a^2*b^2)) - (a^2*b*cos(c + d*x)^2)/(2*(a^2 - b^2)^2) - (a*co
s(c + d*x)*(5*a^2 - b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^4
 - 2*cos(c + d*x)^2 + 1)) + (log(cos(c + d*x) - 1)*(3/(16*(a + b)) - (5*b)/
(16*(a + b)^2) + b^2/(8*(a + b)^3)))/d - (log(cos(c + d*x) + 1)*(b^2/(8*(a
 - b)^3) + (5*b)/(16*(a - b)^2) + 3/(16*(a - b))))/d + (a^4*b*log(b + a*co
s(c + d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.203.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} - \frac{2(a-b)^{5/2}b(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d} + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} + \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d}$$

output

```
1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*x/a^7-2*(a-b)^(5/2)*b*(a+b)^(5/2
)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d+1/16*(16*b*(a^
2-b^2)^2-a*(5*a^4-14*a^2*b^2+8*b^4)*cos(d*x+c))*sin(d*x+c)/a^6/d+1/24*(8*b
*(a^2-b^2)-a*(5*a^2-6*b^2)*cos(d*x+c))*sin(d*x+c)^3/a^4/d+1/30*(6*b-5*a*co
s(d*x+c))*sin(d*x+c)^5/a^2/d
```

3.203.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.17

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{300a^6c - 1800a^4b^2c + 2400a^2b^4c - 960b^6c + 300a^6dx - 1800a^4b^2dx + 2400a^2b^4dx - 960b^6dx + 1920b(a^2$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output
```
(300*a^6*c - 1800*a^4*b^2*c + 2400*a^2*b^4*c - 960*b^6*c + 300*a^6*d*x - 1
800*a^4*b^2*d*x + 2400*a^2*b^4*d*x - 960*b^6*d*x + 1920*b*(a^2 - b^2)^(5/2
)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(11*a^4 -
 18*a^2*b^2 + 8*b^4)*Sin[c + d*x] - 15*(15*a^6 - 32*a^4*b^2 + 16*a^2*b^4)*
Sin[2*(c + d*x)] - 140*a^5*b*Sin[3*(c + d*x)] + 80*a^3*b^3*Sin[3*(c + d*x)
] + 45*a^6*Sin[4*(c + d*x)] - 30*a^4*b^2*Sin[4*(c + d*x)] + 12*a^5*b*Sin[5
*(c + d*x)] - 5*a^6*Sin[6*(c + d*x)])/(960*a^7*d)
```

3.203.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 25, 3042, 3344, 27, 3042, 3344, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^6(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$
$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^6(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^6(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^6}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{\int -\frac{(ab-(5a^2-6b^2)\cos(c+dx))\sin^4(c+dx)}{b+a\cos(c+dx)}dx}{6a^2}+\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}$$

$$\downarrow 25$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}-\frac{\int \frac{(ab-(5a^2-6b^2)\cos(c+dx))\sin^4(c+dx)}{b+a\cos(c+dx)}dx}{6a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}-\frac{\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4\left(ab+\left(6b^2-5a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}$$

$$\downarrow 3344$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}-\frac{\frac{\int \frac{3\left(ab\left(3a^2-2b^2\right)-\left(5a^4-14b^2a^2+8b^4\right)\cos(c+dx)\right)\sin^2(c+dx)}{b+a\cos(c+dx)}dx}{4a^2}-\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{4a^2d}}{6a^2}$$

$$\downarrow 27$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}-\frac{\frac{3\int \frac{\left(ab\left(3a^2-2b^2\right)-\left(5a^4-14b^2a^2+8b^4\right)\cos(c+dx)\right)\sin^2(c+dx)}{b+a\cos(c+dx)}dx}{4a^2}-\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{4a^2d}}{6a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}-\frac{\frac{3\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(ab\left(3a^2-2b^2\right)+\left(-5a^4+14b^2a^2-8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}-\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{4a^2d}}{6a^2}$$

$$\downarrow 3344$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{\int \frac{ab\left(11a^4-18b^2a^2+8b^4\right)-\left(5a^6-30b^2a^4+40b^4a^2-16b^6\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{2a^2} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)}{2a^2d}\right)}{4a^2} - \frac{\sin^3(c+dx)(8b}{6a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{\int \frac{ab\left(11a^4-18b^2a^2+8b^4\right)+\left(-5a^6+30b^2a^4-40b^4a^2+16b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)}{2a^2d}\right)}{4a^2} - \frac{\sin^3(c+d}{6a^2}$$

$$\downarrow 3214$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{16b\left(a^2-b^2\right)^3\int\frac{1}{b+a\cos(c+dx)}dx}{a} - \frac{x\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)}{a} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)}{2a^2d}\right)}{4a^2} - \frac{\sin^3(c+dx)(8b(a^2}{6a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{16b\left(a^2-b^2\right)^3\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a} - \frac{x\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)}{a} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)}{2a^2d}\right)}{4a^2} - \frac{\sin^3(c+dx)(8}{6a^2}$$

$$\downarrow 3138$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{32b\left(a^2-b^2\right)^3\int\frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad} - \frac{x\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)}{a} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+\right.}{2a^2d}\right)}{4a^2} - \frac{}{6a^2}$$

$$\downarrow 221$$

$$\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} -$$

$$\frac{3\left(\frac{\frac{32b\left(a^2-b^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{2a^2} - \frac{x\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)}{a} - \frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)}{2a^2d}\right)}{4a^2} - \frac{\sin}{6a^2}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output
```
((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30*a^2*d) - (-1/4*((8*b*(a^2 -
b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])*Sin[c + d*x]^3)/(a^2*d) + (3*((-(((
5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/a) + (32*b*(a^2 - b^2)^3*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b
]*d))/(2*a^2) - ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[
c + d*x])*Sin[c + d*x])/(2*a^2*d)))/(4*a^2))/(6*a^2)
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.203.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(213) = 426$.

Time = 1.56 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.92

method	result
derivativedivides	$-\frac{2b(a+b)^3(a-b)^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^7\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{5}{16}a^6+a^5b-\frac{7}{8}a^4b^2-2a^3b^3+\frac{1}{2}a^2b^4+ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{19}{3}a^5b-\frac{29}{8}a^4b\right.\right.}{}$
default	$-\frac{2b(a+b)^3(a-b)^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^7\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{5}{16}a^6+a^5b-\frac{7}{8}a^4b^2-2a^3b^3+\frac{1}{2}a^2b^4+ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{19}{3}a^5b-\frac{29}{8}a^4b\right.\right.}{}$
risch	$\frac{3\sin(4dx+4c)}{64ad}-\frac{15xb^2}{8a^3}+\frac{5xb^4}{2a^5}-\frac{xb^6}{a^7}-\frac{\sin(6dx+6c)}{192da}-\frac{15\sin(2dx+2c)}{64da}-\frac{\sin(4dx+4c)b^2}{32a^3d}-\frac{7b\sin(3dx+3c)}{48a^2d}$

input
```
int(sin(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b*(a+b)^3*(a-b)^3/a^7/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a-b)*(a+b))^(1/2))+2/a^7*(((5/16*a^6+a^5*b-7/8*a^4*b^2-2*a^3*b^
3+1/2*a^2*b^4+a*b^5)*tan(1/2*d*x+1/2*c)^11+(19/3*a^5*b-29/8*a^4*b^2+3/2*a^
2*b^4+5*a*b^5+85/48*a^6-34/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(86/5*a^5*b-11/
4*a^4*b^2-24*a^3*b^3+a^2*b^4+10*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/
8*a^6+11/4*a^4*b^2-a^2*b^4+86/5*a^5*b-24*a^3*b^3+10*a*b^5)*tan(1/2*d*x+1/2
*c)^5+(19/3*a^5*b+29/8*a^4*b^2-34/3*a^3*b^3-3/2*a^2*b^4+5*a*b^5-85/48*a^6)
*tan(1/2*d*x+1/2*c)^3+(a^5*b-2*a^3*b^3+a*b^5-5/16*a^6+7/8*a^4*b^2-1/2*a^2*
b^4)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-30*a^4*b^2
+40*a^2*b^4-16*b^6)*arctan(tan(1/2*d*x+1/2*c))))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.40

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{15\,(5\,a^6-30\,a^4b^2+40\,a^2b^4-16\,b^6)dx+120\,(a^4b-2\,a^2b^3+b^5)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+}{a^2\cos(d\right.\right.$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x + 120*(a^4*b - 2
*a^2*b^3 + b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^6*cos(d*x
 + c)^5 - 48*a^5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 -
10*(13*a^6 - 6*a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x
 + c)^2 + 15*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))
/(a^7*d), 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x - 240*(
a^4*b - 2*a^2*b^3 + b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(
d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (40*a^6*cos(d*x + c)^5 - 48*a^
5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(13*a^6 - 6*
a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^2 + 15*(1
1*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d)]
```

3.203.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(212) = 424$.

Time $= 0.35$ (sec) , antiderivative size $= 781$, normalized size of antiderivative $= 3.40$

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output

```
1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*(d*x + c)/a^7 - 480*(
a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn
(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^7) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^
11 + 240*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 210*a^3*b^2*tan(1/2*d*x + 1/2*c)^
11 - 480*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^
11 + 240*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 15
20*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 272
0*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200
*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 4128*a^4*b*
tan(1/2*d*x + 1/2*c)^7 - 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a^2*b^3
*tan(1/2*d*x + 1/2*c)^7 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*b^5*tan(
1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 4128*a^4*b*tan(1/2*d
*x + 1/2*c)^5 + 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*a^2*b^3*tan(1/2*
d*x + 1/2*c)^5 - 240*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*b^5*tan(1/2*d*x +
 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 1520*a^4*b*tan(1/2*d*x + 1/2*
c)^3 + 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 2720*a^2*b^3*tan(1/2*d*x + 1/2
*c)^3 - 360*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 1200*b^5*tan(1/2*d*x + 1/2*c)^3
 - 75*a^5*tan(1/2*d*x + 1/2*c) + 240*a^4*b*tan(1/2*d*x + 1/2*c) + 210*a^3*
b^2*tan(1/2*d*x + 1/2*c) - 480*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4...
```

3.203.9 Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 3341, normalized size of antiderivative = 14.53

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input

```
int(sin(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

output

```
(atan((((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6
+ 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2
+ (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i +
 a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i -
 a^4*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 2
5*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*
a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1
136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i +
a^2*b^4*40i - a^4*b^2*30i)*1i)/(16*a^7) - (((((42*a^21*b - 10*a^22 + 32*a^
14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19
*b^3 + 6*a^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 -
 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19)
)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d
*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 512
0*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7
 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b
^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(16*a^7)
)/(((25*a^19*b)/4 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 760*a^3*b^17 + 15
44*a^4*b^16 - 2628*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13 + 2890*a^8*b^1
2 - 6359*a^9*b^11 - 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (937*a^12*b^8...
```

3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.204.1 Optimal result 1502
3.204.2 Mathematica [A] (verified) 1503
3.204.3 Rubi [A] (verified) 1503
3.204.4 Maple [A] (verified) 1506
3.204.5 Fricas [A] (verification not implemented) 1507
3.204.6 Sympy [F] 1508
3.204.7 Maxima [F(-2)] 1508
3.204.8 Giac [B] (verification not implemented) 1508
3.204.9 Mupad [B] (verification not implemented) 1509

3.204.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5 d} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4 d} + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2 d}$$

output

```
1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctanh((a-
b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/d+1/8*(8*b*(a^2-b^2)-a*(3*a^2
-4*b^2)*cos(d*x+c))*sin(d*x+c)/a^4/d+1/12*(4*b-3*a*cos(d*x+c))*sin(d*x+c)^
3/a^2/d
```

3.204.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{36a^4c - 144a^2b^2c + 96b^4c + 36a^4dx - 144a^2b^2dx + 96b^4dx + 192b(a^2-b^2)^{3/2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{96a}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(36*a^4*c - 144*a^2*b^2*c + 96*b^4*c + 36*a^4*d*x - 144*a^2*b^2*d*x + 96*b
^4*d*x + 192*b*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]] + 24*a*b*(5*a^2 - 4*b^2)*Sin[c + d*x] - 24*(a^4 - a^2*b^2)*Sin
[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5
*d)
```

3.204.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 25, 3042, 3344, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}\,dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^4(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}\,dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos(c+dx)\sin^4(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$\downarrow 25$$

$$\int \frac{\sin^4(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^4}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{\int -\frac{(ab-(3a^2-4b^2)\cos(c+dx))\sin^2(c+dx)}{b+a\cos(c+dx)}dx}{4a^2}+\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}$$

$$\downarrow 25$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}-\frac{\int \frac{(ab-(3a^2-4b^2)\cos(c+dx))\sin^2(c+dx)}{b+a\cos(c+dx)}dx}{4a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}-\frac{\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2(ab+(4b^2-3a^2)\sin\left(c+dx+\frac{\pi}{2}\right))}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}$$

$$\downarrow 3344$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}-\frac{\frac{\int \frac{ab\left(5a^2-4b^2\right)-\left(3a^4-12b^2a^2+8b^4\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))}{2a^2d}}{4a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}-\frac{\frac{\int \frac{ab\left(5a^2-4b^2\right)+\left(-3a^4+12b^2a^2-8b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))}{2a^2d}}{4a^2}$$

$$\downarrow 3214$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}-\frac{\frac{8b\left(a^2-b^2\right)^2\int \frac{1}{b+a\cos(c+dx)}dx}{a}-\frac{x\left(3a^4-12a^2b^2+8b^4\right)}{a}}{2a^2}-\frac{\sin(c+dx)(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))}{2a^2d}}{4a^2}$$

$$\downarrow 3042$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d} - \frac{\frac{8b\left(a^2-b^2\right)^2\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a} - \frac{x\left(3a^4-12a^2b^2+8b^4\right)}{a}}{2a^2} - \frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{2a^2d}}{4a^2}$$

$$\downarrow 3138$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d} - \frac{\frac{16b\left(a^2-b^2\right)^2\int\frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad} - \frac{x\left(3a^4-12a^2b^2+8b^4\right)}{a}}{2a^2} - \frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{2a^2d}}{4a^2}$$

$$\downarrow 221$$

$$\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d} - \frac{\frac{16b\left(a^2-b^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} - \frac{x\left(3a^4-12a^2b^2+8b^4\right)}{a}}{2a^2} - \frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{2a^2d}}{4a^2}$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((4*b - 3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d) - ((-(((3*a^4 - 12*a^
2*b^2 + 8*b^4)*x)/a) + (16*b*(a^2 - b^2)^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(2*a^2) - ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(2*a^2*d))/(4*a^2)
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.204.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-\frac{2(a-b)^2(a+b)^2 b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{3}{8}a^4+a^3b-\frac{1}{2}a^2b^2-ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{13}{3}a^3b-\frac{1}{2}a^2b^2-3ab^3+\frac{11}{8}a^4\right)$ ta
default	$-\frac{2(a-b)^2(a+b)^2 b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{3}{8}a^4+a^3b-\frac{1}{2}a^2b^2-ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{13}{3}a^3b-\frac{1}{2}a^2b^2-3ab^3+\frac{11}{8}a^4\right)$ ta
risch	$\frac{3x}{8a}-\frac{3xb^2}{2a^3}+\frac{xb^4}{a^5}-\frac{5ib\,e^{i(dx+c)}}{8da^2}+\frac{ib^3e^{i(dx+c)}}{2da^4}+\frac{5ib\,e^{-i(dx+c)}}{8da^2}-\frac{ib^3e^{-i(dx+c)}}{2da^4}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}+$

input
```
int(sin(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(a-b)^2*(a+b)^2*b/a^5/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a-b)*(a+b))^(1/2))+2/a^5*(((3/8*a^4+a^3*b-1/2*a^2*b^2-a*b^3)*ta
n(1/2*d*x+1/2*c)^7+(13/3*a^3*b-1/2*a^2*b^2-3*a*b^3+11/8*a^4)*tan(1/2*d*x+1
/2*c)^5+(-11/8*a^4+1/2*a^2*b^2+13/3*a^3*b-3*a*b^3)*tan(1/2*d*x+1/2*c)^3+(a
^3*b-a*b^3-3/8*a^4+1/2*a^2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^
2)^4+1/8*(3*a^4-12*a^2*b^2+8*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.44

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{3\,(3\,a^4-12\,a^2b^2+8\,b^4)dx-12\,(a^2b-b^3)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.\right.$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 12*(a^2*b - b^3)*sqrt(a^2 - b^
2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b
^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2)) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)
^2 + 32*a^3*b - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c
))/(a^5*d), 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 24*(a^2*b - b^3)*sq
rt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*
sin(d*x + c))) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 + 32*a^3*b
 - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a^5*d)]
```

3.204.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.204.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(145) = 290$.

Time $= 0.31$ (sec) , antiderivative size $= 407$, normalized size of antiderivative $= 2.53$

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{3\left(3a^4-12a^2b^2+8b^4\right)(dx+c)}{a^5} - \frac{48\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^5} + \frac{2\left(9a^3}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 - 48*(a^4*b - 2*a^2*b^3
 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
+ b^2)*a^5) + 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2
*c)^7 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*b^3*tan(1/2*d*x + 1/2*c)^7 +
33*a^3*tan(1/2*d*x + 1/2*c)^5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^
2*tan(1/2*d*x + 1/2*c)^5 - 72*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*
d*x + 1/2*c)^3 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x +
 1/2*c)^3 - 72*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 2
4*a^2*b*tan(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) - 24*b^3*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

3.204.9 Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.97

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{5b\sin(c+dx)}{4}-\frac{b\sin(3c+3dx)}{12}}{a^2 d} - \frac{3b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) - \frac{b^2\sin(2c+2dx)}{4}}{a^3 d}$$
$$+ \frac{\frac{3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4} - \frac{\sin(2c+2dx)}{4} + \frac{\sin(4c+4dx)}{32}}{a\,d} - \frac{b^3\sin(c+dx)}{a^4 d} + \frac{2b^4\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^5 d}$$
$$- \frac{2b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^6-3a^4b^2+3a^2b^4-b^6}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}\right)\sqrt{a^6-3a^4b^2+3a^2b^4-b^6}}{a^5 d}$$

input
```
int(sin(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

output
```
((5*b*sin(c + d*x))/4 - (b*sin(3*c + 3*d*x))/12)/(a^2*d) - (3*b^2*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a^3*d) +
((3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - sin(2*c + 2*d*x)/4 +
sin(4*c + 4*d*x)/32)/(a*d) - (b^3*sin(c + d*x))/(a^4*d) + (2*b^4*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^5*d) - (2*b*atanh((sin(c/2 + (d*x)/
2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - b^
3*cos(c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + a^2*b*cos(c/2 + (d*x)/2)
))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^5*d)
```

3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.205.1 Optimal result 1510
3.205.2 Mathematica [A] (verified) 1510
3.205.3 Rubi [A] (verified) 1511
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3.205.8 Giac [B] (verification not implemented) 1515
3.205.9 Mupad [B] (verification not implemented) 1516

3.205.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(a^2-2b^2)\,x}{2a^3} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3 d} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2 d}$$

output
```
1/2*(a^2-2*b^2)*x/a^3+1/2*(2*b-a*cos(d*x+c))*sin(d*x+c)/a^2/d-2*b*arctanh(
(a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3/d
```

3.205.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2(a^2-2b^2)(c+dx) + 8b\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + 4ab\sin(c+dx) - a^2\sin(2(c+dx))}{4a^3 d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(2*(a^2 - 2*b^2)*(c + d*x) + 8*b*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]] + 4*a*b*Sin[c + d*x] - a^2*Sin[2*(c + d*x)])/(
4*a^3*d)
```

3.205.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^2(c+dx)\cos(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sin^2(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^2(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^2}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{\int -\frac{ab-\left(a^2-2b^2\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{2a^2}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}$$

$$\downarrow 25$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}-\frac{\int \frac{ab-\left(a^2-2b^2\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{2a^2}$$

$$\downarrow 3042$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}-\frac{\int \frac{ab+\left(2b^2-a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}$$

$$\downarrow 3214$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d} - \frac{\frac{2b(a^2-b^2)\int \frac{1}{b+a\cos(c+dx)}dx}{a} - \frac{x(a^2-2b^2)}{a}}{2a^2}$$

$$\downarrow 3042$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d} - \frac{\frac{2b(a^2-b^2)\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a} - \frac{x(a^2-2b^2)}{a}}{2a^2}$$

$$\downarrow 3138$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d} - \frac{\frac{4b(a^2-b^2)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad} - \frac{x(a^2-2b^2)}{a}}{2a^2}$$

$$\downarrow 221$$

$$\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d} - \frac{\frac{4b(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} - \frac{x(a^2-2b^2)}{a}}{2a^2}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
-1/2*(-(((a^2 - 2*b^2)*x)/a) + (4*b*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/a^2 + ((2*b - a*
Cos[c + d*x])*Sin[c + d*x])/(2*a^2*d)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.205.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2(a+b)b(a-b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(ab-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2-2b^2\right)\arctan\left(\tan\left(\frac{d}{2}\right.\right.}{a^3}}{d}$
default	$\frac{-\frac{2(a+b)b(a-b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(ab-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2-2b^2\right)\arctan\left(\tan\left(\frac{d}{2}\right.\right.}{a^3}}{d}$
risch	$\frac{x}{2a}-\frac{xb^2}{a^3}-\frac{ib\,e^{i(dx+c)}}{2da^2}+\frac{ib\,e^{-i(dx+c)}}{2da^2}-\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}+\frac{b+i\sqrt{a^2-b^2}}{a}\right)}{da^3}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2}}{}\right.}{da^3}$

input
```
int(sin(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(a+b)*b*(a-b)/a^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a-b)*(a+b))^(1/2))+2/a^3*(((1/2*a^2+a*b)*tan(1/2*d*x+1/2*c)^3+(a*b-
1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-2*b^2)*ar
ctan(tan(1/2*d*x+1/2*c))))
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.58

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{(a^2-2\,b^2)dx+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-(a^2\right.$$
$$\left.\frac{}{2\,a^3d}\right.$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*((a^2 - 2*b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^
2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d), 1/2*((a^2 - 2*b^2)*d*x - 2*
sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - (a^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d)]
```

3.205.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{(a^2-2b^2)(dx+c)}{a^3} - \frac{4\,(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^3} + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\right.}{}}{2d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((a^2 - 2*b^2)*(d*x + c)/a^3 - 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) + 2*(a*tan(1/2*d*x
+ 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan
(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.205.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\frac{\sin(2c+2dx)}{4}}{ad} - \frac{2b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^3 d} + \frac{b\sin(c+dx)}{a^2 d} - \frac{2b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a^3 d}$$

input
```
int(sin(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

output
```
(atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - sin(2*c + 2*d*x)/4)/(a*d) -
 (2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d) + (b*sin(c +
d*x))/(a^2*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2
 + (d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a^3*d)
```

3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.206.1 Optimal result 1517
3.206.2 Mathematica [A] (verified) 1517
3.206.3 Rubi [A] (verified) 1518
3.206.4 Maple [A] (verified) 1520
3.206.5 Fricas [A] (verification not implemented) 1521
3.206.6 Sympy [F] 1521
3.206.7 Maxima [F(-2)] 1522
3.206.8 Giac [A] (verification not implemented) 1522
3.206.9 Mupad [B] (verification not implemented) 1523

3.206.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}$$

output
```
-2*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+
b)^(3/2)/d+(b-a*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)/d
```

3.206.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}(b-a\cos(c+dx)) + 2ab\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx\right.}{2(a-b)(a+b)\sqrt{a^2-b^2}d}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Sqrt[a^2 - b^2]*(b - a*Cos[c + d*x]) +
 2*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x]))
/(2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d)
```

3.206.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)} - \frac{\int \frac{ab}{b+a\cos(c+dx)}dx}{a^2-b^2}$$

$$\downarrow 27$$

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)} - \frac{ab\int \frac{1}{b+a\cos(c+dx)}dx}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)} - \frac{ab\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\downarrow 3138$$

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)} - \frac{2ab\int \frac{1}{-((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right))+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)} - \frac{2ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*(a^2 - b^2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b
^2)*d)
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.206.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ab\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	96
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ab\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	96
risch	$-\frac{2i\left(-b\,\mathrm{e}^{i(dx+c)}+a\right)}{d(a^2-b^2)\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{ba\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}+\frac{ba\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$	209

input
```
int(csc(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a-b)-2*a/(a-b)/(a+b)*b/((a-b)*(a+b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+
1/2*c))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.57

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{a^2-b^2}ab\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^2b}{2\,(a^4-2\,a^2b^2+b^4)d\sin(dx+c)}\right.$$
$$\left. -\frac{\sqrt{-a^2+b^2}ab\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^2b+b^3+(a^3-ab^2)\cos(dx+c)}{(a^4-2\,a^2b^2+b^4)d\sin(dx+c)}\right]$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a^2 - b^2)*a*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^
2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^2*b
 + 2*b^3 + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*
x + c)), -(sqrt(-a^2 + b^2)*a*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)
*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x + c))]
```

3.206.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a-b} + \frac{1}{(a+b)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/
2*d*x + 1/2*c)))/d
```

3.206.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(2a-2b)} - \frac{a-b}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a+b)\,(2a-2b)} - \frac{2ab\,\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)}{(a+b)^{3/2}\sqrt{a-b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(d*(2*a - 2*b)) - (a - b)/(d*tan(c/2 + (d*x)/2)*(a + b)
*(2*a - 2*b)) - (2*a*b*atanh((tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/
2)*(a - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.207.1 Optimal result 1524
3.207.2 Mathematica [A] (verified) 1524
3.207.3 Rubi [A] (verified) 1525
3.207.4 Maple [A] (verified) 1528
3.207.5 Fricas [A] (verification not implemented) 1528
3.207.6 Sympy [F] 1529
3.207.7 Maxima [F(-2)] 1529
3.207.8 Giac [B] (verification not implemented) 1530
3.207.9 Mupad [B] (verification not implemented) 1530

3.207.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2a^3b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(3a^2b-a(2a^2+b^2)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d}$$

output
```
-2*a^3*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(
a+b)^(5/2)/d+1/3*(3*a^2*b-a*(2*a^2+b^2)*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2
/d+1/3*(b-a*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^2)/d
```

3.207.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{24a^3b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}(10a^2b-4b^3+(-6a^3+3ab^2)\cos(c+dx)-6a^2b\cos(2(c+$$
$$12(a-b)^2(a+b)^2\sqrt{a^2-b^2}d$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(24*a^3*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + Sqrt[a^2
- b^2]*(10*a^2*b - 4*b^3 + (-6*a^3 + 3*a*b^2)*Cos[c + d*x] - 6*a^2*b*Cos[2
*(c + d*x)] + 2*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)])*Csc[c + d*x
]^3)/(12*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2]*d)
```

3.207.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 3042, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^3(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^3(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\int \frac{(ab-2a^2\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{3\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\int \frac{2\sin\left(c+dx-\frac{\pi}{2}\right)a^2+ba}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}$$

$$\downarrow 3345$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\frac{\int \frac{3a^3b}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\frac{3a^3b\int \frac{1}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\frac{3a^3b\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)}$$

$$\downarrow 3138$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\frac{6a^3b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\frac{6a^3b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^2)*d) - ((6*a^3*b*ArcTan
h[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a
^2 - b^2)*d) - ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a
^2 - b^2)*d))/(3*(a^2 - b^2))
```

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.207.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}-\frac{2a^3 b\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}-\frac{2a^3 b\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$
risch	$\frac{2i\left(3a^2 b\,e^{5i(dx+c)}-3a\,b^2 e^{4i(dx+c)}-10a^2 b\,e^{3i(dx+c)}+4b^3 e^{3i(dx+c)}+6a^3 e^{2i(dx+c)}+3a^2 b\,e^{i(dx+c)}-2a^3-a\,b^2\right)}{3d(-a^2+b^2)^2\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\,a^3\ln($

input
```
int(csc(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/(a-b)^2*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+1/2*c)^3*b+3*
tan(1/2*d*x+1/2*c)*a-tan(1/2*d*x+1/2*c)*b)-2/(a-b)^2/(a+b)^2*a^3*b/((a-b)*
(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/24/(a
+b)/tan(1/2*d*x+1/2*c)^3-1/8*(3*a+b)/(a+b)^2/tan(1/2*d*x+1/2*c))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 3.99

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{8\,a^4 b-10\,a^2 b^3+2\,b^5+2\left(2\,a^5-a^3 b^2-a b^4\right)\cos(dx+c)^3-3\left(a^3 b\cos(dx+c)^2-a^3 b\right)\sqrt{a^2-b^2}\log\left(\right.}{6\left(\left(a^6-3\,a^4 b^2+3\,a^2 b^4-b\right.\right.}\right.$$
$$\left.-\frac{4\,a^4 b-5\,a^2 b^3+b^5+\left(2\,a^5-a^3 b^2-a b^4\right)\cos(dx+c)^3+3\left(a^3 b\cos(dx+c)^2-a^3 b\right)\sqrt{-a^2+b^2}\arctan\left(\right.}{3\left(\left(a^6-3\,a^4 b^2+3\,a^2 b^4-b^6\right)d\cos(dx+c)^2-(a\right.}\right]$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/6*(8*a^4*b - 10*a^2*b^3 + 2*b^5 + 2*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x
+ c)^3 - 3*(a^3*b*cos(d*x + c)^2 - a^3*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d
*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
 + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + b^2))*sin(d*x + c) - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 6*(a^5 - a^3*
b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2
 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c)), -1/3*(4*a^4*b - 5*
a^2*b^3 + b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3 + 3*(a^3*b*cos(d*
x + c)^2 - a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c
) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4*b - a^2*b^3)*cos(
d*x + c)^2 - 3*(a^5 - a^3*b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x
 + c))]
```

3.207.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(128) = 256$.

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3b}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^3-3}}{24\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3*b/((a^4
- 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b
*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x +
 1/2*c) - 12*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(a^3 -
 3*a^2*b + 3*a*b^2 - b^3) - (9*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*d*x
+ 1/2*c)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/2*c)^3))/d
```

3.207.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2}{8a-8b}+\frac{8a+8b}{(8a-8b)^2}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3d\,(8a-8b)} - \frac{\frac{a^2-2ab+b^2}{3(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(3a^3-5a^2b+ab^2+b^3)}{(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(8a^2-16ab+8b^2)} - \frac{2a^3b\,\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

output
```
(tan(c/2 + (d*x)/2)*(2/(8*a - 8*b) + (8*a + 8*b)/(8*a - 8*b)^2))/d + tan(c
/2 + (d*x)/2)^3/(3*d*(8*a - 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) + (ta
n(c/2 + (d*x)/2)^2*(a*b^2 - 5*a^2*b + 3*a^3 + b^3))/(a + b)^2)/(d*tan(c/2
+ (d*x)/2)^3*(8*a^2 - 16*a*b + 8*b^2)) - (2*a^3*b*atanh((tan(c/2 + (d*x)/2
)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2
)*(a - b)^(5/2))
```

3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$

3.208.1 Optimal result 1532
3.208.2 Mathematica [A] (verified) 1533
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3.208.8 Giac [B] (verification not implemented) 1539
3.208.9 Mupad [B] (verification not implemented) 1540

3.208.1 Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = &-\frac{2a^5 b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &+\frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d}\\ &+\frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2 d}\\ &+\frac{\left(b-a\cos(c+dx)\right)\csc^5(c+dx)}{5\left(a^2-b^2\right)d}\end{aligned}$$

output

```
-2*a^5*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(
a+b)^(7/2)/d+1/15*(15*a^4*b-a*(8*a^4+9*a^2*b^2-2*b^4)*cos(d*x+c))*csc(d*x+
c)/(a^2-b^2)^3/d+1/15*(5*a^2*b-a*(4*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^3/(a^2
-b^2)^2/d+1/5*(b-a*cos(d*x+c))*csc(d*x+c)^5/(a^2-b^2)/d
```

3.208.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.38

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{960a^5b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{2(64a^2+43ab+9b^2)\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{8(19a-9b)\csc}{}\right.}{4}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]*((960*a^5*b*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - (2*(64*a^2 + 43*a*b + 9*b^2
)*Cot[(c + d*x)/2])/(a + b)^3 + (8*(19*a - 9*b)*Csc[c + d*x]^3*Sin[(c + d*
x)/2]^4)/(a - b)^2 + (96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6)/(a - b) - ((19
*a + 9*b)*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(2*(a + b)^2) - (3*Csc[(c + d*x
)/2]^6*Sin[c + d*x])/(2*(a + b)) + (2*(64*a^2 - 43*a*b + 9*b^2)*Tan[(c + d
*x)/2])/(a - b)^3))/(480*d*(a + b*Sec[c + d*x]))
```

3.208.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 3042, 3345, 3042, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot(c+dx)\csc^5(c+dx)}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot(c+dx)\csc^5(c+dx)}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cot(c+dx)\csc^5(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^6\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^6\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}-\frac{\int \frac{(ab-4a^2\cos(c+dx))\csc^4(c+dx)}{b+a\cos(c+dx)}dx}{5\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}-\frac{\int \frac{4\sin\left(c+dx-\frac{\pi}{2}\right)a^2+ba}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5\left(a^2-b^2\right)}$$

$$\downarrow 3345$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}-\frac{\frac{\int \frac{\left(ab\left(7a^2-2b^2\right)-2a^2\left(4a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{3\left(a^2-b^2\right)}-\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}-\frac{\frac{\int \frac{2\left(4a^2+b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)a^2+b\left(7a^2-2b^2\right)a}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}-\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 3345$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} - \frac{\frac{\int \frac{15a^5b}{b+a\cos(c+dx)}\,dx}{a^2-b^2} - \frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)} - \frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} - \frac{\frac{15a^5b\int \frac{1}{b+a\cos(c+dx)}\,dx}{a^2-b^2} - \frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)} - \frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} - \frac{\frac{15a^5b\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}\,dx}{a^2-b^2} - \frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)} - \frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 3138$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} - \frac{\frac{30a^5b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}\,d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)} - \frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)} - \frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} - \frac{\frac{30a^5b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)} - \frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{3\left(a^2-b^2\right)} - \frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output
```
((b - a*Cos[c + d*x])*Csc[c + d*x]^5)/(5*(a^2 - b^2)*d) - (-1/3*((5*a^2*b
- a*(4*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^3)/((a^2 - b^2)*d) + ((30*a^5
*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*(a^2 - b^2)*d) - ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c +
 d*x])*Csc[c + d*x])/((a^2 - b^2)*d))/(3*(a^2 - b^2)))/(5*(a^2 - b^2))
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.208.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10a^2\tan\left(\frac{dx}{2}+$
default	$\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10a^2\tan\left(\frac{dx}{2}+$
risch	$-\frac{2i\left(15a^4b\,e^{9i(dx+c)}-15a^3b^2e^{8i(dx+c)}-80a^4b\,e^{7i(dx+c)}+20a^2b^3e^{7i(dx+c)}+90a^3b^2e^{6i(dx+c)}-30a\,b^4e^{6i(dx+c)}+178a^4b$

input

```
int(csc(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/32/(a-b)^3*(1/5*a^2*tan(1/2*d*x+1/2*c)^5-2/5*a*b*tan(1/2*d*x+1/2*c)
^5+1/5*b^2*tan(1/2*d*x+1/2*c)^5+5/3*a^2*tan(1/2*d*x+1/2*c)^3-8/3*a*b*tan(1
/2*d*x+1/2*c)^3+b^2*tan(1/2*d*x+1/2*c)^3+10*a^2*tan(1/2*d*x+1/2*c)-8*a*b*t
an(1/2*d*x+1/2*c)+2*b^2*tan(1/2*d*x+1/2*c))-2/(a-b)^3/(a+b)^3*a^5*b/((a-b)
*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/160/
(a+b)/tan(1/2*d*x+1/2*c)^5-1/96*(5*a+3*b)/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/3
2/(a+b)^3*(10*a^2+8*a*b+2*b^2)/tan(1/2*d*x+1/2*c))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(187) = 374.

Time = 0.32 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.28

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \left[\frac{46\,a^6b-68\,a^4b^3+28\,a^2b^5-6\,b^7-2\left(8\,a^7+a^5b^2-11\,a^3b^4+2\,ab^6\right)\cos\left(dx+c\right)^5+30\left(a^6b-a^4b^3\right)\cos\left(d\right.\right.}{\left.\right.}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/30*(46*a^6*b - 68*a^4*b^3 + 28*a^2*b^5 - 6*b^7 - 2*(8*a^7 + a^5*b^2 - 1
1*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 30*(a^6*b - a^4*b^3)*cos(d*x + c)^4
+ 10*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*cos(
d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x +
 c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2))*sin(d*x + c) - 10*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)
^2 - 30*(a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a
^2*b^6 + b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*d)*sin(d*x + c)), 1/15*(23*a^6*b - 34*a^4*b^3 + 14*a^2*b^5 - 3*b^7 - (8*
a^7 + a^5*b^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 15*(a^6*b - a^4*b^3
)*cos(d*x + c)^4 + 5*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3
- 15*(a^5*b*cos(d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
)))*sin(d*x + c) - 5*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 15*(
a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(
d*x + c))]
```

3.208.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.208.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(187) = 374$.

Time = 0.34 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.69

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$\frac{960\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^5b}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{-a^2+b^2}} - \frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18a}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/480*(960*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^5*b/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^4*tan(1/2*d*x
 + 1/2*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 +
 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 90*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2
*b^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*b^4*tan
(1/2*d*x + 1/2*c)^3 + 150*a^4*tan(1/2*d*x + 1/2*c) - 420*a^3*b*tan(1/2*d*x
 + 1/2*c) + 420*a^2*b^2*tan(1/2*d*x + 1/2*c) - 180*a*b^3*tan(1/2*d*x + 1/2
*c) + 30*b^4*tan(1/2*d*x + 1/2*c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5) + (150*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*a*b*tan(1/2*d*x
 + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^
2 + 40*a*b*tan(1/2*d*x + 1/2*c)^2 + 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2
+ 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^5))
/d
```

3.208.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.93

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5\,d\,(32\,a-32\,b)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4}{3\,(32\,a-32\,b)}+\frac{32\,a+32\,b}{3\,(32\,a-32\,b)^2}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5}{32\,a-32\,b}+\frac{\left(\frac{4}{32\,a-32\,b}+\frac{32\,a+32\,b}{(32\,a-32\,b)^2}\right)(32\,a+32\,b)}{32\,a-32\,b}\right)}{d} - \frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{5\,(a+b)} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-5\,a^5+11\,a^4\,b-4\,a^3\,b^2-4\,a^2\,b^3+a\,b^4+b^5\right)}{(a+b)^3} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(5\,a^4-12\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3-3\,b^4\right)}{3\,(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(32\,a^3-96\,a^2\,b+96\,a\,b^2-32\,b^3\right)} - \frac{2\,a^5\,b\,\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{(a+b)^{7/2}\,(a-b)^{5/2}}\right)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

input
```
int(1/(sin(c + d*x)^6*(a + b/cos(c + d*x))),x)
```

output

```
tan(c/2 + (d*x)/2)^5/(5*d*(32*a - 32*b)) + (tan(c/2 + (d*x)/2)^3*(4/(3*(32
*a - 32*b)) + (32*a + 32*b)/(3*(32*a - 32*b)^2)))/d + (tan(c/2 + (d*x)/2)*
(5/(32*a - 32*b) + ((4/(32*a - 32*b) + (32*a + 32*b)/(32*a - 32*b)^2)*(32*
a + 32*b))/(32*a - 32*b)))/d - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(5*(a + b)
) - (2*tan(c/2 + (d*x)/2)^4*(a*b^4 + 11*a^4*b - 5*a^5 + b^5 - 4*a^2*b^3 -
4*a^3*b^2))/(a + b)^3 + (tan(c/2 + (d*x)/2)^2*(4*a*b^3 - 12*a^3*b + 5*a^4
- 3*b^4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*tan(c/2 + (d*x)/2)^5*(96*a*b^2 - 9
6*a^2*b + 32*a^3 - 32*b^3)) - (2*a^5*b*atanh((tan(c/2 + (d*x)/2)*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(5/2))))/(d*(a + b)^(7/
2)*(a - b)^(7/2))
```

3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.209.1 Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{(a^2-7b^2)\left(a^2-b^2\right)^2\cos(c+dx)}{a^8 d}-\frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7 d}\\ &+\frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6 d}+\frac{b(3a^2-2b^2)\cos^4(c+dx)}{2a^5 d}\\ &-\frac{3(a^2-b^2)\cos^5(c+dx)}{5a^4 d}-\frac{b\cos^6(c+dx)}{3a^3 d}\\ &+\frac{\cos^7(c+dx)}{7a^2 d}+\frac{b^2(a^2-b^2)^3}{a^9 d(b+a\cos(c+dx))}\\ &+\frac{2b(a^2-4b^2)\left(a^2-b^2\right)^2\log(b+a\cos(c+dx))}{a^9 d}\end{aligned}$$

output

```
-(a^2-7*b^2)*(a^2-b^2)^2*cos(d*x+c)/a^8/d-3*b*(a^2-b^2)^2*cos(d*x+c)^2/a^7
/d+1/3*(3*a^4-9*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^6/d+1/2*b*(3*a^2-2*b^2)*cos(
d*x+c)^4/a^5/d-3/5*(a^2-b^2)*cos(d*x+c)^5/a^4/d-1/3*b*cos(d*x+c)^6/a^3/d+1
/7*cos(d*x+c)^7/a^2/d+b^2*(a^2-b^2)^3/a^9/d/(b+a*cos(d*x+c))+2*b*(a^2-4*b^
2)*(a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^9/d
```

3.209.2 Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.56

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-3675a^8+61320a^6b^2-132720a^4b^4+87360a^2b^6-13440b^8-140(21a^8-228a^6b^2+400a^4b^4-192a^2b^6)}{}$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-3675*a^8 + 61320*a^6*b^2 - 132720*a^4*b^4 + 87360*a^2*b^6 - 13440*b^8 -
140*(21*a^8 - 228*a^6*b^2 + 400*a^4*b^4 - 192*a^2*b^6)*Cos[2*(c + d*x)] -
3780*a^7*b*Cos[3*(c + d*x)] + 8400*a^5*b^3*Cos[3*(c + d*x)] - 4480*a^3*b^5
*Cos[3*(c + d*x)] + 588*a^8*Cos[4*(c + d*x)] - 1848*a^6*b^2*Cos[4*(c + d*x
)] + 1120*a^4*b^4*Cos[4*(c + d*x)] + 476*a^7*b*Cos[5*(c + d*x)] - 336*a^5*
b^3*Cos[5*(c + d*x)] - 132*a^8*Cos[6*(c + d*x)] + 112*a^6*b^2*Cos[6*(c + d
*x)] - 40*a^7*b*Cos[7*(c + d*x)] + 15*a^8*Cos[8*(c + d*x)] + 26880*a^6*b^2
*Log[b + a*Cos[c + d*x]] - 161280*a^4*b^4*Log[b + a*Cos[c + d*x]] + 241920
*a^2*b^6*Log[b + a*Cos[c + d*x]] - 107520*b^8*Log[b + a*Cos[c + d*x]] + 16
80*a*b*Cos[c + d*x]*(-8*a^6 + 67*a^4*b^2 - 116*a^2*b^4 + 56*b^6 + 16*(a^2
- 4*b^2)*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]]))/(13440*a^9*d*(b + a*Cos[c
 + d*x]))
```

3.209.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^7(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^7}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^9d}$$

$$\downarrow 522$$

$$-\frac{\int\left(-a^6\cos^6(c+dx)+2a^5b\cos^5(c+dx)+3a^4\left(a^2-b^2\right)\cos^4(c+dx)+2a^3b\left(2b^2-3a^2\right)\cos^3(c+dx)-a^2\left(3a^4\right.\right.}{}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{1}{3}a^6b\cos^6(c+dx)+3a^2b\left(a^2-b^2\right)^2\cos^2(c+dx)+a\left(a^2-7b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)-\frac{b}{a}}{}$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a*(a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x] + 3*a^2*b*(a^2 - b^2)^2*Cos[
c + d*x]^2 - (a^3*(3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/3 - (a^4*b*(
3*a^2 - 2*b^2)*Cos[c + d*x]^4)/2 + (3*a^5*(a^2 - b^2)*Cos[c + d*x]^5)/5 +
(a^6*b*Cos[c + d*x]^6)/3 - (a^7*Cos[c + d*x]^7)/7 - (b^2*(a^2 - b^2)^3)/(b
 + a*Cos[c + d*x]) - 2*b*(a^2 - 4*b^2)*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x
]])/(a^9*d))
```

3.209. $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.209.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.209.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{3}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{3a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4a^5b}{2}-\cos(dx+c)^4a^3b^3+\cos(dx+c)^3a^6-3\cos(d$
default	$\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{3}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{3a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4a^5b}{2}-\cos(dx+c)^4a^3b^3+\cos(dx+c)^3a^6-3\cos(d$
parallelrisch	$2240b(a-2b)(a+2b)(a-b)^2(a+b)^2(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-2240b(a-2b)(a+2b)(a-b)^2(a+b)^2($
risch	$\frac{7\cos(3dx+3c)}{64da^2}-\frac{35e^{i(dx+c)}}{128a^2d}-\frac{35e^{-i(dx+c)}}{128a^2d}+\frac{\cos(7dx+7c)}{448a^2d}-\frac{2b^2\left(-a^6+3a^4b^2-3a^2b^4+b^6\right)e^{i(dx+c)}}{a^9d\left(ae^{2i(dx+c)}+2be^{i(dx+c)}+a\right)}+\frac{24ib^3c}{a^5d}$
norman	$\frac{\left(96a^7+96ba^6+380a^5b^2-1596a^4b^3-1188b^4a^3+3220b^5a^2+720ab^6-1680b^7\right)(a+b)}{210a^8db}-\frac{\left(32a^8-160a^7b-32a^6b^2+172a^5b^3-644a^4b^4-19\right.}{5c}$

input
```
int(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^8*(1/7*cos(d*x+c)^7*a^6-1/3*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)
^5+3/5*a^4*b^2*cos(d*x+c)^5+3/2*cos(d*x+c)^4*a^5*b-cos(d*x+c)^4*a^3*b^3+co
s(d*x+c)^3*a^6-3*cos(d*x+c)^3*a^4*b^2+5/3*cos(d*x+c)^3*a^2*b^4-3*cos(d*x+c
)^2*a^5*b+6*cos(d*x+c)^2*a^3*b^3-3*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a^6+9*cos
(d*x+c)*a^4*b^2-15*cos(d*x+c)*a^2*b^4+7*b^6*cos(d*x+c))+b^2*(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/a^9/(b+a*cos(d*x+c))+2/a^9*b*(a^6-6*a^4*b^2+9*a^2*b^4-4*b^
6)*ln(b+a*cos(d*x+c)))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{120\,a^8\cos(dx+c)^8-160\,a^7b\cos(dx+c)^7+1715\,a^6b^2-4725\,a^4b^4+3780\,a^2b^6-840\,b^8-56\,(9\,a^8-4\,a^6$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/840*(120*a^8*cos(d*x + c)^8 - 160*a^7*b*cos(d*x + c)^7 + 1715*a^6*b^2 -
4725*a^4*b^4 + 3780*a^2*b^6 - 840*b^8 - 56*(9*a^8 - 4*a^6*b^2)*cos(d*x + c
)^6 + 84*(9*a^7*b - 4*a^5*b^3)*cos(d*x + c)^5 + 140*(6*a^8 - 9*a^6*b^2 + 4
*a^4*b^4)*cos(d*x + c)^4 - 280*(6*a^7*b - 9*a^5*b^3 + 4*a^3*b^5)*cos(d*x +
 c)^3 - 840*(a^8 - 6*a^6*b^2 + 9*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^2 + 35*
(a^7*b + 153*a^5*b^3 - 324*a^3*b^5 + 168*a*b^7)*cos(d*x + c) + 1680*(a^6*b
^2 - 6*a^4*b^4 + 9*a^2*b^6 - 4*b^8 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*
b^7)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^10*d*cos(d*x + c) + a^9*b*d
)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.01

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{210\,(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)}{a^{10}\cos(dx+c)+a^9b} + \frac{30\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-126\,(a^6-a^4b^2)\cos(dx+c)^5+105\,(3\,a^5b-2\,a^3b^3)\cos(dx+c)^4+70\,(3\,a^6-}{a^8}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/210*(210*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)/(a^10*cos(d*x + c) + a^
9*b) + (30*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 126*(a^6 - a^4*b
^2)*cos(d*x + c)^5 + 105*(3*a^5*b - 2*a^3*b^3)*cos(d*x + c)^4 + 70*(3*a^6
 - 9*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 630*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c)^2 - 210*(a^6 - 9*a^4*b^2 + 15*a^2*b^4 - 7*b^6)*cos(d*x + c))/
a^8 + 420*(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(a*cos(d*x + c) + b)/
a^9)/d
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. 2(257) = 514.

Time = 0.37 (sec) , antiderivative size = 1861, normalized size of antiderivative = 6.97

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/210*(420*(a^7*b - a^6*b^2 - 6*a^5*b^3 + 6*a^4*b^4 + 9*a^3*b^5 - 9*a^2*b^
6 - 4*a*b^7 + 4*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - a^9*b) - 420*(a^6*b
 - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1))/a^9 - 420*(a^7*b - 7*a^5*b^3 - 4*a^4*b^4 + 11*a^3*b^5 + 8*a^2
*b^6 - 5*a*b^7 - 4*b^8 + a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^6
*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^5*b^3*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 6*a^4*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a
^3*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*b^6*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 4*a*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b
^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^9) + (192*a
^7 - 1089*a^6*b - 2772*a^5*b^2 + 6534*a^4*b^3 + 5600*a^3*b^4 - 9801*a^2*b^
5 - 2940*a*b^6 + 4356*b^7 - 1344*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18144*a^5*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 49098*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 35000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 71127*
a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 17640*a*b^6*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 30492*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 4032*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos...
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.20

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\cos(c+dx)^4\left(\frac{b^3}{2a^5}+\frac{b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{2a}\right)}{d}$$

$$-\frac{\cos(c+dx)^2\left(\frac{b^2\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{2a^2}+\frac{b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{d}$$

$$-\frac{\cos(c+dx)^5\left(\frac{3}{5a^2}-\frac{3b^2}{5a^4}\right)}{d}+\frac{\cos(c+dx)^7}{7a^2d}$$

$$-\frac{\cos(c+dx)\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a^2}-\frac{2b\left(\frac{b^2\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a^2}+\frac{2b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{a}\right)}{d}$$

$$+\frac{\cos(c+dx)^3\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{3a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{3a}\right)}{d}$$

$$-\frac{b\cos(c+dx)^6}{3a^3d}-\frac{-a^6b^2+3a^4b^4-3a^2b^6+b^8}{ad\left(\cos(c+dx)\,a^9+ba^8\right)}$$

$$+\frac{\ln\left(b+a\cos(c+dx)\right)\left(2a^6b-12a^4b^3+18a^2b^5-8b^7\right)}{a^9d}$$

input
```
int(sin(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

output
```
(cos(c + d*x)^4*(b^3/(2*a^5) + (b*(3/a^2 - (3*b^2)/a^4))/(2*a)))/d - (cos(
c + d*x)^2*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a))/(2*a^2) +
(b*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/
a^2 - (3*b^2)/a^4))/a))/a))/a))/d - (cos(c + d*x)^5*(3/(5*a^2) - (3*b^2)/(
5*a^4)))/d + cos(c + d*x)^7/(7*a^2*d) - (cos(c + d*x)*(1/a^2 + (b^2*(3/a^2
 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*
b^2)/a^4))/a))/a))/a^2 - (2*b*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a
^4))/a))/a^2 + (2*b*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^
3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(
1/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/(3*a^2) - (2*b*((2*b^3)/a^5 + (2*b*(3/
a^2 - (3*b^2)/a^4))/a))/(3*a)))/d - (b*cos(c + d*x)^6)/(3*a^3*d) - (b^8 -
3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)/(a*d*(a^9*cos(c + d*x) + a^8*b)) + (log(b
 + a*cos(c + d*x))*(2*a^6*b - 8*b^7 + 18*a^2*b^5 - 12*a^4*b^3))/(a^9*d)
```

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.210.1 Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6d} - \frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d} + \frac{b\cos^4(c+dx)}{2a^3d} - \frac{\cos^5(c+dx)}{5a^2d} + \frac{b^2(a^2-b^2)^2}{a^7d(b+a\cos(c+dx))} + \frac{2b(a^4-4a^2b^2+3b^4)\log(b+a\cos(c+dx))}{a^7d}$$

output

```
-(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)/a^6/d-2*b*(a^2-b^2)*cos(d*x+c)^2/a^5/d+1
/3*(2*a^2-3*b^2)*cos(d*x+c)^3/a^4/d+1/2*b*cos(d*x+c)^4/a^3/d-1/5*cos(d*x+c
)^5/a^2/d+b^2*(a^2-b^2)^2/a^7/d/(b+a*cos(d*x+c))+2*b*(a^4-4*a^2*b^2+3*b^4)
*ln(b+a*cos(d*x+c))/a^7/d
```

3.210.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.44

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-150a^6+1740a^4b^2-2160a^2b^4+480b^6-5(25a^6-168a^4b^2+144a^2b^4)\cos(2(c+dx))-115a^5b\cos(3(c+$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-150*a^6 + 1740*a^4*b^2 - 2160*a^2*b^4 + 480*b^6 - 5*(25*a^6 - 168*a^4*b^
2 + 144*a^2*b^4)*Cos[2*(c + d*x)] - 115*a^5*b*Cos[3*(c + d*x)] + 120*a^3*b
^3*Cos[3*(c + d*x)] + 22*a^6*Cos[4*(c + d*x)] - 30*a^4*b^2*Cos[4*(c + d*x)
] + 9*a^5*b*Cos[5*(c + d*x)] - 3*a^6*Cos[6*(c + d*x)] + 960*a^4*b^2*Log[b
+ a*Cos[c + d*x]] - 3840*a^2*b^4*Log[b + a*Cos[c + d*x]] + 2880*b^6*Log[b
+ a*Cos[c + d*x]] + 120*a*b*Cos[c + d*x]*(-4*a^4 + 23*a^2*b^2 - 20*b^4 + 8
*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]]))/(480*a^7*d*(b + a*Cos
[c + d*x]))
```

3.210.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}\,dx$$
$$\downarrow 4360$$
$$\int \frac{\sin^5(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}\,dx$$
$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^5}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^5d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^7d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\cos^4(c+dx)a^4+\left(\frac{5b^4-6a^2b^2}{a^4}+1\right)a^4-2b\cos^3(c+dx)a^3-\left(2a^2-3b^2\right)\cos^2(c+dx)a^2-4b\left(b^2-a^2\right)\cos(c+\right.}{a^7d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)-\frac{1}{2}a^4b\cos^4(c+dx)+2a^2b\left(a^2-b^2\right)\cos^2(c+dx)-\frac{b^2\left(a^2-b^2\right)^2}{a\cos(c+dx)+b}+a\left(a^4-6a^2b^2+5b^4\right)\cos(c+}{a^7d}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a*(a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x] + 2*a^2*b*(a^2 - b^2)*Cos[c +
d*x]^2 - (a^3*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/3 - (a^4*b*Cos[c + d*x]^4)/2
 + (a^5*Cos[c + d*x]^5)/5 - (b^2*(a^2 - b^2)^2)/(b + a*Cos[c + d*x]) - 2*b
*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]])/(a^7*d))
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.210.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{2}-\frac{2\cos(dx+c)^3a^4}{3}+\cos(dx+c)^3a^2b^2+2\cos(dx+c)^2a^3b-2\cos(dx+c)^2ab^3+\cos(dx+c)a^4-6\cos(dx+c)}{a^6}}{d}$
default	$-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{2}-\frac{2\cos(dx+c)^3a^4}{3}+\cos(dx+c)^3a^2b^2+2\cos(dx+c)^2a^3b-2\cos(dx+c)^2ab^3+\cos(dx+c)a^4-6\cos(dx+c)}{a^6}}{d}$
parallelrisch	$\frac{960b^2(a-b)(a+b)(a^2-3b^2)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-960b^2(a-b)(a+b)(a^2-3b^2)(b+a\cos(dx+c)}{}$
risch	$-\frac{4ibc}{a^3d}+\frac{16ib^3c}{a^5d}-\frac{12ib^5c}{a^7d}+\frac{5\cos(3dx+3c)}{48da^2}+\frac{2b^2\left(a^4-2a^2b^2+b^4\right)e^{i(dx+c)}}{a^7d\left(ae^{2i(dx+c)}+2be^{i(dx+c)}+a\right)}-\frac{5e^{i(dx+c)}}{16a^2d}+\frac{8ib^3x}{a^5}-\frac{6ib^5}{a^7}$
norman	$\frac{\left(32a^5+32a^4b+96a^3b^2-360a^2b^3-144ab^4+360b^5\right)(a+b)}{60a^6bd}-\frac{\left(32a^6-128a^5b+120a^3b^3-408a^2b^4-72ab^5+360b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{12a^6bd}+\frac{(32a}{}$

input
```
int(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^6*(1/5*cos(d*x+c)^5*a^4-1/2*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*
a^4+cos(d*x+c)^3*a^2*b^2+2*cos(d*x+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^3+cos(d*x
+c)*a^4-6*cos(d*x+c)*a^2*b^2+5*cos(d*x+c)*b^4)+b^2*(a^4-2*a^2*b^2+b^4)/a^7
/(b+a*cos(d*x+c))+2/a^7*b*(a^4-4*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c)))
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{48\,a^6\cos(dx+c)^6-72\,a^5b\cos(dx+c)^5-435\,a^4b^2+720\,a^2b^4-240\,b^6-40\left(4\,a^6-3\,a^4b^2\right)\cos(dx+c}{}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/240*(48*a^6*cos(d*x + c)^6 - 72*a^5*b*cos(d*x + c)^5 - 435*a^4*b^2 + 72
0*a^2*b^4 - 240*b^6 - 40*(4*a^6 - 3*a^4*b^2)*cos(d*x + c)^4 + 80*(4*a^5*b
 - 3*a^3*b^3)*cos(d*x + c)^3 + 240*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*cos(d*x +
c)^2 + 15*(3*a^5*b - 80*a^3*b^3 + 80*a*b^5)*cos(d*x + c) - 480*(a^4*b^2 -
4*a^2*b^4 + 3*b^6 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*log(a*cos(
d*x + c) + b))/(a^8*d*cos(d*x + c) + a^7*b*d)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{30\,(a^4b^2-2\,a^2b^4+b^6)}{a^8\cos(dx+c)+a^7b} - \frac{6\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-10\,(2\,a^4-3\,a^2b^2)\cos(dx+c)^3+60\,(a^3b-ab^3)\cos(dx+c)^2+30\,(a^4-6\,a^2b^2+5\,b^4)\,\cos(dx+c)}{a^6}}{30\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*(30*(a^4*b^2 - 2*a^2*b^4 + b^6)/(a^8*cos(d*x + c) + a^7*b) - (6*a^4*c
os(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 10*(2*a^4 - 3*a^2*b^2)*cos(d*x +
 c)^3 + 60*(a^3*b - a*b^3)*cos(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + 5*b^4)*c
os(d*x + c))/a^6 + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(a*cos(d*x + c) + b)/
a^7)/d
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(188) = 376.

Time = 0.34 (sec) , antiderivative size = 1102, normalized size of antiderivative = 5.68

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(60*(a^5*b - a^4*b^2 - 4*a^3*b^3 + 4*a^2*b^4 + 3*a*b^5 - 3*b^6)*log(a
bs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1)))/(a^8 - a^7*b) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^7 - 60*(a^5*b - 5*a^3*b^3
 - 3*a^2*b^4 + 4*a*b^5 + 3*b^6 + a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 3*a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*b^6*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^7) + (32*a^5 - 137*a^4*b - 30
0*a^3*b^2 + 548*a^2*b^3 + 300*a*b^4 - 411*b^5 - 160*a^5*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 13
20*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2980*a^2*b^3*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 1200*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + 2055*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^5*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 - 1920*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 62
00*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1800*a*b^4*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4110*b^5*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 1970*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1...
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.30

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\cos(c+dx)^3\left(\frac{2}{3a^2}-\frac{b^2}{a^4}\right)}{d} - \frac{\cos(c+dx)^2\left(\frac{b^3}{a^5}+\frac{b\left(\frac{2}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{d} - \frac{\cos(c+dx)\left(\frac{1}{a^2}+\frac{b^2\left(\frac{2}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{2}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{d} - \frac{\cos(c+dx)^5}{5a^2d} + \frac{b\cos(c+dx)^4}{2a^3d} + \frac{\ln(b+a\cos(c+dx))\,(2a^4b-8a^2b^3+6b^5)}{a^7d} + \frac{a^4b^2-2a^2b^4+b^6}{ad\,(\cos(c+dx)\,a^7+ba^6)}$$

input
```
int(sin(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

output
```
(cos(c + d*x)^3*(2/(3*a^2) - b^2/a^4))/d - (cos(c + d*x)^2*(b^3/a^5 + (b*(
2/a^2 - (3*b^2)/a^4))/a))/d - (cos(c + d*x)*(1/a^2 + (b^2*(2/a^2 - (3*b^2)
/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(2/a^2 - (3*b^2)/a^4))/a))/a))/d - c
os(c + d*x)^5/(5*a^2*d) + (b*cos(c + d*x)^4)/(2*a^3*d) + (log(b + a*cos(c
+ d*x))*(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b^6 - 2*a^2*b^4 + a^4*b^
2)/(a*d*(a^7*cos(c + d*x) + a^6*b))
```

3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.211.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(a^2-3b^2)\cos(c+dx)}{a^4 d} - \frac{b\cos^2(c+dx)}{a^3 d} + \frac{\cos^3(c+dx)}{3a^2 d} + \frac{b^2(a^2-b^2)}{a^5 d(b+a\cos(c+dx))} + \frac{2b(a^2-2b^2)\log(b+a\cos(c+dx))}{a^5 d}$$

output

```
-(a^2-3*b^2)*cos(d*x+c)/a^4/d-b*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^2/d+
b^2*(a^2-b^2)/a^5/d/(b+a*cos(d*x+c))+2*b*(a^2-2*b^2)*ln(b+a*cos(d*x+c))/a^
5/d
```

3.211.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-9a^4+60a^2b^2-24b^4-8(a^4-3a^2b^2)\cos(2(c+dx))-4a^3b\cos(3(c+dx))+a^4\cos(4(c+dx))+48a^2b^2}{24a^5}$$

input

```
Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-9*a^4 + 60*a^2*b^2 - 24*b^4 - 8*(a^4 - 3*a^2*b^2)*Cos[2*(c + d*x)] - 4*a
^3*b*Cos[3*(c + d*x)] + a^4*Cos[4*(c + d*x)] + 48*a^2*b^2*Log[b + a*Cos[c
+ d*x]] - 96*b^4*Log[b + a*Cos[c + d*x]] + 24*a*b*Cos[c + d*x]*(-a^2 + 3*b
^2 + 2*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]]))/(24*a^5*d*(b + a*Cos[c + d*
x]))
```

3.211.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^3(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^3}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)(a^2-a^2\cos^2(c+dx))}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+\left(1-\frac{3b^2}{a^2}\right)a^2+2b\cos(c+dx)a+\frac{2b(2b^2-a^2)}{b+a\cos(c+dx)}-\frac{b^2(b^2-a^2)}{(b+a\cos(c+dx))^2}\right)d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a\left(a^2-3b^2\right)\cos(c+dx)-\frac{b^2(a^2-b^2)}{a\cos(c+dx)+b}-2b\left(a^2-2b^2\right)\log(a\cos(c+dx)+b)+a^2b\cos^2(c+}{a^5 d}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a*(a^2 - 3*b^2)*Cos[c + d*x] + a^2*b*Cos[c + d*x]^2 - (a^3*Cos[c + d*x]
^3)/3 - (b^2*(a^2 - b^2))/(b + a*Cos[c + d*x]) - 2*b*(a^2 - 2*b^2)*Log[b +
 a*Cos[c + d*x]])/(a^5*d))
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.211.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^3a^2}{3}-ab\cos(dx+c)^2-\cos(dx+c)a^2+3\cos(dx+c)b^2}{a^4}+\frac{b^2\left(a^2-b^2\right)}{a^5(b+a\cos(dx+c))}+\frac{2b\left(a^2-2b^2\right)\ln(b+a\cos(dx+c))}{a^5}$ / d
default	$\frac{\frac{\cos(dx+c)^3a^2}{3}-ab\cos(dx+c)^2-\cos(dx+c)a^2+3\cos(dx+c)b^2}{a^4}+\frac{b^2\left(a^2-b^2\right)}{a^5(b+a\cos(dx+c))}+\frac{2b\left(a^2-2b^2\right)\ln(b+a\cos(dx+c))}{a^5}$ / d
parallelrisch	$\frac{48b^2\left(a^2-2b^2\right)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-48b^2\left(a^2-2b^2\right)(b+a\cos(dx+c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+(-}{24da^5b(b+}$
risch	$-\frac{2ibx}{a^3}+\frac{4ib^3x}{a^5}-\frac{b\mathrm{e}^{2i(dx+c)}}{4a^3d}-\frac{3\mathrm{e}^{i(dx+c)}}{8a^2d}+\frac{3\mathrm{e}^{i(dx+c)}b^2}{2a^4d}-\frac{3\mathrm{e}^{-i(dx+c)}}{8a^2d}+\frac{3\mathrm{e}^{-i(dx+c)}b^2}{2a^4d}-\frac{b\mathrm{e}^{-2i(dx+c)}}{4a^3d}+$
norman	$-\frac{\left(4a^4-8a^3b+12a^2b^2+16ab^3-24b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{6da^4b}+\frac{\left(4a^3+4a^2b+8ab^2-24b^3\right)(a+b)}{6a^4bd}-\frac{\left(4a^4-12a^3b+4a^2b^2+8ab^3-24b^4\right)\tan\left(\frac{dx}{2}\right.}{3a^4bd}$ / $\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)$

input
```
int(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^4*(1/3*cos(d*x+c)^3*a^2-a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+3*cos(d*x
+c)*b^2)+b^2*(a^2-b^2)/a^5/(b+a*cos(d*x+c))+2/a^5*b*(a^2-2*b^2)*ln(b+a*cos
(d*x+c)))
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{2\,a^4\cos(dx+c)^4 - 4\,a^3b\cos(dx+c)^3 + 9\,a^2b^2 - 6\,b^4 - 6\,(a^4 - 2\,a^2b^2)\cos(dx+c)^2 - 3\,(a^3b - 6\,ab^3)\cos}{6\,(a^6d\cos(dx+c) + a^5bd)}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*a^4*cos(d*x + c)^4 - 4*a^3*b*cos(d*x + c)^3 + 9*a^2*b^2 - 6*b^4 - 6
*(a^4 - 2*a^2*b^2)*cos(d*x + c)^2 - 3*(a^3*b - 6*a*b^3)*cos(d*x + c) + 12*
(a^2*b^2 - 2*b^4 + (a^3*b - 2*a*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b)
)/(a^6*d*cos(d*x + c) + a^5*b*d)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{3\,(a^2b^2-b^4)}{a^6\cos(dx+c)+a^5b} + \frac{a^2\cos(dx+c)^3 - 3\,ab\cos(dx+c)^2 - 3\,(a^2-3\,b^2)\cos(dx+c)}{a^4} + \frac{6\,(a^2b-2\,b^3)\log(a\cos(dx+c)+b)}{a^5}}{3\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*(3*(a^2*b^2 - b^4)/(a^6*cos(d*x + c) + a^5*b) + (a^2*cos(d*x + c)^3 -
3*a*b*cos(d*x + c)^2 - 3*(a^2 - 3*b^2)*cos(d*x + c))/a^4 + 6*(a^2*b - 2*b^
3)*log(a*cos(d*x + c) + b)/a^5)/d
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{2\left(a^2b-2b^3\right)\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^5d} + \frac{a^2b^2-b^4}{\left(a\cos\left(dx+c\right)+b\right)a^5d}$$
$$+ \frac{a^4d^5\cos\left(dx+c\right)^3 - 3\,a^3bd^5\cos\left(dx+c\right)^2 - 3\,a^4d^5\cos\left(dx+c\right) + 9\,a^2b^2d^5\cos\left(dx+c\right)}{3\,a^6d^6}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(a^2*b - 2*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^5*d) + (a^2*b^2 - b^4)/
((a*cos(d*x + c) + b)*a^5*d) + 1/3*(a^4*d^5*cos(d*x + c)^3 - 3*a^3*b*d^5*c
os(d*x + c)^2 - 3*a^4*d^5*cos(d*x + c) + 9*a^2*b^2*d^5*cos(d*x + c))/(a^6*
d^6)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{\cos\left(c+d\,x\right)\left(\frac{1}{a^2}-\frac{3\,b^2}{a^4}\right) - \frac{\cos(c+d\,x)^3}{3\,a^2} + \frac{b\cos(c+d\,x)^2}{a^3} - \frac{\ln(b+a\,\cos(c+d\,x))\left(2\,a^2\,b-4\,b^3\right)}{a^5} + \frac{b^4-a^2\,b^2}{a\left(\cos(c+d\,x)\,a^5+b\,a^4\right)}}{d}$$

input
```
int(sin(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

output
```
-(cos(c + d*x)*(1/a^2 - (3*b^2)/a^4) - cos(c + d*x)^3/(3*a^2) + (b*cos(c +
 d*x)^2)/a^3 - (log(b + a*cos(c + d*x))*(2*a^2*b - 4*b^3))/a^5 + (b^4 - a^
2*b^2)/(a*(a^5*cos(c + d*x) + a^4*b)))/d
```

3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.212.9 Mupad [B] (verification not implemented) . . . 1570

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{a^2 d} + \frac{b^2}{a^3 d(b+a\cos(c+dx))} + \frac{2b\log(b+a\cos(c+dx))}{a^3 d}$$

output
```
-cos(d*x+c)/a^2/d+b^2/a^3/d/(b+a*cos(d*x+c))+2*b*ln(b+a*cos(d*x+c))/a^3/d
```

3.212.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-a^2\cos^2(c+dx) + ab\cos(c+dx)(-1+2\log(b+a\cos(c+dx))) + b^2(1+2\log(b+a\cos(c+dx)))}{a^3 d(b+a\cos(c+dx))}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*Cos[c + d*x]^2) + a*b*Cos[c + d*x]*(-1 + 2*Log[b + a*Cos[c + d*x]])
 + b^2*(1 + 2*Log[b + a*Cos[c + d*x]]))/(a^3*d*(b + a*Cos[c + d*x]))
```

3.212.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4360, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos^2(c+dx)}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))^2}d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{b^2}{(b+a\cos(c+dx))^2}-\frac{2b}{b+a\cos(c+dx)}+1\right)d(a\cos(c+dx))}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^2}{a\cos(c+dx)+b}-2b\log(a\cos(c+dx)+b)+a\cos(c+dx)}{a^3d}$$

input
```
Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a*Cos[c + d*x] - b^2/(b + a*Cos[c + d*x]) - 2*b*Log[b + a*Cos[c + d*x]]
)/(a^3*d))
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.212.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{-\frac{1}{a^2\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^3}-\frac{b}{a^2(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^3}}{d}$
default	$\frac{-\frac{1}{a^2\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^3}-\frac{b}{a^2(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^3}}{d}$
risch	$-\frac{2ibx}{a^3}-\frac{e^{i(dx+c)}}{2a^2d}-\frac{e^{-i(dx+c)}}{2a^2d}-\frac{4ibc}{a^3d}+\frac{2b^2e^{i(dx+c)}}{a^3d(ae^{2i(dx+c)}+2be^{i(dx+c)}+a)}+\frac{2b\ln\left(e^{2i(dx+c)}+\frac{2be^{i(dx+c)}}{a}+1\right)}{a^3d}$
parallelrisch	$\frac{4\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)ab^2\cos(dx+c)-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)ab^2\cos(dx+c)-2a^3\cos(dx+c)-4\cos(dx+c)ab^2+}{2a^3db(b+a\cos(dx+c))}$
norman	$\frac{\frac{2a^2+4ab+4b^2}{2da^2b}-\frac{\left(2a^2-4ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2da^2b}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{2b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}+\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-t}{a^3}$

input
```
int(sin(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2/sec(d*x+c)-2/a^3*b*ln(sec(d*x+c))-b/a^2/(a+b*sec(d*x+c))+2/a^3
*b*ln(a+b*sec(d*x+c)))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\cos(dx+c)^2+ab\cos(dx+c)-b^2-2\left(ab\cos(dx+c)+b^2\right)\log\left(a\cos(dx+c)+b\right)}{a^4d\cos(dx+c)+a^3bd}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*cos(d*x + c)^2 + a*b*cos(d*x + c) - b^2 - 2*(a*b*cos(d*x + c) + b^2)
*log(a*cos(d*x + c) + b))/(a^4*d*cos(d*x + c) + a^3*b*d)
```

3.212.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{b^2}{a^4\cos(dx+c)+a^3b} - \frac{\cos(dx+c)}{a^2} + \frac{2\,b\log(a\cos(dx+c)+b)}{a^3}}{d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(b^2/(a^4*cos(d*x + c) + a^3*b) - cos(d*x + c)/a^2 + 2*b*log(a*cos(d*x + c
) + b)/a^3)/d
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\cos\left(dx+c\right)}{a^2d} + \frac{2\,b\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^3d} + \frac{b^2}{(a\cos\left(dx+c\right)+b)a^3d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a^2*d) + 2*b*log(abs(-a*cos(d*x + c) - b))/(a^3*d) + b^2/((
a*cos(d*x + c) + b)*a^3*d)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{d\,(\cos(c+d\,x)\,a^4+b\,a^3)} - \frac{\cos(c+d\,x)}{a^2\,d} + \frac{2\,b\,\ln(b+a\,\cos(c+d\,x))}{a^3\,d}$$

input
```
int(sin(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

output
```
b^2/(d*(a^4*cos(c + d*x) + a^3*b)) - cos(c + d*x)/(a^2*d) + (2*b*log(b + a
*cos(c + d*x)))/(a^3*d)
```

3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.213.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d} + \frac{2ab\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^2 d}$$

output
```
b^2/a/(a^2-b^2)/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^2/d-1/2*ln(1
+cos(d*x+c))/(a-b)^2/d+2*a*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

3.213.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-a^2\cos(c+dx)\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2ab\log(b+a\cos(c+dx)) - (a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{a(a-b)^2}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*Cos[c + d*x]*((a + b)^2*Log[Cos[(c + d*x)/2]] - 2*a*b*Log[b + a*Cos
[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]])) + b*(-(a*(a + b)^2*Log[Cos[
(c + d*x)/2]]) + 2*a^2*b*Log[b + a*Cos[c + d*x]] + (a - b)*(b*(a + b) + a*
(a - b)*Log[Sin[(c + d*x)/2]])))/(a*(a - b)^2*(a + b)^2*d*(b + a*Cos[c + d
*x]))
```

3.213.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4360, 3042, 3316, 27, 603, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3316$$

$$\frac{a\int \frac{\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))}d(-a\cos(c+dx))}{ad}$$

$$\downarrow 603$$

$$\frac{\frac{b^2}{(a^2-b^2)(a\cos(c+dx)+b)} - \frac{\int \frac{a^2(b-a\cos(c+dx))}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)} d(-a\cos(c+dx))}{a^2-b^2}}{ad}$$

$$\downarrow 27$$

$$\frac{\frac{b^2}{(a^2-b^2)(a\cos(c+dx)+b)} - \frac{a^2\int \frac{b-a\cos(c+dx)}{(b+a\cos(c+dx))\left(a^2-a^2\cos^2(c+dx)\right)} d(-a\cos(c+dx))}{a^2-b^2}}{ad}$$

$$\downarrow 657$$

$$\frac{\frac{b^2}{(a^2-b^2)(a\cos(c+dx)+b)} - \frac{a^2\int \left(\frac{-a-b}{2a(a-b)(\cos(c+dx)a+a)} + \frac{b-a}{2a(a+b)(a-a\cos(c+dx))} + \frac{2b}{(a-b)(a+b)(b+a\cos(c+dx))}\right) d(-a\cos(c+dx))}{a^2-b^2}}{ad}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2}{(a^2-b^2)(a\cos(c+dx)+b)} - \frac{a^2\left(-\frac{2b\log(a\cos(c+dx)+b)}{a^2-b^2} - \frac{(a-b)\log(a-a\cos(c+dx))}{2a(a+b)} + \frac{(a+b)\log(a\cos(c+dx)+a)}{2a(a-b)}\right)}{a^2-b^2}}{ad}$$

input
```
Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2/((a^2 - b^2)*(b + a*Cos[c + d*x])) - (a^2*(-1/2*((a - b)*Log[a - a*Co
s[c + d*x]])/(a*(a + b)) + ((a + b)*Log[a + a*Cos[c + d*x]])/(2*a*(a - b))
 - (2*b*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)))/(a^2 - b^2))/(a*d)
```

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.213.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
parallelrisch	$\frac{(2a^2b\cos(dx+c)+2ab^2)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+(a-b)^2(b+a\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^2(\cos(dx+c)+1)(}{d(a-b)^2(a+b)^2(b+a\cos(dx+c))}$
norman	$-\frac{2b^2}{d(a^3-a^2b-ab^2+b^3)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2+2ab+b^2)}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan(}{d(a^4-2a^2b^2+}$
risch	$\frac{ix}{a^2-2ab+b^2}+\frac{ic}{d(a^2-2ab+b^2)}-\frac{ix}{a^2+2ab+b^2}-\frac{ic}{d(a^2+2ab+b^2)}-\frac{4iabx}{a^4-2a^2b^2+b^4}-\frac{4iabc}{d(a^4-2a^2b^2+b^4)}-\frac{}{ad(-a^}$

input
```
int(csc(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a-b)^2*ln(cos(d*x+c)+1)+1/2/(a+b)^2*ln(cos(d*x+c)-1)+b^2/(a+b)/
(a-b)/a/(b+a*cos(d*x+c))+2*a*b/(a+b)^2/(a-b)^2*ln(b+a*cos(d*x+c)))
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.93

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{2\,a^2b^2 - 2\,b^4 + 4\,(a^3b\cos(dx+c) + a^2b^2)\log(a\cos(dx+c)+b) - (a^3b + 2\,a^2b^2 + ab^3 + (a^4 + 2\,a^3b + a^2$$
$$2\,((a^6 - 2\,a^4b^2 + a^2b^4$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*b^2 - 2*b^4 + 4*(a^3*b*cos(d*x + c) + a^2*b^2)*log(a*cos(d*x +
c) + b) - (a^3*b + 2*a^2*b^2 + a*b^3 + (a^4 + 2*a^3*b + a^2*b^2)*cos(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) + (a^3*b - 2*a^2*b^2 + a*b^3 + (a^4 - 2*a
^3*b + a^2*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^6 - 2*a^4*
b^2 + a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)
```

3.213.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4ab\log(a\cos(dx+c)+b)}{a^4-2a^2b^2+b^4}+\frac{2b^2}{a^3b-ab^3+(a^4-a^2b^2)\cos(dx+c)}-\frac{\log(\cos(dx+c)+1)}{a^2-2ab+b^2}+\frac{\log(\cos(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) + 2*b^2/(a^3*b
- a*b^3 + (a^4 - a^2*b^2)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^2 - 2*a
*b + b^2) + log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(105) = 210.

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.95

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4ab\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2a^2b^2+b^4}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2}-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^3+a^2b-ab^2-b^3)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{2d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + log(abs(-c
os(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a*b + b^2
 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 + a^2*b - a*b^2 - b^3)
*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))))/d
```

3.213.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln(\cos(c+dx)-1)}{2d(a+b)^2} - \frac{\ln(\cos(c+dx)+1)}{2d(a-b)^2} + \frac{b^2}{ad\,(a^2-b^2)\,(b+a\cos(c+dx))} + \frac{2ab\ln(b+a\cos(c+dx))}{d(a^2-b^2)^2}$$

input
```
int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

output
```
log(cos(c + d*x) - 1)/(2*d*(a + b)^2) - log(cos(c + d*x) + 1)/(2*d*(a - b)
^2) + b^2/(a*d*(a^2 - b^2)*(b + a*cos(c + d*x))) + (2*a*b*log(b + a*cos(c
+ d*x)))/(d*(a^2 - b^2)^2)
```

3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.214.1 Optimal result . . . 1578
3.214.2 Mathematica [A] (verified) . . . 1578
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3.214.9 Mupad [B] (verification not implemented) . . . 1585

3.214.1 Optimal result

Integrand size = 21, antiderivative size = 168

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{ab^2}{\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))}\\ &+\frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d}\\ &+\frac{(a-b)\log(1-\cos(c+dx))}{4(a+b)^3 d}-\frac{(a+b)\log(1+\cos(c+dx))}{4(a-b)^3 d}\\ &+\frac{2ab(a^2+b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^3 d}\end{aligned}$$

output

```
a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/2*(2*a*b-(a^2+b^2)*cos(d*x+c))*csc(
d*x+c)^2/(a^2-b^2)^2/d+1/4*(a-b)*ln(1-cos(d*x+c))/(a+b)^3/d-1/4*(a+b)*ln(1
+cos(d*x+c))/(a-b)^3/d+2*a*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

3.214.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

$$\begin{aligned}&\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx\\ &=\frac{(b+a\cos(c+dx))\left(\frac{8ab^2}{(a-b)^2(a+b)^2}-\frac{(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{4(a+b)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3}+\frac{16ab(a^2+b^2}{8d(a+b\sec}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*((8*a*b^2)/((a - b)^2*(a + b)^2) - ((b + a*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a + b)*(b + a*Cos[c + d*x])*Log[C
os[(c + d*x)/2]])/(-a + b)^3 + (16*a*b*(a^2 + b^2)*(b + a*Cos[c + d*x])*Lo
g[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (4*(a - b)*(b + a*Cos[c + d*x])*Log
[Sin[(c + d*x)/2]])/(a + b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a - b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 3042, 3316, 27, 601, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3316$$

$$\frac{a^3\int \frac{\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a\int \frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))^2}d(-a\cos(c+dx))}{d}$$

$$\downarrow 601$$

$$\frac{a\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{2\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)}-\frac{\int\frac{-\frac{\left(a^2+b^2\right)\cos^2(c+dx)a^4}{\left(a^2-b^2\right)^2}+\frac{2b\cos(c+dx)a^3}{a^2-b^2}+\frac{b^2\left(a^2+b^2\right)a^2}{\left(a^2-b^2\right)^2}}{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{2a^2}\right)}{d}$$

$$\downarrow 2160$$

$$\frac{a\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{2\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)}-\frac{\int\left(\frac{4b\left(a^2+b^2\right)a^2}{(a-b)^3(a+b)^3(b+a\cos(c+dx))}-\frac{2b^2a^2}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}-\frac{(a-b)a}{2(a+b)^3(a-a\cos(c+dx))}-\frac{(a+b)a}{2(a-b)^3(\cos(c+dx)}\right.}{2a^2}\right)}{d}$$

$$\downarrow 2009$$

$$\frac{a\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{2\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)}-\frac{-\frac{2a^2b^2}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^3}-\frac{a(a-b)\log(a-a\cos(c+dx))}{2(a+b)^3}+\frac{a(a+b)\log(a\cos(c+d}{2(a-b)^3}}{2a^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(a*((2*a^2*b - a*(a^2 + b^2)*Cos[c + d*x])/(2*(a^2 - b^2)^2*(a^2 - a^2*Cos
[c + d*x]^2)) - ((-2*a^2*b^2)/((a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (a*(a
 - b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^3) + (a*(a + b)*Log[a + a*Cos[c
+ d*x]])/(2*(a - b)^3) - (4*a^2*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^
2 - b^2)^3)/(2*a^2)))/d
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.214.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(-a-b)\ln(\cos(dx+c)+1)}{4(a-b)^3}+\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(a-b)\ln(\cos(dx+c)-1)}{4(a+b)^3}+\frac{b^2a}{(a+b)^2(a-b)^2(b+a\cos(dx+}}{d}$
default	$\frac{\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(-a-b)\ln(\cos(dx+c)+1)}{4(a-b)^3}+\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(a-b)\ln(\cos(dx+c)-1)}{4(a+b)^3}+\frac{b^2a}{(a+b)^2(a-b)^2(b+a\cos(dx+}}{d}$
norman	$\frac{\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d(a-b)}-\frac{\left(a^4+14a^2b^2+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4d\left(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^3+3a^2b+3ab^2+b^3)}+\frac{2ab(a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{}{}\right.\right.}{d(a^6-3a^4b}$
parallelrisch	$\frac{256ba(a^2+b^2)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+64(a-b)^4(b+a\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(((-8a^5}{}$
risch	$-\frac{iac}{2d(a^3+3a^2b+3ab^2+b^3)}-\frac{4iab^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{iax}{2a^3-6a^2b+6ab^2-2b^3}-\frac{4ia^3bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{}{d(a^6-3a}$

input
```
int(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a-b)^2/(cos(d*x+c)+1)+1/4/(a-b)^3*(-a-b)*ln(cos(d*x+c)+1)+1/4/(a
+b)^2/(cos(d*x+c)-1)+1/4*(a-b)/(a+b)^3*ln(cos(d*x+c)-1)+b^2/(a+b)^2*a/(a-b
)^2/(b+a*cos(d*x+c))+2*a*b*(a^2+b^2)/(a-b)^3/(a+b)^3*ln(b+a*cos(d*x+c)))
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(162) = 324.

Time = 0.36 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.75

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{8a^3b^2-8ab^4-2\left(a^5+2a^3b^2-3ab^4\right)\cos(dx+c)^2+2\left(a^4b-2a^2b^3+b^5\right)\cos(dx+c)+8\left(a^3b^2+ab^4\right.}{}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a^3*b^2 - 8*a*b^4 - 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 +
 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 8*(a^3*b^2 + a*b^4 - (a^4*b +
a^2*b^3)*cos(d*x + c)^3 - (a^3*b^2 + a*b^4)*cos(d*x + c)^2 + (a^4*b + a^2*
b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 4*a^3*b^2 + 6*a^2*b^
3 + 4*a*b^4 + b^5 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*
x + c)^3 - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*cos(d*x + c)^2
 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5 - (a^5
 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 4*a^3
*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*
b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3 + (a^6*b - 3*a^4*b^3 +
3*a^2*b^5 - b^7)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*
d*cos(d*x + c) - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)
```

3.214.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.63

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,(a^3b+ab^3)\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(a+b)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(a-b)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(4\,ab^2-(a^3+3\,ab^2)\,\cos\right.}{a^4b-2\,a^2b^3+b^5-(a^5-2\,a^3b^2+ab^4)\cos(dx+c)^3-(}}{4\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(8*(a^3*b + a*b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - (a + b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
 (a - b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b^
2 - (a^3 + 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - b^3)*cos(d*x + c))/(a^4*b -
2*a^2*b^3 + b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 2*a^
2*b^3 + b^5)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)))/d
```

3.214.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(162) = 324$.

Time $= 0.36$ (sec) , antiderivative size $= 456$, normalized size of antiderivative $= 2.71$

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{2\,(a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{16\,(a^3b+ab^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{a^3-a^2b-ab^2+b^3-\frac{8\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab^2(}{\cos}}{(a^4-2\,a^2b^2+b^4)\left(\frac{a}{}\right.}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(2*(a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 16*(a^3*b + a*b^3)*log(abs(-a - b - a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a^3 - a^2*b - a*b^2 + b^3 - 8*a^2*b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a^2*b*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^4 - 2*a
^2*b^2 + b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/((a^2 - 2*
a*b + b^2)*(cos(d*x + c) + 1)))/d
```

3.214.9 Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.36

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2ab^2}{(a^2-b^2)^2} + \frac{b\cos(c+dx)}{2(a^2-b^2)} - \frac{\cos(c+dx)^2(a^3+3ab^2)}{2(a^4-2a^2b^2+b^4)}}{d\left(-a\cos(c+dx)^3 - b\cos(c+dx)^2 + a\cos(c+dx) + b\right)} - \frac{\ln(\cos(c+dx)-1)\left(\frac{b}{2(a+b)^3} - \frac{1}{4(a+b)^2}\right)}{d} + \frac{\ln(b+a\cos(c+dx))(2a^3b+2ab^3)}{d(a^6-3a^4b^2+3a^2b^4-b^6)} - \frac{\ln(\cos(c+dx)+1)(a+b)}{4d(a-b)^3}$$

input
```
int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

output
```
((2*a*b^2)/(a^2 - b^2)^2 + (b*cos(c + d*x))/(2*(a^2 - b^2)) - (cos(c + d*x
)^2*(3*a*b^2 + a^3))/(2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) -
 a*cos(c + d*x)^3 - b*cos(c + d*x)^2)) - (log(cos(c + d*x) - 1)*(b/(2*(a +
 b)^3) - 1/(4*(a + b)^2)))/d + (log(b + a*cos(c + d*x))*(2*a*b^3 + 2*a^3*b
))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(cos(c + d*x) + 1)*(a + b
))/(4*d*(a - b)^3)
```

3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.215.1 Optimal result

Integrand size = 21, antiderivative size = 259

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {}& \frac{a^3b^2}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} \\ &+ \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} \\ &+ \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^2 d} \\ &+ \frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16(a+b)^4 d} \\ &- \frac{(3a^2+4ab-b^2)\log(1+\cos(c+dx))}{16(a-b)^4 d} \\ &+ \frac{2a^3b(a^2+2b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4 d}\end{aligned}$$

output

```
a^3*b^2/(a^2-b^2)^3/d/(b+a*cos(d*x+c))+1/8*(8*a*b*(a^2+b^2)-(3*a^4+12*a^2*
b^2+b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^3/d+1/4*(2*a*b-(a^2+b^2)*cos(d
*x+c))*csc(d*x+c)^4/(a^2-b^2)^2/d+1/16*(3*a^2-4*a*b-b^2)*ln(1-cos(d*x+c))/
(a+b)^4/d-1/16*(3*a^2+4*a*b-b^2)*ln(1+cos(d*x+c))/(a-b)^4/d+2*a^3*b*(a^2+2
*b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.215.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.24

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{(b+a\cos(c+dx))\left(\frac{64a^3b^2}{(a-b)^3(a+b)^3} + \frac{2(-3a+b)(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} - \frac{(b+a\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{8(-3a^2-4ab}{}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*((64*a^3*b^2)/((a - b)^3*(a + b)^3) + (2*(-3*a + b)*
(b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])
*Csc[(c + d*x)/2]^4)/(a + b)^2 + (8*(-3*a^2 - 4*a*b + b^2)*(b + a*Cos[c +
d*x])*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (128*a^3*b*(a^2 + 2*b^2)*(b + a*C
os[c + d*x])*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 + (8*(3*a^2 - 4*a*b -
b^2)*(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]])/(a + b)^4 + (2*(3*a + b)*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 + ((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])
^2)
```

3.215.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4360, 3042, 3316, 27, 601, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow \text{4360}$$
$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3316$$

$$\frac{a^5\int\frac{\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))^3}d(-a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$\frac{a^3\int\frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))^2(a^2-a^2\cos^2(c+dx))^3}d(-a\cos(c+dx))}{d}$$

$$\downarrow 601$$

$$\frac{a^3\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{4\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)^2}-\frac{\int\frac{-\frac{3\left(a^2+b^2\right)\cos^2(c+dx)a^4}{\left(a^2-b^2\right)^2}+\frac{2b\left(a^2-3b^2\right)\cos(c+dx)a^3}{\left(a^2-b^2\right)^2}+\frac{b^2\left(a^2+b^2\right)a^2}{\left(a^2-b^2\right)^2}}{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)^2}d(-a\cos(c+dx))}{4a^2}\right)}{d}$$

$$\downarrow 2178$$

$$\frac{a^3\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{4\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)^2}-\frac{\int\frac{-\frac{\left(3a^4+12b^2a^2+b^4\right)\cos^2(c+dx)a^4}{\left(a^2-b^2\right)^3}+\frac{2b\left(5a^2+b^2\right)\cos(c+dx)a^3}{\left(a^2-b^2\right)^2}+\frac{b^2\left(5a^4+12b^2a^2-b^4\right)a^2}{\left(a^2-b^2\right)^3}}{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)}d(-a\cos(c+dx))}{2a^2}-\frac{8a^2}{4a^2}\right)}{d}$$

$$\downarrow 2160$$

$$\frac{a^3\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{4\left(a^2-b^2\right)^2\left(a^2-a^2\cos^2(c+dx)\right)^2}-\frac{\int\left(\frac{16b\left(a^2+2b^2\right)a^4}{\left(a^2-b^2\right)^4(b+a\cos(c+dx))}-\frac{8b^2a^4}{\left(a^2-b^2\right)^3(b+a\cos(c+dx))^2}-\frac{\left(3a^2-4ba-b^2\right)a}{2(a+b)^4(a-a\cos(c+dx))}-\frac{\left(3a^2+4ba-b^2\right)a}{2(a-b)^4(\cos(c+dx)a+a)}\right.}{2a^2}}{4a^2}\right)}{d}$$

$$\downarrow 2009$$

$$a^3\left(\frac{2a^2b-a\left(a^2+b^2\right)\cos(c+dx)}{4(a^2-b^2)^2(a^2-a^2\cos^2(c+dx))^2}-\frac{-\frac{a\left(3a^2-4ab-b^2\right)\log(a-a\cos(c+dx))}{2(a+b)^4}+\frac{a\left(3a^2+4ab-b^2\right)\log(a\cos(c+dx)+a)}{2(a-b)^4}-\frac{8a^4b^2}{\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{16a^4b}{}}{2a^2}}{4a^2}\right)$$

$$d$$

input
```
Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^3*((2*a^2*b - a*(a^2 + b^2)*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a^2 - a^2*C
os[c + d*x]^2)^2) - (-1/2*(8*a^2*b*(a^2 + b^2) - a*(3*a^4 + 12*a^2*b^2 + b
^4)*Cos[c + d*x])/((a^2 - b^2)^3*(a^2 - a^2*Cos[c + d*x]^2)) + ((-8*a^4*b^
2)/((a^2 - b^2)^3*(b + a*Cos[c + d*x])) - (a*(3*a^2 - 4*a*b - b^2)*Log[a -
 a*Cos[c + d*x]])/(2*(a + b)^4) + (a*(3*a^2 + 4*a*b - b^2)*Log[a + a*Cos[c
 + d*x]])/(2*(a - b)^4) - (16*a^4*b*(a^2 + 2*b^2)*Log[b + a*Cos[c + d*x]])
/(a^2 - b^2)^4)/(2*a^2))/(4*a^2)))/d
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.215.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^3b^2}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}+\frac{2a^3b\left(a^2+2b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}-\frac{1}{16(a+b)^2(\cos(dx+c)-1)^2}-\frac{-3a+b}{16(a+b)^3(\cos(dx+c)-1)}+\frac{\left(3a^2-4\right.}{d}$
default	$\frac{a^3b^2}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}+\frac{2a^3b\left(a^2+2b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}-\frac{1}{16(a+b)^2(\cos(dx+c)-1)^2}-\frac{-3a+b}{16(a+b)^3(\cos(dx+c)-1)}+\frac{\left(3a^2-4\right.}{d}$
norman	$\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64d(a-b)}+\frac{(7a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d\left(a^2-2ab+b^2\right)}+\frac{(7a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{64d\left(a^2+2ab+b^2\right)}-\frac{\left(a^6+18a^4b^2+5a^2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d\left(a^7-b\,a^6-3a^5b^2+3a^4b^3+3b^4a^3-3b^5a^2-a\,b^6+b^7\right)}$ $\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)$
parallelrisch	$8192b\,a^3\left(a^2+2b^2\right)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+1536(b+a\cos(dx+c))\left(a^2-\frac{4}{3}ab-\frac{1}{3}b^2\right)(a-b)^4\ln\left(\tan\right.$
risch	Expression too large to display

input
```
int(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2*a^3*b*(a^2+2*b^2)/(a+b)^4/
(a-b)^4*ln(b+a*cos(d*x+c))-1/16/(a+b)^2/(cos(d*x+c)-1)^2-1/16*(-3*a+b)/(a+
b)^3/(cos(d*x+c)-1)+1/16*(3*a^2-4*a*b-b^2)/(a+b)^4*ln(cos(d*x+c)-1)+1/16/(
a-b)^2/(cos(d*x+c)+1)^2-1/16*(-3*a-b)/(a-b)^3/(cos(d*x+c)+1)+1/16/(a-b)^4*
(-3*a^2-4*a*b+b^2)*ln(cos(d*x+c)+1))
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(251) = 502$.

Time $= 0.49$ (sec) , antiderivative size $= 1205$, normalized size of antiderivative $= 4.65$

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(40*a^5*b^2 - 32*a^3*b^4 - 8*a*b^6 + 2*(3*a^7 + 17*a^5*b^2 - 19*a^3*b
^4 - a*b^6)*cos(d*x + c)^4 - 2*(5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos
(d*x + c)^3 - 2*(5*a^7 + 31*a^5*b^2 - 29*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2
 + 2*(7*a^6*b - 15*a^4*b^3 + 9*a^2*b^5 - b^7)*cos(d*x + c) + 32*(a^5*b^2 +
 2*a^3*b^4 + (a^6*b + 2*a^4*b^3)*cos(d*x + c)^5 + (a^5*b^2 + 2*a^3*b^4)*co
s(d*x + c)^4 - 2*(a^6*b + 2*a^4*b^3)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b
^4)*cos(d*x + c)^2 + (a^6*b + 2*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c)
+ b) - (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7
+ (3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*
x + c)^5 + (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 -
b^7)*cos(d*x + c)^4 - 2*(3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a
^3*b^4 - a*b^6)*cos(d*x + c)^3 - 2*(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32
*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 + 16*a^6*b + 33*a^5*b
^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) +
 1/2) + (3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7
 + (3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d
*x + c)^5 + (3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 -
 b^7)*cos(d*x + c)^4 - 2*(3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*
a^3*b^4 - a*b^6)*cos(d*x + c)^3 - 2*(3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 3
2*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 - 16*a^6*b + 33*a...
```

3.215.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(251) = 502$.

Time $= 0.22$ (sec) , antiderivative size $= 511$, normalized size of antiderivative $= 1.97$

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{32\,(a^5b+2\,a^3b^3)\log(a\cos(dx+c)+b)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{(3\,a^2+4\,ab-b^2)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} + \frac{(3\,a^2-4\,ab-b^2)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7+}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/16*(32*(a^5*b + 2*a^3*b^3)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8) - (3*a^2 + 4*a*b - b^2)*log(cos(d*x + c) + 1)/(
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*a^2 - 4*a*b - b^2)*log(cos
(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(20*a^3*b^2
 + 4*a*b^4 + (3*a^5 + 20*a^3*b^2 + a*b^4)*cos(d*x + c)^4 - (5*a^4*b - 4*a^
2*b^3 - b^5)*cos(d*x + c)^3 - (5*a^5 + 36*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^
2 + (7*a^4*b - 8*a^2*b^3 + b^5)*cos(d*x + c))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (a^6*b -
 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*
b^4 - a*b^6)*cos(d*x + c)^3 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(
d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)))/d
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(251) = 502$.

Time = 0.40 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.74

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{4\left(3a^2-4ab-b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{128\left(a^5b+2a^3b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{\frac{8a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{8ab(\cos(dx+}{\cos(dx+c)}}{}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/64*(4*(3*a^2 - 4*a*b - b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c)
+ 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 128*(a^5*b + 2*a^3*b^3
)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^
8) - (8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*
a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 +
2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 24*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*b^2*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) - 128*(a^6*b + a^4*b^3 +
2*a^3*b^4 + a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^5*b^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 2*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))))/d
```

3.215.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.73

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln(\cos(c+dx)-1)\left(\frac{3}{16(a+b)^2}-\frac{5b}{8(a+b)^3}+\frac{3b^2}{8(a+b)^4}\right)}{d}$$
$$-\frac{\ln(\cos(c+dx)+1)\left(\frac{3b^2}{8(a-b)^4}+\frac{5b}{8(a-b)^3}+\frac{3}{16(a-b)^2}\right)}{d}$$
$$+\frac{\frac{\cos(c+dx)^4\left(3a^5+20a^3b^2+ab^4\right)}{8\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{\cos(c+dx)\left(7a^2b-b^3\right)}{8\left(a^4-2a^2b^2+b^4\right)}-\frac{\cos(c+dx)^3\left(5a^2b+b^3\right)}{8\left(a^4-2a^2b^2+b^4\right)}-\frac{\cos(c+dx)^2\left(5a^5+36a^3b^2+7ab^4\right)}{8\left(a^2-b^2\right)\left(a^4-2a^2b^2+b^4\right)}+\frac{b}{2(a^2-b}}{d\left(a\cos(c+dx)^5+b\cos(c+dx)^4-2a\cos(c+dx)^3-2b\cos(c+dx)^2+a\cos(c+dx)\right.}$$
$$+\frac{\ln(b+a\cos(c+dx))\left(2a^5b+4a^3b^3\right)}{d\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}$$

input
```
int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

output
```
(log(cos(c + d*x) - 1)*(3/(16*(a + b)^2) - (5*b)/(8*(a + b)^3) + (3*b^2)/(
8*(a + b)^4)))/d - (log(cos(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) + (5*b)/(
8*(a - b)^3) + 3/(16*(a - b)^2)))/d + ((cos(c + d*x)^4*(a*b^4 + 3*a^5 + 20
*a^3*b^2))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)*(7*a^2*
b - b^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^3*(5*a^2*b + b^3))/(
8*(a^4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^2*(7*a*b^4 + 5*a^5 + 36*a^3*b^2
))/(8*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*(a*b^3 + 5*a^3*b))/(2*(a^2
 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - 2*a*cos(c + d*x
)^3 + a*cos(c + d*x)^5 - 2*b*cos(c + d*x)^2 + b*cos(c + d*x)^4)) + (log(b
+ a*cos(c + d*x))*(2*a^5*b + 4*a^3*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4
*b^4 - 4*a^6*b^2))
```

3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.216.1 Optimal result

Integrand size = 21, antiderivative size = 473

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8}\\ &\quad-\frac{2(a-b)^{3/2}b(a+b)^{3/2}\,(2a^2-7b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d}\\ &\quad+\frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d}\\ &\quad-\frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d}\\ &\quad+\frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd}\\ &\quad-\frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d}\\ &\quad-\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))}+\frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))}\\ &\quad+\frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))}\\ &\quad+\frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))}-\frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))}\end{aligned}$$

output
```
1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*x/a^8-2*(a-b)^(3/2)*b*(a+b)^(3
/2)*(2*a^2-7*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^8/
d+1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*sin(d*x+c)/a^7/d-1/16*(27*a^4-86*a^2
*b^2+56*b^4)*cos(d*x+c)*sin(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b^4)*c
os(d*x+c)^2*sin(d*x+c)/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cos(d*x+c)^
3*sin(d*x+c)/a^4/b^2/d-1/3*cos(d*x+c)^3*sin(d*x+c)/b/d/(b+a*cos(d*x+c))+1/
6*a*cos(d*x+c)^4*sin(d*x+c)/b^2/d/(b+a*cos(d*x+c))+1/10*(5*a^4-20*a^2*b^2+
14*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))+7/30*b*cos(d*x+
c)^5*sin(d*x+c)/a^2/d/(b+a*cos(d*x+c))-1/6*cos(d*x+c)^6*sin(d*x+c)/a/d/(b+
a*cos(d*x+c))
```

3.216.2 Mathematica [A] (verified)

Time = 6.32 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.85

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{3840b(2a^2-7b^2)\left(a^2-b^2\right)^{3/2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \frac{600a^6bc-10800a^4b^3c+24000a^2b^5c-13440b^7c+600a^6bdx-108}{}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

output
```
(3840*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]] + (600*a^6*b*c - 10800*a^4*b^3*c + 24000*a^2*b^5*c -
13440*b^7*c + 600*a^6*b*d*x - 10800*a^4*b^3*d*x + 24000*a^2*b^5*d*x - 1344
0*b^7*d*x + 120*a*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(c + d*x)*C
os[c + d*x] - 15*a*(15*a^6 - 576*a^4*b^2 + 1488*a^2*b^4 - 896*b^6)*Sin[c +
 d*x] + 1910*a^6*b*Sin[2*(c + d*x)] - 5440*a^4*b^3*Sin[2*(c + d*x)] + 3360
*a^2*b^5*Sin[2*(c + d*x)] - 180*a^7*Sin[3*(c + d*x)] + 790*a^5*b^2*Sin[3*(
c + d*x)] - 560*a^3*b^4*Sin[3*(c + d*x)] - 166*a^6*b*Sin[4*(c + d*x)] + 14
0*a^4*b^3*Sin[4*(c + d*x)] + 40*a^7*Sin[5*(c + d*x)] - 42*a^5*b^2*Sin[5*(c
 + d*x)] + 14*a^6*b*Sin[6*(c + d*x)] - 5*a^7*Sin[7*(c + d*x)])/(b + a*Cos[
c + d*x]))/(1920*a^8*d)
```

3.216.3 Rubi [A] (verified)

Time = 3.64 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.22, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4360, 3042, 3375, 27, 3042, 3526, 3042, 3528, 27, 3042, 3528, 25, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\sin^6(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3375$$

$$\frac{\int \frac{12\cos^4(c+dx)\left(-\left(\left(20a^4-65b^2a^2+42b^4\right)\cos^2(c+dx)\right)+ab\left(5a^2-2b^2\right)\cos(c+dx)+5\left(3a^4-10b^2a^2+7b^4\right)\right)}{(b+a\cos(c+dx))^2}dx}{360a^2b^2}+$$
$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-$$
$$\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 27$$

$$\frac{\int \frac{\cos^4(c+dx)\left(-\left(\left(20a^4-65b^2a^2+42b^4\right)\cos^2(c+dx)\right)+ab\left(5a^2-2b^2\right)\cos(c+dx)+5\left(3a^4-10b^2a^2+7b^4\right)\right)}{(b+a\cos(c+dx))^2}dx}{30a^2b^2}+$$
$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-$$
$$\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(\left(-20a^4+65b^2a^2-42b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\left(5a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+5\left(3a^4-10b^2a^2+7b^4\right)\right)}{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{30a^2b^2}+$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3526$$

$$\frac{\int \frac{\cos^3(c+dx)\left(-5\left(16a^6-77b^2a^4+103b^4a^2-42b^6\right)\cos^2(c+dx)+ab\left(10a^4-17b^2a^2+7b^4\right)\cos(c+dx)+12\left(5a^6-25b^2a^4+34b^4a^2-14b^6\right)\right)}{b+a\cos(c+dx)}dx}{a(a^2-b^2)}+\frac{3\left(5a^4-20a^2b^2+14\right.}{ad(a\,c}$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(-5\left(16a^6-77b^2a^4+103b^4a^2-42b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\left(10a^4-17b^2a^2+7b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+12\left(5a^6-25b^2a^4+34b^4a^2-14b^6\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{3\left(5a^4\right.}{}$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3528$$

$$\frac{\int -\frac{3\cos^2(c+dx)\left(a\left(15a^4-29b^2a^2+14b^4\right)\cos(c+dx)b^2-8\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)\cos^2(c+dx)b+5\left(16a^6-77b^2a^4+103b^4a^2-42b^6\right)b\right)}{b+a\cos(c+dx)}dx}{4a}-\frac{5\left(16a^6-77a^4b^2+\right.}{a(a^2-b^2)}$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 27$$

$$-\frac{3\int\frac{\cos^2(c+dx)\left(a\left(15a^4-29b^2a^2+14b^4\right)\cos(c+dx)b^2-8\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)\cos^2(c+dx)b+5\left(16a^6-77b^2a^4+103b^4a^2-42b^6\right)b\right)}{b+a\cos(c+dx)}dx}{4a}-\frac{5\left(16a^6-77a^4b^2-}{a(a^2-b^2)}$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{30a^2b^2}{\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}}$$

$$\downarrow 3042$$

$$-\frac{3\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\left(15a^4-29b^2a^2+14b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2-8\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+5\left(16a^6-77b^2a^4+103b^4a^2-42b^6\right)b\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a}-\frac{5\left(}{a(a^2-b^2)}$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{30a^2b^2}{\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}}$$

$$\downarrow 3528$$

$$-\frac{3\left(\int-\frac{\cos(c+dx)\left(a\left(83a^4-153b^2a^2+70b^4\right)\cos(c+dx)b^3-15\left(27a^6-113b^2a^4+142b^4a^2-56b^6\right)\cos^2(c+dx)b^2+16\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)b^2\right)}{b+a\cos(c+dx)}dx}{3a}-\frac{8b\left(15a^6\right.}{4a}\right)}{a(a^2-b^2)}$$

$$30a$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{30a^2b^2}{\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}}$$

$$\downarrow 25$$

$$-\frac{3\left(-\int\frac{\cos(c+dx)\left(a\left(83a^4-153b^2a^2+70b^4\right)\cos(c+dx)b^3-15\left(27a^6-113b^2a^4+142b^4a^2-56b^6\right)\cos^2(c+dx)b^2+16\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)b^2\right)}{b+a\cos(c+dx)}dx}{3a}-\frac{8b\left(15a^6\right.}{4a}\right)}{a(a^2-b^2)}$$

$$30a$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{30a^2b^2}{\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}}$$

$$\downarrow 3042$$

$$3\left(-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\left(83a^4-153b^2a^2+70b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^3-15\left(27a^6-113b^2a^4+142b^4a^2-56b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+16\left(15a^6-67b^2a^4+87b^4a^2-35b^6\right)b^2\right.}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}{3a}\right.$$

$$-\frac{}{4a}$$

$$\left.-\frac{}{a(a^2-b^2)}\right)$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3528$$

$$3\left(-\frac{\int -\frac{-16\left(61a^6-231b^2a^4+275b^4a^2-105b^6\right)\cos^2(c+dx)b^3+15\left(27a^6-113b^2a^4+142b^4a^2-56b^6\right)b^3-a\left(75a^6-449b^2a^4+654b^4a^2-280b^6\right)\cos(c+dx)b^2}{b+a\cos(c+dx)}dx}{2a}-\frac{}{3a}\right.$$

$$-\frac{}{4a}$$

$$\left.-\frac{}{}\right)$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 25$$

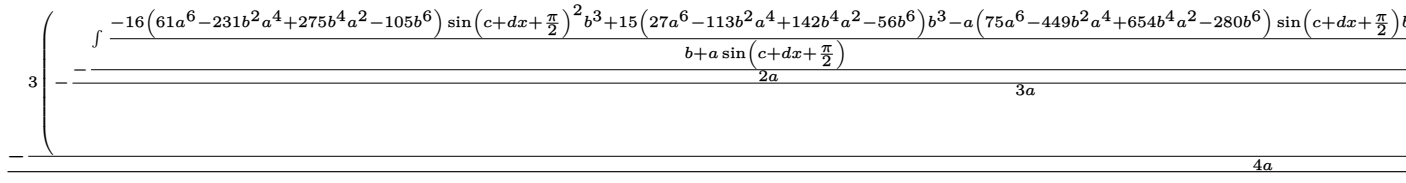
$$3\left(-\frac{-\frac{\int \frac{-16\left(61a^6-231b^2a^4+275b^4a^2-105b^6\right)\cos^2(c+dx)b^3+15\left(27a^6-113b^2a^4+142b^4a^2-56b^6\right)b^3-a\left(75a^6-449b^2a^4+654b^4a^2-280b^6\right)\cos(c+dx)b^2}{b+a\cos(c+dx)}dx}{2a}}{3a}\right.$$

$$-\frac{}{4a}$$

$$\left.-\frac{}{}\right)$$

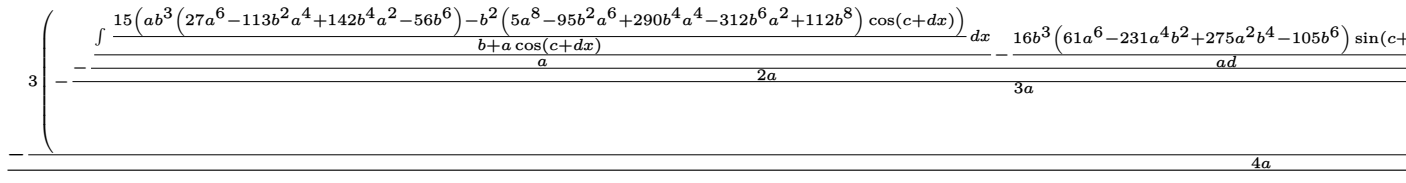
$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$



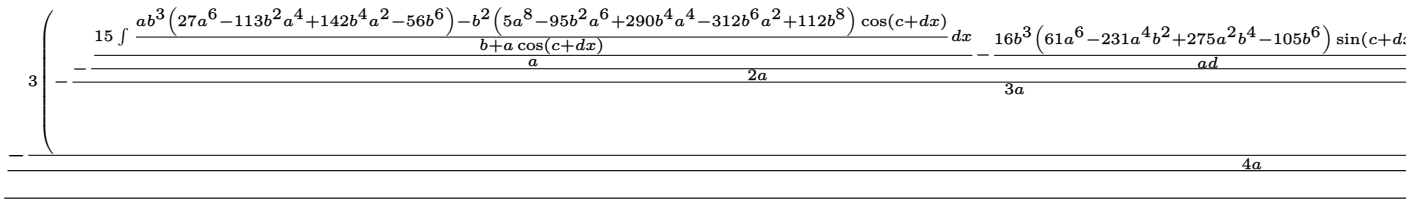
$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3502$$



$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 27$$



$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

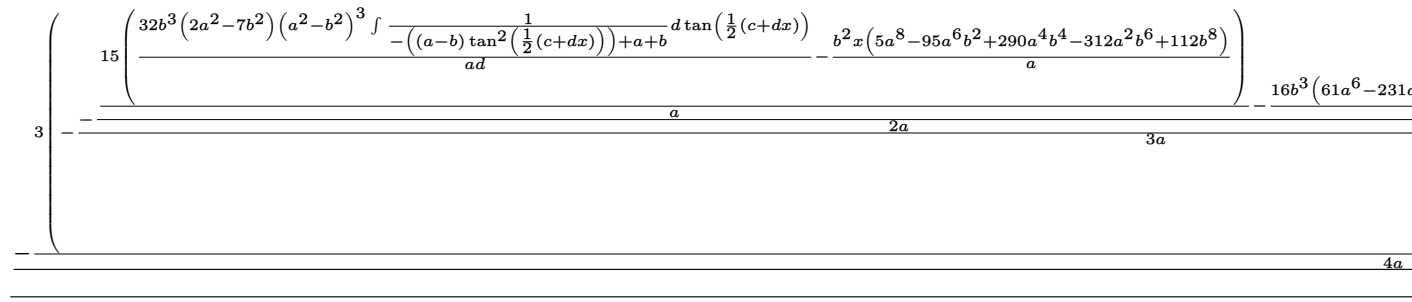
$$\downarrow 3214$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

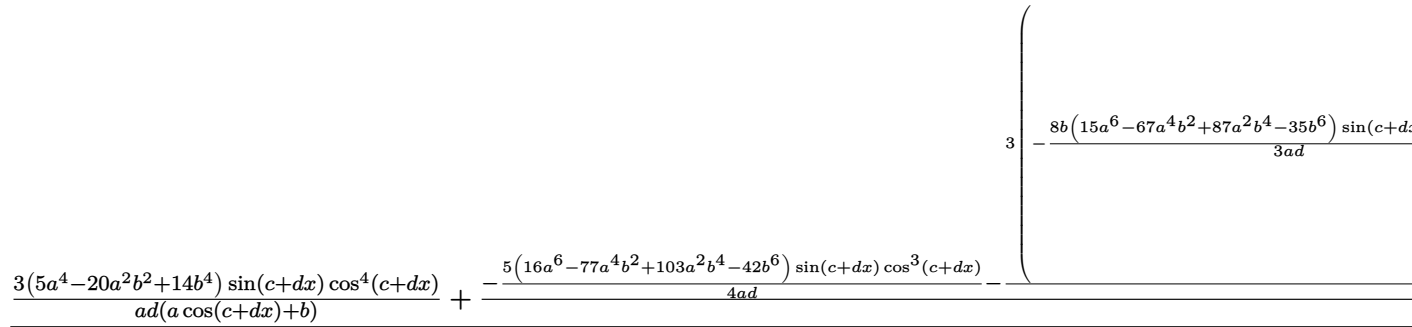
$$\downarrow 3138$$



$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

$$\downarrow 221$$

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}+$$



$$\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^3*Sin[c + d*x])/(b*d*(b + a*Cos[c + d*x])) + (a*Cos[c +
 d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + (7*b*Cos[c + d*x]^5
*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x])) - (Cos[c + d*x]^6*Sin[c + d
*x])/(6*a*d*(b + a*Cos[c + d*x])) + ((3*(5*a^4 - 20*a^2*b^2 + 14*b^4)*Cos[
c + d*x]^4*Sin[c + d*x])/(a*d*(b + a*Cos[c + d*x])) + ((-5*(16*a^6 - 77*a^
4*b^2 + 103*a^2*b^4 - 42*b^6)*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (3*((
-8*b*(15*a^6 - 67*a^4*b^2 + 87*a^2*b^4 - 35*b^6)*Cos[c + d*x]^2*Sin[c + d*
x])/(3*a*d) - ((-15*b^2*(27*a^6 - 113*a^4*b^2 + 142*a^2*b^4 - 56*b^6)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) - ((15*(-((b^2*(5*a^8 - 95*a^6*b^2 + 290*a^
4*b^4 - 312*a^2*b^6 + 112*b^8)*x)/a) + (32*b^3*(2*a^2 - 7*b^2)*(a^2 - b^2)
^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqr
t[a + b]*d)))/a - (16*b^3*(61*a^6 - 231*a^4*b^2 + 275*a^2*b^4 - 105*b^6)*S
in[c + d*x])/(a*d))/(2*a))/(3*a)))/(4*a))/(a*(a^2 - b^2)))/(30*a^2*b^2)
```

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3375

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x]
)^(m + 1)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d
*Sin[e + f*x])^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)
*(m + n + 6))), x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin
[e + f*x])^(m + 1)/(b*d^4*f*(m + n + 6))), x] + Simp[1/(a^2*b^2*d^2*(n + 1)
*(n + 2)*(m + n + 5)*(m + n + 6))   Int[(d*Sin[e + f*x])^(n + 2)*(a + b*Sin
[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5) - a^2*b^2*(n + 2)*(2*
n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m + n + 3)*(m + n + 5)*(m
 + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n + 5)*(m + n + 6))*Sin[e
 + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4*(m + n + 2)*(m + n + 4
)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*(m + n + 5)*(2*n + 2*m
+ 13))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && Ne
Q[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m
 + n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3526

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.216.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{2\left(\left(\frac{5}{16}a^6+2a^5b-\frac{21}{8}a^4b^2-8a^3b^3+\frac{5}{2}a^2b^4+6ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{38}{3}a^5b-\frac{87}{8}a^4b^2+\frac{15}{2}a^2b^4+30ab^5+\frac{85}{48}a^6-\frac{136}{3}a^3b^3\right)\tan\left(\frac{dx}{2}+\right.\right.}{\left.\left.\right.\right.}$
default	$\frac{2\left(\left(\frac{5}{16}a^6+2a^5b-\frac{21}{8}a^4b^2-8a^3b^3+\frac{5}{2}a^2b^4+6ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{38}{3}a^5b-\frac{87}{8}a^4b^2+\frac{15}{2}a^2b^4+30ab^5+\frac{85}{48}a^6-\frac{136}{3}a^3b^3\right)\tan\left(\frac{dx}{2}+\right.\right.}{\left.\left.\right.\right.}$
risch	$\frac{2\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,a^4}-\frac{9\sqrt{a^2-b^2}\,b^3\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,a^6}+\frac{7\sqrt{a^2-b^2}\,b^5\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2}}{d\,a^8}\right.\right.}$

input

```
int(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^8*(((5/16*a^6+2*a^5*b-21/8*a^4*b^2-8*a^3*b^3+5/2*a^2*b^4+6*a*b^5)
*tan(1/2*d*x+1/2*c)^11+(38/3*a^5*b-87/8*a^4*b^2+15/2*a^2*b^4+30*a*b^5+85/4
8*a^6-136/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(172/5*a^5*b-33/4*a^4*b^2-96*a^3
*b^3+5*a^2*b^4+60*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6+33/4*a^4
*b^2-5*a^2*b^4+172/5*a^5*b-96*a^3*b^3+60*a*b^5)*tan(1/2*d*x+1/2*c)^5+(38/3
*a^5*b+87/8*a^4*b^2-136/3*a^3*b^3-15/2*a^2*b^4+30*a*b^5-85/48*a^6)*tan(1/2
*d*x+1/2*c)^3+(2*a^5*b-8*a^3*b^3+6*a*b^5-5/16*a^6+21/8*a^4*b^2-5/2*a^2*b^4
)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-90*a^4*b^2+20
0*a^2*b^4-112*b^6)*arctan(tan(1/2*d*x+1/2*c)))+2*(a-b)^2*(a+b)^2*b/a^8*(-a
*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-
(2*a^2-7*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*
(a+b))^(1/2))))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.68

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \left[\frac{15\,(5\,a^7 - 90\,a^5b^2 + 200\,a^3b^4 - 112\,ab^6)dx\cos\left(dx+c\right) + 15\,(5\,a^6b - 90\,a^4b^3 + 200\,a^2b^5 - 112\,b^7)dx + 12}{\right.$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
 + 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x + 120*(2*a^4*b^2
- 9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
 c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos
(d*x + c)^5 - 976*a^5*b^2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5
*b^2)*cos(d*x + c)^4 + 2*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^
7 - 458*a^5*b^2 + 280*a^3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3
+ 840*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d),
 1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
 + 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x - 240*(2*a^4*b^2
- 9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x + c)^5 - 976*a^5*b^
2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*cos(d*x + c)^4 + 2
*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*a^5*b^2 + 280*a^
3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x +
 c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d)]
```

3.216.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

3.216.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.84

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(d*x + c)/a^8 - 480
*(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^8) - 480*(a^4*b^2*tan(1/2*d*
x + 1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c))/((
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^7) + 2*(75*
a^5*tan(1/2*d*x + 1/2*c)^11 + 480*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 630*a^3*
b^2*tan(1/2*d*x + 1/2*c)^11 - 1920*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 600*a
*b^4*tan(1/2*d*x + 1/2*c)^11 + 1440*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*
tan(1/2*d*x + 1/2*c)^9 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 2610*a^3*b^2*
tan(1/2*d*x + 1/2*c)^9 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a*b^4
*tan(1/2*d*x + 1/2*c)^9 + 7200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/
2*d*x + 1/2*c)^7 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a^3*b^2*tan(1/
2*d*x + 1/2*c)^7 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a*b^4*tan(1
/2*d*x + 1/2*c)^7 + 14400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x
 + 1/2*c)^5 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 1200*a*b^4*tan(1/2*d*
x + 1/2*c)^5 + 14400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/
2*c)^3 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a^3*b^2*tan(1/2*d*x + 1/
2*c)^3 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 1800*a*b^4*tan(1/2*d*x ...
```

3.216.9 Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 3692, normalized size of antiderivative = 7.81

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

output
```
(atan(((((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b
^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan
(c/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*
112i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b
^4*200i - a^4*b^2*90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*
a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a
^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7
 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^
13*b^2))/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(1
6*a^8) - (((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18
*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (t
an(c/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^
6*112i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2
*b^4*200i - a^4*b^2*90i))/(16*a^8) - (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 7
5*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720
*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b
^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*
a^13*b^2))/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/
(16*a^8))/((((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^
18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21...
```

3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.217.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {}& \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} \\ & - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,(2a^2-5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6 d} \\ & + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5 d} \\ & - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4 d} \\ & + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3 bd} \\ & + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2 d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2 bd(b+a\cos(c+dx))}\end{aligned}$$

output

```
1/8*(3*a^4-36*a^2*b^2+40*b^4)*x/a^6+1/3*b*(11*a^2-15*b^2)*sin(d*x+c)/a^5/d
-1/8*(13*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^4/d+1/3*(3*a^2-5*b^2)*cos(d*x
+c)^2*sin(d*x+c)/a^3/b/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d-(a^2-b^2)*cos(d
*x+c)^3*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))-2*b*(2*a^2-5*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^6/d
```

3.217.2 Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.08

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{384b(2a^4-7a^2b^2+5b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{72a^4bc-864a^2b^3c+960b^5c+72a^4bdx-864a^2b^3dx+960b^5dx+24a(3a^4-36a^2b^2+4}{}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((384*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (72*a^4*b*c - 864*a^2*b^3*c + 960*b^5*c
+ 72*a^4*b*d*x - 864*a^2*b^3*d*x + 960*b^5*d*x + 24*a*(3*a^4 - 36*a^2*b^2
+ 40*b^4)*(c + d*x)*Cos[c + d*x] - 24*a*(a^4 - 31*a^2*b^2 + 40*b^4)*Sin[c
+ d*x] + 176*a^4*b*Sin[2*(c + d*x)] - 240*a^2*b^3*Sin[2*(c + d*x)] - 21*a^
5*Sin[3*(c + d*x)] + 40*a^3*b^2*Sin[3*(c + d*x)] - 10*a^4*b*Sin[4*(c + d*x
)] + 3*a^5*Sin[5*(c + d*x)])/(b + a*Cos[c + d*x]))/(192*a^6*d)
```

3.217.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.15, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4360, 3042, 3371, 3042, 3528, 25, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4360$$
$$\int \frac{\sin^4(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2 \cos\left(c+dx+\frac{\pi}{2}\right)^4}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3371$$

$$-\frac{\int \frac{\cos^2(c+dx)\left(8a^2+b\cos(c+dx)a-15b^2-4\left(3a^2-5b^2\right)\cos^2(c+dx)\right)}{b+a\cos(c+dx)}dx}{4a^2b} - \frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(8a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a-15b^2-4\left(3a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2b} - \frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3528$$

$$-\frac{\frac{\int -\frac{\cos(c+dx)\left(5a\cos(c+dx)b^2-3\left(13a^2-20b^2\right)\cos^2(c+dx)b+8\left(3a^2-5b^2\right)b\right)}{b+a\cos(c+dx)}dx}{3a} - \frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}}{4a^2b} - \frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 25$$

$$-\frac{-\frac{\int \frac{\cos(c+dx)\left(5a\cos(c+dx)b^2-3\left(13a^2-20b^2\right)\cos^2(c+dx)b+8\left(3a^2-5b^2\right)b\right)}{b+a\cos(c+dx)}dx}{3a} - \frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}}{4a^2b} - \frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3042$$

$$-\frac{-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(5a\sin\left(c+dx+\frac{\pi}{2}\right)b^2-3\left(13a^2-20b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+8\left(3a^2-5b^2\right)b\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3a} - \frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}}{4a^2b} - \frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3528$$

$$-\frac{\int -\frac{-8\left(11a^2-15b^2\right)\cos^2(c+dx)b^2+3\left(13a^2-20b^2\right)b^2-a\left(9a^2-20b^2\right)\cos(c+dx)b}{b+a\cos(c+dx)}dx}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos}{3ad}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{4a^2b}{\sin(c+dx)\cos^3(c+dx)}$$

$$\downarrow 25$$

$$-\frac{\int \frac{-8\left(11a^2-15b^2\right)\cos^2(c+dx)b^2+3\left(13a^2-20b^2\right)b^2-a\left(9a^2-20b^2\right)\cos(c+dx)b}{b+a\cos(c+dx)}dx}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos}{3ad}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{4a^2b}{\sin(c+dx)\cos^3(c+dx)}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{-8\left(11a^2-15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+3\left(13a^2-20b^2\right)b^2-a\left(9a^2-20b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin(c+}{3a}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{4a^2b}{\sin(c+dx)\cos^3(c+dx)}$$

$$\downarrow 3502$$

$$-\frac{\int \frac{3\left(ab^2\left(13a^2-20b^2\right)-b\left(3a^4-36b^2a^2+40b^4\right)\cos(c+dx)\right)}{b+a\cos(c+dx)}dx}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin(c}{3}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{4a^2b}{\sin(c+dx)\cos^3(c+dx)}$$

$$\downarrow 27$$

$$-\frac{\frac{3\int \frac{ab^2\left(13a^2-20b^2\right)-b\left(3a^4-36b^2a^2+40b^4\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin(c+}{3ad}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{4a^2b}{\sin(c+dx)\cos^3(c+dx)}$$

$$\downarrow 3042$$

$$-\frac{-\frac{3\int\frac{ab^2\left(13a^2-20b^2\right)-b\left(3a^4-36b^2a^2+40b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a^2-5b^2\right)\sin$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3214$$

$$-\frac{-\frac{3\left(\frac{8b^2\left(2a^4-7a^2b^2+5b^4\right)\int\frac{1}{b+a\cos(c+dx)}dx}{a}-\frac{bx\left(3a^4-36a^2b^2+40b^4\right)}{a}\right)}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4\left(3a$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3042$$

$$-\frac{-\frac{3\left(\frac{8b^2\left(2a^4-7a^2b^2+5b^4\right)\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{bx\left(3a^4-36a^2b^2+40b^4\right)}{a}\right)}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{3a}-\frac{4}{}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 3138$$

$$-\frac{-\frac{3\left(\frac{16b^2\left(2a^4-7a^2b^2+5b^4\right)\int\frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}-\frac{bx\left(3a^4-36a^2b^2+40b^4\right)}{a}\right)}{a}-\frac{8b^2\left(11a^2-15b^2\right)\sin(c+dx)}{ad}}{2a}-\frac{3b\left(13a^2-20b^2\right.}{}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}$$

$$\downarrow 221$$

$$-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-$$

$$-\frac{4\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{-\frac{3b\left(13a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3\left(\frac{16b^2\left(2a^4-7a^2b^2+5b^4\right)\arctanh\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(3a^4-\right.}{a}\right)}{2a}}{3a}$$

$$4a^2b$$

3.217. $\int\frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

input
```
Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos[c + d*x]^3*Sin[
c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x])) - ((-4*(3*a^2 - 5*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(3*a*d) - ((-3*b*(13*a^2 - 20*b^2)*Cos[c + d*x]*Sin[c +
 d*x])/(2*a*d) - ((3*(-((b*(3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/a) + (16*b^2*(
2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a - (8*b^2*(11*a^2 - 15*b^2)*Sin[c +
 d*x])/(a*d))/(2*a))/(3*a))/(4*a^2*b)
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3371

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*
x])^(n + 1)/(b^2*d*f*(m + n + 4))), x] - Simp[1/(a*b^2*(m + 1)*(m + n + 4))
   Int[(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n
+ 3) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2
)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; Fre
eQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && Lt
Q[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.217.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2a^3b-\frac{3}{2}a^2b^2-4\right.\right.}{}$
default	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2a^3b-\frac{3}{2}a^2b^2-4\right.\right.}{}$
risch	$\frac{3x}{8a^2}-\frac{9xb^2}{2a^4}+\frac{5xb^4}{a^6}-\frac{ie^{-2i(dx+c)}}{8a^2d}-\frac{5ibe^{i(dx+c)}}{4a^3d}-\frac{2ib^3e^{-i(dx+c)}}{a^5d}+\frac{ie^{2i(dx+c)}}{8a^2d}+\frac{2ib^3e^{i(dx+c)}}{a^5d}-\frac{3ie^{2i(dx}}{8a^4}$

input
```
int(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(a-b)*(a+b)*b/a^6*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-5*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^6*(((3/8*a^4+2*a^3*b-3/2*a^
2*b^2-4*a*b^3)*tan(1/2*d*x+1/2*c)^7+(26/3*a^3*b-3/2*a^2*b^2-12*a*b^3+11/8*
a^4)*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3/2*a^2*b^2+26/3*a^3*b-12*a*b^3)*tan(
1/2*d*x+1/2*c)^3+(2*a^3*b-4*a*b^3-3/8*a^4+3/2*a^2*b^2)*tan(1/2*d*x+1/2*c))
/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4-36*a^2*b^2+40*b^4)*arctan(tan(1/2*d
*x+1/2*c))))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.23

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{3\left(3a^5-36a^3b^2+40ab^4\right)dx\cos(dx+c)+3\left(3a^4b-36a^2b^3+40b^5\right)dx-12\left(2a^2b^2-5b^4+\left(2a^3b-5\right.\right.}{}\right.$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x + c) + 3*(3*a^4*b - 3
6*a^2*b^3 + 40*b^5)*d*x - 12*(2*a^2*b^2 - 5*b^4 + (2*a^3*b - 5*a*b^3)*cos(
d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^5*cos(d*x + c)^4
 - 10*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2
)*cos(d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^
7*d*cos(d*x + c) + a^6*b*d), 1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*c
os(d*x + c) + 3*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*d*x - 24*(2*a^2*b^2 - 5*b^
4 + (2*a^3*b - 5*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^5*cos(d*x +
c)^4 - 10*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3
*b^2)*cos(d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))
/(a^7*d*cos(d*x + c) + a^6*b*d)]
```

3.217.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.217.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.85

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{3\,(3\,a^4-36\,a^2b^2+40\,b^4)\,(dx+c)}{a^6} - \frac{48\,(2\,a^4b-7\,a^2b^3+5\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^6} - \frac{4}{(a}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^4 - 36*a^2*b^2 + 40*b^4)*(d*x + c)/a^6 - 48*(2*a^4*b - 7*a^2*
b^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(
-a^2 + b^2)*a^6) - 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^5)
+ 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 48*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*a
*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*tan(1
/2*d*x + 1/2*c)^5 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*a*b^2*tan(1/2*d*
x + 1/2*c)^5 - 288*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1/2*c
)^3 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 -
 288*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 48*a^2*b*ta
n(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^5))/d
```

3.217.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 2804, normalized size of antiderivative = 10.74

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

output

```
((tan(c/2 + (d*x)/2)^5*(33*a^4 - 360*b^4 + 244*a^2*b^2))/(6*a^5) - (tan(c/
2 + (d*x)/2)^3*(60*a*b^3 - 59*a^3*b + 12*a^4 + 240*b^4 - 176*a^2*b^2))/(6*
a^5) - (tan(c/2 + (d*x)/2)^7*(59*a^3*b - 60*a*b^3 + 12*a^4 + 240*b^4 - 176
*a^2*b^2))/(6*a^5) + (tan(c/2 + (d*x)/2)^9*(a - b)*(20*a*b^2 - 16*a^2*b -
3*a^3 + 40*b^3))/(4*a^5) + (tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^2 + 16*a^2*
b - 3*a^3 - 40*b^3))/(4*a^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) +
tan(c/2 + (d*x)/2)^2*(3*a + 5*b) + tan(c/2 + (d*x)/2)^4*(2*a + 10*b) - tan
(c/2 + (d*x)/2)^8*(3*a - 5*b) - tan(c/2 + (d*x)/2)^6*(2*a - 10*b))) + (ata
n(((((((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 -
 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i
- a^2*b^2*36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4
*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*a*b^10 -
 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*
b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2
))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(8*a^6) - (((((76*a^17*b
 - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 2
8*a^16*b^2)/a^15 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(1
28*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a
^2*b^2*36i))/(8*a^6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^
11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5...
```

3.218 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.218.1 Optimal result . . . 1622
3.218.2 Mathematica [A] (verified) . . . 1622
3.218.3 Rubi [A] (verified) . . . 1623
3.218.4 Maple [A] (verified) . . . 1628
3.218.5 Fricas [A] (verification not implemented) . . . 1628
3.218.6 Sympy [F] . . . 1629
3.218.7 Maxima [F(-2)] . . . 1629
3.218.8 Giac [A] (verification not implemented) . . . 1630
3.218.9 Mupad [B] (verification not implemented) . . . 1630

3.218.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2-6b^2)\,x}{2a^4} - \frac{2b(2a^2-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))}$$

output
```
1/2*(a^2-6*b^2)*x/a^4+3*b*sin(d*x+c)/a^3/d-3/2*cos(d*x+c)*sin(d*x+c)/a^2/d
+cos(d*x+c)^2*sin(d*x+c)/a/d/(b+a*cos(d*x+c))-2*b*(2*a^2-3*b^2)*arctanh((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{16b(2a^2-3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4a^2bc-24b^3c+4a^2bdx-24b^3dx+4a(a^2-6b^2)(c+dx)\cos(c+dx)-a(a^2-24b^2)\sin(c+dx)+6}{b+a\cos(c+dx)}$$

$$= \frac{}{8a^4d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
((16*b*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + (4*a^2*b*c - 24*b^3*c + 4*a^2*b*d*x - 24*b^3*d*x + 4*
a*(a^2 - 6*b^2)*(c + d*x)*Cos[c + d*x] - a*(a^2 - 24*b^2)*Sin[c + d*x] + 6
*a^2*b*Sin[2*(c + d*x)] - a^3*Sin[3*(c + d*x)])/(b + a*Cos[c + d*x]))/(8*a
^4*d)
```

3.218.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4360, 3042, 3368, 3042, 3527, 25, 3042, 3529, 25, 3042, 3502, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4360$$
$$\int \frac{\sin^2(c+dx)\cos^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\cos\left(c+dx+\frac{\pi}{2}\right)^2}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$
$$\downarrow 3368$$
$$\int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(-a\cos(c+dx)-b)^2}dx$$
$$\downarrow 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3527$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)} - \frac{\int -\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{b+a\cos(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{b+a\cos(c+dx)}dx}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3529$$

$$\frac{\frac{\int -\frac{-6b\left(a^2-b^2\right)\cos^2(c+dx)-a\left(a^2-b^2\right)\cos(c+dx)+3b\left(a^2-b^2\right)}{b+a\cos(c+dx)}dx}{2a} - \frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{-6b\left(a^2-b^2\right)\cos^2(c+dx)-a\left(a^2-b^2\right)\cos(c+dx)+3b\left(a^2-b^2\right)}{b+a\cos(c+dx)}dx}{2a} - \frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{-\frac{\int \frac{-6b\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3b\left(a^2-b^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a} - \frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3502$$

$$\frac{-\frac{\frac{\int \frac{3ab\left(a^2-b^2\right)-\left(a^2-6b^2\right)\left(a^2-b^2\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{a} - \frac{6b\left(a^2-b^2\right)\sin(c+dx)}{ad}}{2a} - \frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}}{a\left(a^2-b^2\right)} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3214$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 3138$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

$$\downarrow 221$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

3.218. $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

output
```
(Cos[c + d*x]^2*Sin[c + d*x])/(a*d*(b + a*Cos[c + d*x])) + ((-3*(a^2 - b^2
)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (((-(((a^2 - 6*b^2)*(a^2 - b^2)*x)/a
) + (4*b*(2*a^2 - 3*b^2)*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/a - (6*b*(a^2 - b^2)*Sin[c
+ d*x])/(a*d))/(2*a))/(a*(a^2 - b^2))
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3527

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3529

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.218.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-3b^2\right)\arctanh\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4}+\frac{2\left(\left(\frac{1}{2}a^2+2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(2ab-\right.\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
default	$\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-3b^2\right)\arctanh\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4}+\frac{2\left(\left(\frac{1}{2}a^2+2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(2ab-\right.\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
risch	$\frac{x}{2a^2}-\frac{3xb^2}{a^4}+\frac{ie^{2i(dx+c)}}{8a^2d}-\frac{ibe^{i(dx+c)}}{a^3d}+\frac{ibe^{-i(dx+c)}}{a^3d}-\frac{ie^{-2i(dx+c)}}{8a^2d}+\frac{2ib^2\left(be^{i(dx+c)}+a\right)}{a^4d\left(ae^{2i(dx+c)}+2be^{i(dx+c)}+a\right)}-$

input
```
int(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^4*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)-(2*a^2-3*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^4*(((1/2*a^2+2*a*b)*tan(1/2*d*x+1/2*c)^
3+(2*a*b-1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-
6*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.62

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{(a^5-7\,a^3b^2+6\,ab^4)dx\cos(dx+c)+(a^4b-7\,a^2b^3+6\,b^5)dx-(2\,a^2b^2-3\,b^4+(2\,a^3b-3\,ab^3)\cos(dx+\right.$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 +
6*b^5)*d*x - (2*a^2*b^2 - 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
 c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^3*b^2 - 6*a*b^4 - (a^5 - a^3*b^2
)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7 -
 a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d), 1/2*((a^5 - 7*a^3*b^2 + 6
*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6*b^5)*d*x - 2*(2*a^2*b^2
- 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^3*b^2
- 6*a*b^4 - (a^5 - a^3*b^2)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x +
 c))*sin(d*x + c))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)]
```

3.218.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.218.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.58

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{\frac{4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)a^3}-\frac{(a^2-6b^2)(dx+c)}{a^4}+\frac{4\left(2a^2b-3b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx\right.\right.\right.}{\sqrt{-a^2+b^2}a^4}}{2d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*
x + 1/2*c)^2 - a - b)*a^3) - (a^2 - 6*b^2)*(d*x + c)/a^4 + 4*(2*a^2*b - 3*
b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b
^2)*a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 - a*ta
n(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 +
1)^2*a^3))/d
```

3.218.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 1655, normalized size of antiderivative = 10.89

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

output

```
((2*tan(c/2 + (d*x)/2)^3*(a^2 + 6*b^2))/a^3 + (tan(c/2 + (d*x)/2)*(3*a*b -
 a^2 + 6*b^2))/a^3 - (tan(c/2 + (d*x)/2)^5*(3*a*b + a^2 - 6*b^2))/a^3)/(d*
(a + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) -
 tan(c/2 + (d*x)/2)^6*(a - b))) + (atan((8*tan(c/2 + (d*x)/2))/((8*b)/a +
(24*b^2)/a^2 - (24*b^3)/a^3 + (144*b^4)/a^4 - (144*b^5)/a^5 - 8) + (8*b*ta
n(c/2 + (d*x)/2))/(8*a - 8*b - (24*b^2)/a + (24*b^3)/a^2 - (144*b^4)/a^3 +
 (144*b^5)/a^4) - (24*b^2*tan(c/2 + (d*x)/2))/(8*a*b - 8*a^2 + 24*b^2 - (2
4*b^3)/a + (144*b^4)/a^2 - (144*b^5)/a^3) + (24*b^3*tan(c/2 + (d*x)/2))/(2
4*a*b^2 + 8*a^2*b - 8*a^3 - 24*b^3 + (144*b^4)/a - (144*b^5)/a^2) + (144*b
^4*tan(c/2 + (d*x)/2))/(24*a*b^3 - 8*a^3*b + 8*a^4 - 144*b^4 - 24*a^2*b^2
+ (144*b^5)/a) + (144*b^5*tan(c/2 + (d*x)/2))/(144*a*b^4 + 8*a^4*b - 8*a^5
 - 144*b^5 - 24*a^2*b^3 + 24*a^3*b^2))*(a^2*1i - b^2*6i)*1i)/(a^4*d) - (b*
atan(((b*((a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(1
44*a*b^6 - 3*a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 +
 7*a^5*b^2))/a^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12
*a^8*b^4 + 18*a^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b
)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6
*(a^6 - a^4*b^2)))*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2) +
 (b*((a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*
b^6 - 3*a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7...
```

3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.219.1 Optimal result 1632
3.219.2 Mathematica [A] (verified) 1633
3.219.3 Rubi [A] (verified) 1633
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3.219.1 Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{4a^2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}$$

output

```
-4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(
a+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-
b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x
+c)/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c
))
```

3.219.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4b(2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/(
(a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

3.219.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4359, 3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4359$$
$$\int \frac{\cot^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$
$$\downarrow 3042$$
$$\int \frac{\tan\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3210$$

$$\int \left(-\frac{2a^2b}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{b^2}{\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}-\frac{1}{2(a+b)^2(\cos(c+dx)-1)}+\frac{}{2(a-b)^2(co}\right.$$

$$\downarrow 2009$$

$$-\frac{4a^2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-$$

$$\frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5
/2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1
- Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^
2*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

3.219.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.219.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{2i\left(-2a^2b\,e^{3i(dx+c)}-b^3e^{3i(dx+c)}+a^3e^{2i(dx+c)}-4a\,b^2e^{2i(dx+c)}+3b^3e^{i(dx+c)}+a^3+2a\,b^2\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2\left(e^{2i(dx+c)}-1\right)d}-\frac{2b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2}{\sqrt{a}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(}$

input
```
int(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+
b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1
/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.59

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{6\,a^3b^2-6\,ab^4+\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\sqrt{}}{a^2\cos(dx+c)^2+2}\right.\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(d\right.\right.}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*
b^2 - 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/
(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 +
 b^4 + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^
5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

3.219.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.219.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{4\left(2a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}} + \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2} - \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}d\right.\right.}}{2d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 -
 2*a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)
^2 + 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a
^2*b + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b
*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))
))/d
```

3.219.9 Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{a^2-2ab+b^2}{a+b} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^3-3a^2b+7ab^2-b^3\right)}{(a+b)^2}}{d\left(\left(2a^3-6a^2b+6ab^2-2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(-2a^3+2a^2b+2ab^2-2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d(a-b)^2} + \frac{b\operatorname{atan}\left(\frac{1i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^4-2i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^2+1i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^4}{(a+b)^{5/2}(a-b)^{3/2}}\right)\left(2a^2+b^2\right)2i}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

output

```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b +
a^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan
(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4
*tan(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a
- b)^(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.220.1 Optimal result

Integrand size = 21, antiderivative size = 343

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{2a^2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{4a^2b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2}-\frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))}\\ &-\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))}\\ &+\frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2}\\ &+\frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))}+\frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))}\\ &+\frac{a^3b^2\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\end{aligned}$$

output

```
-2*a^2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)
/(a+b)^(7/2)/d-4*a^2*b*(a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+
c))^2-1/4*(a-b)*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^
2/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2+1/12*sin(d*x
+c)/(a-b)^2/d/(1+cos(d*x+c))+1/4*(a+b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))
+a^3*b^2*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

3.220.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.82

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{48a^2b(2a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{(a^2-b^2)^{7/2}} - \frac{4(2a-b)(b+a\cos(c+dx)}{(a+b)^3}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((48*a^2*b*(2*a^2 + 3*b^2)*ArcTanh[((
-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^
2)^(7/2) - (4*(2*a - b)*(b + a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 -
 ((b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (2
4*a^3*b^2*Sin[c + d*x])/((a - b)^3*(a + b)^3) + (4*(2*a + b)*(b + a*Cos[c
+ d*x])*Tan[(c + d*x)/2])/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/(a - b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

3.220.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4360, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3376$$

$$\int\left(\frac{2a^2b\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3(-a\cos(c+dx)-b)}+\frac{a^2b^2}{\left(a^2-b^2\right)^2(-a\cos(c+dx)-b)^2}+\frac{-a-b}{4(a-b)^3(-\cos(c+dx)-1)}+\frac{4(a+b}{}\right.$$

$$\downarrow 2009$$

$$-\frac{2a^2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4a^2b\left(a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+$$
$$\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))}-\frac{(a-b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))}+$$
$$\frac{(a+b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2}+$$
$$\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^
(7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(1
2*(a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*
d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) +
 Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a
- b)^2*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 +
Cos[c + d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*
x]))
```

3.220.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.220.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a-b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^2b\left(-\frac{}{\tan\left(}\right.}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a-b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^2b\left(-\frac{}{\tan\left(}\right.}{d}$
risch	$\frac{2i\left(-6a^4b\,e^{7i(dx+c)}-9a^2b^3e^{7i(dx+c)}-6a^3b^2e^{6i(dx+c)}-9ab^4e^{6i(dx+c)}+14a^4b\,e^{5i(dx+c)}+25a^2b^3e^{5i(dx+c)}+6b^5e^{5i(dx+c)}\right.}{}$

input
```
int(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x
+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)*a+tan(1/2*d*x+1/2*c)*b)-1/24/(a+b)^2/tan(
1/2*d*x+1/2*c)^3-1/8*(3*a-b)/(a+b)^3/tan(1/2*d*x+1/2*c)+2*a^2*b/(a-b)^3/(a
+b)^3*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)-(2*a^2+3*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a-b)*(a+b))^(1/2))))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1040, normalized size of antiderivative = 3.03

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(22*a^5*b^2 - 14*a^3*b^4 - 8*a*b^6 + 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*
b^4 - a*b^6)*cos(d*x + c)^4 - 2*(4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*co
s(d*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x +
c)^3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(
d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^7 +
 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2 + 10*(a^6*b - 2*a^4*b^3 +
 a^2*b^5)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8
)*d*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*c
os(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*
x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d)*sin(d*x + c)
), -1/3*(11*a^5*b^2 - 7*a^3*b^4 - 4*a*b^6 + (2*a^7 + 10*a^5*b^2 - 11*a^3*b
^4 - a*b^6)*cos(d*x + c)^4 - (4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d
*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^
3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x
 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*
a*b^6)*cos(d*x + c)^2 + 5*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))/(((a
^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^8...
```

3.220.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.220.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{48\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{48\left(2\,a^4b+3\,a^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx-\right.\right.\right.}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/24*(48*a^3*b^2*tan(1/2*d*x + 1/2*c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(2*a
^4*b + 3*a^2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arcta
n((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1
/2*c)^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^
3 - 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*ta
n(1/2*d*x + 1/2*c) - 24*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*
x + 1/2*c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*
a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*
tan(1/2*d*x + 1/2*c)^2 + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d
*x + 1/2*c)^3))/d
```

3.220.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.17

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d\,(a-b)^2}$$
$$+\frac{\frac{a^3-3a^2b+3ab^2-b^3}{3\,(a+b)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4a^4-13a^3b+15a^2b^2-7ab^3+b^4\right)}{3\,(a+b)^2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3a^5-13a^4b+38a^3b^2-18a^2b^3+7ab^4-\right.}{(a+b)^3}$$
$$+\frac{}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(8a^4-32a^3b+48a^2b^2-32ab^3+8b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(8a^4-16a^3b+16ab^3-\right.\right.}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16a^2-16b^2}{64\,(a-b)^4}+\frac{1}{8\,(a-b)^2}\right)}{d}$$
$$+\frac{a^2b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^6-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^2+3\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^4-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^6}{(a+b)^{7/2}\,(a-b)^{5/2}}\right)\left(2a^2+3b^2\right)\,2\mathrm{i}}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

input
```
int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

output

```
tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(
3*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4 - 13*a^3*b - 7*a*b^3 + b^4 + 1
5*a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 13*a^4*b + 3*
a^5 - b^5 - 18*a^2*b^3 + 38*a^3*b^2))/(a + b)^3)/(d*(tan(c/2 + (d*x)/2)^5*
(8*a^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^3*
(16*a*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x)/2)*((16*a^2 - 1
6*b^2)/(64*(a - b)^4) + 1/(8*(a - b)^2)))/d + (a^2*b*atan((a^6*tan(c/2 + (
d*x)/2)*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2 + (d*x)/2)*3i - a
^4*b^2*tan(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)))*(2*a^2 + 3*b^
2)*2i)/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

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3.221.1 Optimal result

Integrand size = 21, antiderivative size = 329

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9d}\\ &-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8d}\\ &+\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7d}+\frac{b(9a^2-10b^2)\cos^4(c+dx)}{4a^6d}\\ &-\frac{3(a^2-2b^2)\cos^5(c+dx)}{5a^5d}-\frac{b\cos^6(c+dx)}{2a^4d}+\frac{\cos^7(c+dx)}{7a^3d}\\ &-\frac{b^3(a^2-b^2)^3}{2a^{10}d(b+a\cos(c+dx))^2}+\frac{3b^2(a^2-3b^2)(a^2-b^2)^2}{a^{10}d(b+a\cos(c+dx))}\\ &+\frac{3b(a^2-b^2)(a^4-9a^2b^2+12b^4)\log(b+a\cos(c+dx))}{a^{10}d}\end{aligned}$$

output

```
-(a^6-18*a^4*b^2+45*a^2*b^4-28*b^6)*cos(d*x+c)/a^9/d-3/2*b*(3*a^4-10*a^2*b
^2+7*b^4)*cos(d*x+c)^2/a^8/d+(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^7/d+1/4*
b*(9*a^2-10*b^2)*cos(d*x+c)^4/a^6/d-3/5*(a^2-2*b^2)*cos(d*x+c)^5/a^5/d-1/2
*b*cos(d*x+c)^6/a^4/d+1/7*cos(d*x+c)^7/a^3/d-1/2*b^3*(a^2-b^2)^3/a^10/d/(b
+a*cos(d*x+c))^2+3*b^2*(a^2-3*b^2)*(a^2-b^2)^2/a^10/d/(b+a*cos(d*x+c))+3*b
*(a^2-b^2)*(a^4-9*a^2*b^2+12*b^4)*ln(b+a*cos(d*x+c))/a^10/d
```

3.221.2 Mathematica [A] (verified)

Time = 5.50 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.67

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-7945a^8b + 164080a^6b^3 - 502320a^4b^5 + 425600a^2b^7 - 76160b^9 - 784a^9\cos(3(c+dx)) + 17528a^7b^2\cos(3$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-7945*a^8*b + 164080*a^6*b^3 - 502320*a^4*b^5 + 425600*a^2*b^7 - 76160*b^
9 - 784*a^9*Cos[3*(c + d*x)] + 17528*a^7*b^2*Cos[3*(c + d*x)] - 43680*a^5*
b^4*Cos[3*(c + d*x)] + 26880*a^3*b^6*Cos[3*(c + d*x)] - 1456*a^8*b*Cos[4*(
c + d*x)] + 4872*a^6*b^3*Cos[4*(c + d*x)] - 3360*a^4*b^5*Cos[4*(c + d*x)]
+ 152*a^9*Cos[5*(c + d*x)] - 840*a^7*b^2*Cos[5*(c + d*x)] + 672*a^5*b^4*Co
s[5*(c + d*x)] + 174*a^8*b*Cos[6*(c + d*x)] - 168*a^6*b^3*Cos[6*(c + d*x)]
 - 39*a^9*Cos[7*(c + d*x)] + 48*a^7*b^2*Cos[7*(c + d*x)] - 15*a^8*b*Cos[8*
(c + d*x)] + 5*a^9*Cos[9*(c + d*x)] + 13440*a^8*b*Log[b + a*Cos[c + d*x]]
- 107520*a^6*b^3*Log[b + a*Cos[c + d*x]] + 13440*a^4*b^5*Log[b + a*Cos[c +
 d*x]] + 403200*a^2*b^7*Log[b + a*Cos[c + d*x]] - 322560*b^9*Log[b + a*Cos
[c + d*x]] + 70*a^2*b*Cos[2*(c + d*x)]*(-137*a^6 + 1896*a^4*b^2 - 4656*a^2
*b^4 + 2912*b^6 + 192*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*C
os[c + d*x]]) - 70*a*Cos[c + d*x]*(49*a^8 - 1472*a^6*b^2 + 3216*a^4*b^4 +
576*a^2*b^6 - 2432*b^8 - 768*b^2*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*
Log[b + a*Cos[c + d*x]]))/(8960*a^10*d*(b + a*Cos[c + d*x])^2)
```

3.221.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\downarrow \text{3042}$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^7}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^7(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^7(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^7(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^7\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^3(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^7 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^3}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^{10}d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^6(c+dx)a^6+\left(\frac{-28b^6+45a^2b^4-18a^4b^2}{a^6}+1\right)a^6+3b\cos^5(c+dx)a^5+3\left(a^2-2b^2\right)\cos^4(c+dx)a^4+b\left(10b^2\right.\right.}{}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}a^7\cos^7(c+dx)+\frac{1}{2}a^6b\cos^6(c+dx)-\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{a\cos(c+dx)+b}+\frac{b^3\left(a^2-b^2\right)^3}{2(a\cos(c+dx)+b)^2}+\frac{3}{5}a^5\left(a^2-2b^2\right)\cos^5(c+dx)-\frac{1}{4}}{}$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

output
```
-((a*(a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x] + (3*a^2*b*(3*a
^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/2 - a^3*(a^4 - 6*a^2*b^2 + 5*b^4)
*Cos[c + d*x]^3 - (a^4*b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/4 + (3*a^5*(a^2
- 2*b^2)*Cos[c + d*x]^5)/5 + (a^6*b*Cos[c + d*x]^6)/2 - (a^7*Cos[c + d*x]^
7)/7 + (b^3*(a^2 - b^2)^3)/(2*(b + a*Cos[c + d*x])^2) - (3*b^2*(a^2 - 3*b^
2)*(a^2 - b^2)^2)/(b + a*Cos[c + d*x]) - 3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2
+ 12*b^4)*Log[b + a*Cos[c + d*x]])/(a^10*d))
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.221.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{2}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{6a^4b^2\cos(dx+c)^5}{5}+\frac{9\cos(dx+c)^4a^5b}{4}-\frac{5\cos(dx+c)^4a^3b^3}{2}+\cos(dx+c)^3a^6-6\cos($
default	$\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{2}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{6a^4b^2\cos(dx+c)^5}{5}+\frac{9\cos(dx+c)^4a^5b}{4}-\frac{5\cos(dx+c)^4a^3b^3}{2}+\cos(dx+c)^3a^6-6\cos($
parallelrisch	$1680b(a-b)(a+b)(a^4-9a^2b^2+12b^4)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-1680b($
risch	Expression too large to display

input
```
int(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^9*(1/7*cos(d*x+c)^7*a^6-1/2*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)
^5+6/5*a^4*b^2*cos(d*x+c)^5+9/4*cos(d*x+c)^4*a^5*b-5/2*cos(d*x+c)^4*a^3*b^
3+cos(d*x+c)^3*a^6-6*cos(d*x+c)^3*a^4*b^2+5*cos(d*x+c)^3*a^2*b^4-9/2*cos(d
*x+c)^2*a^5*b+15*cos(d*x+c)^2*a^3*b^3-21/2*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a
^6+18*cos(d*x+c)*a^4*b^2-45*cos(d*x+c)*a^2*b^4+28*b^6*cos(d*x+c))+3/a^10*b
^2*(a^6-5*a^4*b^2+7*a^2*b^4-3*b^6)/(b+a*cos(d*x+c))-1/2*b^3*(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/a^10/(b+a*cos(d*x+c))^2+3/a^10*b*(a^6-10*a^4*b^2+21*a^2*b^
4-12*b^6)*ln(b+a*cos(d*x+c)))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.36

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{80\,a^9\cos(dx+c)^9 - 120\,a^8b\cos(dx+c)^8 + 2275\,a^6b^3 - 11235\,a^4b^5 + 13860\,a^2b^7 - 4760\,b^9 - 48\,(7\,a^9 - 4$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/560*(80*a^9*cos(d*x + c)^9 - 120*a^8*b*cos(d*x + c)^8 + 2275*a^6*b^3 - 1
1235*a^4*b^5 + 13860*a^2*b^7 - 4760*b^9 - 48*(7*a^9 - 4*a^7*b^2)*cos(d*x +
 c)^7 + 84*(7*a^8*b - 4*a^6*b^3)*cos(d*x + c)^6 + 56*(10*a^9 - 21*a^7*b^2
+ 12*a^5*b^4)*cos(d*x + c)^5 - 140*(10*a^8*b - 21*a^6*b^3 + 12*a^4*b^5)*co
s(d*x + c)^4 - 560*(a^9 - 10*a^7*b^2 + 21*a^5*b^4 - 12*a^3*b^6)*cos(d*x +
c)^3 - 35*(7*a^8*b - 399*a^6*b^3 + 1116*a^4*b^5 - 728*a^2*b^7)*cos(d*x + c
)^2 + 70*(41*a^7*b^2 - 81*a^5*b^4 - 108*a^3*b^6 + 152*a*b^8)*cos(d*x + c)
+ 1680*(a^6*b^3 - 10*a^4*b^5 + 21*a^2*b^7 - 12*b^9 + (a^8*b - 10*a^6*b^3 +
 21*a^4*b^5 - 12*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 10*a^5*b^4 + 21*a^
3*b^6 - 12*a*b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^12*d*cos(d*x +
 c)^2 + 2*a^11*b*d*cos(d*x + c) + a^10*b^2*d)
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.99

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{70\left(5a^6b^3-27a^4b^5+39a^2b^7-17b^9+6\left(a^7b^2-5a^5b^4+7a^3b^6-3ab^8\right)\cos(dx+c)\right)}{a^{12}\cos(dx+c)^2+2a^{11}b\cos(dx+c)+a^{10}b^2} + \frac{20a^6\cos(dx+c)^7-70a^5b\cos(dx+c)^6-84\left(a^6-2a^4b^2\right)\cos(d}{}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/140*(70*(5*a^6*b^3 - 27*a^4*b^5 + 39*a^2*b^7 - 17*b^9 + 6*(a^7*b^2 - 5*a
^5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c))/(a^12*cos(d*x + c)^2 + 2*a^11*
b*cos(d*x + c) + a^10*b^2) + (20*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c
)^6 - 84*(a^6 - 2*a^4*b^2)*cos(d*x + c)^5 + 35*(9*a^5*b - 10*a^3*b^3)*cos(
d*x + c)^4 + 140*(a^6 - 6*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5
*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 - 140*(a^6 - 18*a^4*b^2 + 45*a^2
*b^4 - 28*b^6)*cos(d*x + c))/a^9 + 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 -
12*b^7)*log(a*cos(d*x + c) + b)/a^10)/d
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(317) = 634$.

Time $= 0.46$ (sec) , antiderivative size $= 2150$, normalized size of antiderivative $= 6.53$

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/140*(420*(a^7*b - a^6*b^2 - 10*a^5*b^3 + 10*a^4*b^4 + 21*a^3*b^5 - 21*a^
2*b^6 - 12*a*b^7 + 12*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^11 - a^10*b) - 420
*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)*log(abs(-(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1))/a^10 - 70*(9*a^8*b + 6*a^7*b^2 - 105*a^6*b^3 - 148*
a^5*b^4 + 187*a^4*b^5 + 390*a^3*b^6 + 17*a^2*b^7 - 248*a*b^8 - 108*b^9 + 1
8*a^8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^7*b^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - 202*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 56*a^5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566*a^4*b^5*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 76*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 598*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b^8*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 216*b^9*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 9*a^8*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^7*b^2*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 81*a^6*b^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 180*a^5*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 99*a^4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 378*a^3*b^6*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 81*a^2*b^7*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 216*a*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 - 108*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1...
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.32

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\cos(c+dx)^4\left(\frac{2b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{4a}\right)}{d}$$
$$-\frac{\frac{-5a^6b^3+27a^4b^5-39a^2b^7+17b^9}{2a}+\cos(c+dx)\left(-3a^6b^2+15a^4b^4-21a^2b^6+9b^8\right)}{d\left(a^{11}\cos(c+dx)^2+2a^{10}b\cos(c+dx)+a^9b^2\right)}$$
$$-\frac{\cos(c+dx)^5\left(\frac{3}{5a^3}-\frac{6b^2}{5a^5}\right)}{d}$$
$$-\frac{\cos(c+dx)^2\left(\frac{3b^2\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{2a^2}-\frac{b^3\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{2a^3}+\frac{3b\left(\frac{3}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{2a}\right)}{d}$$
$$+\frac{\cos(c+dx)^7}{7a^3d}$$
$$-\frac{\cos(c+dx)\left(\frac{1}{a^3}+\frac{b^3\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a^3}+\frac{3b^2\left(\frac{3}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{a^2}-\frac{3b\left(\frac{3b^2\left(\frac{8}{a}\right.\right.}{}\right.}{}$$
$$+\frac{\cos(c+dx)^3\left(\frac{1}{a^3}+\frac{b^4}{a^7}+\frac{b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{d}$$
$$+\frac{b\cos(c+dx)^6\ln(b+a\cos(c+dx))\left(3a^6b-30a^4b^3+63a^2b^5-36b^7\right)}{a^{10}d}$$

3.221. $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

input
```
int(sin(c + d*x)^7/(a + b/cos(c + d*x))^3,x)
```

output
```
(cos(c + d*x)^4*((2*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/(4*a)))/d - ((1
7*b^9 - 39*a^2*b^7 + 27*a^4*b^5 - 5*a^6*b^3)/(2*a) + cos(c + d*x)*(9*b^8 -
 21*a^2*b^6 + 15*a^4*b^4 - 3*a^6*b^2))/(d*(a^11*cos(c + d*x)^2 + a^9*b^2 +
 2*a^10*b*cos(c + d*x))) - (cos(c + d*x)^5*(3/(5*a^3) - (6*b^2)/(5*a^5)))/
d - (cos(c + d*x)^2*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))
/(2*a^2) - (b^3*(3/a^3 - (6*b^2)/a^5))/(2*a^3) + (3*b*(3/a^3 + (3*b^4)/a^7
 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(3/a^3 - (
6*b^2)/a^5))/a))/a))/(2*a)))/d + cos(c + d*x)^7/(7*a^3*d) - (cos(c + d*x)*
(1/a^3 + (b^3*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a^3 + (3*b^2*
(3/a^3 + (3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a
^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a))/a^2 - (3*b*((3*b^2*((8*b^3)/a^6 +
 (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a^2 - (b^3*(3/a^3 - (6*b^2)/a^5))/a^3 + (
3*b*(3/a^3 + (3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^
3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(
1/a^3 + b^4/a^7 + (b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (b*((8*b^3)/a^6 + (3*b
*(3/a^3 - (6*b^2)/a^5))/a))/a))/d - (b*cos(c + d*x)^6)/(2*a^4*d) + (log(b
+ a*cos(c + d*x))*(3*a^6*b - 36*b^7 + 63*a^2*b^5 - 30*a^4*b^3))/(a^10*d)
```

3.222 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.222.1 Optimal result 1656
3.222.2 Mathematica [A] (verified) 1657
3.222.3 Rubi [A] (verified) 1657
3.222.4 Maple [A] (verified) 1660
3.222.5 Fricas [A] (verification not implemented) 1660
3.222.6 Sympy [F(-1)] 1661
3.222.7 Maxima [A] (verification not implemented) 1661
3.222.8 Giac [B] (verification not implemented) 1662
3.222.9 Mupad [B] (verification not implemented) 1663

3.222.1 Optimal result

Integrand size = 21, antiderivative size = 239

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d}-\frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d}\\&+\frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d}+\frac{3b\cos^4(c+dx)}{4a^4d}-\frac{\cos^5(c+dx)}{5a^3d}\\&-\frac{b^3(a^2-b^2)^2}{2a^8d(b+a\cos(c+dx))^2}+\frac{b^2(3a^4-10a^2b^2+7b^4)}{a^8d(b+a\cos(c+dx))}\\&+\frac{b(3a^4-20a^2b^2+21b^4)\log(b+a\cos(c+dx))}{a^8d}\end{aligned}$$

output

```
-(a^4-12*a^2*b^2+15*b^4)*cos(d*x+c)/a^7/d-b*(3*a^2-5*b^2)*cos(d*x+c)^2/a^6
/d+2/3*(a^2-3*b^2)*cos(d*x+c)^3/a^5/d+3/4*b*cos(d*x+c)^4/a^4/d-1/5*cos(d*x
+c)^5/a^3/d-1/2*b^3*(a^2-b^2)^2/a^8/d/(b+a*cos(d*x+c))^2+b^2*(3*a^4-10*a^2
*b^2+7*b^4)/a^8/d/(b+a*cos(d*x+c))+b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(
d*x+c))/a^8/d
```

3.222.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.62

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-1740a^6b + 26160a^4b^3 - 46080a^2b^5 + 12480b^7 - 206a^7\cos(3(c+dx)) + 2780a^5b^2\cos(3(c+dx)) - 3360a$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-1740*a^6*b + 26160*a^4*b^3 - 46080*a^2*b^5 + 12480*b^7 - 206*a^7*Cos[3*(
c + d*x)] + 2780*a^5*b^2*Cos[3*(c + d*x)] - 3360*a^3*b^4*Cos[3*(c + d*x)]
- 274*a^6*b*Cos[4*(c + d*x)] + 420*a^4*b^3*Cos[4*(c + d*x)] + 38*a^7*Cos[5
*(c + d*x)] - 84*a^5*b^2*Cos[5*(c + d*x)] + 21*a^6*b*Cos[6*(c + d*x)] - 6*
a^7*Cos[7*(c + d*x)] + 2880*a^6*b*Log[b + a*Cos[c + d*x]] - 13440*a^4*b^3*
Log[b + a*Cos[c + d*x]] - 18240*a^2*b^5*Log[b + a*Cos[c + d*x]] + 40320*b^
7*Log[b + a*Cos[c + d*x]] + 5*a^2*b*Cos[2*(c + d*x)]*(-407*a^4 + 3888*a^2*
b^2 - 4800*b^4 + 192*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]
) - 10*a*Cos[c + d*x]*(85*a^6 - 1728*a^4*b^2 + 1584*a^2*b^4 + 1536*b^6 - 3
84*b^2*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]))/(1920*a^8*d
*(b + a*Cos[c + d*x])^2)
```

3.222.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^5}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4360$$

$$\int -\frac{\sin^5(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^5(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^5(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^3(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^5 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)\left(a^2-a^2\cos^2(c+dx)\right)^2}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^8 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\cos^4(c+dx)a^4+\left(\frac{3(5b^2-4a^2)b^2}{a^4}+1\right)a^4-3b\cos^3(c+dx)a^3-2\left(a^2-3b^2\right)\cos^2(c+dx)a^2-2b\left(5b^2-3a^2\right)\cos(c+dx)a+\left(\frac{b^3\left(a^2-b^2\right)^2}{a^4(a\cos(c+dx)+b)^2}+\frac{3b^2-4a^2}{a^2}\right)b^2\right)d(a\cos(c+dx))}{a^8 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^5\cos^5(c+dx)-\frac{3}{4}a^4b\cos^4(c+dx)+a^2b\left(3a^2-5b^2\right)\cos^2(c+dx)+\frac{b^3\left(a^2-b^2\right)^2}{2(a\cos(c+dx)+b)^2}+a\left(a^4-12a^2b^2+15b^4\right)\cos(c+dx)-\frac{b\left(a^2-b^2\right)\left(a^4-10a^2b^2+5b^4\right)}{a^4(a\cos(c+dx)+b)}-5b\left(a^4-3a^2b^2+b^4\right)\log(a\cos(c+dx)+b)}{a^8}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

3.222. $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}dx$

output
```
-((a*(a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x] + a^2*b*(3*a^2 - 5*b^2)*Cos[
c + d*x]^2 - (2*a^3*(a^2 - 3*b^2)*Cos[c + d*x]^3)/3 - (3*a^4*b*Cos[c + d*x
]^4)/4 + (a^5*Cos[c + d*x]^5)/5 + (b^3*(a^2 - b^2)^2)/(2*(b + a*Cos[c + d*
x])^2) - (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(b + a*Cos[c + d*x]) - b*(3*a^
4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]])/(a^8*d))
```

3.222.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.222.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{3b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+2\cos(dx+c)^3a^2b^2+3\cos(dx+c)^2a^3b-5\cos(dx+c)^2ab^3+\cos(dx+c)a^4-12\cos(}{a^7}}{d}$
default	$-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{3b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+2\cos(dx+c)^3a^2b^2+3\cos(dx+c)^2a^3b-5\cos(dx+c)^2ab^3+\cos(dx+c)a^4-12\cos(}{a^7}}{d}$
parallelrisch	$\frac{2880\left(a^4-\frac{20}{3}a^2b^2+7b^4\right)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-2880\left(a^4-\frac{20}{3}a^2b^2+\right.}{}$
risch	$\frac{3b\cos(4dx+4c)}{32da^4}-\frac{\cos(5dx+5c)}{80a^3d}-\frac{6ibc}{a^4d}+\frac{40ib^3c}{a^6d}-\frac{42ib^5c}{a^8d}+\frac{2b^2\left(3a^5e^{3i(dx+c)}-10a^3b^2e^{3i(dx+c)}+7ab^4e^{3i(dx+c)}\right.}{}$
norman	$\frac{\left(6ba^6-12a^5b^2-34a^4b^3+80b^4a^3+2b^5a^2-84ab^6+42b^7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{a^7d(a-b)}+\frac{\left(18a^7b-72a^6b^2-30a^5b^3+444a^4b^4-474a^3b^5-264a^2b^6+6\right.}{a^7d\left(a^2-2ab+b^2\right)}$

input
```
int(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^7*(1/5*cos(d*x+c)^5*a^4-3/4*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*
a^4+2*cos(d*x+c)^3*a^2*b^2+3*cos(d*x+c)^2*a^3*b-5*cos(d*x+c)^2*a*b^3+cos(d
*x+c)*a^4-12*cos(d*x+c)*a^2*b^2+15*cos(d*x+c)*b^4)-1/2*b^3*(a^4-2*a^2*b^2+
b^4)/a^8/(b+a*cos(d*x+c))^2+1/a^8*b^2*(3*a^4-10*a^2*b^2+7*b^4)/(b+a*cos(d*
x+c))+1/a^8*b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+c)))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.38

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx =$$
$$-\frac{96\,a^7\cos(dx+c)^7-168\,a^6b\cos(dx+c)^6-1785\,a^4b^3+5520\,a^2b^5-3120\,b^7-16\,(20\,a^7-21\,a^5b^2)\cos($$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/480*(96*a^7*cos(d*x + c)^7 - 168*a^6*b*cos(d*x + c)^6 - 1785*a^4*b^3 +
5520*a^2*b^5 - 3120*b^7 - 16*(20*a^7 - 21*a^5*b^2)*cos(d*x + c)^5 + 40*(20
*a^6*b - 21*a^4*b^3)*cos(d*x + c)^4 + 160*(3*a^7 - 20*a^5*b^2 + 21*a^3*b^4
)*cos(d*x + c)^3 + 15*(25*a^6*b - 592*a^4*b^3 + 800*a^2*b^5)*cos(d*x + c)^
2 - 30*(71*a^5*b^2 - 48*a^3*b^4 - 128*a*b^6)*cos(d*x + c) - 480*(3*a^4*b^3
 - 20*a^2*b^5 + 21*b^7 + (3*a^6*b - 20*a^4*b^3 + 21*a^2*b^5)*cos(d*x + c)^
2 + 2*(3*a^5*b^2 - 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c))*log(a*cos(d*x + c)
 + b))/(a^10*d*cos(d*x + c)^2 + 2*a^9*b*d*cos(d*x + c) + a^8*b^2*d)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{30\,(5\,a^4b^3-18\,a^2b^5+13\,b^7+2\,(3\,a^5b^2-10\,a^3b^4+7\,ab^6)\cos(dx+c))}{a^{10}\cos(dx+c)^2+2\,a^9b\cos(dx+c)+a^8b^2} - \frac{12\,a^4\cos(dx+c)^5-45\,a^3b\cos(dx+c)^4-40\,(a^4-3\,a^2b^2)\cos(dx+c)^3+60\,(3\,a}{a^7}}{60\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(30*(5*a^4*b^3 - 18*a^2*b^5 + 13*b^7 + 2*(3*a^5*b^2 - 10*a^3*b^4 + 7*
a*b^6)*cos(d*x + c))/(a^10*cos(d*x + c)^2 + 2*a^9*b*cos(d*x + c) + a^8*b^2
) - (12*a^4*cos(d*x + c)^5 - 45*a^3*b*cos(d*x + c)^4 - 40*(a^4 - 3*a^2*b^2
)*cos(d*x + c)^3 + 60*(3*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 12*a^
2*b^2 + 15*b^4)*cos(d*x + c))/a^7 + 60*(3*a^4*b - 20*a^2*b^3 + 21*b^5)*log
(a*cos(d*x + c) + b)/a^8)/d
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. $2(231) = 462$.

Time = 0.42 (sec) , antiderivative size = 1337, normalized size of antiderivative = 5.59

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*(3*a^5*b - 3*a^4*b^2 - 20*a^3*b^3 + 20*a^2*b^4 + 21*a*b^5 - 21*b^
6)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 60*(3*a^4*b - 20*a^2*b^3 + 21
*b^5)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^8 - 30*(9*a^6
*b + 6*a^5*b^2 - 75*a^4*b^3 - 108*a^3*b^4 + 51*a^2*b^5 + 150*a*b^6 + 63*b^
7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^5*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 142*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 36*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 250*a^2*b^5*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 24*a*b^6*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 126*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*b*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 51*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
120*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b^5*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 126*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 63*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b
+ a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))^2*a^8) + (64*a^5 - 411*a^4*b - 1200*a^3*b^2 + 2740*a^2*b^3 + 18
00*a*b^4 - 2877*b^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2415
*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5280*a^3*b^2*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 14900*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c...
```

3.222.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.32

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{\cos(c+dx)^3\left(\frac{2}{3a^3}-\frac{2b^2}{a^5}\right)}{d} - \frac{\cos(c+dx)^2\left(\frac{4b^3}{a^6}+\frac{3b\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{2a}\right)}{d}$$
$$+ \frac{\cos(c+dx)\left(3a^4b^2-10a^2b^4+7b^6\right)+\frac{5a^4b^3-18a^2b^5+13b^7}{2a}}{d\left(a^9\cos(c+dx)^2+2a^8b\cos(c+dx)+a^7b^2\right)}$$
$$- \frac{\cos(c+dx)\left(\frac{1}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{d} - \frac{\cos(c+dx)^5}{5a^3d}$$
$$+ \frac{3b\cos(c+dx)^4}{4a^4d} + \frac{\ln(b+a\cos(c+dx))\left(3a^4b-20a^2b^3+21b^5\right)}{a^8d}$$

input
```
int(sin(c + d*x)^5/(a + b/cos(c + d*x))^3,x)
```

output
```
(cos(c + d*x)^3*(2/(3*a^3) - (2*b^2)/a^5))/d - (cos(c + d*x)^2*((4*b^3)/a^
6 + (3*b*(2/a^3 - (6*b^2)/a^5))/(2*a)))/d + (cos(c + d*x)*(7*b^6 - 10*a^2*
b^4 + 3*a^4*b^2) + (13*b^7 - 18*a^2*b^5 + 5*a^4*b^3)/(2*a))/(d*(a^9*cos(c
+ d*x)^2 + a^7*b^2 + 2*a^8*b*cos(c + d*x))) - (cos(c + d*x)*(1/a^3 + (3*b^
4)/a^7 + (3*b^2*(2/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(2/a
^3 - (6*b^2)/a^5))/a))/a))/d - cos(c + d*x)^5/(5*a^3*d) + (3*b*cos(c + d*x
)^4)/(4*a^4*d) + (log(b + a*cos(c + d*x))*(3*a^4*b + 21*b^5 - 20*a^2*b^3))
/(a^8*d)
```

3.223 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.223.1 Optimal result 1664
3.223.2 Mathematica [A] (verified) 1664
3.223.3 Rubi [A] (verified) 1665
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3.223.9 Mupad [B] (verification not implemented) 1670

3.223.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{(a^2-6b^2)\cos(c+dx)}{a^5 d} - \frac{3b\cos^2(c+dx)}{2a^4 d} + \frac{\cos^3(c+dx)}{3a^3 d} - \frac{b^3(a^2-b^2)}{2a^6 d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-5b^2)}{a^6 d(b+a\cos(c+dx))} + \frac{b(3a^2-10b^2)\log(b+a\cos(c+dx))}{a^6 d}$$

output
```
-(a^2-6*b^2)*cos(d*x+c)/a^5/d-3/2*b*cos(d*x+c)^2/a^4/d+1/3*cos(d*x+c)^3/a^
3/d-1/2*b^3*(a^2-b^2)/a^6/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-5*b^2)/a^6/d/(b+
a*cos(d*x+c))+b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c))/a^6/d
```

3.223.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(b+a\cos(c+dx))\left(9a^4(b+2a\cos(c+dx)) - (b+a\cos(c+dx))^2\left(72a(a^2-8b^2)\cos(c+dx) + \frac{-9a^4b+48}{(b+a\cos}\right.\right.$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*(9*a^4*(b + 2*a*Cos[c + d*x]) - (b + a*Cos[c + d*x])
^2*(72*a*(a^2 - 8*b^2)*Cos[c + d*x] + (-9*a^4*b + 48*a^2*b^3 - 48*b^5)/(b
+ a*Cos[c + d*x])^2 + (6*(3*a^4 - 48*a^2*b^2 + 80*b^4))/(b + a*Cos[c + d*x
]) + 72*a^2*b*Cos[2*(c + d*x)] - 8*a^3*Cos[3*(c + d*x)] + 96*(-3*a^2*b + 1
0*b^3)*Log[b + a*Cos[c + d*x]]))*Sec[c + d*x]^3)/(96*a^6*d*(a + b*Sec[c +
d*x])^3)
```

3.223.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^3}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^3(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$
$$\downarrow 25$$
$$-\int -\frac{\cos^3(c+dx)\sin^3(c+dx)}{(b+a\cos(c+dx))^3}dx$$
$$\downarrow 25$$
$$\int \frac{\sin^3(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$
$$\downarrow 3042$$
$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$
$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3 \sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3316$$

$$-\frac{\int \frac{\cos^3(c+dx)(a^2-a^2\cos^2(c+dx))}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^3 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)(a^2-a^2\cos^2(c+dx))}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^6 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-\cos^2(c+dx)a^2+\left(1-\frac{6b^2}{a^2}\right)a^2+3b\cos(c+dx)a+\frac{10b^3-3a^2b}{b+a\cos(c+dx)}+\frac{3a^2b^2-5b^4}{(b+a\cos(c+dx))^2}+\frac{b^5-a^2b^3}{(b+a\cos(c+dx))^3}\right)d(a\cos}{a^6 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)-\frac{b^2(3a^2-5b^2)}{a\cos(c+dx)+b}+a\left(a^2-6b^2\right)\cos(c+dx)-b\left(3a^2-10b^2\right)\log(a\cos(c+dx)+b)+\frac{b^3(a^2-b}{2(a\cos(c+dx}}{a^6 d}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
-((a*(a^2 - 6*b^2)*Cos[c + d*x] + (3*a^2*b*Cos[c + d*x]^2)/2 - (a^3*Cos[c
+ d*x]^3)/3 + (b^3*(a^2 - b^2))/(2*(b + a*Cos[c + d*x])^2) - (b^2*(3*a^2 -
 5*b^2))/(b + a*Cos[c + d*x]) - b*(3*a^2 - 10*b^2)*Log[b + a*Cos[c + d*x]]
)/(a^6*d))
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.223.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{3ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+6\cos(dx+c)b^2}{a^5}-\frac{b^3\left(a^2-b^2\right)}{2a^6(b+a\cos(dx+c))^2}+\frac{b^2\left(3a^2-5b^2\right)}{a^6(b+a\cos(dx+c))}+\frac{b\left(3a^2-10b^2\right)\ln(b+a}{a^6}$
default	$\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{3ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+6\cos(dx+c)b^2}{a^5}-\frac{b^3\left(a^2-b^2\right)}{2a^6(b+a\cos(dx+c))^2}+\frac{b^2\left(3a^2-5b^2\right)}{a^6(b+a\cos(dx+c))}+\frac{b\left(3a^2-10b^2\right)\ln(b+a}{a^6}$
parallelrisch	$72(a+b)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)\left(a^2-\frac{10b^2}{3}\right)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-72(a+b)(\cos(2dx+2c)$
risch	$\frac{20ib^3c}{a^6d}-\frac{3ixb}{a^4}+\frac{e^{3i(dx+c)}}{24a^3d}-\frac{3be^{2i(dx+c)}}{8a^4d}-\frac{3e^{i(dx+c)}}{8a^3d}+\frac{3e^{i(dx+c)}b^2}{a^5d}-\frac{3e^{-i(dx+c)}}{8a^3d}+\frac{3e^{-i(dx+c)}b^2}{a^5d}-\frac{3b}{}$
norman	$\frac{\left(6a^4b-12a^3b^2-14a^2b^3+40ab^4-20b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{a^5d(a-b)}+\frac{-4a^6+62a^4b^2-118a^2b^4+60b^6}{3a^5d\left(a^2-2ab+b^2\right)}-\frac{2\left(2a^6-5a^5b+18a^4b^2-17a^3b^3-48a^2b^4+9}{da^5\left(a^2-2ab+b^2\right.}$ $\left(1+\tan\right.$

input
```
int(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^5*(1/3*cos(d*x+c)^3*a^2-3/2*a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+6*cos
(d*x+c)*b^2)-1/2*b^3*(a^2-b^2)/a^6/(b+a*cos(d*x+c))^2+1/a^6*b^2*(3*a^2-5*b
^2)/(b+a*cos(d*x+c))+1/a^6*b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c)))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.43

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{4\,a^5\cos(dx+c)^5 - 10\,a^4b\cos(dx+c)^4 + 39\,a^2b^3 - 54\,b^5 - 4\,(3\,a^5 - 10\,a^3b^2)\cos(dx+c)^3 - 3\,(5\,a^4b - 42$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*a^5*cos(d*x + c)^5 - 10*a^4*b*cos(d*x + c)^4 + 39*a^2*b^3 - 54*b^5
 - 4*(3*a^5 - 10*a^3*b^2)*cos(d*x + c)^3 - 3*(5*a^4*b - 42*a^2*b^3)*cos(d*
x + c)^2 + 6*(7*a^3*b^2 + 2*a*b^4)*cos(d*x + c) + 12*(3*a^2*b^3 - 10*b^5 +
 (3*a^4*b - 10*a^2*b^3)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 10*a*b^4)*cos(d*x
+ c))*log(a*cos(d*x + c) + b))/(a^8*d*cos(d*x + c)^2 + 2*a^7*b*d*cos(d*x +
 c) + a^6*b^2*d)
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{3\left(5a^2b^3-9b^5+2\left(3a^3b^2-5ab^4\right)\cos(dx+c)\right)}{a^8\cos(dx+c)^2+2a^7b\cos(dx+c)+a^6b^2}+\frac{2a^2\cos(dx+c)^3-9ab\cos(dx+c)^2-6\left(a^2-6b^2\right)\cos(dx+c)}{a^5}+\frac{6\left(3a^2b-10b^3\right)\log(a\cos(dx+c)+}{a^6}}{6d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(3*(5*a^2*b^3 - 9*b^5 + 2*(3*a^3*b^2 - 5*a*b^4)*cos(d*x + c))/(a^8*cos
(d*x + c)^2 + 2*a^7*b*cos(d*x + c) + a^6*b^2) + (2*a^2*cos(d*x + c)^3 - 9*
a*b*cos(d*x + c)^2 - 6*(a^2 - 6*b^2)*cos(d*x + c))/a^5 + 6*(3*a^2*b - 10*b
^3)*log(a*cos(d*x + c) + b)/a^6)/d
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(3a^2b-10b^3)\log\left(|-a\cos(dx+c)-b|\right)}{a^6d}+\frac{5a^2b^3-9b^5+\frac{2\left(3a^3b^2d-5ab^4d\right)\cos(dx+c)}{d}}{2\left(a\cos(dx+c)+b\right)^2a^6d}$$
$$+\frac{2a^6d^8\cos(dx+c)^3-9a^5bd^8\cos(dx+c)^2-6a^6d^8\cos(dx+c)+36a^4b^2d^8\cos(dx+c)}{6a^9d^9}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*a^2*b - 10*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^6*d) + 1/2*(5*a^2*b^3
- 9*b^5 + 2*(3*a^3*b^2*d - 5*a*b^4*d)*cos(d*x + c)/d)/((a*cos(d*x + c) + b
)^2*a^6*d) + 1/6*(2*a^6*d^8*cos(d*x + c)^3 - 9*a^5*b*d^8*cos(d*x + c)^2 -
6*a^6*d^8*cos(d*x + c) + 36*a^4*b^2*d^8*cos(d*x + c))/(a^9*d^9)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\cos(c+dx)^3}{3a^3d} - \frac{\cos(c+dx)\left(5b^4-3a^2b^2\right)+\frac{9b^5-5a^2b^3}{2a}}{d\left(a^7\cos(c+dx)^2+2a^6b\cos(c+dx)+a^5b^2\right)} - \frac{\cos(c+dx)\left(\frac{1}{a^3}-\frac{6b^2}{a^5}\right)}{d} - \frac{3b\cos(c+dx)^2}{2a^4d} + \frac{\ln(b+a\cos(c+dx))\left(3a^2b-10b^3\right)}{a^6d}$$

input
```
int(sin(c + d*x)^3/(a + b/cos(c + d*x))^3,x)
```

output
```
cos(c + d*x)^3/(3*a^3*d) - (cos(c + d*x)*(5*b^4 - 3*a^2*b^2) + (9*b^5 - 5*
a^2*b^3)/(2*a))/(d*(a^7*cos(c + d*x)^2 + a^5*b^2 + 2*a^6*b*cos(c + d*x)))
- (cos(c + d*x)*(1/a^3 - (6*b^2)/a^5))/d - (3*b*cos(c + d*x)^2)/(2*a^4*d)
+ (log(b + a*cos(c + d*x))*(3*a^2*b - 10*b^3))/(a^6*d)
```

3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

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3.224.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{a^3 d} - \frac{b^3}{2a^4 d(b+a\cos(c+dx))^2} + \frac{3b^2}{a^4 d(b+a\cos(c+dx))} + \frac{3b\log(b+a\cos(c+dx))}{a^4 d}$$

output
```
-cos(d*x+c)/a^3/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(d*
x+c))+3*b*ln(b+a*cos(d*x+c))/a^4/d
```

3.224.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-2a^3\cos^3(c+dx) + 2a^2 b\cos^2(c+dx)(-2+3\log(b+a\cos(c+dx))) + 4ab^2\cos(c+dx)(1+3\log(b+a\cos(c+dx))) + b^3(5+6\log(b+a\cos(c+dx)))}{2a^4 d(b+a\cos(c+dx))^2}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-2*a^3*Cos[c + d*x]^3 + 2*a^2*b*Cos[c + d*x]^2*(-2 + 3*Log[b + a*Cos[c +
d*x]]) + 4*a*b^2*Cos[c + d*x]*(1 + 3*Log[b + a*Cos[c + d*x]]) + b^3*(5 + 6
*Log[b + a*Cos[c + d*x]]))/(2*a^4*d*(b + a*Cos[c + d*x])^2)
```

3.224.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3312$$

$$-\frac{\int \frac{\cos^3(c+dx)}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{ad}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))^3}d(a\cos(c+dx))}{a^4d}$$

$$\downarrow 49$$

$$-\frac{\int \left(-\frac{b^3}{(b+a\cos(c+dx))^3}+\frac{3b^2}{(b+a\cos(c+dx))^2}-\frac{3b}{b+a\cos(c+dx)}+1\right) d(a\cos(c+dx))}{a^4 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^3}{2(a\cos(c+dx)+b)^2}-\frac{3b^2}{a\cos(c+dx)+b}-3b\log(a\cos(c+dx)+b)+a\cos(c+dx)}{a^4 d}$$

input
```
Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
-((a*Cos[c + d*x] + b^3/(2*(b + a*Cos[c + d*x])^2) - (3*b^2)/(b + a*Cos[c
+ d*x]) - 3*b*Log[b + a*Cos[c + d*x]])/(a^4*d))
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.224.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{1}{a^3\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^4}-\frac{b}{2a^2(a+b\sec(dx+c))^2}+\frac{3b\ln(a+b\sec(dx+c))}{a^4}-\frac{2b}{a^3(a+b\sec(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^3\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^4}-\frac{b}{2a^2(a+b\sec(dx+c))^2}+\frac{3b\ln(a+b\sec(dx+c))}{a^4}-\frac{2b}{a^3(a+b\sec(dx+c))}}{d}$
risch	$-\frac{3ixb}{a^4}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{6ibc}{a^4d}+\frac{2b^2\left(3a\,e^{3i(dx+c)}+5b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{a^4\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2d}+\frac{3b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(}}{a^4d}$
norman	$\frac{\frac{-2a^4+10a^2b^2-6b^4}{d\,a^3\left(a^2-2ab+b^2\right)}-\frac{\left(2a^3-6a^2b+12a\,b^2-6b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d\,a^3(a-b)}-\frac{\left(-4a^4+8a^3b-18a\,b^3+12b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\,a^3\left(a^2-2ab+b^2\right)}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3b\ln\left(1+\tan\left(\frac{dx}{2}\right.\right.}{a^4d}$
parallelrisch	$\frac{6b(a-b)^2\left(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2\right)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-6b(a-b)^2(\cos(2dx+2c)a^2+4\cos(}{2a^4d}$

input
```
int(sin(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3/sec(d*x+c)-3/a^4*b*ln(sec(d*x+c))-1/2*b/a^2/(a+b*sec(d*x+c))^2
+3/a^4*b*ln(a+b*sec(d*x+c))-2/a^3*b/(a+b*sec(d*x+c)))
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx =$$
$$-\frac{2\,a^3\cos(dx+c)^3+4\,a^2b\cos(dx+c)^2-4\,ab^2\cos(dx+c)-5\,b^3-6\left(a^2b\cos(dx+c)^2+2\,ab^2\cos(dx+c)\right.}{2\left(a^6d\cos(dx+c)^2+2\,a^5bd\cos(dx+c)+a^4b^2d\right)}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*cos(d*x + c)^3 + 4*a^2*b*cos(d*x + c)^2 - 4*a*b^2*cos(d*x + c)
 - 5*b^3 - 6*(a^2*b*cos(d*x + c)^2 + 2*a*b^2*cos(d*x + c) + b^3)*log(a*cos
(d*x + c) + b))/(a^6*d*cos(d*x + c)^2 + 2*a^5*b*d*cos(d*x + c) + a^4*b^2*d
)
```

3.224.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{6\,ab^2\cos(dx+c)+5\,b^3}{a^6\cos(dx+c)^2+2\,a^5b\cos(dx+c)+a^4b^2} - \frac{2\cos(dx+c)}{a^3} + \frac{6\,b\log(a\cos(dx+c)+b)}{a^4}}{2\,d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((6*a*b^2*cos(d*x + c) + 5*b^3)/(a^6*cos(d*x + c)^2 + 2*a^5*b*cos(d*x
+ c) + a^4*b^2) - 2*cos(d*x + c)/a^3 + 6*b*log(a*cos(d*x + c) + b)/a^4)/d
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\cos(dx+c)}{a^3 d} + \frac{3\,b\log\left(\left|-a\cos(dx+c)-b\right|\right)}{a^4 d} + \frac{6\,ab^2\cos(dx+c)+5\,b^3}{2\,(a\cos(dx+c)+b)^2 a^4 d}$$

input
```
integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-cos(d*x + c)/(a^3*d) + 3*b*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/2*(6
*a*b^2*cos(d*x + c) + 5*b^3)/((a*cos(d*x + c) + b)^2*a^4*d)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{3\,b^2\,\cos(c+d\,x)+\frac{5\,b^3}{2\,a}}{d\,\left(a^5\cos(c+d\,x)^2+2\,a^4\,b\,\cos(c+d\,x)+a^3\,b^2\right)} - \frac{\cos(c+d\,x)}{a^3\,d} + \frac{3\,b\,\ln(b+a\,\cos(c+d\,x))}{a^4\,d}$$

input
```
int(sin(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

output
```
(3*b^2*cos(c + d*x) + (5*b^3)/(2*a))/(d*(a^5*cos(c + d*x)^2 + a^3*b^2 + 2*
a^4*b*cos(c + d*x))) - cos(c + d*x)/(a^3*d) + (3*b*log(b + a*cos(c + d*x))
)/(a^4*d)
```

3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.225.1 Optimal result 1677
3.225.2 Mathematica [A] (verified) 1677
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3.225.8 Giac [B] (verification not implemented) 1683
3.225.9 Mupad [B] (verification not implemented) 1684

3.225.1 Optimal result

Integrand size = 19, antiderivative size = 163

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{b^3}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^3 d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^3 d} + \frac{b(3a^2+b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^3 d}$$

output
```
-1/2*b^3/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^
2/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^3/d-1/2*ln(1+cos(d*x+c))/(
a-b)^3/d+b*(3*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

3.225.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.25

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(b+a\cos(c+dx))\left(\frac{b^3}{a^2(-a^2+b^2)} - \frac{2b^2(-3a^2+b^2)(b+a\cos(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{2(b+a\cos(c+dx))^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3} + \frac{2b(3a^2+b^2)(b+a\cos}{2d(a+b\sec(c+dx))^3}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*(b^3/(a^2*(-a^2 + b^2)) - (2*b^2*(-3*a^2 + b^2)*(b +
 a*Cos[c + d*x]))/(a^2*(a - b)^2*(a + b)^2) + (2*(b + a*Cos[c + d*x])^2*Lo
g[Cos[(c + d*x)/2]])/(-a + b)^3 + (2*b*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^
2*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (2*(b + a*Cos[c + d*x])^2*Log[S
in[(c + d*x)/2]])/(a + b)^3)*Sec[c + d*x]^3)/(2*d*(a + b*Sec[c + d*x])^3)
```

3.225.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 603, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos^2(c+dx)\cot(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^2(c+dx)\cot(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3316$$

$$\frac{a\int -\frac{\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a\int \frac{\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{a^2d}$$

$$\downarrow 603$$

$$-\frac{\frac{b^3}{2(a^2-b^2)(a\cos(c+dx)+b)^2}-\frac{\int -\frac{2\left(-b\cos(c+dx)a^3+b^2a^2+\left(a^2-b^2\right)\cos^2(c+dx)a^2\right)}{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2(a^2-b^2)}}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{-b\cos(c+dx)a^3+b^2a^2+\left(a^2-b^2\right)\cos^2(c+dx)a^2}{(b+a\cos(c+dx))^2\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{a^2-b^2}+\frac{b^3}{2(a^2-b^2)(a\cos(c+dx)+b)^2}}{a^2d}$$

$$\downarrow 2160$$

$$-\frac{\frac{\int\left(\frac{(a-b)a^2}{2(a+b)^2(a-a\cos(c+dx))}+\frac{(a+b)a^2}{2(a-b)^2(\cos(c+dx)a+a)}-\frac{b\left(3a^2+b^2\right)a^2}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))}+\frac{b^2\left(b^2-3a^2\right)}{\left(b^2-a^2\right)(b+a\cos(c+dx))^2}\right)d(a\cos(c+dx))}{a^2-b^2}+\frac{b^3}{2(a^2-b^2)(a\cos(c+}}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^2\left(3a^2-b^2\right)}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{a^2b\left(3a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^2}-\frac{a^2(a-b)\log(a-a\cos(c+dx))}{2(a+b)^2}+\frac{a^2(a+b)\log(a\cos(c+dx)+a)}{2(a-b)^2}}{a^2-b^2}+\frac{b^3}{2(a^2-b^2)(a\cos(c+dx)+b)}}{a^2d}$$

input
```
Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

3.225. $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
-((b^3/(2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-((b^2*(3*a^2 - b^2))/((a
^2 - b^2)*(b + a*Cos[c + d*x]))) - (a^2*(a - b)*Log[a - a*Cos[c + d*x]])/(
2*(a + b)^2) + (a^2*(a + b)*Log[a + a*Cos[c + d*x]])/(2*(a - b)^2) - (a^2*
b*(3*a^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^2)/(a^2 - b^2))/(a^2*
d))
```

3.225.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.225.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b\left(3a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2\left(3a^2-b^2\right)}{(a+b)^2(a-b)^2a^2(b+a\cos(dx+c))}-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^3}+\frac{\ln(\cos(}{2(}}{d}$
default	$\frac{-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b\left(3a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2\left(3a^2-b^2\right)}{(a+b)^2(a-b)^2a^2(b+a\cos(dx+c))}-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^3}+\frac{\ln(\cos(}{2(}}{d}$
parallelrisch	$\frac{3\left(a^2+\frac{b^2}{3}\right)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+(a-b)^3(\cos(2dx+2c)a^2+4\cos(}{(a-b)^3(a+b)^3d(\cos(2dx+2c)a^2-}$
norman	$\frac{\frac{6a\,b^2}{d\left(a^4-2a^3b+2a\,b^3-b^4\right)}-\frac{2\left(3a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(a^4-2a^2b^2+b^4\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{b(3a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{d}{2}\right.\right.}{d(a^6-3a^4b^2+3a^2b^4-b^6}$
risch	$\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}+\frac{ic}{d(a^3-3a^2b+3a\,b^2-b^3)}-\frac{ix}{a^3+3a^2b+3a\,b^2+b^3}-\frac{ic}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6ib\,a^2x}{a^6-3a^4b^2+3a^2}$

input
```
int(csc(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-
b)^3*ln(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2/a^2/(b+a*cos(d*x+c
))-1/2/(a-b)^3*ln(cos(d*x+c)+1)+1/2/(a+b)^3*ln(cos(d*x+c)-1))
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(157) = 314.

Time = 0.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.91

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{5\,a^4b^3-6\,a^2b^5+b^7+2\left(3\,a^5b^2-4\,a^3b^4+ab^6\right)\cos(dx+c)+2\left(3\,a^4b^3+a^2b^5+\left(3\,a^6b+a^4b^3\right)\cos(dx+c)\right.}{}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(5*a^4*b^3 - 6*a^2*b^5 + b^7 + 2*(3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d
*x + c) + 2*(3*a^4*b^3 + a^2*b^5 + (3*a^6*b + a^4*b^3)*cos(d*x + c)^2 + 2*
(3*a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 3
*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos
(d*x + c)^2 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*cos(d*x + c))*lo
g(1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 + (
a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*cos(d*x + c)^2 + 2*(a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 - a^3*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10
 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^
3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6
 - a^2*b^8)*d)
```

3.225.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.48

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(3\,a^2b+b^3)\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{5\,a^2b^3-b^5+2\,(3\,a^3b^2-ab^4)\cos(dx+c)}{a^6b^2-2\,a^4b^4+a^2b^6+(a^8-2\,a^6b^2+a^4b^4)\cos(dx+c)^2+2\,(a^7b-2\,a^5b^3+a^3b^5)\cos(dx+c)} - \frac{\log(\cos(dx+c)+}{a^3-3\,a^2b+3\,ab^2-}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(3*a^2*b + b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + (5*a^2*b^3 - b^5 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c))/(a^6*b^2
 - 2*a^4*b^4 + a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^2 + 2*(a
^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^3 - 3
*a^2*b + 3*a*b^2 - b^3) + log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3))/d
```

3.225.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(157) = 314$.

Time $= 0.39$ (sec) , antiderivative size $= 452$, normalized size of antiderivative $= 2.77$

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{2\left(3a^2b+b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{9a^3b+15a^2b^2+3ab^3-3b^4+\frac{18a^3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6a^2}{}}{2a}$$

input
```
integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(3*a^2*b + b^3)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6) + log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) - (9*a^3*b + 15*a^2*b^2 + 3*a*b^3 - 3*b^4 + 18*a^3*b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 10*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^4*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 9*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a*
b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^4*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5
)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))^2))/d
```

3.225.9 Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^3} - \frac{\frac{\cos(c+d\,x)\,\left(b^4-3\,a^2\,b^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)} + \frac{b\,\left(b^4-5\,a^2\,b^2\right)}{2\,a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\cos\left(c+d\,x\right)^2+2\,a\,b\,\cos\left(c+d\,x\right)+b^2\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^3} - \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^3}-\frac{1}{2\,{\left(a-b\right)}^3}\right)}{d}$$

input
```
int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

output
```
log(cos(c + d*x) - 1)/(2*d*(a + b)^3) - ((cos(c + d*x)*(b^4 - 3*a^2*b^2))/
(a*(a^4 + b^4 - 2*a^2*b^2)) + (b*(b^4 - 5*a^2*b^2))/(2*a^2*(a^4 + b^4 - 2*
a^2*b^2)))/(d*(b^2 + a^2*cos(c + d*x)^2 + 2*a*b*cos(c + d*x))) - log(cos(c
 + d*x) + 1)/(2*d*(a - b)^3) - (log(b + a*cos(c + d*x))*(1/(2*(a + b)^3) -
 1/(2*(a - b)^3)))/d
```

3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.226.1 Optimal result 1685
3.226.2 Mathematica [C] (verified) 1686
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3.226.8 Giac [B] (verification not implemented) 1692
3.226.9 Mupad [B] (verification not implemented) 1693

3.226.1 Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}\\ &+\frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}\\ &+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d}\\ &+\frac{(a-2b)\log(1-\cos(c+dx))}{4(a+b)^4 d}-\frac{(a+2b)\log(1+\cos(c+dx))}{4(a-b)^4 d}\\ &+\frac{b(3a^4+8a^2b^2+b^4)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4 d}\end{aligned}$$

output

```
-1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b
+a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(
a^2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d
*x+c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.226.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{2i(3a^4b+8a^2b^3+b^5)\,(c+dx)}{(a-b)^4(a+b)^4d}\\ &-\frac{i(-a-2b)\arctan(\tan(c+dx))}{2(-a+b)^4d}\\ &-\frac{i(a-2b)\arctan(\tan(c+dx))}{2(a+b)^4d}\\ &-\frac{b^3}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2}\\ &-\frac{b^2(3a^2+b^2)}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))}\\ &-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d}+\frac{(-a-2b)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(-a+b)^4d}\\ &+\frac{(3a^4b+8a^2b^3+b^5)\log(b+a\cos(c+dx))}{\left(-a^2+b^2\right)^4d}\\ &+\frac{(a-2b)\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(a+b)^4d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x))/((a - b)^4*(a + b)^4*d) - (
(I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]])/((-a + b)^4*d) - ((I/2)*(a - 2*b)*A
rcTan[Tan[c + d*x]])/((a + b)^4*d) - b^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*
Cos[c + d*x])^2) - (b^2*(3*a^2 + b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[
c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) + ((-a - 2*b)*Log[Cos[(c +
 d*x)/2]^2])/(4*(-a + b)^4*d) + ((3*a^4*b + 8*a^2*b^3 + b^5)*Log[b + a*Cos
[c + d*x]])/((-a^2 + b^2)^4*d) + ((a - 2*b)*Log[Sin[(c + d*x)/2]^2])/(4*(a
 + b)^4*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

3.226.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4359, 25, 25, 3042, 25, 3200, 601, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4359$$

$$\int -\frac{\cot^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)^3}{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3200$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\downarrow 601$$

$$-\frac{-\frac{\int \frac{-\frac{\left(a^2+3b^2\right)\cos^3(c+dx)a^7}{\left(a^2-b^2\right)^3}+\frac{b\left(3a^2-7b^2\right)\cos^2(c+dx)a^6}{\left(a^2-b^2\right)^3}+\frac{b^3\left(a^2+3b^2\right)a^4}{\left(a^2-b^2\right)^3}+\frac{b^2\left(3a^4+3b^2a^2-2b^4\right)\cos(c+dx)a^3}{\left(a^2-b^2\right)^3}}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2a^2}-\frac{a^2\left(b\left(3a^2+b^2\right)-a\left(a\right.\right.}{2(a^2-b^2)^3(a^2-}}{d}$$

$$\downarrow 2160$$

$$-\frac{-\frac{\int\left(\frac{2a^2b^3}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^3}-\frac{2a^2\left(3a^2+b^2\right)b^2}{\left(a^2-b^2\right)^3(b+a\cos(c+dx))^2}+\frac{2a^2\left(3a^4+8b^2a^2+b^4\right)b}{\left(a^2-b^2\right)^4(b+a\cos(c+dx))}-\frac{a^2(a-2b)}{2(a+b)^4(a-a\cos(c+dx))}-\frac{a^2(a+2b)}{2(a-b)^4(\cos(c+dx)a+a)}\right)d(a\cos}{2a^2}}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a^2\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}-\frac{\frac{2a^2b^2\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{a^2b^3}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{a^2(a-2b)\log(a-a\cos(c+dx))}{2(a+b)^4}-\frac{a^2(a+2b)\log(a\,c}{2(a-b}}{2a^2}}{d}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
-((-1/2*(a^2*(b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x]))/((a^2 - b^2
)^3*(a^2 - a^2*Cos[c + d*x]^2)) - (-((a^2*b^3)/((a^2 - b^2)^2*(b + a*Cos[c
 + d*x])^2)) + (2*a^2*b^2*(3*a^2 + b^2))/((a^2 - b^2)^3*(b + a*Cos[c + d*x
])) + (a^2*(a - 2*b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) - (a^2*(a + 2*
b)*Log[a + a*Cos[c + d*x]])/(2*(a - b)^4) + (2*a^2*b*(3*a^4 + 8*a^2*b^2 +
b^4)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4)/(2*a^2))/d)
```

3.226.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

3.226.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{b\left(3a^4+8a^2b^2+b^4\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{(a+}{d}$
default	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{b\left(3a^4+8a^2b^2+b^4\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{(a+}{d}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d(a-b)}+\frac{\left(a^5+22a^3b^2+13ab^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d\left(a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6\right)}-\frac{\left(2a^5+5a^4b+44a^3b^2+18a^2b^3+26ab^4+b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}$
parallelrisch	$\frac{192\left(a^4+\frac{8}{3}a^2b^2+\frac{1}{3}b^4\right)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+32(a-2b)(a-b)^4(\cos$
risch	Expression too large to display

input
```
int(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(a-2*b)/(a+b)^4*ln(cos(d*x+c)-1)-1/2*b
^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+b*(3*a^4+8*a^2*b^2+b^4)/(a+b)^4/(a-b
)^4*ln(b+a*cos(d*x+c))+b^2*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/
4/(a-b)^3/(cos(d*x+c)+1)+1/4/(a-b)^4*(-a-2*b)*ln(cos(d*x+c)+1))
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(221) = 442$.

Time $= 0.45$ (sec) , antiderivative size $= 1071$, normalized size of antiderivative $= 4.68$

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*
a*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x
 + c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3
+ 8*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*
a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2
*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c
))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^
5 + 9*a*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4
 + 2*a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*
b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 +
10*a^4*b^3 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*
(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*
x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 -
16*a^2*b^5 + 9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 +
9*a^3*b^4 - 2*a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3
- 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a
^5*b^2 - 10*a^4*b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x +
c)^2 + 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^
6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*
b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*...
```

3.226.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.90

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{4\,(3\,a^4b+8\,a^2b^3+b^5)\log(a\cos(dx+c)+b)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{(a+2\,b)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} + \frac{(a-2\,b)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8-(a^8-3\,a}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(4*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^
2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + 2*b)*log(cos(d*x + c) + 1)/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (a - 2*b)*log(cos(d*x + c) - 1)/(a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(8*a^2*b^3 + 4*b^5 - (a^5 + 9
*a^3*b^2 + 2*a*b^4)*cos(d*x + c)^3 + (a^4*b - 10*a^2*b^3 - 3*b^5)*cos(d*x
+ c)^2 + (11*a^3*b^2 + a*b^4)*cos(d*x + c))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4 - 2*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*cos(d*x + c)))/d
```

3.226.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(221) = 442$.

Time = 0.44 (sec) , antiderivative size = 800, normalized size of antiderivative = 3.49

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a
bs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a
 + b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^
3*b^4 + 11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b
^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*
b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 27*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b
+ a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d...
```

3.226.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.65

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(b+a\cos\left(c+dx\right)\right)\left(\frac{3b}{4(a+b)^4}-\frac{1}{4(a+b)^3}+\frac{3b}{4(a-b)^4}+\frac{1}{4(a-b)^3}\right)}{d}$$
$$-\frac{\ln\left(\cos\left(c+dx\right)-1\right)\left(\frac{3b}{4(a+b)^4}-\frac{1}{4(a+b)^3}\right)}{d}$$
$$-\frac{\frac{\cos(c+dx)^3\left(a^5+9a^3b^2+2ab^4\right)}{2\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{\cos(c+dx)^2\left(-a^4b+10a^2b^3+3b^5\right)}{2\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{2b\left(2a^2b^2+b^4\right)}{\left(a^2-b^2\right)\left(a^4-2a^2b^2+b^4\right)}-\frac{a\cos(c+dx)\left(11a^2b^2+b^4\right)}{2\left(a^2-b^2\right)\left(a^4-2a^2b^2+b^4\right)}}{d\left(\cos\left(c+dx\right)^2\left(a^2-b^2\right)+b^2-a^2\cos\left(c+dx\right)^4+2ab\cos\left(c+dx\right)-2ab\cos\left(c+dx\right)^3\right)}$$
$$-\frac{\ln\left(\cos\left(c+dx\right)+1\right)\left(a+2b\right)}{4d\left(a-b\right)^4}$$

input
```
int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

output
```
(log(b + a*cos(c + d*x))*((3*b)/(4*(a + b)^4) - 1/(4*(a + b)^3) + (3*b)/(4
*(a - b)^4) + 1/(4*(a - b)^3)))/d - (log(cos(c + d*x) - 1)*((3*b)/(4*(a +
b)^4) - 1/(4*(a + b)^3)))/d - ((cos(c + d*x)^3*(2*a*b^4 + a^5 + 9*a^3*b^2)
)/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)^2*(3*b^5 - a^4*b
 + 10*a^2*b^3))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*b*(b^4 + 2*a^
2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a*cos(c + d*x)*(b^4 + 11*
a^2*b^2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2*(a^2
 - b^2) + b^2 - a^2*cos(c + d*x)^4 + 2*a*b*cos(c + d*x) - 2*a*b*cos(c + d*
x)^3)) - (log(cos(c + d*x) + 1)*(a + 2*b))/(4*d*(a - b)^4)
```

3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.227.1 Optimal result 1694
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3.227.8 Giac [B] (verification not implemented) 1702
3.227.9 Mupad [B] (verification not implemented) 1703

3.227.1 Optimal result

Integrand size = 21, antiderivative size = 313

$$\begin{aligned}
&\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx \\
&= -\frac{a^2b^3}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))^2} + \frac{3a^2b^2(a^2+b^2)}{\left(a^2-b^2\right)^4 d(b+a\cos(c+dx))} \\
&\quad + \frac{(4b(3a^4+8a^2b^2+b^4) - 3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} \\
&\quad + \frac{(b(3a^2+b^2) - a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d} + \frac{3a(a-3b)\log(1-\cos(c+dx))}{16(a+b)^5 d} \\
&\quad - \frac{3a(a+3b)\log(1+\cos(c+dx))}{16(a-b)^5 d} + \frac{3a^2b(a^4+5a^2b^2+2b^4)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^5 d}
\end{aligned}$$

output

```
-1/2*a^2*b^3/(a^2-b^2)^3/d/(b+a*cos(d*x+c))^2+3*a^2*b^2*(a^2+b^2)/(a^2-b^2
)^4/d/(b+a*cos(d*x+c))+1/8*(4*b*(3*a^4+8*a^2*b^2+b^4)-3*a*(a^4+10*a^2*b^2+
5*b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^4/d+1/4*(b*(3*a^2+b^2)-a*(a^2+3*
b^2)*cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)^3/d+3/16*a*(a-3*b)*ln(1-cos(d*x+c)
)/(a+b)^5/d-3/16*a*(a+3*b)*ln(1+cos(d*x+c))/(a-b)^5/d+3*a^2*b*(a^4+5*a^2*b
^2+2*b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^5/d
```

3.227.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.58

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(b+a\cos(c+dx))\left(\frac{32a^2b^3}{(-a+b)^3(a+b)^3}+\frac{192a^2(a-ib)(a+ib)b^2(b+a\cos(c+dx))}{(a-b)^4(a+b)^4}-\frac{384ia^2b(a^4+5a^2b^2+2b^4)(c+dx)(b+a\cos(c+dx))^2}{(a-b)^5(a+b)^5}-\right.}{}$$

input
```
Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*((32*a^2*b^3)/((-a + b)^3*(a + b)^3) + (192*a^2*(a -
 I*b)*(a + I*b)*b^2*(b + a*Cos[c + d*x]))/((a - b)^4*(a + b)^4) - ((384*I)
*a^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x])^2)/((a - b
)^5*(a + b)^5) - ((24*I)*a*(a - 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d
*x])^2)/(a + b)^5 + ((24*I)*a*(a + 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c
 + d*x])^2)/(a - b)^5 + (6*(-a + b)*(b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]
^2)/(a + b)^4 - ((b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^4)/(a + b)^3 - (1
2*a*(a + 3*b)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2]^2])/(a - b)^5 +
(192*a^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[
c + d*x]])/(a^2 - b^2)^5 + (12*a*(a - 3*b)*(b + a*Cos[c + d*x])^2*Log[Sin[
(c + d*x)/2]^2])/(a + b)^5 + (6*(a + b)*(b + a*Cos[c + d*x])^2*Sec[(c + d*
x)/2]^2)/(a - b)^4 + ((b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4)/(a - b)^3
)*Sec[c + d*x]^3)/(64*d*(a + b*Sec[c + d*x])^3)
```

3.227.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3316, 25, 27, 601, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^5 \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc^2(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc^2(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^5 \left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^5 \left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3316$$

$$\frac{a^5 \int -\frac{\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 25$$

$$-\frac{a^5 \int \frac{\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 27$$

$$-\frac{a^2 \int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^3}d(a\cos(c+dx))}{d}$$

$$\downarrow 601$$

$$a^2\left(-\frac{\int\frac{-\frac{3\left(a^2+3b^2\right)\cos^3(c+dx)a^7}{\left(a^2-b^2\right)^3}+\frac{b\left(3a^2-23b^2\right)\cos^2(c+dx)a^6}{\left(a^2-b^2\right)^3}+\frac{b^3\left(a^2+3b^2\right)a^4}{\left(a^2-b^2\right)^3}+\frac{b^2\left(3a^4-3b^2a^2-4b^4\right)\cos(c+dx)a^3}{\left(a^2-b^2\right)^3}}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)^2}d(a\cos(c+dx))}{4a^2}-\frac{a^2\left(b\left(3a^2+b\right.\right.}{4(a^2-b^2}\right.$$
$$\left.\left.\left.

output
```
-((a^2*(-1/4*(a^2*(b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x]))/((a^2
- b^2)^3*(a^2 - a^2*Cos[c + d*x]^2)^2) - ((a^2*(4*b*(3*a^4 + 8*a^2*b^2 + b
^4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x]))/(2*(a^2 - b^2)^4*(a^2
- a^2*Cos[c + d*x]^2)) + ((-4*a^4*b^3)/((a^2 - b^2)^3*(b + a*Cos[c + d*x])
^2) + (24*a^4*b^2*(a^2 + b^2))/((a^2 - b^2)^4*(b + a*Cos[c + d*x])) + (3*a
^3*(a - 3*b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^5) - (3*a^3*(a + 3*b)*Log
[a + a*Cos[c + d*x]])/(2*(a - b)^5) + (24*a^4*b*(a^4 + 5*a^2*b^2 + 2*b^4)*
Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^5)/(2*a^2))/(4*a^2)))/d)
```

3.227.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.227.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{1}{16(a+b)^3(\cos(dx+c)-1)^2}-\frac{-3a+3b}{16(a+b)^4(\cos(dx+c)-1)}+\frac{3a(a-3b)\ln(\cos(dx+c)-1)}{16(a+b)^5}-\frac{b^3a^2}{2(a+b)^3(a-b)^3(b+a\cos(dx+c))^2}+\frac{3a^2b\left(a^4+$
default	$-\frac{1}{16(a+b)^3(\cos(dx+c)-1)^2}-\frac{-3a+3b}{16(a+b)^4(\cos(dx+c)-1)}+\frac{3a(a-3b)\ln(\cos(dx+c)-1)}{16(a+b)^5}-\frac{b^3a^2}{2(a+b)^3(a-b)^3(b+a\cos(dx+c))^2}+\frac{3a^2b\left(a^4+$
norman	$-\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{64d(a-b)}-\frac{(3a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{32d\left(a^2+2ab+b^2\right)}+\frac{(3a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{32d\left(a^2-2ab+b^2\right)}+\frac{\left(15a^7+459a^5b^2+621b^4a^3+57a\,b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{32\left(a^8-2a^7b-2a^6b^2+6a^5b^3-6a^3b^5+2a^2b^6+2a\,b^7-\right.}$ $\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b$
parallelrisch	$\frac{3b\,a^2\left(a^4+5a^2b^2+2b^4\right)\left(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2\right)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+3a(a-3b)(a-b)^5\left(\cos(2$
risch	Expression too large to display

input
```
int(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16/(a+b)^3/(cos(d*x+c)-1)^2-1/16*(-3*a+3*b)/(a+b)^4/(cos(d*x+c)-1)
+3/16*a*(a-3*b)/(a+b)^5*ln(cos(d*x+c)-1)-1/2*b^3/(a+b)^3*a^2/(a-b)^3/(b+a*
cos(d*x+c))^2+3*a^2*b*(a^4+5*a^2*b^2+2*b^4)/(a-b)^5/(a+b)^5*ln(b+a*cos(d*x
+c))+3*a^2*b^2*(a^2+b^2)/(a+b)^4/(a-b)^4/(b+a*cos(d*x+c))+1/16/(a-b)^3/(co
s(d*x+c)+1)^2-1/16*(-3*a-3*b)/(a-b)^4/(cos(d*x+c)+1)-3/16*a*(a+3*b)/(a-b)^
5*ln(cos(d*x+c)+1))
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1803 vs. 2(303) = 606.

Time = 0.56 (sec) , antiderivative size = 1803, normalized size of antiderivative = 5.76

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(76*a^6*b^3 + 36*a^4*b^5 - 108*a^2*b^7 - 4*b^9 + 6*(a^9 + 17*a^7*b^2
- 5*a^5*b^4 - 13*a^3*b^6)*cos(d*x + c)^5 - 12*(a^8*b - 9*a^6*b^3 - a^4*b^5
 + 9*a^2*b^7)*cos(d*x + c)^4 - 2*(5*a^9 + 98*a^7*b^2 - 12*a^5*b^4 - 98*a^3
*b^6 + 7*a*b^8)*cos(d*x + c)^3 + 8*(2*a^8*b - 25*a^6*b^3 - 3*a^4*b^5 + 25*
a^2*b^7 + b^9)*cos(d*x + c)^2 + 2*(55*a^7*b^2 - 9*a^5*b^4 - 51*a^3*b^6 + 5
*a*b^8)*cos(d*x + c) + 48*(a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b + 5*a^
6*b^3 + 2*a^4*b^5)*cos(d*x + c)^6 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*co
s(d*x + c)^5 - (2*a^8*b + 9*a^6*b^3 - a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^4
- 4*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x + c)^3 + (a^8*b + 3*a^6*b^3
- 8*a^4*b^5 - 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b
^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) - 3*(a^7*b^2 + 8*a^6*b^3 + 25*a^
5*b^4 + 40*a^4*b^5 + 35*a^3*b^6 + 16*a^2*b^7 + 3*a*b^8 + (a^9 + 8*a^8*b +
25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x + c
)^6 + 2*(a^8*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c)^5 - (2*a^9 + 16*a^8*b + 49*a^7*b^2 + 72*a^6
*b^3 + 45*a^5*b^4 - 8*a^4*b^5 - 29*a^3*b^6 - 16*a^2*b^7 - 3*a*b^8)*cos(d*x
 + c)^4 - 4*(a^8*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16
*a^3*b^6 + 3*a^2*b^7)*cos(d*x + c)^3 + (a^9 + 8*a^8*b + 23*a^7*b^2 + 24*a^
6*b^3 - 15*a^5*b^4 - 64*a^4*b^5 - 67*a^3*b^6 - 32*a^2*b^7 - 6*a*b^8)*cos(d
*x + c)^2 + 2*(a^8*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5...
```

3.227.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

3.227.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(303) = 606$.

Time = 0.23 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.26

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{48\left(a^6b+5a^4b^3+2a^2b^5\right)\log(a\cos(dx+c)+b)}{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}} - \frac{3\left(a^2+3ab\right)\log(\cos(dx+c)+1)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5} + \frac{3\left(a^2-3ab\right)\log(\cos(dx+c)-1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5} + \frac{}{a^8b^2-4a}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(48*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(a*cos(d*x + c) + b)/(a^10 - 5
*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 + 3*a*b)*l
og(cos(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5) + 3*(a^2 - 3*a*b)*log(cos(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(38*a^4*b^3 + 56*a^2*b^5 + 2*b^7 + 3*(a^7
 + 18*a^5*b^2 + 13*a^3*b^4)*cos(d*x + c)^5 - 6*(a^6*b - 8*a^4*b^3 - 9*a^2*
b^5)*cos(d*x + c)^4 - (5*a^7 + 103*a^5*b^2 + 91*a^3*b^4 - 7*a*b^6)*cos(d*x
 + c)^3 + 4*(2*a^6*b - 23*a^4*b^3 - 26*a^2*b^5 - b^7)*cos(d*x + c)^2 + (55
*a^5*b^2 + 46*a^3*b^4 - 5*a*b^6)*cos(d*x + c))/(a^8*b^2 - 4*a^6*b^4 + 6*a^
4*b^6 - 4*a^2*b^8 + b^10 + (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2
*b^8)*cos(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^
9)*cos(d*x + c)^5 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*
b^8 - b^10)*cos(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7
+ a*b^9)*cos(d*x + c)^3 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^6 + 9*
a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^
3*b^7 + a*b^9)*cos(d*x + c)))/d
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(303) = 606$.

Time = 0.48 (sec) , antiderivative size = 1551, normalized size of antiderivative = 4.96

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/64*(12*(a^2 - 3*a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 192*(a^6*b + 5*
a^4*b^3 + 2*a^2*b^5)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 5*a^8*b^2 + 10*a^6
*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^3*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 3*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b^3*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2)/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
 + b^6) - (a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 +
 2*a*b^7 - b^8 - 6*a^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^7*b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^6*b^2*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 28*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40*a^4*b
^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^5*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 20*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a
*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b^8*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 6*a^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 163*a^7
*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 257*a^6*b^2*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 339*a^5*b^3*(cos(d*x + c) - 1)^2/(cos(d*x ...
```

3.227.9 Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.15

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,(a+b)^3}-\frac{15\,b}{16\,(a+b)^4}+\frac{3\,b^2}{4\,(a+b)^5}\right)}{d}$$
$$+\frac{\frac{19\,a^4\,b^3+28\,a^2\,b^5+b^7}{4\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}-\frac{\cos(c+d\,x)^2\,\left(-2\,a^6\,b+23\,a^4\,b^3+26\,a^2\,b^5+b^7\right)}{2\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}+\frac{3\cos(c+d\,x)^4\,\left(-a^6\,b+8\,a^4\,b^3+9\,a^2\,b^5\right)}{4\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}+\frac{3\cos(c+d}{8\,(a^8-4}}{d\,\left(\cos\left(c+d\,x\right)^2\,\left(a^2-2\,b^2\right)-\cos\left(c+d\,x\right)^4\,\left(2\,a^2-b^2\right)+b^2+a^2\cos\left(c+d\,x\right)\right.}$$
$$-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{4\,(a-b)^5}+\frac{15\,b}{16\,(a-b)^4}+\frac{3}{16\,(a-b)^3}\right)}{d}$$
$$+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(3\,a^6\,b+15\,a^4\,b^3+6\,a^2\,b^5\right)}{d\,\left(a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}\right)}$$

input
```
int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

output

```
(log(cos(c + d*x) - 1)*(3/(16*(a + b)^3) - (15*b)/(16*(a + b)^4) + (3*b^2)
/(4*(a + b)^5)))/d + ((b^7 + 28*a^2*b^5 + 19*a^4*b^3)/(4*(a^8 + b^8 - 4*a^
2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (cos(c + d*x)^2*(b^7 - 2*a^6*b + 26*a^2*
b^5 + 23*a^4*b^3))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (
3*cos(c + d*x)^4*(9*a^2*b^5 - a^6*b + 8*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^
6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*cos(c + d*x)^5*(a^7 + 13*a^3*b^4 + 18*a^5
*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (a*cos(c + d*
x)*(46*a^2*b^4 - 5*b^6 + 55*a^4*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^
4 - 4*a^6*b^2)) - (a*cos(c + d*x)^3*(5*a^6 - 7*b^6 + 91*a^2*b^4 + 103*a^4*
b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(cos(c + d*x
)^2*(a^2 - 2*b^2) - cos(c + d*x)^4*(2*a^2 - b^2) + b^2 + a^2*cos(c + d*x)^
6 + 2*a*b*cos(c + d*x) - 4*a*b*cos(c + d*x)^3 + 2*a*b*cos(c + d*x)^5)) - (
log(cos(c + d*x) + 1)*((3*b^2)/(4*(a - b)^5) + (15*b)/(16*(a - b)^4) + 3/(
16*(a - b)^3)))/d + (log(b + a*cos(c + d*x))*(3*a^6*b + 6*a^2*b^5 + 15*a^4
*b^3))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.228.1 Optimal result 1705
3.228.2 Mathematica [A] (verified) 1706
3.228.3 Rubi [A] (verified) 1707
3.228.4 Maple [A] (verified) 1719
3.228.5 Fricas [A] (verification not implemented) 1720
3.228.6 Sympy [F] 1721
3.228.7 Maxima [F(-2)] 1722
3.228.8 Giac [B] (verification not implemented) 1722
3.228.9 Mupad [B] (verification not implemented) 1723

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 539

$$
\begin{aligned}
&\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx \\
&= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9} \\
&\quad - \frac{\sqrt{a-b}b\sqrt{a+b}(6a^4-47a^2b^2+56b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d} \\
&\quad + \frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d} \\
&\quad - \frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} \\
&\quad + \frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} \\
&\quad - \frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} \\
&\quad + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} \\
&\quad + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} \\
&\quad + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))}
\end{aligned}
$$

output
```
1/16*(5*a^6-180*a^4*b^2+600*a^2*b^4-448*b^6)*x/a^9+1/30*b*(213*a^4-985*a^2
*b^2+840*b^4)*sin(d*x+c)/a^8/d-1/16*(43*a^4-244*a^2*b^2+224*b^4)*cos(d*x+c
)*sin(d*x+c)/a^7/d+1/30*(45*a^4-291*a^2*b^2+280*b^4)*cos(d*x+c)^2*sin(d*x+
c)/a^6/b/d-1/24*(24*a^4-169*a^2*b^2+168*b^4)*cos(d*x+c)^3*sin(d*x+c)/a^5/b
^2/d-1/4*cos(d*x+c)^4*sin(d*x+c)/b/d/(b+a*cos(d*x+c))^2+1/10*a*cos(d*x+c)^
5*sin(d*x+c)/b^2/d/(b+a*cos(d*x+c))^2+1/60*(9*a^4-60*a^2*b^2+56*b^4)*cos(d
*x+c)^5*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))^2+4/15*b*cos(d*x+c)^6*sin(d*
x+c)/a^2/d/(b+a*cos(d*x+c))^2-1/6*cos(d*x+c)^7*sin(d*x+c)/a/d/(b+a*cos(d*x
+c))^2+1/20*(15*a^4-110*a^2*b^2+112*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^4/b^2/d
/(b+a*cos(d*x+c))-b*(6*a^4-47*a^2*b^2+56*b^4)*arctanh((a-b)^(1/2)*tan(1/2*
d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^9/d
```

3.228.2 Mathematica [A] (verified)

Time = 9.15 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.11

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-7680b(-a^2+b^2)^3\left(6a^4-47a^2b^2+56b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2+2(a^2-b^2)^{5/2}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-7680*b*(-a^2 + b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2 + 2*(a^2 - b^2)^(5
/2)*(600*a^8*c - 20400*a^6*b^2*c + 28800*a^4*b^4*c + 90240*a^2*b^6*c - 107
520*b^8*c + 600*a^8*d*x - 20400*a^6*b^2*d*x + 28800*a^4*b^4*d*x + 90240*a^
2*b^6*d*x - 107520*b^8*d*x + 480*a*b*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 -
448*b^6)*(c + d*x)*Cos[c + d*x] + 120*a^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b
^4 - 448*b^6)*(c + d*x)*Cos[2*(c + d*x)] + 2640*a^7*b*Sin[c + d*x] + 16160
*a^5*b^3*Sin[c + d*x] - 117120*a^3*b^5*Sin[c + d*x] + 107520*a*b^7*Sin[c +
 d*x] - 405*a^8*Sin[2*(c + d*x)] + 24600*a^6*b^2*Sin[2*(c + d*x)] - 99040*
a^4*b^4*Sin[2*(c + d*x)] + 80640*a^2*b^6*Sin[2*(c + d*x)] + 2436*a^7*b*Sin
[3*(c + d*x)] - 10880*a^5*b^3*Sin[3*(c + d*x)] + 8960*a^3*b^5*Sin[3*(c + d
*x)] - 140*a^8*Sin[4*(c + d*x)] + 1164*a^6*b^2*Sin[4*(c + d*x)] - 1120*a^4
*b^4*Sin[4*(c + d*x)] - 188*a^7*b*Sin[5*(c + d*x)] + 224*a^5*b^3*Sin[5*(c
+ d*x)] + 35*a^8*Sin[6*(c + d*x)] - 56*a^6*b^2*Sin[6*(c + d*x)] + 16*a^7*b
*Sin[7*(c + d*x)] - 5*a^8*Sin[8*(c + d*x)]))/(7680*a^9*(a - b)^2*(a + b)^2
*Sqrt[a^2 - b^2]*d*(b + a*Cos[c + d*x])^2)
```

3.228.3 Rubi [A] (verified)

Time = 4.10 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.26, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {3042, 4360, 25, 25, 3042, 3375, 27, 3042, 3526, 27, 3042, 3526, 27, 3042, 3528, 27, 3042, 3528, 25, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^6}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^6(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^6(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^6(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 3375$$

$$\frac{\int \frac{10\cos^5(c+dx)\left(-2\left(12a^4-65b^2a^2+56b^4\right)\cos^2(c+dx)+3ab\left(3a^2-2b^2\right)\cos(c+dx)+3\left(6a^4-35b^2a^2+32b^4\right)\right)}{(b+a\cos(c+dx))^3}dx}{600a^2b^2}+$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{\cos^5(c+dx)\left(-2\left(12a^4-65b^2a^2+56b^4\right)\cos^2(c+dx)+3ab\left(3a^2-2b^2\right)\cos(c+dx)+3\left(6a^4-35b^2a^2+32b^4\right)\right)}{(b+a\cos(c+dx))^3}dx}{60a^2b^2}+$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^5\left(-2\left(12a^4-65b^2a^2+56b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+3ab\left(3a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(6a^4-35b^2a^2+32b^4\right)\right)}{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{60a^2b^2}+$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3526$$

$$\frac{\int \frac{2\cos^4(c+dx)\left(-2\left(30a^6-215b^2a^4+353b^4a^2-168b^6\right)\cos^2(c+dx)+ab\left(15a^4-31b^2a^2+16b^4\right)\cos(c+dx)+5\left(9a^6-69b^2a^4+116b^4a^2-56b^6\right)\right)}{(b+a\cos(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{(9a^4-60a^2b^2-}{ad}$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{\cos^4(c+dx)\left(-2\left(30a^6-215b^2a^4+353b^4a^2-168b^6\right)\cos^2(c+dx)+ab\left(15a^4-31b^2a^2+16b^4\right)\cos(c+dx)+5\left(9a^6-69b^2a^4+116b^4a^2-56b^6\right)\right)}{(b+a\cos(c+dx))^2}dx}{a(a^2-b^2)}+\frac{(9a^4-60a^2b^2+}{ad(}$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(-2\left(30a^6-215b^2a^4+353b^4a^2-168b^6\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\left(15a^4-31b^2a^2+16b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+5\left(9a^6-69b^2a^4+116b^4a^2-56b^6\right)\right)}{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{a(a^2-b^2)}+\frac{(9a}{}$$
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

3.228. $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\downarrow 3526$$

$$\frac{\int \frac{2\cos^3(c+dx)\left(-5\left(24a^4-169b^2a^2+168b^4\right)\cos^2(c+dx)\left(a^2-b^2\right)^2+6\left(15a^4-110b^2a^2+112b^4\right)\left(a^2-b^2\right)^2+ab\left(15a^2-28b^2\right)\cos(c+dx)\left(a^2-b^2\right)^2\right)}{b+a\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{3\left(a^2-b^2\right)\left(}{a(a^2-b^2)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{60a^2b^2}{\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}}$$

$$\downarrow 27$$

$$\frac{2\int \frac{\cos^3(c+dx)\left(-5\left(24a^4-169b^2a^2+168b^4\right)\cos^2(c+dx)\left(a^2-b^2\right)^2+6\left(15a^4-110b^2a^2+112b^4\right)\left(a^2-b^2\right)^2+ab\left(15a^2-28b^2\right)\cos(c+dx)\left(a^2-b^2\right)^2\right)}{b+a\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{3\left(a^2-b^2\right)\left(}{a(a^2-b^2)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{60a^2b^2}{\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(-5\left(24a^4-169b^2a^2+168b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2-b^2\right)^2+6\left(15a^4-110b^2a^2+112b^4\right)\left(a^2-b^2\right)^2+ab\left(15a^2-28b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}-\frac{}{a(a^2-b^2)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{60a^2b^2}{\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}}$$

$$\downarrow 3528$$

$$\frac{2\left(\int -\frac{3\cos^2(c+dx)\left(-4b\left(45a^4-291b^2a^2+280b^4\right)\cos^2(c+dx)\left(a^2-b^2\right)^2+5b\left(24a^4-169b^2a^2+168b^4\right)\left(a^2-b^2\right)^2+7ab^2\left(5a^2-8b^2\right)\cos(c+dx)\left(a^2-b^2\right)^2\right)}{b+a\cos(c+dx)}dx}{4a}-\frac{5\left(a^2\right.}{a\left(a^2-b^2\right)}\right)}{a(a^2-b^2)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{}{\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}}$$

$$\downarrow 27$$

$$2\left(-\frac{3\int \frac{\cos^2(c+dx)\left(-4b\left(45a^4-291b^2a^2+280b^4\right)\cos^2(c+dx)\left(a^2-b^2\right)^2+5b\left(24a^4-169b^2a^2+168b^4\right)\left(a^2-b^2\right)^2+7ab^2\left(5a^2-8b^2\right)\cos(c+dx)\left(a^2-b^2\right)^2\right)}{b+a\cos(c+dx)}dx}{4a}-\frac{5\left(a^2\right.}{a\left(a^2-b^2\right)}\right)$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$2\left(-\frac{3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-4b\left(45a^4-291b^2a^2+280b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2-b^2\right)^2+5b\left(24a^4-169b^2a^2+168b^4\right)\left(a^2-b^2\right)^2+7ab^2\left(5a^2-8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}{4a}\right)$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3528$$

$$2\left(-\frac{3\left(\int -\frac{\cos(c+dx)\left(a\left(207a^2-280b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)b^3-15\left(a^2-b^2\right)^2\left(43a^4-244b^2a^2+224b^4\right)\cos^2(c+dx)b^2+8\left(a^2-b^2\right)^2\left(45a^4-291b^2a^2+280b^4\right)b^2\right)}{b+a\cos(c+dx)}dx\right.}{4a}\right)$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 25$$

$$\frac{2\left(-\frac{3\left(-\frac{\int \frac{\cos(c+dx)\left(a\left(207a^2-280b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)b^3-15\left(a^2-b^2\right)^2\left(43a^4-244b^2a^2+224b^4\right)\cos^2(c+dx)b^2+8\left(a^2-b^2\right)^2\left(45a^4-291b^2a^2+280b^4\right)b^2\right)}{b+a\cos(c+dx)}dx}{3a}\right)}{4a}\right)}{a\left(a^2-b^2\right)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{2\left(-\frac{3\left(-\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(a\left(207a^2-280b^2\right)\left(a^2-b^2\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)b^3-15\left(a^2-b^2\right)^2\left(43a^4-244b^2a^2+224b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+8\left(a^2-b^2\right)^2\left(45a^4-291b^2a^2+\right.\right.}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}{3a}\right)}{4a}\right)}{a\left(a^2-b^2\right)}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3528$$

$$\frac{2\left(-\frac{3\left(-\frac{\int -\frac{-8\left(a^2-b^2\right)^2\left(213a^4-985b^2a^2+840b^4\right)\cos^2(c+dx)b^3+15\left(a^2-b^2\right)^2\left(43a^4-244b^2a^2+224b^4\right)b^3-a\left(a^2-b^2\right)^2\left(75a^4-996b^2a^2+1120b^4\right)\cos(c+dx)b^2}{b+a\cos(c+dx)}}{2a}}{3a}\right)}{4a}\right)}{}$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

3.228. $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

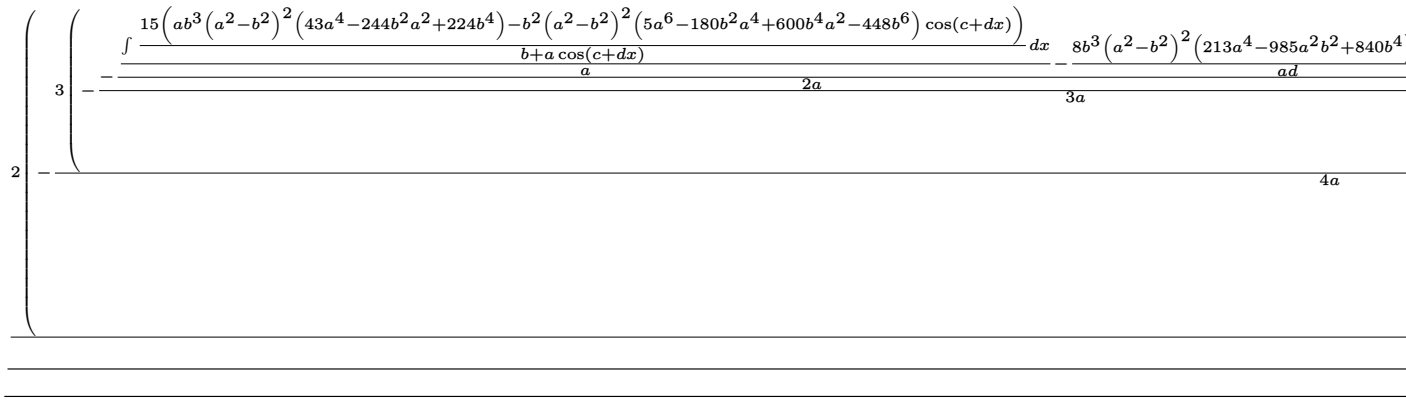
$$\downarrow 25$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

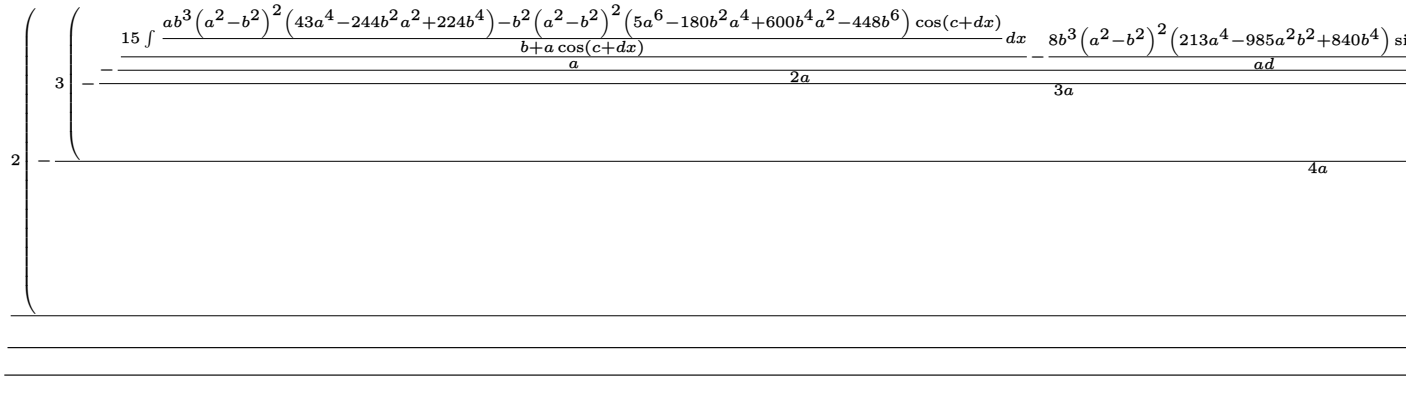
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3502$$



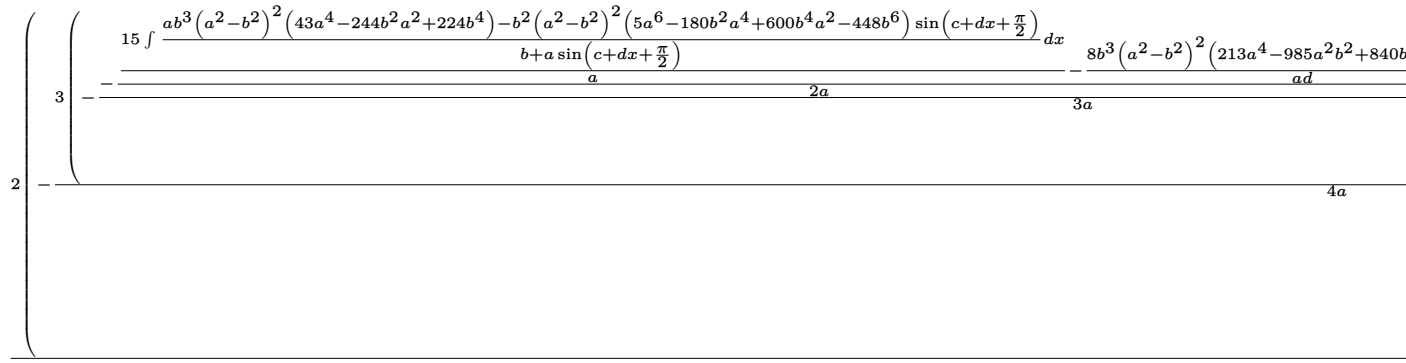
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 27$$



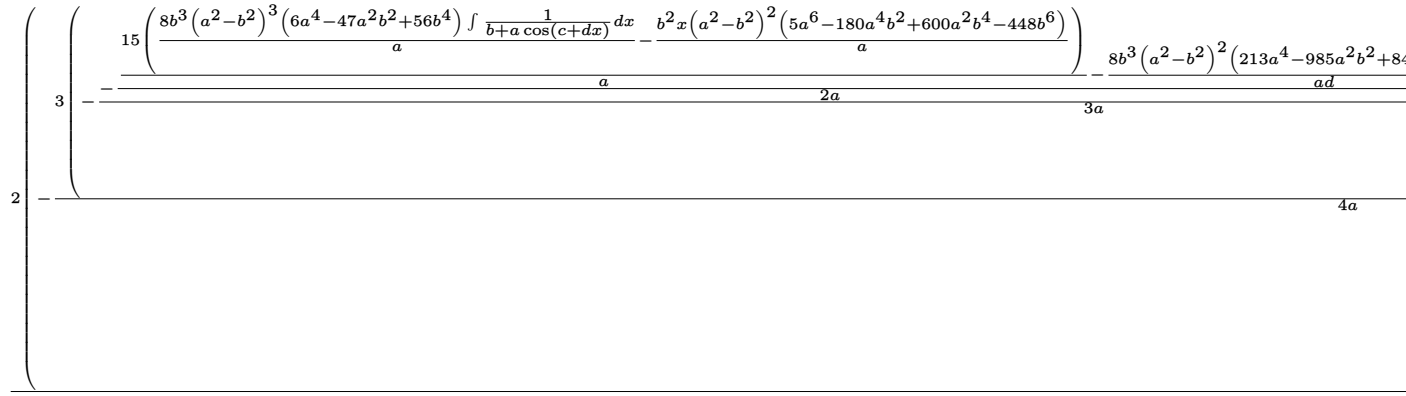
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$



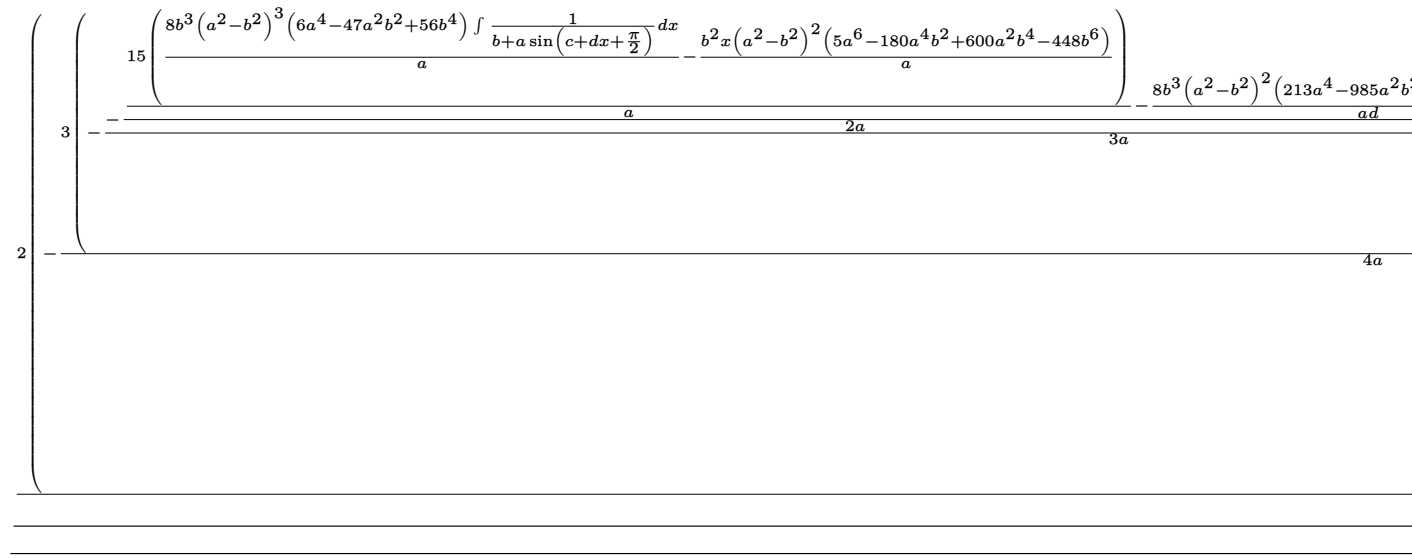
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3214$$



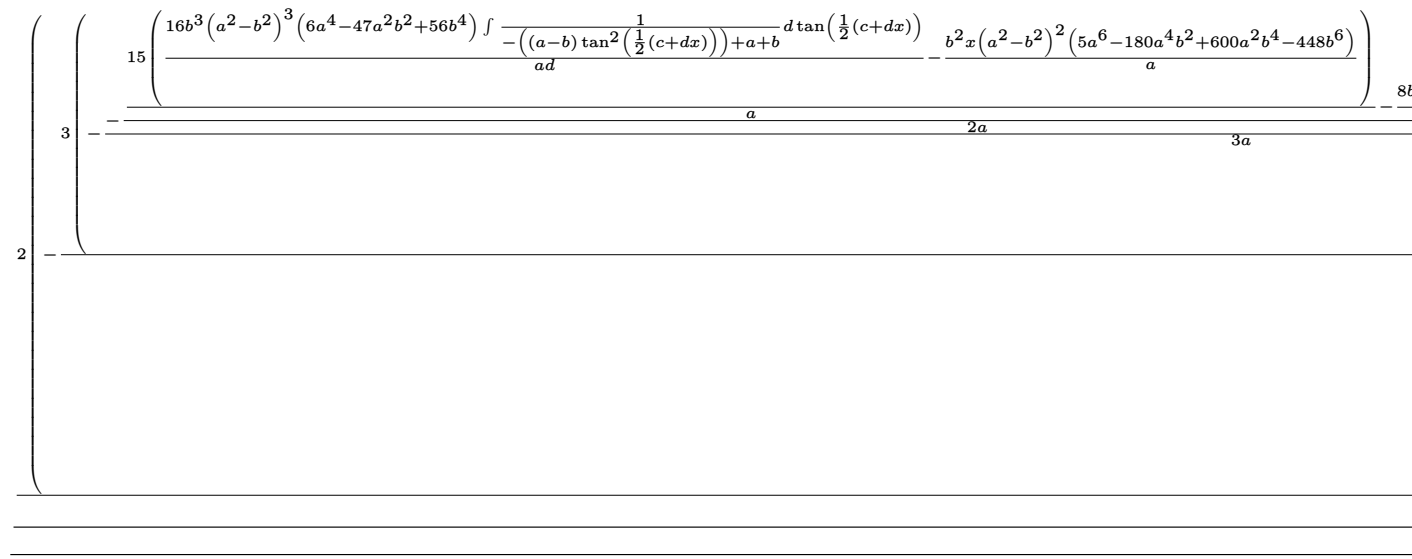
$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$



$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3138$$



$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 221$$

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{(9a^4-60a^2b^2+56b^4)\sin(c+dx)\cos^5(c+dx)}{ad(a\cos(c+dx)+b)^2}+\frac{\frac{3\left(a^2-b^2\right)\left(15a^4-110a^2b^2+112b^4\right)\sin(c+dx)\cos^4(c+dx)}{ad(a\cos(c+dx)+b)}+2\left(-\frac{5\left(a^2-b^2\right)^2\left(24a^4-169a^2b^2+168b^4\right)\sin(c+d}{4ad}\right.}{}$$

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/4*(Cos[c + d*x]^4*Sin[c + d*x])/(b*d*(b + a*Cos[c + d*x])^2) + (a*Cos[c
 + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d
*x]^6*Sin[c + d*x])/(15*a^2*d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Si
n[c + d*x])/(6*a*d*(b + a*Cos[c + d*x])^2) + (((9*a^4 - 60*a^2*b^2 + 56*b^
4)*Cos[c + d*x]^5*Sin[c + d*x])/(a*d*(b + a*Cos[c + d*x])^2) + ((3*(a^2 -
b^2)*(15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(a*d*(b
 + a*Cos[c + d*x])) + (2*((-5*(a^2 - b^2)^2*(24*a^4 - 169*a^2*b^2 + 168*b^
4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (3*((-4*b*(a^2 - b^2)^2*(45*a^4
- 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((-15*b^2*
(a^2 - b^2)^2*(43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c + d*x])/
(2*a*d) - ((15*(-((b^2*(a^2 - b^2)^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 -
448*b^6)*x)/a) + (16*b^3*(a^2 - b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTa
nh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]
*d)))/a - (8*b^3*(a^2 - b^2)^2*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c + d
*x])/(a*d))/(2*a))/(3*a)))/(4*a)))/(a*(a^2 - b^2)))/(a*(a^2 - b^2)))/(60*a
^2*b^2)
```

3.228. $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.228.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3375

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x]
)^(m + 1)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d
*Sin[e + f*x])^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)
*(m + n + 6))), x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin
[e + f*x])^(m + 1)/(b*d^4*f*(m + n + 6))), x] + Simp[1/(a^2*b^2*d^2*(n + 1)
*(n + 2)*(m + n + 5)*(m + n + 6))   Int[(d*Sin[e + f*x])^(n + 2)*(a + b*Sin
[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5) - a^2*b^2*(n + 2)*(2*
n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m + n + 3)*(m + n + 5)*(m
 + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n + 5)*(m + n + 6))*Sin[e
 + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4*(m + n + 2)*(m + n + 4
)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*(m + n + 5)*(2*n + 2*m
+ 13))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && Ne
Q[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m
 + n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3526

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.228.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{2\left(\left(\frac{5}{16}a^6+3a^5b-\frac{21}{4}a^4b^2-20a^3b^3+\frac{15}{2}a^2b^4+21ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(19a^5b-\frac{87}{4}a^4b^2+\frac{45}{2}a^2b^4+105ab^5+\frac{85}{48}a^6-\frac{340}{3}a^3b^3\right)\tan\left(\frac{d}{2}\right.\right.}{}$
default	$\frac{2\left(\left(\frac{5}{16}a^6+3a^5b-\frac{21}{4}a^4b^2-20a^3b^3+\frac{15}{2}a^2b^4+21ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(19a^5b-\frac{87}{4}a^4b^2+\frac{45}{2}a^2b^4+105ab^5+\frac{85}{48}a^6-\frac{340}{3}a^3b^3\right)\tan\left(\frac{d}{2}\right.\right.}{}$
risch	$\frac{5x}{16a^3}-\frac{45xb^2}{4a^5}+\frac{75xb^4}{2a^7}-\frac{28xb^6}{a^9}-\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(4dx+4c)}{64da^3}+\frac{3b\sin(5dx+5c)}{80a^4d}-\frac{3\sin(4dx+4c)b^2}{16da^5}+$

input

```
int(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^9*(((5/16*a^6+3*a^5*b-21/4*a^4*b^2-20*a^3*b^3+15/2*a^2*b^4+21*a*b
^5)*tan(1/2*d*x+1/2*c)^11+(19*a^5*b-87/4*a^4*b^2+45/2*a^2*b^4+105*a*b^5+85
/48*a^6-340/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(258/5*a^5*b-33/2*a^4*b^2-240*
a^3*b^3+15*a^2*b^4+210*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6+33/
2*a^4*b^2-15*a^2*b^4+258/5*a^5*b-240*a^3*b^3+210*a*b^5)*tan(1/2*d*x+1/2*c)
^5+(19*a^5*b+87/4*a^4*b^2-340/3*a^3*b^3-45/2*a^2*b^4+105*a*b^5-85/48*a^6)*
tan(1/2*d*x+1/2*c)^3+(3*a^5*b-20*a^3*b^3+21*a*b^5-5/16*a^6+21/4*a^4*b^2-15
/2*a^2*b^4)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-180
*a^4*b^2+600*a^2*b^4-448*b^6)*arctan(tan(1/2*d*x+1/2*c)))+2*b*(a+b)*(a-b)/
a^9*(((5/2*a^3*b^2-7*a*b^4-3*a^4*b+15/2*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(5/2
*a^3*b^2-7*a*b^4+3*a^4*b-15/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4-47*a^2*b^2+56*b^4)/((a-b
)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.96

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x +
 c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x +
 c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x + 60*(6*a^4
*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x +
 c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + b^2)) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c
)^6 - 1704*a^5*b^3 + 7880*a^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*c
os(d*x + c)^5 + 4*(67*a^7*b - 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*
a^6*b^2 + 560*a^4*b^4)*cos(d*x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120
*a^3*b^5)*cos(d*x + c)^2 - (2763*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*
cos(d*x + c))*sin(d*x + c))/(a^11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x +
c) + a^9*b^2*d), 1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^
6)*d*x*cos(d*x + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^
7)*d*x*cos(d*x + c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)
*d*x - 120*(6*a^4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a
^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 ...
```

3.228.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**3, x)
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.228.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(508) = 1016$.

Time $= 0.49$ (sec) , antiderivative size $= 1030$, normalized size of antiderivative $= 1.91$

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output

```
1/240*(15*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*(d*x + c)/a^9 - 24
0*(6*a^6*b - 53*a^4*b^3 + 103*a^2*b^5 - 56*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^9) - 240*(6*a^5*b^2*tan(1
/2*d*x + 1/2*c)^3 - 5*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 21*a^3*b^4*tan(1/2*
d*x + 1/2*c)^3 + 19*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 15*a*b^6*tan(1/2*d*x
+ 1/2*c)^3 - 14*b^7*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c
) - 5*a^4*b^3*tan(1/2*d*x + 1/2*c) + 21*a^3*b^4*tan(1/2*d*x + 1/2*c) + 19*
a^2*b^5*tan(1/2*d*x + 1/2*c) - 15*a*b^6*tan(1/2*d*x + 1/2*c) - 14*b^7*tan(
1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
a - b)^2*a^8) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 + 720*a^4*b*tan(1/2*d*x
+ 1/2*c)^11 - 1260*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 - 4800*a^2*b^3*tan(1/2*
d*x + 1/2*c)^11 + 1800*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 5040*b^5*tan(1/2*d*
x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 4560*a^4*b*tan(1/2*d*x +
1/2*c)^9 - 5220*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 27200*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^9 + 5400*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 25200*b^5*tan(1/2*d*x +
1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 12384*a^4*b*tan(1/2*d*x + 1/2*
c)^7 - 3960*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 57600*a^2*b^3*tan(1/2*d*x + 1
/2*c)^7 + 3600*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 50400*b^5*tan(1/2*d*x + 1/2*
c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 12384*a^4*b*tan(1/2*d*x + 1/2*c...
```

3.228.9 Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 3975, normalized size of antiderivative = 7.37

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(sin(c + d*x)^6/(a + b/cos(c + d*x))^3,x)
```

output

```
((tan(c/2 + (d*x)/2)^3*(10080*a*b^6 + 454*a^6*b - 55*a^7 + 9408*b^7 - 9688
*a^2*b^5 - 12212*a^3*b^4 + 608*a^4*b^3 + 2969*a^5*b^2))/(24*a^8) + (tan(c/
2 + (d*x)/2)^13*(454*a^6*b - 10080*a*b^6 + 55*a^7 + 9408*b^7 - 9688*a^2*b^
5 + 12212*a^3*b^4 + 608*a^4*b^3 - 2969*a^5*b^2))/(24*a^8) + (tan(c/2 + (d*
x)/2)^5*(90720*a*b^6 + 2154*a^6*b - 215*a^7 + 141120*b^7 - 163240*a^2*b^5
- 107220*a^3*b^4 + 32224*a^4*b^3 + 22673*a^5*b^2))/(120*a^8) + (tan(c/2 +
(d*x)/2)^11*(2154*a^6*b - 90720*a*b^6 + 215*a^7 + 141120*b^7 - 163240*a^2*
b^5 + 107220*a^3*b^4 + 32224*a^4*b^3 - 22673*a^5*b^2))/(120*a^8) + (tan(c/
2 + (d*x)/2)^7*(50400*a*b^6 - 4994*a^6*b + 2545*a^7 + 235200*b^7 - 287000*
a^2*b^5 - 58820*a^3*b^4 + 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) - (tan
(c/2 + (d*x)/2)^9*(50400*a*b^6 + 4994*a^6*b + 2545*a^7 - 235200*b^7 + 2870
00*a^2*b^5 - 58820*a^3*b^4 - 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) + (
tan(c/2 + (d*x)/2)^15*(a - b)*(224*a*b^5 + 43*a^5*b + 5*a^6 - 448*b^6 + 60
0*a^2*b^4 - 244*a^3*b^3 - 180*a^4*b^2))/(8*a^8) - (tan(c/2 + (d*x)/2)*(2*a
*b + a^2 + b^2)*(224*a*b^4 - 48*a^4*b + 5*a^5 - 448*b^5 + 376*a^2*b^3 - 13
2*a^3*b^2))/(8*a^8))/(d*(2*a*b - tan(c/2 + (d*x)/2)^8*(10*a^2 - 70*b^2) +
tan(c/2 + (d*x)/2)^2*(12*a*b + 4*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^14*(4*a
^2 - 12*a*b + 8*b^2) + tan(c/2 + (d*x)/2)^4*(28*a*b + 4*a^2 + 28*b^2) + ta
n(c/2 + (d*x)/2)^12*(4*a^2 - 28*a*b + 28*b^2) + tan(c/2 + (d*x)/2)^6*(28*a
*b - 4*a^2 + 56*b^2) - tan(c/2 + (d*x)/2)^10*(28*a*b + 4*a^2 - 56*b^2) ...
```

3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.229.1 Optimal result 1725
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3.229.9 Mupad [B] (verification not implemented) 1738

3.229.1 Optimal result

Integrand size = 21, antiderivative size = 333

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = {}& \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} \\ & - \frac{3b(2a^4-11a^2b^2+10b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\sqrt{a+b}d} \\ & + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} \\ & - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\ & + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} \\ & - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\ & - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} \\ & + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))}\end{aligned}$$

output
```
3/8*(a^4-24*a^2*b^2+40*b^4)*x/a^7+1/2*b*(13*a^2-30*b^2)*sin(d*x+c)/a^6/d-3
/8*(7*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^5/d+1/2*(3*a^2-10*b^2)*cos(d*x+c
)^2*sin(d*x+c)/a^4/b/d-1/4*(4*a^2-15*b^2)*cos(d*x+c)^3*sin(d*x+c)/a^3/b^2/
d-1/2*(a^2-b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))^2+1/2*(2*
a^2-7*b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b^2/d/(b+a*cos(d*x+c))-3*b*(2*a^4-1
1*a^2*b^2+10*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/
d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.229.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1178 vs. $2(333) = 666$.

Time $= 6.86$ (sec) , antiderivative size $= 1178$, normalized size of antiderivative $= 3.54$

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{6\left(8(c+dx)+\frac{2b\left(15a^4-20a^2b^2+8b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{ab\left(3a^2-4b^2\right)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^2}-\frac{3a\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}\right)}{a^3}+\frac{6\left(6c\right.}{}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output

```
((-6*(8*(c + d*x) + (2*b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[((-a + b)*T
an[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(3*a^2 - 4*b^2
)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) - (3*a*(2*a^4 - 7
*a^2*b^2 + 4*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))
))/a^3 + (6*((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c
+ d*x])/(b + a*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2) - (2*(-24*(a^2 - 8*
b^2)*(c + d*x) + (6*b*(-35*a^6 + 140*a^4*b^2 - 168*a^2*b^4 + 64*b^6)*ArcTa
nh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 96*a*
b*Sin[c + d*x] + (a*b*(-5*a^4 + 20*a^2*b^2 - 16*b^4)*Sin[c + d*x])/((a - b
)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*(10*a^6 - 115*a^4*b^2 + 220*a^2*b^4
 - 112*b^6)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + 8*a
^2*Sin[2*(c + d*x)]))/a^5 + ((12*b*(105*a^8 - 840*a^6*b^2 + 2016*a^4*b^4 -
 1920*a^2*b^6 + 640*b^8)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(5/2) + (48*a^10*c - 960*a^8*b^2*c + 1776*a^6*b^4*c + 297
6*a^4*b^6*c - 7680*a^2*b^8*c + 3840*b^10*c + 48*a^10*d*x - 960*a^8*b^2*d*x
 + 1776*a^6*b^4*d*x + 2976*a^4*b^6*d*x - 7680*a^2*b^8*d*x + 3840*b^10*d*x
+ 192*a*b*(a^2 - b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[c + d*x]
 + 48*(a^3 - a*b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[2*(c + d*x
)] + 114*a^9*b*Sin[c + d*x] + 788*a^7*b^3*Sin[c + d*x] - 5696*a^5*b^5*S...
```

3.229.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.13, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4360, 25, 25, 3042, 3370, 25, 3042, 3528, 27, 3042, 3528, 25, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^4}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\sin^4(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^4(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^4(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 3370$$

$$\frac{-\int -\frac{\cos^3(c+dx)\left(-2\left(4a^2-15b^2\right)\cos^2(c+dx)+ab\cos(c+dx)+6\left(a^2-4b^2\right)\right)}{b+a\cos(c+dx)}dx}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos^3(c+dx)\left(-2\left(4a^2-15b^2\right)\cos^2(c+dx)+ab\cos(c+dx)+6\left(a^2-4b^2\right)\right)}{b+a\cos(c+dx)}dx}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(-2\left(4a^2-15b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\sin\left(c+dx+\frac{\pi}{2}\right)+6\left(a^2-4b^2\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3528$$

$$\frac{\frac{\int -\frac{6\cos^2(c+dx)\left(a\cos(c+dx)b^2-2\left(3a^2-10b^2\right)\cos^2(c+dx)b+\left(4a^2-15b^2\right)b\right)}{b+a\cos(c+dx)}dx}{4a}-\frac{\left(4a^2-15b^2\right)\sin(c+dx)\cos^3(c+dx)}{2ad}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 27$$

$$\frac{-\frac{3\int\frac{\cos^2(c+dx)\left(a\cos(c+dx)b^2-2\left(3a^2-10b^2\right)\cos^2(c+dx)b+\left(4a^2-15b^2\right)b\right)}{b+a\cos(c+dx)}dx}{2a}-\frac{\left(4a^2-15b^2\right)\sin(c+dx)\cos^3(c+dx)}{2ad}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{3\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)b^2-2\left(3a^2-10b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b+\left(4a^2-15b^2\right)b\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}-\frac{\left(4a^2-15b^2\right)\sin(c+dx)\cos^3(c+dx)}{2ad}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3528$$

$$\frac{-\frac{3\left(\int-\frac{\cos(c+dx)\left(5a\cos(c+dx)b^3-3\left(7a^2-20b^2\right)\cos^2(c+dx)b^2+4\left(3a^2-10b^2\right)b^2\right)}{b+a\cos(c+dx)}dx}{3a}-\frac{2b\left(3a^2-10b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}\right)}{2a}-\frac{\left(4a^2-15b^2\right)\sin(c+dx)c}{2ad}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 25$$

$$\frac{-\frac{3\left(-\frac{\int\frac{\cos(c+dx)\left(5a\cos(c+dx)b^3-3\left(7a^2-20b^2\right)\cos^2(c+dx)b^2+4\left(3a^2-10b^2\right)b^2\right)}{b+a\cos(c+dx)}dx}{3a}-\frac{2b\left(3a^2-10b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}\right)}{2a}-\frac{\left(4a^2-15b^2\right)\sin(c+dx)c}{2ad}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{3\left(-\frac{\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(5a\sin\left(c+dx+\frac{\pi}{2}\right)b^3-3\left(7a^2-20b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^2+4\left(3a^2-10b^2\right)b^2\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}-\frac{2b\left(3a^2-10b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}\right)}{2a}-\frac{\left(4a^2-15b^2\right)}{}}{2a^2b^2}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3528$$

$$-\frac{3\left(-\frac{\int -\frac{-4\left(13a^2-30b^2\right)\cos^2(c+dx)b^3+3\left(7a^2-20b^2\right)b^3-a\left(3a^2-20b^2\right)\cos(c+dx)b^2}{b+a\cos(c+dx)}dx}{2a}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\sin(c+dx)\cos}{3ad}}{3a}\right)}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 25$$

$$-\frac{3\left(-\frac{\int \frac{-4\left(13a^2-30b^2\right)\cos^2(c+dx)b^3+3\left(7a^2-20b^2\right)b^3-a\left(3a^2-20b^2\right)\cos(c+dx)b^2}{b+a\cos(c+dx)}dx}{2a}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\sin(c+dx)\cos}{3ad}}{3a}\right)}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$-\frac{3\left(-\frac{\int \frac{-4\left(13a^2-30b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2b^3+3\left(7a^2-20b^2\right)b^3-a\left(3a^2-20b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\sin}{3a}\right)}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3502$$

$$-\frac{3\left(-\frac{\int \frac{3\left(ab^3\left(7a^2-20b^2\right)-b^2\left(a^4-24b^2a^2+40b^4\right)\cos(c+dx)\right)}{b+a\cos(c+dx)}dx}{2a}-\frac{4b^3\left(13a^2-30b^2\right)\sin(c+dx)}{ad}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\sin}{3a}\right)}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 27$$

$$-\frac{3\left(-\frac{3\int\frac{ab^3\left(7a^2-20b^2\right)-b^2\left(a^4-24b^2a^2+40b^4\right)\cos(c+dx)}{b+a\cos(c+dx)}dx}{a}-\frac{4b^3\left(13a^2-30b^2\right)\sin(c+dx)}{ad}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\sin(c+}{3ad}\right)}{2a}}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$-\frac{3\left(-\frac{3\int\frac{ab^3\left(7a^2-20b^2\right)-b^2\left(a^4-24b^2a^2+40b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{4b^3\left(13a^2-30b^2\right)\sin(c+dx)}{ad}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b\left(3a^2-10b^2\right)\si}{3a}\right)}{2a}}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3214$$

$$-\frac{3\left(-\frac{3\left(\frac{4b^3\left(2a^4-11a^2b^2+10b^4\right)\int\frac{1}{b+a\cos(c+dx)}dx}{a}-\frac{b^2x\left(a^4-24a^2b^2+40b^4\right)}{a}\right)}{a}-\frac{4b^3\left(13a^2-30b^2\right)\sin(c+dx)}{ad}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{2b}{}\right)}{2a}}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

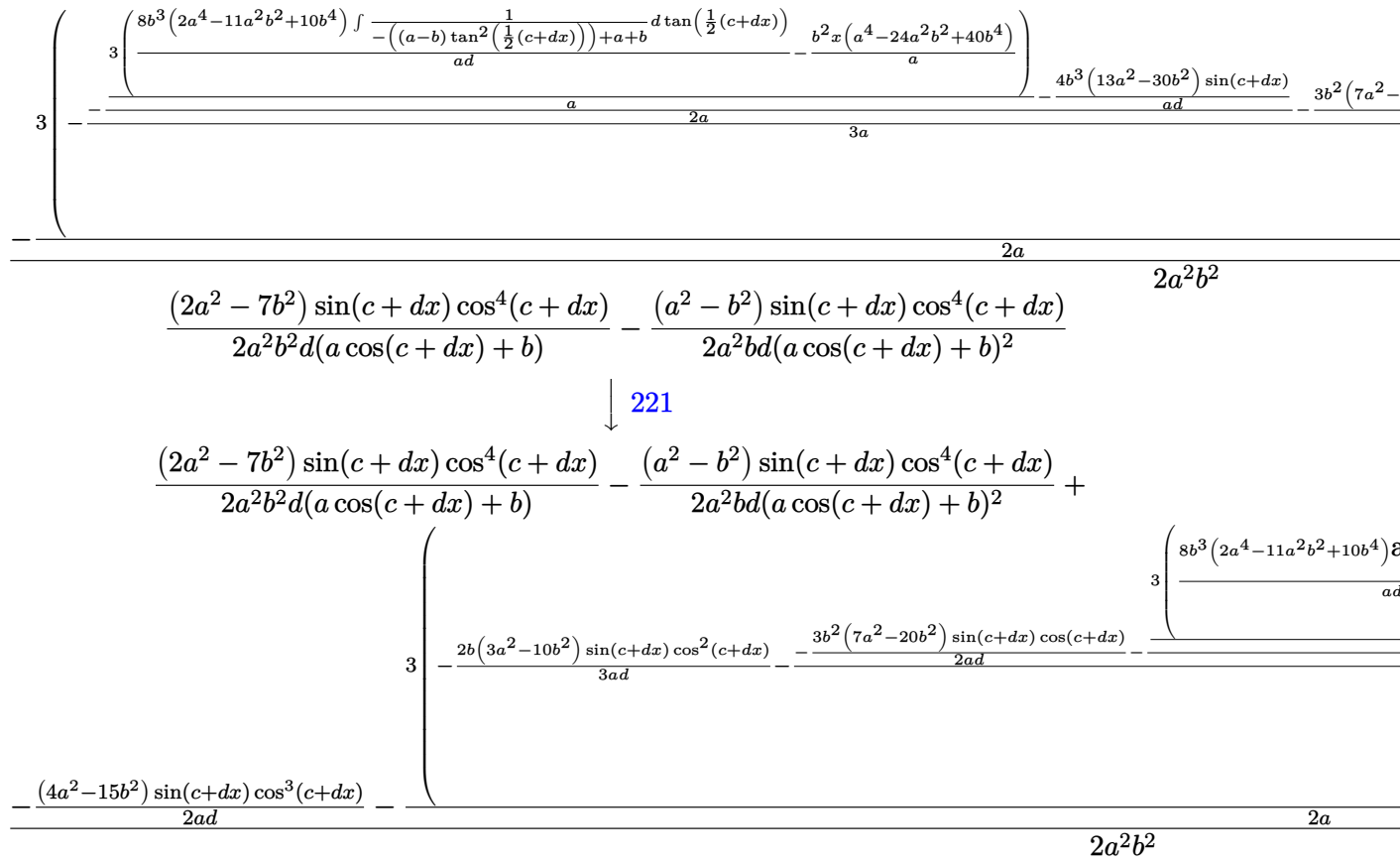
$$\downarrow 3042$$

$$-\frac{3\left(-\frac{3\left(\frac{4b^3\left(2a^4-11a^2b^2+10b^4\right)\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{b^2x\left(a^4-24a^2b^2+40b^4\right)}{a}\right)}{a}-\frac{4b^3\left(13a^2-30b^2\right)\sin(c+dx)}{ad}-\frac{3b^2\left(7a^2-20b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}\right)}{2a}}{2a}$$

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 3138$$

3.229. $\int\frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}dx$



$$\frac{(2a^2-7b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{(a^2-b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}$$

$$\downarrow 221$$

$$\frac{(2a^2-7b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{(a^2-b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}+$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x
])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b^2*d*(b + a*
Cos[c + d*x])) + (-1/2*((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(a*d
) - (3*((-2*b*(3*a^2 - 10*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((-3
*b^2*(7*a^2 - 20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((3*(-((b^2*(a^
4 - 24*a^2*b^2 + 40*b^4)*x)/a) + (8*b^3*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b
]*d)))/a - (4*b^3*(13*a^2 - 30*b^2)*Sin[c + d*x])/(a*d))/(2*a))/(3*a)))/(2
*a))/(2*a^2*b^2)
```

3.229. $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.229.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3370
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (Simp[(a^2*(n - m + 1) - b^2*(m + n + 2))*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(a^2*b^2*d*f*(m + 1)*(m + 2)
)), x] - Simp[1/(a^2*b^2*(m + 1)*(m + 2))   Int[(a + b*Sin[e + f*x])^(m + 2
)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 3)
 + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m + n
 + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4360

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.229.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{2\left(\left(\frac{3}{8}a^4+3a^3b-3a^2b^2-10ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(13a^3b-3a^2b^2-30ab^3+\frac{11}{8}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{11}{8}a^4+3a^2b^2+13a^3b-30ab^3\right)\tan$ $\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4$ a^7
default	$\frac{2\left(\left(\frac{3}{8}a^4+3a^3b-3a^2b^2-10ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(13a^3b-3a^2b^2-30ab^3+\frac{11}{8}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{11}{8}a^4+3a^2b^2+13a^3b-30ab^3\right)\tan$ $\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4$ a^7
risch	$\frac{3x}{8a^3}-\frac{9xb^2}{a^5}+\frac{15xb^4}{a^7}-\frac{ib\mathrm{e}^{-3i(dx+c)}}{8a^4d}-\frac{3i\mathrm{e}^{2i(dx+c)}b^2}{4a^5d}-\frac{15ib\mathrm{e}^{i(dx+c)}}{8a^4d}-\frac{ib^2\left(-7a^3b\mathrm{e}^{3i(dx+c)}+12ab^3\mathrm{e}^{3i(dx+c}$

input
```
int(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^7*(((3/8*a^4+3*a^3*b-3*a^2*b^2-10*a*b^3)*tan(1/2*d*x+1/2*c)^7+(13
*a^3*b-3*a^2*b^2-30*a*b^3+11/8*a^4)*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3*a^2*
b^2+13*a^3*b-30*a*b^3)*tan(1/2*d*x+1/2*c)^3+(3*a^3*b-10*a*b^3-3/8*a^4+3*a^
2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+3/8*(a^4-24*a^2*b^2+
40*b^4)*arctan(tan(1/2*d*x+1/2*c)))+2*b/a^7*(((5/2*a^3*b^2-5*a*b^4-3*a^4*b
+11/2*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(5/2*a^3*b^2-5*a*b^4+3*a^4*b-11/2*a^2*
b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^2-3/2*(2*a^4-11*a^2*b^2+10*b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1
/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1041, normalized size of antiderivative = 3.13

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output

```
[1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x + c)^2 +
6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b
^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x + 6*(2*a^4*b^3 - 11*a^2*b^5 + 1
0*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^5*b^2
 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 b^2)) + (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x
 + c)^5 - 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b
^4)*cos(d*x + c)^3 + 2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2
 + (83*a^6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((
a^11 - a^9*b^2)*d*cos(d*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (
a^9*b^2 - a^7*b^4)*d), 1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)
*d*x*cos(d*x + c)^2 + 6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*c
os(d*x + c) + 3*(a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x - 12*(2*a
^4*b^3 - 11*a^2*b^5 + 10*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x
 + c)^2 + 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x +
c))) + (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x +
 c)^5 - 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4...
```

3.229.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input

```
integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

output

```
Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.229.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.75

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{3\,(a^4-24\,a^2b^2+40\,b^4)\,(dx+c)}{a^7} - \frac{24\,(2\,a^4b-11\,a^2b^3+10\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^7} - \frac{8\,($$

input
```
integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output

```
1/8*(3*(a^4 - 24*a^2*b^2 + 40*b^4)*(d*x + c)/a^7 - 24*(2*a^4*b - 11*a^2*b^
3 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-
a^2 + b^2)*a^7) - 8*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 - 11*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 10*b^5*tan(1/2*d*x + 1/
2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) +
 11*a*b^4*tan(1/2*d*x + 1/2*c) + 10*b^5*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a^6) + 2*(3*a^3*tan(1
/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*a*b^2*tan(1/2*d*x
 + 1/2*c)^7 - 80*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2*d*x + 1/2*c)^
5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
40*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 + 104*a^2*b*
tan(1/2*d*x + 1/2*c)^3 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*b^3*tan(1/2
*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*
c) + 24*a*b^2*tan(1/2*d*x + 1/2*c) - 80*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/
2*d*x + 1/2*c)^2 + 1)^4*a^6))/d
```

3.229.9 Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 3255, normalized size of antiderivative = 9.77

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(sin(c + d*x)^4/(a + b/cos(c + d*x))^3,x)
```

output

```
(atan(((((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^1
6*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i
+ b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a
^19))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(576
00*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8
 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*
b^3 + 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7)
 - (((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^
4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^
4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19)
)*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a
*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 2
2752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3
+ 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7))/((
324000*a*b^13 + 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^1
1 - 195480*a^4*b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 244
08*a^8*b^6 + (62181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a
^12*b^2)/a^18 + (3*((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^
5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/
2)*(a^4*1i + b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^...
```

3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.230.1 Optimal result 1740
3.230.2 Mathematica [A] (verified) 1741
3.230.3 Rubi [A] (verified) 1741
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3.230.5 Fricas [A] (verification not implemented) 1747
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3.230.8 Giac [B] (verification not implemented) 1749
3.230.9 Mupad [B] (verification not implemented) 1750

3.230.1 Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = {}& \frac{(a^2-12b^2)\,x}{2a^5} \\ & - \frac{b(6a^4-19a^2b^2+12b^4)\arctanh\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} \\ & + \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} \\ & - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} \\ & + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} \\ & + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))}\end{aligned}$$

output

```
1/2*(a^2-12*b^2)*x/a^5-b*(6*a^4-19*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b*(11*a^2-1
2*b^2)*sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)/
a^3/(a^2-b^2)/d+1/2*cos(d*x+c)^3*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1/2*(3*
a^2-4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

3.230.2 Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.06

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{4b\left(6a^4-19a^2b^2+12b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4ab\left(a^4-13a^2b^2+12b^4\right)(c+dx)\cos(c+dx)-2a^4\left(a^2-b^2\right)\cos^3(c+dx)\sin(c+dx)+2a}{4a^5(a-$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (4*a*b*(a^4 - 13*a^2*b^2 + 12*b^4)*(c +
d*x)*Cos[c + d*x] - 2*a^4*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x] + 2*a^2*
(a^2 - b^2)*Cos[c + d*x]^2*((a^2 - 12*b^2)*(c + d*x) + 4*a*b*Sin[c + d*x])
 + b^2*(2*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*x) + (22*a^3*b - 24*a*b^3)*Si
n[c + d*x] + (17*a^4 - 18*a^2*b^2)*Sin[2*(c + d*x)]))/(b + a*Cos[c + d*x])
^2)/(4*a^5*(a - b)*(a + b)*d)
```

3.230.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.15, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$, Rules used = {3042, 4360, 25, 25, 3042, 3368, 3042, 3527, 3042, 3527, 3042, 3528, 27, 3042, 3502, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^2}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$
$$\downarrow 4360$$
$$\int -\frac{\sin^2(c+dx)\cos^3(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)\sin^2(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\sin^2(c+dx)\cos^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\cos\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 3368$$

$$\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 3527$$

$$\frac{\int \frac{\cos^2(c+dx)\left(3\left(a^2-b^2\right)-4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(3\left(a^2-b^2\right)-4\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3527$$

$$\frac{\int \frac{\cos(c+dx)\left(-2\left(5a^2-6b^2\right)\left(a^2-b^2\right)\cos^2(c+dx)+ab\left(a^2-b^2\right)\cos(c+dx)+2\left(3a^4-7b^2a^2+4b^4\right)\right)}{b+a\cos(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{\left(3a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}}{2a\left(a^2-b^2\right)}+\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-2\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2+ab\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)+2\left(3a^4-7b^2a^2+4b^4\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}+$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\Big\downarrow 3528$$

$$\frac{\int -\frac{2\left(-b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\cos^2(c+dx)-a\left(a^4-3b^2a^2+2b^4\right)\cos(c+dx)+b\left(5a^4-11b^2a^2+6b^4\right)\right)}{b+a\cos(c+dx)}dx}{2a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-4b^2}{ad}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{\int \frac{-b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\cos^2(c+dx)-a\left(a^4-3b^2a^2+2b^4\right)\cos(c+dx)+b\left(5a^4-11b^2a^2+6b^4\right)}{b+a\cos(c+dx)}dx}{a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-4b^2)\,s}{ad(a}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\Big\downarrow 3042$$

$$\frac{-\frac{\int \frac{-b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^4-3b^2a^2+2b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)+b\left(5a^4-11b^2a^2+6b^4\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2}{}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\Big\downarrow 3502$$

$$\frac{-\frac{\int \frac{ab\left(5a^4-11b^2a^2+6b^4\right)-\left(a^2-12b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)}{b+a\cos(c+dx)}dx}{a}-\frac{b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{ad}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-4b^2}{ad(}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{\int \frac{ab\left(5a^4-11b^2a^2+6b^4\right)-\left(a^2-12b^2\right)\left(a^2-b^2\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{ad}}{a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-}{2a\left(a^2-b^2\right)}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3214$$

$$\frac{\frac{\frac{b\left(6a^6-25a^4b^2+31a^2b^4-12b^6\right)\int\frac{1}{b+a\cos(c+dx)}dx}{a}-\frac{x\left(a^2-12b^2\right)\left(a^2-b^2\right)^2}{a}}{a}-\frac{b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{ad}}{a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}}{a(a^2-b^2)}$$

$$2a\left(a^2-b^2\right)$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3042$$

$$\frac{\frac{\frac{b\left(6a^6-25a^4b^2+31a^2b^4-12b^6\right)\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{x\left(a^2-12b^2\right)\left(a^2-b^2\right)^2}{a}}{a}-\frac{b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{ad}}{a}-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+}{ad}}{a(a^2-b^2)}$$

$$2a\left(a^2-b^2\right)$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 3138$$

$$\frac{\frac{\frac{2b\left(6a^6-25a^4b^2+31a^2b^4-12b^6\right)\int\frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}-\frac{x\left(a^2-12b^2\right)\left(a^2-b^2\right)^2}{a}}{a}-\frac{b\left(11a^2-12b^2\right)\left(a^2-b^2\right)\sin(c+dx)}{ad}}{a}-\frac{\left(5a^2-6b^2\right.}{a}}{a(a^2-b^2)}$$

$$2a\left(a^2-b^2\right)$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

$$\downarrow 221$$

$$\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}+\frac{-\frac{\left(5a^2-6b^2\right)\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{2b\left(6a^6-25a^4b^2+31a^2b^4-12b^6\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}-x}{a}}{a(a^2-b^2)}$$

$$2a\left(a^2-b^2\right)$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

3.230. $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

input
```
Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(Cos[c + d*x]^3*Sin[c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + (((3*a^2 -
4*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(a*d*(b + a*Cos[c + d*x])) + (-(((5*a^
2 - 6*b^2)*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(a*d)) - ((-(((a^2 - 12*
b^2)*(a^2 - b^2)^2*x)/a) + (2*b*(6*a^6 - 25*a^4*b^2 + 31*a^2*b^4 - 12*b^6)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[
a + b]*d))/a - (b*(11*a^2 - 12*b^2)*(a^2 - b^2)*Sin[c + d*x])/(a*d))/a)/(a
*(a^2 - b^2)))/(2*a*(a^2 - b^2))
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.230.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{\left(6a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}+\frac{\left(6a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(6a^4-19a^2b^2+12b^4\right)\arctanh\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^2-b^2\right)\sqrt{(a-b)(a+b)}}\right)}{a^5}+$
default	$\frac{2b\left(\frac{-\frac{\left(6a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}+\frac{\left(6a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(6a^4-19a^2b^2+12b^4\right)\arctanh\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^2-b^2\right)\sqrt{(a-b)(a+b)}}\right)}{a^5}+$
risch	$\frac{x}{2a^3}-\frac{6xb^2}{a^5}+\frac{ie^{2i(dx+c)}}{8a^3d}-\frac{3ibe^{i(dx+c)}}{2a^4d}+\frac{3ibe^{-i(dx+c)}}{2a^4d}-\frac{ie^{-2i(dx+c)}}{8a^3d}+\frac{ib^2\left(-7a^3be^{3i(dx+c)}+8ab^3e^{3i(dx+c)}\right.}{}$

input
```
int(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^5*((-1/2*(6*a^2+a*b-6*b^2)*a*b/(a+b)*tan(1/2*d*x+1/2*c)^3+1/2*(
6*a^2-a*b-6*b^2)*a*b/(a-b)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4-19*a^2*b^2+12*b^4)/(a^2-b^2)/((a-b)*
(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^5*
(((1/2*a^2+3*a*b)*tan(1/2*d*x+1/2*c)^3+(3*a*b-1/2*a^2)*tan(1/2*d*x+1/2*c))
/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-12*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.69

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{2\left(a^8-14a^6b^2+25a^4b^4-12a^2b^6\right)dx\cos(dx+c)^2+4\left(a^7b-14a^5b^3+25a^3b^5-12ab^7\right)dx\cos(dx+c)\right.}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 +
4*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b
^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*
b^7 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 -
19*a^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x +
c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)
*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^
2)) + 2*(11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*
cos(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*
b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*
b^2 + a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d
*x + c) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d), 1/2*((a^8 - 14*a^6*b^2 + 25*
a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*
b^5 - 12*a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12
*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7 + (6*a^6*b - 19*a^4*b^3 + 12*
a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
 b^2)*sin(d*x + c))) + (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*
b^2 + a^4*b^4)*cos(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x +
c)^2 + (17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c...
```

3.230.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.230.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(248) = 496$.

Time $= 0.49$ (sec) , antiderivative size $= 1193$, normalized size of antiderivative $= 4.47$

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*((a^11 - 7*a^10*b - 14*a^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5
- 12*a^5*b^6 + 24*a^4*b^7 - a^4*abs(-a^7 + a^5*b^2) - 5*a^3*b*abs(-a^7 + a
^5*b^2) + 13*a^2*b^2*abs(-a^7 + a^5*b^2) + 6*a*b^3*abs(-a^7 + a^5*b^2) - 1
2*b^4*abs(-a^7 + a^5*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(
1/2*d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 + sqrt((a^7 + a^6*b - a^5*b^2 - a^
4*b^3)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^
6*b - a^5*b^2 + a^4*b^3))))/(a^6*b*abs(-a^7 + a^5*b^2) - a^4*b^3*abs(-a^7
+ a^5*b^2) + (a^7 - a^5*b^2)^2) + ((a^4 + 5*a^3*b - 13*a^2*b^2 - 6*a*b^3 +
 12*b^4)*sqrt(-a^2 + b^2)*abs(-a^7 + a^5*b^2)*abs(-a + b) + (a^11 - 7*a^10
*b - 14*a^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 - 12*a^5*b^6 + 24*a
^4*b^7)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) +
arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 - sqrt((a^7 + a^6*b - a
^5*b^2 - a^4*b^3)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2)
)/(a^7 - a^6*b - a^5*b^2 + a^4*b^3))))/((a^7 - a^5*b^2)^2*(a^2 - 2*a*b + b
^2) - (a^8*b - 2*a^7*b^2 + 2*a^5*b^4 - a^4*b^5)*abs(-a^7 + a^5*b^2)) + 2*(
a^5*tan(1/2*d*x + 1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*t
an(1/2*d*x + 1/2*c)^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/
2*d*x + 1/2*c)^7 - 12*b^5*tan(1/2*d*x + 1/2*c)^7 - 3*a^5*tan(1/2*d*x + 1/2
*c)^5 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 ...
```

3.230.9 Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 4026, normalized size of antiderivative = 15.08

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

output

```
((tan(c/2 + (d*x)/2)*(6*a*b^3 - 5*a^3*b + a^4 + 12*b^4 - 13*a^2*b^2))/(a^4
*b - a^5) + (tan(c/2 + (d*x)/2)^3*(18*a*b^4 + 4*a^4*b - 3*a^5 + 36*b^5 - 3
7*a^2*b^3 - 14*a^3*b^2))/((a^4*b - a^5)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(
4*a^4*b - 18*a*b^4 + 3*a^5 + 36*b^5 - 37*a^2*b^3 + 14*a^3*b^2))/((a^4*b -
a^5)*(a + b)) + (tan(c/2 + (d*x)/2)^7*(5*a^3*b - 6*a*b^3 + a^4 + 12*b^4 -
13*a^2*b^2))/(a^4*(a + b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^
2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b -
4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan(((
(a^2*1i - b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 +
 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15
- a^12*b^3 - a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*
b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^
5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*t
an(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 6
24*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*
b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2))*1i)/(2*a^5) - ((a^2*1i - b^2*12
i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 -
56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^1
3*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 +
 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a...
```

3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.231.1 Optimal result . . . 1752
3.231.2 Mathematica [A] (verified) . . . 1753
3.231.3 Rubi [A] (verified) . . . 1753
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3.231.5 Fricas [A] (verification not implemented) . . . 1756
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3.231.1 Optimal result

Integrand size = 21, antiderivative size = 376

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{2b^3(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{b^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))}\\ &-\frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2}\\ &+\frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\\ &+\frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\end{aligned}$$

output
```
-2*b^3*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(
a-b)^(7/2)/(a+b)^(7/2)/d-2*a*b*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x
+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-b^3*(a^2+2*b^2)*arctanh((a-
b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(7/2)/(a+b)^(7/2)/d-1/2*s
in(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))
-1/2*b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*b^4*sin(d*x+c)/(a
^2-b^2)^3/d/(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(b+a
*cos(d*x+c))
```

3.231.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.61

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(\frac{6ab(2a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{(a^2-b^2)^{7/2}} - \frac{(b+a\cos(c+dx))^2\cot\left(\frac{1}{2}(}{(a+b)^3}\right.}{2d(a+b\sec(c+dx))^3}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^3*((6*a*b*(2*a^2 + 3*b^2)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2
)^(7/2) - ((b + a*Cos[c + d*x])^2*Cot[(c + d*x)/2])/(a + b)^3 - (b^3*Sin[c
 + d*x])/((a - b)^2*(a + b)^2) + (b^2*(6*a^2 + b^2)*(b + a*Cos[c + d*x])*S
in[c + d*x])/((a - b)^3*(a + b)^3) + ((b + a*Cos[c + d*x])^2*Tan[(c + d*x)
/2])/(a - b)^3))/(2*d*(a + b*Sec[c + d*x])^3)
```

3.231.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)\cot^2(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3376$$

$$-\int \left(-\frac{b^3}{a\left(b^2-a^2\right)(b+a\cos(c+dx))^3}+\frac{1}{2(a-b)^3(-\cos(c+dx)-1)}-\frac{1}{2(a+b)^3(1-\cos(c+dx))}-\frac{}{(a^2-b^2)}\right.$$

$$\downarrow 2009$$

$$-\frac{2ab\left(3a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{2b^3\left(3a^2-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}}-$$
$$\frac{b^3\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+$$
$$\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-$$
$$\frac{\sin(c+dx)}{2d(a+b)^3(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^3(\cos(c+dx)+1)}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^
3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*
x])) + Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x]
)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^
2 - b^2)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^
2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.231.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6a^2b+6ab^2-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{\left(-3a^3b+\frac{5}{2}a^2b^2-\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(3a^3b+\frac{5}{2}a^2b^2+\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{d}{}\right.\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}$ $\frac{}{(a-b)^3(a+b)^3}$ $\frac{}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6a^2b+6ab^2-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{\left(-3a^3b+\frac{5}{2}a^2b^2-\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(3a^3b+\frac{5}{2}a^2b^2+\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{d}{}\right.\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}$ $\frac{}{(a-b)^3(a+b)^3}$ $\frac{}{d}$
risch	$-\frac{i\left(6a^5be^{5i(dx+c)}+9a^3b^3e^{5i(dx+c)}-2a^6e^{4i(dx+c)}+24a^4b^2e^{4i(dx+c)}+21a^2b^4e^{4i(dx+c)}+2b^6e^{4i(dx+c)}+4a^5be^{3i(dx+c)}+}{a(a}$

input
```
int(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^3-3*a^2*b+3*a*b^2-b^3)-1/2/(a+b)^3/tan(1/2*
d*x+1/2*c)+2*b/(a-b)^3/(a+b)^3*(((-3*a^3*b+5/2*a^2*b^2-1/2*a*b^3+b^4)*tan(
1/2*d*x+1/2*c)^3+(3*a^3*b+5/2*a^2*b^2+1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c))/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/2*(2*a^2+3*b^2)*a/(
(a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.24

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{22\,a^4b^3-14\,a^2b^5-8\,b^7-2\left(2\,a^7+10\,a^5b^2-11\,a^3b^4-ab^6\right)\cos(dx+c)^3-3\left(2\,a^3b^3+3\,ab^5+\left(2\,a^5b+3\right.\right.}{4\left((a^{10}-4\,a^8b^2\right.}\right.$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 - 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4
 - a*b^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*
cos(d*x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2))*sin(d*x + c) + 2*(2*a^6*b - 17*a^4*b^3 + 13*a^2*b^
5 + 2*b^7)*cos(d*x + c)^2 + 2*(16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x +
c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^
2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) +
 (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)*sin(d*x + c)), 1/
2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 - (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b
^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*cos(d*
x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan
(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*
x + c) + (2*a^6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + (16*
a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^
4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b
^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*d)*sin(d*x + c))]
```

3.231.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.231.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{6\left(2a^3b+3ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}} + \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3-3a^2b+3ab^2-b^3} - \frac{2\left(6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\right.}{}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(6*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b
) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b
^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + tan(1/2*d*x
 + 1/2*c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 2*(6*a^3*b^2*tan(1/2*d*x + 1/2
*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + a*b^4*tan(1/2*d*x + 1/2*c)^3 -
2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*
tan(1/2*d*x + 1/2*c) - a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/
2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)^2) - 1/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan
(1/2*d*x + 1/2*c)))/d
```

3.231.9 Mupad [B] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\,(a-b)^3}$$

$$-\frac{\frac{a^3-3a^2b+3ab^2-b^3}{a+b}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a^5-5a^4b+22a^3b^2-20a^2b^3+}{(a+b)^3}}{d\left(\left(2a^5-10a^4b+20a^3b^2-20a^2b^3+10ab^4-2b^5\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-4a^5+12a^4b-8a^3b^2-8a\right.\right.}$$

$$+\frac{ab\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^6-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^2+3\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^4-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^6}{(a+b)^{7/2}\,(a-b)^{5/2}}\right)\,\left(2a^2+3b^2\right)\,3\mathrm{i}}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

output
```
tan(c/2 + (d*x)/2)/(2*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(a +
 b) + (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 5*a^4*b + a^5 - 5*b^5 - 20*a^2*b^3
+ 22*a^3*b^2))/(a + b)^3 - (2*tan(c/2 + (d*x)/2)^2*(a^4 - 4*a^3*b - 5*a*b^
3 + 3*b^4 + 12*a^2*b^2))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)*(2*a*b^4 - 2*a^
4*b + 2*a^5 - 2*b^5 + 4*a^2*b^3 - 4*a^3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^5
 - 12*a^4*b - 12*a*b^4 + 4*b^5 + 8*a^2*b^3 + 8*a^3*b^2) + tan(c/2 + (d*x)/
2)^5*(10*a*b^4 - 10*a^4*b + 2*a^5 - 2*b^5 - 20*a^2*b^3 + 20*a^3*b^2))) + (
a*b*atan((a^6*tan(c/2 + (d*x)/2)*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*
tan(c/2 + (d*x)/2)*3i - a^4*b^2*tan(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a -
 b)^(5/2)))*(2*a^2 + 3*b^2)*3i)/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

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3.232.1 Optimal result

Integrand size = 21, antiderivative size = 515

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{2ab^3(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}\\
&-\frac{ab^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}\\
&-\frac{2ab(3a^4+8a^2b^2+b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}\\
&-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2}-\frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))}\\
&-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))}\\
&+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2}\\
&+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))}+\frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))}\\
&-\frac{a^2b^3\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))^2}\\
&+\frac{3a^2b^4\sin(c+dx)}{2\left(a^2-b^2\right)^4d(b+a\cos(c+dx))}\\
&+\frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{\left(a^2-b^2\right)^4d(b+a\cos(c+dx))}\end{aligned}$$

output
```
-2*a*b^3*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(
a-b)^(9/2)/(a+b)^(9/2)/d-a*b^3*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x
+1/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d-2*a*b*(3*a^4+8*a^2*b^2+b^4)
*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/
2)/d-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))^2-1/4*(a-2*b)*sin(d*x+c)/(a+
b)^4/d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*
x+c)/(a-b)^3/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1
/4*(a+2*b)*sin(d*x+c)/(a-b)^4/d/(1+cos(d*x+c))-1/2*a^2*b^3*sin(d*x+c)/(a^2
-b^2)^3/d/(b+a*cos(d*x+c))^2+3/2*a^2*b^4*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos
(d*x+c))+a^2*b^2*(3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))
```

3.232.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.75

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{(b+a\cos(c+dx))\left(\frac{96ab\left(6a^4+23a^2b^2+6b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{\sqrt{a^2-b^2}} + (36a^6b + 154a^4b^3 + 424c\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*((96*a*b*(6*a^4 + 23*a^2*b^2 + 6*b^4)*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/Sqrt[a^2 -
b^2] + (36*a^6*b + 154*a^4*b^3 + 424*a^2*b^5 + 16*b^7 - 2*a*(16*a^6 - 94*a
^4*b^2 - 35*a^2*b^4 + 8*b^6)*Cos[c + d*x] + 8*(2*a^6*b - 45*a^4*b^3 - 56*a
^2*b^5 - 6*b^7)*Cos[2*(c + d*x)] - 4*a^7*Cos[3*(c + d*x)] - 154*a^5*b^2*Co
s[3*(c + d*x)] - 205*a^3*b^4*Cos[3*(c + d*x)] + 48*a*b^6*Cos[3*(c + d*x)]
- 20*a^6*b*Cos[4*(c + d*x)] + 110*a^4*b^3*Cos[4*(c + d*x)] + 120*a^2*b^5*C
os[4*(c + d*x)] + 4*a^7*Cos[5*(c + d*x)] + 62*a^5*b^2*Cos[5*(c + d*x)] + 3
9*a^3*b^4*Cos[5*(c + d*x)])*Csc[c + d*x]^3)*Sec[c + d*x]^3)/(96*(a^2 - b^2
)^4*d*(a + b*Sec[c + d*x])^3)
```

3.232.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cot^3(c+dx)\csc(c+dx)}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cot^3(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cot^3(c+dx)\csc(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\downarrow 3376$$

$$-\int \left(-\frac{ab^3}{\left(a^2-b^2\right)^2(-b-a\cos(c+dx))^3}-\frac{a\left(3a^2+b^2\right)b^2}{\left(a^2-b^2\right)^3(-b-a\cos(c+dx))^2}-\frac{a\left(3a^4+8b^2a^2+b^4\right)b}{\left(a^2-b^2\right)^4(-b-a\cos(c+dx))}+\right.$$

$$\downarrow 2009$$

$$-\frac{2ab^3\left(3a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}-\frac{ab^3\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}+$$
$$\frac{a^2b^2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^4\left(a\cos(c+dx)+b\right)}+\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4\left(a\cos(c+dx)+b\right)}-$$
$$\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3\left(a\cos(c+dx)+b\right)^2}-\frac{2ab\left(3a^4+8a^2b^2+b^4\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}-$$
$$\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))}-\frac{(a-2b)\sin(c+dx)}{4d(a+b)^4(1-\cos(c+dx))}+\frac{(a+2b)\sin(c+dx)}{4d(a-b)^4(\cos(c+dx)+1)}+$$
$$\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))^2}+\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)^2}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*
a*b*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1
 - Cos[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c +
 d*x])) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/
(12*(a - b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 +
Cos[c + d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x]
)) - (a^2*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (
3*a^2*b^4*Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^
2*(3*a^2 + b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```

3.232.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.232.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^3-3a^2b+3ab^2-b^3\right)(a-b)}-\frac{1}{24(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a-3b}{8(a+b)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2ab\left(\frac{\left(\frac{5}{2}a^3\right.}{}\right.}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^3-3a^2b+3ab^2-b^3\right)(a-b)}-\frac{1}{24(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a-3b}{8(a+b)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2ab\left(\frac{\left(\frac{5}{2}a^3\right.}{}\right.}$
risch	Expression too large to display

input
```
int(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*t
an(1/2*d*x+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)*a+3*tan(1/2*d*x+1/2*c)*b)-1/24/
(a+b)^3/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(3*a-3*b)/tan(1/2*d*x+1/2*c)+2*a*
b/(a-b)^4/(a+b)^4*(((5/2*a^3*b^2+3*a*b^4-3*a^4*b-5/2*a^2*b^3)*tan(1/2*d*x+
1/2*c)^3+(5/2*a^3*b^2+3*a*b^4+3*a^4*b+5/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4+23*a^2*b^2+6
*b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(
1/2))))
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1550, normalized size of antiderivative = 3.01

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/12*(78*a^6*b^3 + 46*a^4*b^5 - 116*a^2*b^7 - 8*b^9 + 2*(4*a^9 + 58*a^7*
b^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x + c)^5 - 10*(2*a^8*b - 13*a^6*b^3 -
 a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4
 - 56*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 + 3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b
^7 - (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*cos(d*x + c)^4 - 2*(6*a^6*b^2 + 23
*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a^7*b + 17*a^5*b^3 - 17*a^3*b^5
 - 6*a*b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x
 + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) + 4*(6*a^8*b
 - 56*a^6*b^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 10*(12*a^7
*b^2 - a^5*b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2
 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^
11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x
 + c)^3 - (a^12 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^
2*b^10 + b^12)*d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*
a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*
a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c)), -1/6*(39*a^6*b
^3 + 23*a^4*b^5 - 58*a^2*b^7 - 4*b^9 + (4*a^9 + 58*a^7*b^2 - 23*a^5*b^4 -
39*a^3*b^6)*cos(d*x + c)^5 - 5*(2*a^8*b - 13*a^6*b^3 - a^4*b^5 + 12*a^2...
```

3.232.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.232.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.38

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{24\left(6a^5b+23a^3b^3+6ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\sqrt{-a^2+b^2}} + \frac{a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{\cdots}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*(24*(6*a^5*b + 23*a^3*b^3 + 6*a*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(-a^2 + b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt
(-a^2 + b^2)) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b*tan(1/2*d*x + 1/2*c)
^3 + 15*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 20*a^3*b^3*tan(1/2*d*x + 1/2*c)^3
 + 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*tan(1/2*d*x + 1/2*c)^3 + b^
6*tan(1/2*d*x + 1/2*c)^3 + 9*a^6*tan(1/2*d*x + 1/2*c) - 36*a^5*b*tan(1/2*d
*x + 1/2*c) + 45*a^4*b^2*tan(1/2*d*x + 1/2*c) - 45*a^2*b^4*tan(1/2*d*x + 1
/2*c) + 36*a*b^5*tan(1/2*d*x + 1/2*c) - 9*b^6*tan(1/2*d*x + 1/2*c))/(a^9 -
 9*a^8*b + 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^
6 - 36*a^2*b^7 + 9*a*b^8 - b^9) - 24*(6*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 5
*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^2
*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a^4*b^3*t
an(1/2*d*x + 1/2*c) - 5*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*a^2*b^5*tan(1/2*d
*x + 1/2*c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (9*a*tan(1/2*d*x +
1/2*c)^2 - 9*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*tan(1/2*d*x + 1/2*c)^3))/d
```

3.232.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.14

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d\,(a-b)^3}$$
$$+\frac{\frac{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}{3\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,(3\,a^7-21\,a^6\,b+111\,a^5\,b^2-145\,a^4\,b^3+145\,a^3\,b^4-111\,a^2\,b^5+21\,a}{(a+b)^4}}{d\,\left(\left(-8\,a^6+48\,a^5\,b-120\,a^4\,b^2+160\,a^3\,b^3-120\,a^2\,b^4+48\,a\,b^5-8\,b^6\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+(16\,a^6-64\,a^5\,b$$
$$+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a+b)}{8\,d\,(a-b)^4}$$
$$+\frac{a\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^8-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^6\,b^2+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^4\,b^4-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^2\,b^6+1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,b^8}{(a+b)^{9/2}\,(a-b)^{7/2}}\right)\,(6\,a^4+23\,a^2\,b^2}{d\,(a+b)^{9/2}\,(a-b)^{9/2}}$$

input
```
int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

output

```
tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^3) + ((a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)/(3*(a + b)) + (tan(c/2 + (d*x)/2)^6*(21*a*b^6 - 21*a^6*b + 3*a^
7 - 3*b^7 - 111*a^2*b^5 + 145*a^3*b^4 - 145*a^4*b^3 + 111*a^5*b^2))/(a + b
)^4 - (tan(c/2 + (d*x)/2)^4*(17*a^6 - 102*a^5*b - 102*a*b^5 + 17*b^6 + 399
*a^2*b^4 - 364*a^3*b^3 + 399*a^4*b^2))/(3*(a + b)^3) + (7*tan(c/2 + (d*x)/
2)^2*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))/(3*(a + b)^2))/(
d*(tan(c/2 + (d*x)/2)^3*(16*a*b^5 + 16*a^5*b - 8*a^6 - 8*b^6 + 8*a^2*b^4 -
 32*a^3*b^3 + 8*a^4*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^6 - 48*a^5*b - 48*a*b
^5 + 8*b^6 + 120*a^2*b^4 - 160*a^3*b^3 + 120*a^4*b^2) + tan(c/2 + (d*x)/2)
^5*(64*a*b^5 - 64*a^5*b + 16*a^6 - 16*b^6 - 80*a^2*b^4 + 80*a^4*b^2))) + (
3*tan(c/2 + (d*x)/2)*(a + b))/(8*d*(a - b)^4) + (a*b*atan((a^8*tan(c/2 + (
d*x)/2)*1i + b^8*tan(c/2 + (d*x)/2)*1i - a^2*b^6*tan(c/2 + (d*x)/2)*4i + a
^4*b^4*tan(c/2 + (d*x)/2)*6i - a^6*b^2*tan(c/2 + (d*x)/2)*4i)/((a + b)^(9/
2)*(a - b)^(7/2)))*(6*a^4 + 6*b^4 + 23*a^2*b^2)*1i)/(d*(a + b)^(9/2)*(a -
b)^(9/2))
```

3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$

3.233.1 Optimal result 1769
3.233.2 Mathematica [C] (warning: unable to verify) 1770
3.233.3 Rubi [A] (warning: unable to verify) 1771
3.233.4 Maple [A] (verified) 1785
3.233.5 Fricas [F(-1)] 1786
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3.233.7 Maxima [F] 1787
3.233.8 Giac [F] 1787
3.233.9 Mupad [F(-1)] 1787

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 516

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = -\frac{b(a^2-b^2)^{5/4}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$-\frac{b(a^2-b^2)^{5/4}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$+\frac{2(5a^4-28a^2b^2+21b^4)e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2(a^2-b^2)^2e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2(a^2-b^2)^2e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d}$$
$$+\frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}$$

output
```
-b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(
1/4)/e^(1/2))/a^(9/2)/d-b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d
*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+2/35*e*(7*b-5*a*cos(d*x+c)
)*(e*sin(d*x+c))^(5/2)/a^2/d-2/21*(5*a^4-28*a^2*b^2+21*b^4)*e^4*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(e*sin(d*x+c))^(1/2)-b^2*(a^
2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin
(d*x+c)^(1/2)/a^5/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(
a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*s
in(d*x+c)^(1/2)/a^5/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/2
1*e^3*(21*b*(a^2-b^2)-a*(5*a^2-7*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/a^4
/d
```

3.233.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 47.84 (sec) , antiderivative size = 2049, normalized size of antiderivative = 3.97

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

output

```
((b + a*Cos[c + d*x])*(-1/42*((23*a^2 - 28*b^2)*Cos[c + d*x])/a^3 - (b*Cos
[2*(c + d*x)])/(5*a^2) + Cos[3*(c + d*x)]/(14*a))*Csc[c + d*x]^3*Sec[c + d
*x]*(e*Sin[c + d*x])^(7/2))/(d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x
])*Sec[c + d*x]*(e*Sin[c + d*x])^(7/2)*((2*(-100*a^3 + 98*a*b^2)*Cos[c + d
*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]
*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sq
rt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-
a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin
[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4
, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a
^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)
))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(89*a^2*b - 70*b^3)*
Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqr...
```

3.233.3 Rubi [A] (warning: unable to verify)

Time = 2.49 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.99, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 25, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{2e^2\int -\frac{(2ab-(5a^2-7b^2)\cos(c+dx))(e\sin(c+dx))^{3/2}}{2(b+a\cos(c+dx))}dx}{7a^2}+\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}$$

$$\downarrow 27$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{e^2\int \frac{(2ab-(5a^2-7b^2)\cos(c+dx))(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}dx}{7a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(2ab+\left(7b^2-5a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{7a^2}$$

$$\downarrow 3344$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{e^2\left(\frac{2e^2\int \frac{2ab\left(8a^2-7b^2\right)-\left(5a^4-28b^2a^2+21b^4\right)\cos(c+dx)}{2(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3a^2}-\frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)}{3a^2d}\right)}{7a^2}$$

$$\downarrow 27$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{e^2\left(\frac{e^2\int \frac{2ab\left(8a^2-7b^2\right)-\left(5a^4-28b^2a^2+21b^4\right)\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3a^2}-\frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)}{3a^2d}\right)}{7a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} - \frac{e^2\left(\frac{e^2\int\frac{2ab\left(8a^2-7b^2\right)-\left(-5a^4+28b^2a^2-21b^4\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3a^2} - \frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)}{3a^2d}\right)}{7a^2}$$

$$\downarrow 3346$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} - \frac{e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a} - \frac{\left(5a^4-28a^2b^2+21b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{a}\right)}{3a^2} - \frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos\right.}{3a^2d}\right)}{7a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} - \frac{e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a} - \frac{\left(5a^4-28a^2b^2+21b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{a}\right)}{3a^2} - \frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)-a\left(5a^2\right.\right.}{3a^2d}\right)}{7a^2}$$

$$\downarrow 3121$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} - \frac{e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a} - \frac{\left(5a^4-28a^2b^2+21b^4\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}}\right)}{3a^2} - \frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right.\right.}{3a^2}\right)}{7a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}-\frac{\left(5a^4-28a^2b^2+21b^4\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}}\right)}{3a^2}-\frac{2e\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right.\right.}{3a^2}\right)$$

$$\frac{}{7a^2}$$

$$\downarrow 3120$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2\left(5a^4-28a^2b^2+21b^4\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}}\right)}{3a^2}-\frac{2e\sqrt{e\sin(c+dx)}(2}{3a^2}\right)$$

$$\frac{}{7a^2}$$

$$\downarrow 3181$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(-\frac{ae\int-\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int}{\sqrt{e\sin(c+dx)}}\right.\right.}{a}\right.$$

$$\frac{}{3a^2}$$

$$\frac{}{7}$$

$$\downarrow 25$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}\right)}{a}\right.$$

$$\left.\frac{}{3a^2}\right)$$

$$\frac{}{7a}$$

$$\downarrow 266$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(\frac{2ae\int\frac{1}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}\right)}{a}\right.$$

$$\left.\frac{}{3a^2}\right)$$

$$\frac{}{7a^2}$$

$$\downarrow 756$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{e^2}{3a^2}\left(\frac{21b\left(a^2-b^2\right)^2}{a}\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}\right)}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c\right.}}{2\sqrt{a^2-b^2}}\right.\right.\right.\right.$$

↓ 218

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2e-ae^2\sin^2(c+dx)}}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{}{\sqrt{}}}{}}{a}\right)}{3a^2}\right)$$

$$\downarrow 221$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{}{d}\right.\right.\right.}{a}\right.$$

$$\left.\left.\left.\right.\right.\right)$$

$$e^2\left(\frac{}{3a^2}\right.$$

$$\left.\right)$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

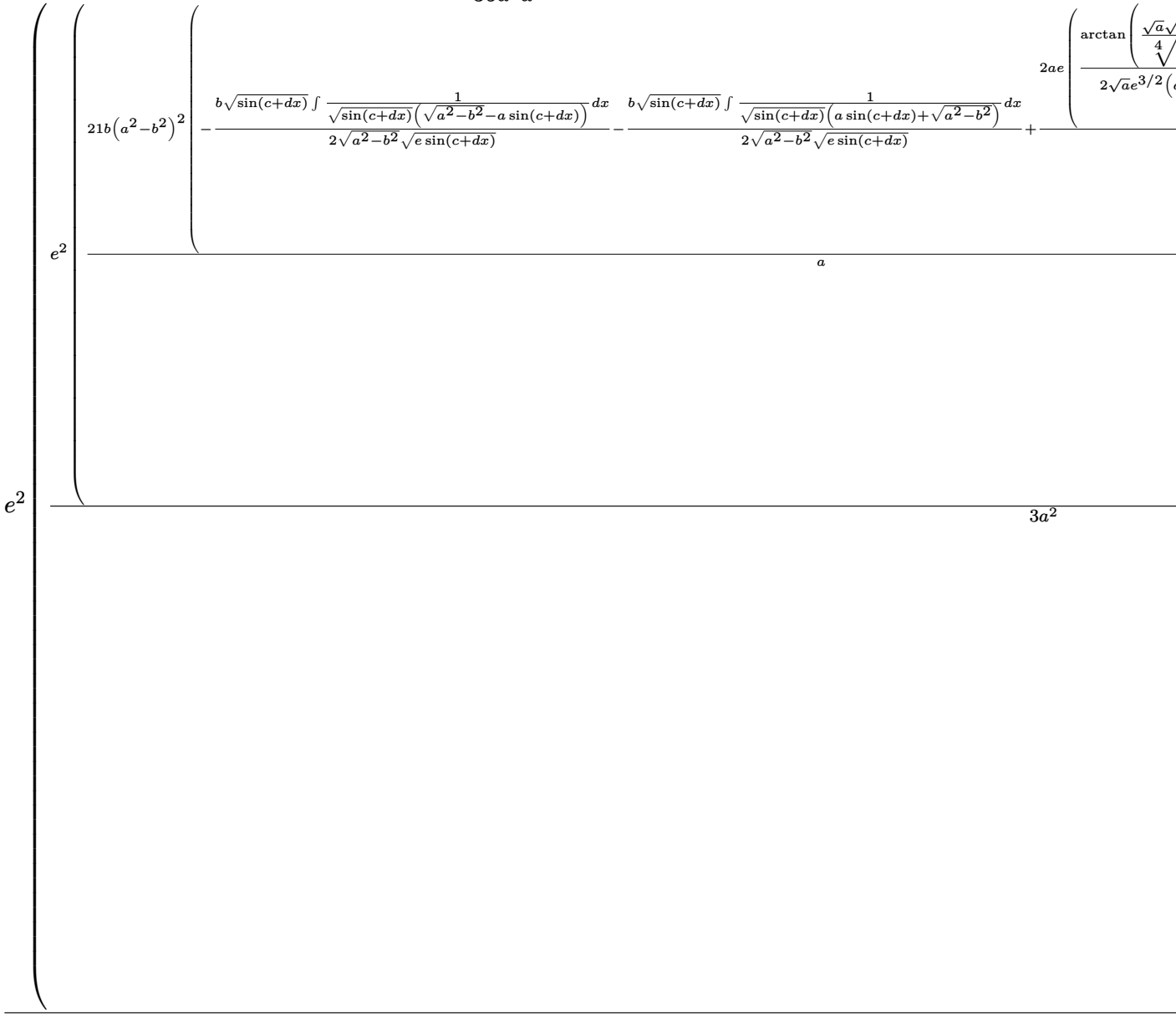
$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\text{arc}}{d}\right.\right.}{a}\right.$$

$$\left.\left.\right)}{3a^2}\right)$$

$$\left.\right)$$

$$\downarrow 3286$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$



$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{}}{4\sqrt{}}\right.}{2\sqrt{a}e^{3/2}\left(\right.}\right.}{a}\right.}{3a^2}\right.$$

$$\downarrow 3284$$

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d} -$$

$$\left(e^2\left(\frac{21b\left(a^2-b^2\right)^2\left(\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}+\frac{b\sqrt{\sin(c+dx)}\,\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{a^2-b^2}\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(}}{a}\right.\right.\right.$$

$$\left.\left.\left.\right.\right.\right)$$

$$\left.\left.\right.\right)$$

$$\left.\right)$$

$$e^2$$

$$3a^2$$

input `Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]`

output
```
(2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*a^2*d) - (e^2*((
-2*e*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*
x]])/(3*a^2*d) + (e^2*((-2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*EllipticF[(c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c + d*x]]) + (21*b*(a^
2 - b^2)^2*((2*a*e*(ArcTan[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4
)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2)) + ArcTanh[(Sqrt[a]*Sqrt[e]*Sin[c
+ d*x])/(a^2 - b^2)^(1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2))))/d + (b*
EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(Sqrt[a^2 - b^2]*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) -
(b*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
))/a))/(3*a^2)))/(7*a^2)
```

3.233.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.233.4 Maple [A] (verified)

Time = 16.19 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.50

method	result
default	$2eb\left(-\frac{\sqrt{e\sin(dx+c)}\,e^2\left(\cos(dx+c)^2a^2-6a^2+5b^2\right)}{5a^4}+\frac{e^4\left(a^4-2a^2b^2+b^4\right)\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+2\arctan\left(-$

input
```
int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(2*e*b*(-1/5/a^4*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*a^2-6*a^2+5*b^2)+1
/4*e^4*(a^4-2*a^2*b^2+b^4)/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2
)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/
2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2
)/a^2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^4*(-1/21/a^6/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))*a^4-28*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+21*b^4
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c)-14*a^2*b^
2*cos(d*x+c)^2*sin(d*x+c))-b^2*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2/(a^2-b^2)^(1/
2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1
+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.233.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

3.233.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x)), x)
```

3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

3.234.1 Optimal result . . . 1788
3.234.2 Mathematica [C] (warning: unable to verify) . . . 1789
3.234.3 Rubi [A] (warning: unable to verify) . . . 1790
3.234.4 Maple [A] (verified) . . . 1799
3.234.5 Fricas [F(-1)] . . . 1800
3.234.6 Sympy [F(-1)] . . . 1800
3.234.7 Maxima [F] . . . 1800
3.234.8 Giac [F] . . . 1801
3.234.9 Mupad [F(-1)] . . . 1801

3.234.1 Optimal result

Integrand size = 25, antiderivative size = 430

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx &= \frac{b(a^2-b^2)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}\\ &- \frac{b(a^2-b^2)^{3/4}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}\\ &- \frac{b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}\\ &- \frac{b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}\\ &+ \frac{2(3a^2-5b^2)e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}}\\ &+ \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d}\end{aligned}$$

output
```
b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(7/2)/d-b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*
x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+2/15*e*(5*b-3*a*cos(d*x+c))
*(e*sin(d*x+c))^(3/2)/a^2/d+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a
/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a-(a^2-b^2)^(1/2))/(
e*sin(d*x+c))^(1/2)+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2
-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*
x+c))^(1/2)-2/5*(3*a^2-5*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(
d*x+c))^(1/2)/a^3/d/sin(d*x+c)^(1/2)
```

3.234.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 36.32 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.98

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx =$$

$$(b+a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{(-3a^2+5b^2)\cos^2(c+dx)\left(3\sqrt{2}b(-a^2+b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx}}{\sqrt[4]{-a^2+b^2}}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\right.\right.\right.\right)$$

$$+\frac{(b+a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{2b\sin(c+dx)}{3a^2}-\frac{\sin(2(c+dx))}{5a}\right)}{d(a+b\sec(c+dx))}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

output

```
-1/5*((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(((-3*a^2 +
 5*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt
[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2
]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] +
 Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x
]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 -
 Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c
 + d*x]^2)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*S
in[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4,
 Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*
(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqr
t[1 - Sin[c + d*x]^2])))/(a^2*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(5/2)) +
 ((b + a*Cos[c + d*x])*Csc[c + d*x]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*
((2*b*Sin[c + d*x])/(3*a^2) - Sin[2*(c + d*x)]/(5*a)))/(d*(a + b*Sec[c ...
```

3.234.3 Rubi [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.97, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 25, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{2e^2\int -\frac{(2ab-(3a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(b+a\cos(c+dx))}dx}{5a^2}+\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}$$

$$\downarrow 27$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}-\frac{e^2\int \frac{(2ab-(3a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx}{5a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}-\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2ab+\left(5b^2-3a^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2}$$

$$\downarrow 3346$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}-\frac{e^2\left(\frac{5b\left(a^2-b^2\right)\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx}{a}-\frac{\left(3a^2-5b^2\right)\int \sqrt{e\sin(c+dx)}dx}{a}\right)}{5a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}-\frac{e^2\left(\frac{5b\left(a^2-b^2\right)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}-\frac{\left(3a^2-5b^2\right)\int \sqrt{e\sin(c+dx)}dx}{a}\right)}{5a^2}$$

$$\downarrow 3121$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} - \frac{e^2\left(\frac{5b(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a} - \frac{(3a^2-5b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{a\sqrt{\sin(c+dx)}}\right)}{5a^2}$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} - \frac{e^2\left(\frac{5b(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a} - \frac{(3a^2-5b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{a\sqrt{\sin(c+dx)}}\right)}{5a^2}$$

$$\downarrow 3119$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} - \frac{e^2\left(\frac{5b(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a} - \frac{2(3a^2-5b^2)E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}\right)}{5a^2}$$

$$\downarrow 3180$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} - \frac{e^2\left(\frac{5b(a^2-b^2)\left(-\frac{ae\int-\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right.}\right.}{2a}\right.}{a}\right.}{5a^2}$$

$$\downarrow 25$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} - \frac{e^2\left(\frac{5b(a^2-b^2)\left(\frac{ae\int\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right.}\right.}{2a}\right.}{a}\right.}{5a^2}$$

3.234. $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

$$\downarrow 266$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b\left(a^2-b^2\right)\left(\frac{2ae\int\frac{e^2\sin^2(c+dx)}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b}\right.}}{2a}\right.}{a}\right.$$

$$\left.\left.\right)}{5a^2}$$

$$\downarrow 827$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b\left(a^2-b^2\right)\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a}-\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2a}\right)}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right.}}{2a}\right.}{a}\right.$$

$$\left.\left.\right)}{5a^2}$$

$$\downarrow 218$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b\left(a^2-b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(\right.\right.\right.}}{d}\right.}{a}\right.$$

$$\left.\left.\left.\right)}{5a^2}$$

$$\downarrow 221$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b(a^2-b^2)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\text{arc}}{2}\right.}{d}\right.}{a}\right.$$

$$\left.\left.\right)}{5a^2}\right)$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b(a^2-b^2)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\text{arc}}{2}\right.}{d}\right.}{a}\right.$$

$$\left.\left.\right)}{5a^2}\right)$$

$$\downarrow 3286$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b(a^2-b^2)\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}}{\sqrt[4]{}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{}}\right.\right.\right.}{a}\right.$$

$$\left.\left.\left.\right.\right.$$

$$\left.\right)$$

$$5a^2$$

$$\downarrow 3042$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b(a^2-b^2)\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}}{\sqrt[4]{}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{}}\right.\right.\right.}{a}\right.$$

$$\left.\left.\left.\right.\right.$$

$$\left.\right)$$

$$5a^2$$

$$\downarrow 3284$$

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} -$$

$$e^2\left(\frac{5b(a^2-b^2)\left(\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a}}\right.}{a}\right.}{5a^2}\right.$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*e*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^2*d) - (e^2*((
-2*(3*a^2 - 5*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/
(a*d*Sqrt[Sin[c + d*x]]) + (5*b*(a^2 - b^2)*((2*a*e*(-1/2*ArcTan[(Sqrt[a]*
Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(a^(3/2)*(a^2 - b^2)^(1/4)*Sqrt[e
]) + ArcTanh[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(2*a^(3/2)*
(a^2 - b^2)^(1/4)*Sqrt[e])))/d + (b*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2
]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a - Sqrt[a^2 - b^2])*d*
Sqrt[e*Sin[c + d*x]]) + (b*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Si
n[c + d*x]])))/a))/(5*a^2)
```

3.234.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.234.4 Maple [A] (verified)

Time = 13.67 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.60

method	result
default	$2eb\left(\frac{(e\sin(dx+c))^{\frac{3}{2}}}{3a^2}+\frac{e^2\left(a^2-b^2\right)\left(2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{4a^4\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+\frac{\sqrt{\cos(dx+c)^2e\sin(dx+c)}\,a\,e^3$

input
```
int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(2*e*b*(1/3*(e*sin(d*x+c))^(3/2)/a^2+1/4*e^2*(a^2-b^2)/a^4/(e^2*(a^2-b^2)/
a^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((
e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*
(a^2-b^2)/a^2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^3*(-1/5/a^2
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*cos(d*x+c)^2)+b^2/a^4*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))-b^2*(a^2-b^2)/a^4*(-1/2/a^2*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1
-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2
)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^
(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.234.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.234.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.234.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\,\sin(c+dx))^{5/2}}{b+a\,\cos(c+dx)}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.235 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

3.235.1 Optimal result 1802
3.235.2 Mathematica [C] (warning: unable to verify) 1803
3.235.3 Rubi [A] (warning: unable to verify) 1804
3.235.4 Maple [A] (verified) 1813
3.235.5 Fricas [F(-1)] 1813
3.235.6 Sympy [F] 1814
3.235.7 Maxima [F] 1814
3.235.8 Giac [F] 1814
3.235.9 Mupad [F(-1)] 1815

3.235.1 Optimal result

Integrand size = 25, antiderivative size = 444

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\frac{b\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d}$$
$$-\frac{b\sqrt[4]{a^2-b^2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d}$$
$$+\frac{2(a^2-3b^2)e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2(a^2-b^2)e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2(a^2-b^2)e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d}$$

output
```
-b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(
1/4)/e^(1/2))/a^(5/2)/d-b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d
*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(5/2)/d-2/3*(a^2-3*b^2)*e^2*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(e*sin(d*x+c))^(1/2)-b^2
*(a^2-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*s
in(d*x+c)^(1/2)/a^3/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2
*(a^2-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*s
in(d*x+c)^(1/2)/a^3/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/3
*e*(3*b-a*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/a^2/d
```

3.235.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 24.66 (sec) , antiderivative size = 1959, normalized size of antiderivative = 4.41

$$\int \frac{(e \sin(c+dx))^{3/2}}{a+b \sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

output

```
(-2*(b + a*Cos[c + d*x])*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(3*a*d*(a +
b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(3/
2)*((4*a*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1
- (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
 b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c +
d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5
*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1
 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (2*b*
Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a
^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq...
```

3.235.3 Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.98, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 25, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e \sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e \cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e \sin(c+dx))^{3/2}}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3344$$

$$\frac{2e^2\int -\frac{2ab-\left(a^2-3b^2\right)\cos(c+dx)}{2(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3a^2}+\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}$$

$$\downarrow 27$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}-\frac{e^2\int \frac{2ab-\left(a^2-3b^2\right)\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3a^2}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}-\frac{e^2\int \frac{2ab-\left(3b^2-a^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3a^2}$$

$$\downarrow 3346$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}-\frac{e^2\left(\frac{3b\left(a^2-b^2\right)\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a}-\frac{\left(a^2-3b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{a}\right)}{3a^2}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}-\frac{e^2\left(\frac{3b\left(a^2-b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}-\frac{\left(a^2-3b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{a}\right)}{3a^2}$$

$$\downarrow 3121$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(\frac{3b(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a} - \frac{(a^2-3b^2)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}}\right)}{3a^2}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(\frac{3b(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a} - \frac{(a^2-3b^2)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}}\right)}{3a^2}$$

$$\downarrow 3120$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(\frac{3b(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a} - \frac{2(a^2-3b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}}\right)}{3a^2}$$

$$\downarrow 3181$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(\frac{3b(a^2-b^2)\left(-\frac{ae\int-\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin\right.}}{2\sqrt{a^2}}\right.}{a}\right.}{3a^2}$$

$$\downarrow 25$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(\frac{3b(a^2-b^2)\left(\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+\right.}}{2\sqrt{a^2-b^2}}\right.}{a}\right.}{3a^2}$$

$$\downarrow 266$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(\frac{2ae\int\frac{1}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}}{2\sqrt{a^2-b^2}}\right.}{a}\right.$$

$$\left.\left.\right)}{3a^2}\right)$$

$$\downarrow 756$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}\right)}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}}{2\sqrt{a^2-b^2}}\right.}{a}\right.$$

$$\left.\left.\right)}{3a^2}\right)$$

$$\downarrow 218$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(}}\right.}{a}\right.$$

$$\left.\left.\right)}{3a^2}\right)$$

$$\downarrow 221$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\text{arctanh}}{2\sqrt{ae}}\right.}{d}\right.}{a}\right.$$

$$\left.\left.\right)\right)$$

$$3a^2$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\text{arctanh}}{2\sqrt{ae}}\right.}{d}\right.}{a}\right.$$

$$\left.\left.\right)\right)$$

$$3a^2$$

$$\downarrow 3286$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(}{\sqrt[4]{a^2-}}\right.\right.}{2\sqrt{ae}^{3/2}\left(a^2-b^2\right.}\right.}{a}\right.}{3a^2}$$

$$\downarrow 3042$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} -$$

$$e^2\left(\frac{3b\left(a^2-b^2\right)\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(}{\sqrt[4]{a^2-}}\right.\right.}{2\sqrt{ae}^{3/2}\left(a^2-b^2\right.}\right.}{a}\right.}{3a^2}$$

$$\downarrow 3284$$

$$\frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d} - \frac{e^2\left(3b\left(a^2-b^2\right)\left(\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}+\frac{b\sqrt{\sin(c+dx)}\,\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{a^2-b^2}\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx)}}{d\sqrt{}}\right.}{a}\right.}{3a^2}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*e*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) - (e^2*((-2*(a
^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt
[e*Sin[c + d*x]]) + (3*b*(a^2 - b^2)*((2*a*e*(ArcTan[(Sqrt[a]*Sqrt[e]*Sin[
c + d*x])/(a^2 - b^2)^(1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2)) + ArcTa
nh[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(2*Sqrt[a]*(a^2 - b^2
)^(3/4)*e^(3/2))))/d + (b*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[a^2 - b^2]*(a - Sqrt[a^2 - b^2])*
d*Sqrt[e*Sin[c + d*x]]) - (b*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]))/a))/(3*a^2)
```

3.235.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.235.4 Maple [A] (verified)

Time = 10.10 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.43

method	result
default	$\frac{eb\left(-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)-2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{a^2e^2-b^2e^2}{a^2}\right)^{\frac{1}{4}}}\right)+4\sqrt{e\sin(dx+c)}\right)\sqrt{\cos(dx+c)^2}}{2a^2}+$

input
```
int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(1/2*e*b*(-(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^
2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-2*(e^2*(a
^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/((a^2*e^2-b^2*e^2)/a^2)^(1/
4))+4*(e*sin(d*x+c))^(1/2))/a^2+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^2*(-
1/3/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)+2*cos(d*x+c)^2*sin(d*x+c))+b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))-b^2*(a^2-b^2)/a^4*(-1/2/(a^2-b^2)^(1/2)/a*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.235.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.235.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{a+b\sec(c+dx)}\,dx$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

3.235.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.235.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{3/2}}{b+a\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.236.1 Optimal result 1816
3.236.2 Mathematica [C] (warning: unable to verify) 1817
3.236.3 Rubi [A] (warning: unable to verify) 1817
3.236.4 Maple [A] (verified) 1824
3.236.5 Fricas [F(-1)] 1824
3.236.6 Sympy [F] 1825
3.236.7 Maxima [F] 1825
3.236.8 Giac [F] 1825
3.236.9 Mupad [F(-1)] 1826

3.236.1 Optimal result

Integrand size = 25, antiderivative size = 356

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b^2 e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2 e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

output

```
b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(
3/2)/(a^2-b^2)^(1/4)/d-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))*e^(1/2)/a^(3/2)/(a^2-b^2)^(1/4)/d+b^2*e*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a-(a^2-b^2)^(
1/2))/(e*sin(d*x+c))^(1/2)+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)
^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))
^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x
+c)^(1/2)
```

3.236.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(3\sqrt{2}b(-a^2+b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\arctan\left(\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
((b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(3*Sqrt[2]*b*(-a^2 + b^
2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(
1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2
+ b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c +
 d*x]^(3/2)))/(12*a^(3/2)*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Sqrt[Sin[c +
d*x]])
```

3.236.3 Rubi [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4360, 25, 25, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 25, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3346$$

$$\frac{\int \sqrt{e\sin(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{e\sin(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3121$$

$$\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{a\sqrt{\sin(c+dx)}} - \frac{b\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{a\sqrt{\sin(c+dx)}} - \frac{b\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3119$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} - \frac{b\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3180$$

$$\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(-\frac{ae\int -\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d} - \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a} + \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)$$

$$a$$

$$\downarrow 25$$

$$\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(\frac{ae\int \frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d} - \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a} + \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)$$

$$a$$

$$\downarrow 266$$

$$\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(\frac{2ae\int \frac{e^2\sin^2(c+dx)}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d} - \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a} + \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)$$

$$a$$

$$\downarrow 827$$

$$\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(\frac{2ae\left(\frac{\int \frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a} - \frac{\int \frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2a}\right)}{d} - \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a} + \frac{be}{}\right)$$

$$a$$

$$\downarrow 218$$

$$\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a} + \frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a} + \frac{2ae\left(\frac{\int \frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a} - \right.}{d}\right)$$

$$a$$

$$\downarrow 221$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt[4]{}}\right.}{2a^{3/2}\sqrt{e}}\right.}{d}\right.$$

$$a$$

$$\downarrow 3042$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt[4]{}}\right.}{2a^{3/2}\sqrt{e}}\right.}{d}\right.$$

$$a$$

$$\downarrow 3286$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$b\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin}{\sqrt[4]{a^2}}\right.\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2}}\right.}{d}\right.$$

$$a$$

$$\downarrow 3042$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$\frac{b\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}\sin}{\sqrt[4]{a^2}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2}}\right.}{a}$$

$$\downarrow 3284$$

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} -$$

$$\frac{b\left(\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2}}\right.}{a}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c
+ d*x]]) - (b*((2*a*e*(-1/2*ArcTan[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b
^2)^(1/4)]/(a^(3/2)*(a^2 - b^2)^(1/4)*Sqrt[e]) + ArcTanh[(Sqrt[a]*Sqrt[e]*
Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(2*a^(3/2)*(a^2 - b^2)^(1/4)*Sqrt[e])))/d
 + (b*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(a*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b*e*E
lliticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])))/a
```

3.236.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.236.4 Maple [A] (verified)

Time = 6.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.23

method	result
default	$\frac{eb\left(2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{2a^2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}-\frac{e\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sqrt{\sin(dx+c)}\,b^2\left(\text{EllipticPi}\left(\sqrt{-}\right.\right.}{}$

input
```
int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(1/2*e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2
*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))
/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))-1/2*e*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2*(EllipticPi((-sin(d*x+c)
+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*(a^2-b^2)^(1/2)-EllipticPi((
-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)-4*
EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a+2*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))*a+EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)
-a),1/2*2^(1/2))*a+EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),
1/2*2^(1/2))*a)/a^2/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

3.236.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.236.6 Sympy [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+b \sec(c+dx)}\,dx = \int \frac{\sqrt{e \sin(c+dx)}}{a+b \sec(c+dx)}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.236.7 Maxima [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+b \sec(c+dx)}\,dx = \int \frac{\sqrt{e \sin(dx+c)}}{b \sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.236.8 Giac [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+b \sec(c+dx)}\,dx = \int \frac{\sqrt{e \sin(dx+c)}}{b \sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,\sqrt{e\,\sin(c+dx)}}{b+a\,\cos(c+dx)}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

3.237.1 Optimal result 1827
3.237.2 Mathematica [C] (warning: unable to verify) 1828
3.237.3 Rubi [A] (warning: unable to verify) 1829
3.237.4 Maple [A] (verified) 1835
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3.237.7 Maxima [F] 1837
3.237.8 Giac [F] 1837
3.237.9 Mupad [F(-1)] 1837

3.237.1 Optimal result

Integrand size = 25, antiderivative size = 370

$$\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} \\
&\quad + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}
\end{aligned}$$

output
```
-b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^
(3/4)/d/a^(1/2)/e^(1/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(
1/4)/e^(1/2))/(a^2-b^2)^(3/4)/d/a^(1/2)/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2-a*(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^
2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e
*sin(d*x+c))^(1/2)
```

3.237.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.37 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{2\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{\sin(c+dx)}\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-\right.\right.\right.}{}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(2*(b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]]*((b*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
- Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((-a^2 + b^2 + a^2*S
in[c + d*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2,
(a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, S
in[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c
+ d*x]^2))))/(d*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

3.237.3 Rubi [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3346, 3042, 3121, 3042, 3120, 3181, 25, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}(-a\cos(c+dx)-b)}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}(a\cos(c+dx)+b)}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3346$$

$$\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{a} - \frac{b\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\downarrow 3121$$

$$\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}} - \frac{b\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{a\sqrt{e\sin(c+dx)}} - \frac{b\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\downarrow 3120$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} - \frac{b\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\downarrow 3181$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$b\left(-\frac{ae\int -\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)\right.}}{2\sqrt{a^2-b^2}}\right.$$

$$a$$

$$\downarrow 25$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$b\left(\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a}\right.}}{2\sqrt{a^2-b^2}}\right.$$

$$a$$

$$\downarrow 266$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$b\left(\frac{2ae\int \frac{1}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}\right)$$

$$a$$

$$\downarrow 756$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} - b\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}\right)}{d} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int}{}\right)$$

$$a$$

$$\downarrow 218$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} - b\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int}{2\sqrt{}}\right)$$

$$a$$

$$\downarrow 221$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} - b\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt[4]{}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-\right.}\right.}{d}\right.$$

$$a$$

$$\downarrow 3042$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$\frac{b\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt[4]{}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-\right.}\right.}{d}\right.}{a}$$

$$\downarrow 3286$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$\frac{b\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right.}{a}$$

$$\downarrow 3042$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$\frac{b\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right.}{a}$$

$$\downarrow 3284$$

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}} -$$

$$b\left(\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}+\frac{b\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{a^2-b^2}\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx)}\,\text{E}}{d\sqrt{a^2-}}\right)$$

$$a$$

input
```
Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) - (b*((2*a*e*(ArcTan[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(
1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2)) + ArcTanh[(Sqrt[a]*Sqrt[e]*Sin
[c + d*x])/(a^2 - b^2)^(1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2))))/d +
(b*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[a^2 - b^2]*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
 - (b*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x
]])))/a
```

3.237.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.237.4 Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.34

method	result
default	$\frac{be\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{-2a^2e^2+2b^2e^2}-\frac{\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\sqrt{\sin(dx+c)}\left($

input
```
int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^
(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)
^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/a*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*(a^2-b^2)^
(3/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(a^2-b^2)^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+(a^2-b^2)^(1/2)*EllipticPi((-s
in(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*b^2+(a^2-b^2)^(1/2)
*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2+E
llipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-
EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*a*b^2)
/(a^2-b^2)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin
(d*x+c))^(1/2))/d
```

3.237.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.237.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))), x)
```

3.237.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.237.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)}{\sqrt{e\,\sin(c+d\,x)}\,(b+a\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

3.238.1 Optimal result . . . 1838
3.238.2 Mathematica [C] (warning: unable to verify) . . . 1839
3.238.3 Rubi [A] (warning: unable to verify) . . . 1840
3.238.4 Maple [A] (verified) . . . 1847
3.238.5 Fricas [F(-1)] . . . 1848
3.238.6 Sympy [F] . . . 1848
3.238.7 Maxima [F(-1)] . . . 1849
3.238.8 Giac [F] . . . 1849
3.238.9 Mupad [F(-1)] . . . 1849

3.238.1 Optimal result

Integrand size = 25, antiderivative size = 430

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}}$$
$$-\frac{\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}}+\frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$

output

```
b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^
2-b^2)^(5/4)/d/e^(3/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(
1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2
*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/s
in(d*x+c)^(1/2)
```

3.238.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.80 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = (b+a\cos(c+dx))\left(-\frac{\sqrt{a}\left(b+a\sqrt{\cos^2(c+dx)}\right)\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\left(\cos(\right.\right.$$

input

```
Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output

```
((b + a*Cos[c + d*x])*(-((Sqrt[a]*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x
]^2*Sin[c + d*x]^(3/2)*(Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*Ar
cTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt
[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*S
in[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*S
qrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) +
 (2 + 2*I)*b*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + L
og[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
 + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))))/((a -
b)*(a + b)*(b + a*Cos[c + d*x]))) + 24*(b - a*Cos[c + d*x])*Tan[c + d*x]))
/(12*(a^2 - b^2)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2))
```

3.238.3 Rubi [A] (warning: unable to verify)

Time = 1.97 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 25, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}\,dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}(-a\cos(c+dx)-b)}\,dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}(a\cos(c+dx)+b)}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{3/2}\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int\frac{(\cos(c+dx)a^2+2ba)\sqrt{e\sin(c+dx)}}{2(b+a\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{(\cos(c+dx)a^2+2ba)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2ba\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3346$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{ab\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx+a\int\sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{ab\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+a\int\sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3121$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{ab\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{ab\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3119$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{ab\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3180$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$ab\left(-\frac{ae\int-\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}}{2a}\right.$$

$$e^2\left(a^2-b^2\right)$$

$$\downarrow 25$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$ab\left(\frac{ae\int\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)$$

$$e^2\left(a^2-b^2\right)$$

$$\downarrow 266$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$ab\left(\frac{2ae\int\frac{e^2\sin^2(c+dx)}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right.$$

$$e^2\left(a^2-b^2\right)$$

$$\downarrow 827$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$ab\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a}-\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2a}\right)}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\right.$$

$$e^2\left(a^2-b^2\right)$$

$$\downarrow 218$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\right.}{2a^{3/2}\sqrt{}}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\right.}{2a^{3/2}\sqrt{}}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3286$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2}}\right.}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2}}\right.}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 3284$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$\frac{ab\left(\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{ad\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{ad\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{}}{\sqrt[4]{}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{}}\right.}{e^2\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - ((2*a*El
lipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]
) + a*b*((2*a*e*(-1/2*ArcTan[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1
/4)]/(a^(3/2)*(a^2 - b^2)^(1/4)*Sqrt[e]) + ArcTanh[(Sqrt[a]*Sqrt[e]*Sin[c
+ d*x])/(a^2 - b^2)^(1/4)]/(2*a^(3/2)*(a^2 - b^2)^(1/4)*Sqrt[e])))/d + (b*
e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(a*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b*e*Ellipti
cPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(a*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2)
```

3.238.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.238.4 Maple [A] (verified)

Time = 8.16 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.54

method	result
default	$\frac{be\left(\frac{2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)}{2e^2(a-b)(a+b)\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}+\frac{2}{e^2\left(a^2-b^2\right)\sqrt{e\sin(dx+c)}}\right)-\frac{b^2\left(\sqrt{a^2-b^2}\sqrt{-\sin(dx+c)+1}\sqrt{2\sin($

input
```
int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b*e*(1/2/e^2/(a-b)/(a+b)/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c
))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2
)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))+2/e^2/(a^
2-b^2)/(e*sin(d*x+c))^(1/2))-1/2*b^2*((a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))-(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),
a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))+4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a-2*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1
/2)-a),1/2*2^(1/2))+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/
2))-4*cos(d*x+c)^2*a)/e/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a+b)/(a-b
)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.238.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.238.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}\,(a+b\sec(c+dx))}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

3.238.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.238.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{3/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

3.239.1 Optimal result 1850
3.239.2 Mathematica [C] (warning: unable to verify) 1851
3.239.3 Rubi [A] (warning: unable to verify) 1852
3.239.4 Maple [A] (verified) 1859
3.239.5 Fricas [F(-1)] 1860
3.239.6 Sympy [F(-1)] 1861
3.239.7 Maxima [F(-1)] 1861
3.239.8 Giac [F] 1861
3.239.9 Mupad [F(-1)] 1862

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 452

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{a^{3/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}}$$
$$-\frac{a^{3/2}b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}}+\frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}$$
$$+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

output

```
-a^(3/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a
^2-b^2)^(7/4)/d/e^(5/2)-a^(3/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-
b^2)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d
*x+c)^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a
^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2
-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

3.239.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.09 (sec) , antiderivative size = 1233, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx =$$

$$a(b+a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\left(-\frac{2a\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\right.}{}\right.}{}\right.$$

$$-\frac{2(b-a\cos(c+dx))(b+a\cos(c+dx))\tan(c+dx)}{3\left(-a^2+b^2\right)d(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}$$

input

```
Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output

```
-1/3*(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(5/2)*((-2*a*Cos[c
+ d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]
*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*
Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^
2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(
2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*S
in[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]
^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(b
 + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 +
I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1
+ I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Lo
g[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4,...
```

3.239.3 Rubi [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 25, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}(-a\cos(c+dx)-b)}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))(e\sin(c+dx))^{5/2}}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}(a\cos(c+dx)+b)}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{5/2}\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{2\int \frac{2ab-a^2\cos(c+dx)}{2(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{\int \frac{2ab-a^2\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)a^2+2ba}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3346$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{3ab\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}dx-a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{3ab\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3121$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{3ab\int\frac{1}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}(b-a\sin(c+dx-\frac{\pi}{2}))}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{3ab\int\frac{1}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}(b-a\sin(c+dx-\frac{\pi}{2}))}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3120$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{3ab\int\frac{1}{\sqrt{e\cos(c+dx-\frac{\pi}{2})}(b-a\sin(c+dx-\frac{\pi}{2}))}dx-\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{d\sqrt{e\sin(c+dx)}}}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3181$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-3ab\left(-\frac{ae\int-\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+\right.}}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-3ab\left(\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)\right)}d(e\sin(c+dx))}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\right.}}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 266$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-3ab\left(\frac{2ae\int\frac{1}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}}-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\downarrow 756$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} - \frac{3ab\left(\frac{2ae\left(\frac{\int \frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\int \frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}\right)}{d} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 218$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} - \frac{3ab\left(\frac{2ae\left(\frac{\int \frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{a^2-b^2}}+\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}}}{}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} - \frac{3ab\left(-\frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int \frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{}}{2\sqrt{a}e^{3/2}\left(\right.}\right.\right.}{d}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} -$$

$$\frac{3ab\left(-\frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}} - \frac{b\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{}}{2\sqrt{a}e^{3/2}\left(\right.}\right.}{d}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3286$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} -$$

$$\frac{3ab\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c-}{\sqrt[4]{a^2-}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} -$$

$$\frac{3ab\left(-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} + \frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c-}{\sqrt[4]{a^2-}}\right.}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\downarrow 3284$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}} -$$

$$3ab\left(\frac{2ae\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2\sqrt{a}e^{3/2}\left(a^2-b^2\right)^{3/4}}\right)}{d}+\frac{b\sqrt{\sin(c+dx)}\,\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{d\sqrt{a^2-b^2}\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b\sqrt{\sin(c+dx}}{d\sqrt{a}}\right.$$

$$\left.\vphantom{\frac{1}{1}}\right)$$

$$3e^2\left(a^2-b^2\right)$$

input
```
Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) - ((-2
*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c +
d*x]]) + 3*a*b*((2*a*e*(ArcTan[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^
(1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2)) + ArcTanh[(Sqrt[a]*Sqrt[e]*Si
n[c + d*x])/(a^2 - b^2)^(1/4)]/(2*Sqrt[a]*(a^2 - b^2)^(3/4)*e^(3/2))))/d +
 (b*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(Sqrt[a^2 - b^2]*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) - (b*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*
x]])))/(3*(a^2 - b^2)*e^2)
```

3.239.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.239.4 Maple [A] (verified)

Time = 12.04 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.34

method	result
default	$2be\left(\frac{1}{3e^2\left(a^2-b^2\right)(e\sin(dx+c))^{\frac{3}{2}}}+\frac{a^2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{4e^2(a-b)(a+b)\left(-a^2e^2+b^2e^2\right)}\right)+\frac{\sqrt{\cos(dx-}$

input
```
int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*e*(1/3/e^2/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+1/4/e^2/(a-b)/(a+b)*a^2*(e^
2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(
a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*a
rctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(co
s(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/
2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))
-1/(a-b)/(a+b)*b^2*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b
^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*
2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)
*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/c
os(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.239.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.239.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.239.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}\,(b+a\cos(c+dx))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

3.240.1 Optimal result 1863
3.240.2 Mathematica [C] (warning: unable to verify) 1864
3.240.3 Rubi [A] (warning: unable to verify) 1865
3.240.4 Maple [A] (verified) 1873
3.240.5 Fricas [F(-1)] 1874
3.240.6 Sympy [F(-1)] 1875
3.240.7 Maxima [F(-1)] 1875
3.240.8 Giac [F] 1875
3.240.9 Mupad [F(-1)] 1876

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 511

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{a^{5/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}\,de^{7/2}}$$
$$-\frac{a^{5/2}b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}\,de^{7/2}}+\frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}}$$
$$+\frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$
$$-\frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$-\frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}}$$
$$-\frac{2a(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\left(a^2-b^2\right)^2de^4\sqrt{\sin(c+dx)}}$$

output
```
a^(5/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(9/4)/d/e^(7/2)-a^(5/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b
^2)/d/e/(e*sin(d*x+c))^(5/2)+2/5*(5*a^2*b-a*(3*a^2+2*b^2)*cos(d*x+c))/(a^2
-b^2)^2/d/e^3/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a
-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a-(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a+(a^2-b^2)^(1/2)
)/(e*sin(d*x+c))^(1/2)+2/5*a*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

3.240.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.40 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{(b+a\cos(c+dx))\sin^3(c+dx)\left(-\frac{2\left(\left(a^2-b^2\right)(-b+a\cos(c+dx))\csc^2(c+\right.\right.}{}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
((b + a*Cos[c + d*x])*Sin[c + d*x]^3*((-2*((a^2 - b^2)*(-b + a*Cos[c + d*x
])*Csc[c + d*x]^2*Sec[c + d*x] + a*(3*a^2 + 2*b^2 - 5*a*b*Sec[c + d*x])))/
(a^2 - b^2)^2 - ((b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c +
d*x]]*((3*a^3 + 2*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*A
rcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*Arc
Tan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqr
t[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*
Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*
Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))
+ (2 + 2*I)*a*b*(4*a^2 + b^2)*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Ar
cTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4,
1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x
]^(3/2))))/(12*Sqrt[a]*(a - b)^2*(a + b)^2*(a^2 - b^2)*(b + a*Cos[c + d*x]
))))/(5*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2))
```

3.240.3 Rubi [A] (warning: unable to verify)

Time = 2.55 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.97, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 25, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}(-a\cos(c+dx)-b)}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))(e\sin(c+dx))^{7/2}}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}(a\cos(c+dx)+b)}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\left(e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^{7/2}\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3345$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int \frac{2ab-3a^2\cos(c+dx)}{2(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{\int \frac{2ab-3a^2\cos(c+dx)}{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{\int \frac{3\sin(c+dx-\frac{\pi}{2})a^2+2ba}{\left(e\cos(c+dx-\frac{\pi}{2})\right)^{3/2}(b-a\sin(c+dx-\frac{\pi}{2}))}dx}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3345$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int \frac{\left(\left(3a^2+2b^2\right)\cos(c+dx)a^2+2b\left(4a^2+b^2\right)a\right)\sqrt{e\sin(c+dx)}}{2(b+a\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{\int \frac{\left(\left(3a^2+2b^2\right)\cos(c+dx)a^2+2b\left(4a^2+b^2\right)a\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(\left(3a^2+2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(4a^2+b^2\right)a\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)} - \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3346$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}} - \frac{5a^3b\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}dx+a\left(3a^2+2b^2\right)\int\sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)} - \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}} - \frac{5a^3b\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+a\left(3a^2+2b^2\right)\int\sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)} - \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3121$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}} - \frac{5a^3b\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\left(3a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)} - \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}} - \frac{5a^3b\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\left(3a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)} - \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3119$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{5a^3b\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2a\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}-\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3180$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{5a^3b\left(-\frac{ae\int-\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)+}{e^2\left(a^2-b^2\right)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{5a^3b\left(\frac{ae\int\frac{\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)e^2-a^2e^2\sin^2(c+dx)}d(e\sin(c+dx))}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)+\frac{2a\left(}{e^2\left(a^2-b^2\right)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 266$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{5a^3b\left(\frac{2ae\int\frac{e^2\sin^2(c+dx)}{\left(a^2-b^2\right)e^2-a^2e^4\sin^4(c+dx)}d\sqrt{e\sin(c+dx)}}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}\right)+\frac{2a}{e^2\left(a^2-b^2\right)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 827$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{5a^3b\left(\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a}-\frac{\int\frac{1}{ae^2\sin^2(c+dx)+\sqrt{a^2-b^2}e}d\sqrt{e\sin(c+dx)}}{2a}\right)}{d}-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int}{e^2\left(a^2-b^2\right)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 218$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$\frac{5a^3b\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\int\frac{1}{\sqrt{a^2-b^2}e-ae^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2a}-\frac{\arct}{2a}\right.}{d}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$\frac{5a^3b\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\s}{\sqrt[4]{a^2}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a}}\right.}{d}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$\frac{5a^3b\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a}+\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a}+\frac{2ae\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}-\frac{\arctan\left(\frac{\sqrt{a}\sqrt{e}\s}{\sqrt[4]{a^2}}\right.}{2a^{3/2}\sqrt{e}\sqrt[4]{a}}\right.}{d}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3286$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$5a^3b\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}\right.}{d}\right.$$

$$\frac{}{e^2(a^2-b^2)}$$

$$\frac{}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$5a^3b\left(-\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(a\sin(c+dx)+\sqrt{a^2-b^2}\right)}dx}{2a\sqrt{e\sin(c+dx)}}+\frac{2ae\left(\frac{\arctanh\left(\frac{\sqrt{a}\sqrt{e}\sin(c+dx)}{\sqrt[4]{a^2-b^2}}\right)}{2a^{3/2}\sqrt{e}\sqrt[4]{a^2-b^2}}\right.}{d}\right.$$

$$\frac{}{e^2(a^2-b^2)}$$

$$\frac{}{5e^2\left(a^2-b^2\right)}$$

$$\downarrow 3284$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-$$

$$\frac{2a\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+5a^3b\left(\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}+\frac{be\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+d\right.\right.\right.$$

$$\frac{}{e^2(a^2-b^2)}$$

$$\frac{}{5e^2\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) - ((-2
*(5*a^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c
 + d*x]]) + ((2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + 5*a^3*b*((2*a*e*(-1/2*ArcTan[(Sqrt
[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(a^(3/2)*(a^2 - b^2)^(1/4)*Sq
rt[e]) + ArcTanh[(Sqrt[a]*Sqrt[e]*Sin[c + d*x])/(a^2 - b^2)^(1/4)]/(2*a^(3
/2)*(a^2 - b^2)^(1/4)*Sqrt[e])))/d + (b*e*EllipticPi[(2*a)/(a - Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) + (b*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (
c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a + Sqrt[a^2 - b^2])*d*Sqrt[
e*Sin[c + d*x]])))/((a^2 - b^2)*e^2))/(5*(a^2 - b^2)*e^2)
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.240.4 Maple [A] (verified)

Time = 11.09 (sec) , antiderivative size = 892, normalized size of antiderivative = 1.75

method	result	size
default	Expression too large to display	892

input
```
int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*e*(1/5/e^2/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2)+1/e^4/(a-b)^2/(a+b)^2*a^2
/(e*sin(d*x+c))^(1/2)+1/4*a^2/e^4/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4
)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x
+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)
/a^2)^(1/4)))))-1/10/e^3*b^2*(12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+8*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-
4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2
)^(1/2)-a),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-
b^2)^(1/2)-a),1/2*2^(1/2))*a^2-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)
),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/
2)),1/2*2^(1/2))*a^2+12*a^2*cos(d*x+c)^4*sin(d*x+c)+8*b^2*cos(d*x+c)^4*sin
(d*x+c)-16*a^2*cos(d*x+c)^2*sin(d*x+c)-4*b^2*cos(d*x+c)^2*sin(d*x+c))*a...
```

3.240.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.240.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}\,(b+a\cos(c+dx))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*sin(c + d*x))^(7/2)*(b + a*cos(c + d*x))), x)
```

3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$

3.241.1 Optimal result 1878
3.241.2 Mathematica [C] (warning: unable to verify) 1879
3.241.3 Rubi [A] (verified) 1880
3.241.4 Maple [A] (warning: unable to verify) 1883
3.241.5 Fricas [F(-1)] 1883
3.241.6 Sympy [F(-1)] 1884
3.241.7 Maxima [F(-1)] 1884
3.241.8 Giac [F] 1884
3.241.9 Mupad [F(-1)] 1885

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 1070

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = -\frac{7b^3(a^2-b^2)^{3/4}e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d}$$
$$+\frac{2b(a^2-b^2)^{7/4}e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d}$$
$$+\frac{7b^3(a^2-b^2)^{3/4}e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d}$$
$$-\frac{2b(a^2-b^2)^{7/4}e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d}$$
$$+\frac{7b^4(a^2-b^2)e^5\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)^2e^5\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4(a^2-b^2)e^5\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)^2e^5\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{7b^2(3a^2-5b^2)e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{4b^2(8a^2-5b^2)e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d}$$
$$-\frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d}$$
$$+\frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d}$$
$$+\frac{4be(e\sin(c+dx))^{7/2}}{7a^3d}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d}$$
$$+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}$$

output

```
-7/2*b^3*(a^2-b^2)^(3/4)*e^(9/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))/a^(13/2)/d+2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctan(a^(1/2)*
(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d+7/2*b^3*(a^2-b^2)
^(3/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(13/2)/d-2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(
1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d-14/45*e^3*cos(d*x+c)*(e*sin(d*x+c
))^(3/2)/a^2/d-7/15*b^2*e^3*(5*b-3*a*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/
d+4/15*b*e^3*(5*a^2-5*b^2+3*a*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/7
*b*e*(e*sin(d*x+c))^(7/2)/a^3/d-2/9*e*cos(d*x+c)*(e*sin(d*x+c))^(7/2)/a^2/
d+b^2*e*(e*sin(d*x+c))^(7/2)/a^3/d/(b+a*cos(d*x+c))-7/2*b^4*(a^2-b^2)*e^5*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/a^7/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7
/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(
a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+...
```

3.241.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.32 (sec) , antiderivative size = 974, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{9/2}\left(\frac{(14a^4-159a^2b^2+165b^4)\cos^2(c+dx)}{}\right.}{}$$

$$+\frac{(b+a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{9/2}\left(-\frac{b(-37a^2+56b^2)\sin(c+dx)}{21a^5}+\frac{a^2b^2\sin(c+dx)-b^4\sin(}{a^5(b+a\cos(c+dx)}\right.}{d(a+b\sec(c+dx))^2}$$

input

```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

output

```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(((14*a^4 -
159*a^2*b^2 + 165*b^4)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*A
rcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*Arc
Tan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqr
t[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*
Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*
Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*
(b + a*Sqrt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d
*x])*(1 - Sin[c + d*x]^2)) + (2*(-46*a^3*b + 66*a*b^3)*Cos[c + d*x]*(((1/8
 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4
)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a
^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d
*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(30*a^5*d*(a +
b*Sec[c + d*x])^2*Sin[c + d*x]^(9/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*
x]^4*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(-1/21*(b*(-37*a^2 + 56*b^2)...
```

3.241.3 Rubi [A] (verified)

Time = 3.22 (sec) , antiderivative size = 1070, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{9/2}}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2(e\sin(c+dx))^{9/2}}{a^2(a\cos(c+dx)+b)^2}-\frac{2b(e\sin(c+dx))^{9/2}}{a^2(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^{9/2}}{a^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{2b^2\left(a^2-b^2\right)^2\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+$$
$$\frac{7b^4\left(a^2-b^2\right)\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-$$
$$\frac{2b^2\left(a^2-b^2\right)^2\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+$$
$$\frac{7b^4\left(a^2-b^2\right)\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+$$
$$\frac{2b\left(a^2-b^2\right)^{7/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{a^{13/2}d}-\frac{7b^3\left(a^2-b^2\right)^{3/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{2a^{13/2}d}-$$
$$\frac{2b\left(a^2-b^2\right)^{7/4}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{a^{13/2}d}+\frac{7b^3\left(a^2-b^2\right)^{3/4}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{2a^{13/2}d}-$$
$$\frac{7b^2\left(3a^2-5b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{5a^6d\sqrt{\sin(c+dx)}}-$$
$$\frac{4b^2\left(8a^2-5b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{5a^6d\sqrt{\sin(c+dx)}}+$$
$$\frac{14E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{15a^2d\sqrt{\sin(c+dx)}}-\frac{14\cos(c+dx)(e\sin(c+dx))^{3/2}e^3}{45a^2d}-$$
$$\frac{7b^2(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}e^3}{15a^5d}+$$
$$\frac{4b\left(5\left(a^2-b^2\right)+3ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}e^3}{15a^5d}-\frac{2\cos(c+dx)(e\sin(c+dx))^{7/2}e}{9a^2d}+$$
$$\frac{4b(e\sin(c+dx))^{7/2}e}{7a^3d}+\frac{b^2(e\sin(c+dx))^{7/2}e}{a^3d(b+a\cos(c+dx))}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((
a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2
)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(
13/2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(
7/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqr
t[e])])/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a
^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^
2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2
*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7
*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(2*a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2
 - b^2)^2*e^5*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) +
 (14*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
Sqrt[Sin[c + d*x]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x
)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2
 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^
6*d*Sqrt[Sin[c + d*x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/...
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.241.4 Maple [A] (warning: unable to verify)

Time = 44.55 (sec) , antiderivative size = 1725, normalized size of antiderivative = 1.61

method	result	size
default	Expression too large to display	1725

input
```
int((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(4*e*a*b*(-1/21/a^6*(e*sin(d*x+c))^(3/2)*e^2*(3*cos(d*x+c)^2*a^2-10*a^2+14
*b^2)+e^4/a^6*((1/4*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(3/2)/(-a^2*e^2*cos(d*
x+c)^2+b^2*e^2)+1/4*(a^4-15/4*a^2*b^2+11/4*b^4)/a^2/(e^2*(a^2-b^2)/a^2)^(1
/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d
*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^
2)/a^2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*(-1/45/a^6/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)*(10*a^4*cos(d*x+c)^6+42*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/
2*2^(1/2))*a^4-432*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b^2+450*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))*b^4-21*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4+216*a^2
*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-225*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1
/2))-34*cos(d*x+c)^4*a^4+54*cos(d*x+c)^4*a^2*b^2+24*cos(d*x+c)^2*a^4-54*co
s(d*x+c)^2*a^2*b^2)+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*a^2/e/b^2/(a^2-b^2
)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2
/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^...
```

3.241.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.241.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.241.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{9/2}}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(9/2))/(b + a*cos(c + d*x))^2, x)
```

3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$

3.242.1 Optimal result 1887
3.242.2 Mathematica [C] (warning: unable to verify) 1888
3.242.3 Rubi [A] (verified) 1889
3.242.4 Maple [A] (warning: unable to verify) 1892
3.242.5 Fricas [F(-1)] 1892
3.242.6 Sympy [F(-1)] 1893
3.242.7 Maxima [F] 1893
3.242.8 Giac [F] 1893
3.242.9 Mupad [F(-1)] 1894

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 1101

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d}$$
$$-\frac{2b(a^2-b^2)^{5/4}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d}$$
$$+\frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d}$$
$$-\frac{2b(a^2-b^2)^{5/4}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d}$$
$$+\frac{10e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^2(a^2-3b^2)e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{4b^2(4a^2-3b^2)e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4(a^2-b^2)e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2(a^2-b^2)^2e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4(a^2-b^2)e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2(a^2-b^2)^2e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}$$
$$-\frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}$$
$$+\frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}+\frac{4be(e\sin(c+dx))^{5/2}}{5a^3d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d}+\frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}$$

output
```
5/2*b^3*(a^2-b^2)^(1/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(
e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+5/2*b^3*(a^2-b^2)^
(1/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)/a^(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1
/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+4/5*b*e*(e*sin(d*x+c))^(5/2)/a^3/d
-2/7*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(5/2)/a^
3/d/(b+a*cos(d*x+c))-10/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)
^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+5/3*b^2*(a^2-3*b^2)*e^4*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2)+4/3*b^2*(4*a^
2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*si
n(d*x+c))^(1/2)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2
-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(
e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-
(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(...
```

3.242.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.73 (sec) , antiderivative size = 2095, normalized size of antiderivative = 1.90

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*(-1/42*((23*a^2 - 84*b^2)*Cos[c + d*x])/a^4 - (b^2
*(-a^2 + b^2))/(a^5*(b + a*Cos[c + d*x])) - (2*b*Cos[2*(c + d*x)])/(5*a^3)
 + Cos[3*(c + d*x)]/(14*a^2))*Csc[c + d*x]^3*Sec[c + d*x]^2*(e*Sin[c + d*x
])^(7/2))/(d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x
]^2*(e*Sin[c + d*x])^(7/2)*((2*(50*a^4 - 273*a^2*b^2 + 105*b^4)*Cos[c + d*
x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sq
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqr
t[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a
^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[
c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*Appe
llF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*
Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^
2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c
 + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))
)))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-139*a^3*b + 210*a*b
^3)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(
2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - ...
```

3.242.3 Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 1101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4360$$
$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2(e\sin(c+dx))^{7/2}}{a^2(a\cos(c+dx)+b)^2}-\frac{2b(e\sin(c+dx))^{7/2}}{a^2(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^{7/2}}{a^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{5b^2\left(a^2-3b^2\right)\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}-$$
$$\frac{4b^2\left(4a^2-3b^2\right)\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}+$$
$$\frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{21a^2d\sqrt{e\sin(c+dx)}}+$$
$$\frac{2b^2\left(a^2-b^2\right)^2\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{a^6\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-$$
$$\frac{5b^4\left(a^2-b^2\right)\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{2a^6\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+$$
$$\frac{2b^2\left(a^2-b^2\right)^2\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{a^6\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-$$
$$\frac{5b^4\left(a^2-b^2\right)\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{2a^6\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-$$
$$\frac{2b\left(a^2-b^2\right)^{5/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{a^{11/2}d}+\frac{5b^3\sqrt[4]{a^2-b^2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{2a^{11/2}d}-$$
$$\frac{2b\left(a^2-b^2\right)^{5/4}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{a^{11/2}d}+\frac{5b^3\sqrt[4]{a^2-b^2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{2a^{11/2}d}-$$
$$\frac{10\cos(c+dx)\sqrt{e\sin(c+dx)}e^3}{21a^2d}-\frac{5b^2(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}e^3}{3a^5d}+$$
$$\frac{4b\left(3\left(a^2-b^2\right)+ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}e^3}{3a^5d}-\frac{2\cos(c+dx)(e\sin(c+dx))^{5/2}e}{7a^2d}+$$
$$\frac{4b(e\sin(c+dx))^{5/2}e}{5a^3d}+\frac{b^2(e\sin(c+dx))^{5/2}e}{a^3d(b+a\cos(c+dx))}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)
*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(1
1/2)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c +
 d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5
/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x
]]) - (4*b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(2*a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2
*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*
Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b
^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqr
t[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticP
i[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*...
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.242.4 Maple [A] (warning: unable to verify)

Time = 45.23 (sec) , antiderivative size = 1494, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	1494

input
```
int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(4*e*a*b*(-1/5/a^6*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*a^2-6*a^2+10*b^2
)+e^4/a^6*((1/4*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(1/2)/(-a^2*e^2*cos(d*x+c)
^2+b^2*e^2)+1/16*(4*a^4-13*a^2*b^2+9*b^4)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*
e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(
d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e
^2*(a^2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/21/a
^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4-84*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/
2))+105*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c
)-42*a^2*b^2*cos(d*x+c)^2*sin(d*x+c))+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*
a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b
^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2
)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^
(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/...
```

3.242.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.242.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

3.242.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{7/2}}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x))^2, x)
```

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$

3.243.1 Optimal result 1896
3.243.2 Mathematica [C] (warning: unable to verify) 1897
3.243.3 Rubi [A] (verified) 1898
3.243.4 Maple [A] (warning: unable to verify) 1901
3.243.5 Fricas [F(-1)] 1901
3.243.6 Sympy [F(-1)] 1902
3.243.7 Maxima [F] 1902
3.243.8 Giac [F] 1902
3.243.9 Mupad [F(-1)] 1903

3.243.1 Optimal result

Integrand size = 25, antiderivative size = 850

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = -\frac{3b^3e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d}$$
$$+\frac{2b(a^2-b^2)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}+\frac{3b^3e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d}$$
$$-\frac{2b(a^2-b^2)^{3/4}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$+\frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}}+\frac{4be(e\sin(c+dx))^{3/2}}{3a^3d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}+\frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))}$$

output

```
-3/2*b^3*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/
2))/a^(9/2)/(a^2-b^2)^(1/4)/d+2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(
e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+3/2*b^3*e^(5/2)*arc
tanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/(a^2-b^
2)^(1/4)/d-2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2
)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+4/3*b*e*(e*sin(d*x+c))^(3/2)/a^3/d-2/
5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(3/2)/a^3/d
/(b+a*cos(d*x+c))-3/2*b^4*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+2*b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/
2*b^4*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*
x+c)^(1/2)/a^5/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)*
e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/a^5/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-6/5*e^2*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4...
```

3.243.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.51 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.04

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx =$$

$$(b+a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(-\frac{(-6a^2+35b^2)\cos^2(c+dx)\left(3\sqrt{2}b\left(-a^2+b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(}}{\sqrt[4]{-a^2-}}\right.\right.\right.\right.$$

$$+\frac{(b+a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{4b\sin(c+dx)}{3a^3}+\frac{b^2\sin(c+dx)}{a^3(b+a\cos(c+dx))}-\frac{\sin(2(c+dx))}{5a^2}\right)}{d(a+b\sec(c+dx))^2}$$

input

```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output

```
-1/10*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((-6*
a^2 + 35*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 -
 (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d
*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[
c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
 + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sq
rt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (28*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + L
og[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2,
1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/
2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*
x])*Sqrt[1 - Sin[c + d*x]^2])))/(a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
^(5/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*x]^2*Sec[c + d*x]^2*(e*Sin[c +
 d*x])^(5/2)*((4*b*Sin[c + d*x])/(3*a^3) + (b^2*Sin[c + d*x])/(a^3*(b +...
```

3.243.3 Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2(e\sin(c+dx))^{5/2}}{a^2(a\cos(c+dx)+b)^2}-\frac{2b(e\sin(c+dx))^{5/2}}{a^2(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^{5/2}}{a^2}\right)dx$$

$$\downarrow 2009$$

$$\frac{3e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+$$

$$\frac{3e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{3e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{9/2}\sqrt[4]{a^2-b^2}d}+$$

$$\frac{3e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{9/2}\sqrt[4]{a^2-b^2}d}+\frac{e(e\sin(c+dx))^{3/2}b^2}{a^3d(b+a\cos(c+dx))}-$$

$$\frac{7e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^4d\sqrt{\sin(c+dx)}}-$$

$$\frac{2\left(a^2-b^2\right)e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-$$

$$\frac{2\left(a^2-b^2\right)e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{4e(e\sin(c+dx))^{3/2}b}{3a^3d}+$$

$$\frac{2\left(a^2-b^2\right)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{9/2}d}-\frac{2\left(a^2-b^2\right)^{3/4}e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{9/2}d}-$$

$$\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}+\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)
*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9
/2)*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2
)^(1/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4
)*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e
])])/(a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b
^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 -
b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a
 + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a +
Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*E
llipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c +
d*x]]) + (4*b*e*(e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*S
in[c + d*x])^(3/2))/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b +
 a*Cos[c + d*x]))
```

3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.243.4 Maple [A] (warning: unable to verify)

Time = 34.74 (sec) , antiderivative size = 1482, normalized size of antiderivative = 1.74

method	result	size
default	Expression too large to display	1482

input
```
int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(4*e*a*b*(1/3*(e*sin(d*x+c))^(3/2)/a^4+e^2/a^4*(1/4*(e*sin(d*x+c))^(3/2)*b
^2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/4*(a^2-7/4*b^2)/a^2/(e^2*(a^2-b^2)/a^
2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*
sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a
^2-b^2)/a^2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*(-1/5/a^2/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*cos(d*x+c)^2)+3*b^2/a^4*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))-b^2*(3*a^2-5*b^2)/a^4*(-1/2/a^2*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-
b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2
*2^(1/2)))+2*b^4*(a^2-b^2)/a^4*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2...
```

3.243.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.243.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

3.243.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{5/2}}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x))^2, x)
```

3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$

3.244.1 Optimal result 1905
3.244.2 Mathematica [C] (warning: unable to verify) 1906
3.244.3 Rubi [A] (verified) 1907
3.244.4 Maple [A] (warning: unable to verify) 1910
3.244.5 Fricas [F(-1)] 1910
3.244.6 Sympy [F(-1)] 1911
3.244.7 Maxima [F] 1911
3.244.8 Giac [F] 1911
3.244.9 Mupad [F(-1)] 1912

3.244.1 Optimal result

Integrand size = 25, antiderivative size = 882

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{b^3e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}$$
$$-\frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}+\frac{b^3e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}$$
$$-\frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}$$
$$+\frac{2e^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^2e^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^4e^2\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\left(a^2-b^2\right)e^2\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^4e^2\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\left(a^2-b^2\right)e^2\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{4be\sqrt{e\sin(c+dx)}}{a^3d}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))}$$

output
```
1/2*b^3*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(7/2)/(a^2-b^2)^(3/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e
*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+1/2*b^3*e^(3/2)*arct
anh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/(a^2-b^2
)^(3/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)
/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d-2/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+5*b^2*e^2*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/
2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a
^4/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/a^4/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2
*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d
/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(co...
```

3.244.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.51 (sec) , antiderivative size = 2012, normalized size of antiderivative = 2.28

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*((-2*Cos[c + d*x])/(3*a^2) + b^2/(a^3*(b + a*Cos[c
 + d*x])))*Csc[c + d*x]*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/2))/(d*(a + b*S
ec[c + d*x])^2) - ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^
(3/2)*((2*(-2*a^2 + 3*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])
*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4
)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2
+ b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^
2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d
*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*S
in[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/
2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]
^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))))/((b + a*Cos[c + d*x])*(1 - Sin[c
+ d*x]^2)) + (8*a*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
+ I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b
^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)
^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[...
```

3.244.3 Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2(e\sin(c+dx))^{3/2}}{a^2(a\cos(c+dx)+b)^2}-\frac{2b(e\sin(c+dx))^{3/2}}{a^2(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^{3/2}}{a^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-$$

$$\frac{e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}+$$

$$\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}+\frac{e\sqrt{e\sin(c+dx)}b^2}{a^3d(b+a\cos(c+dx))}-$$

$$\frac{5e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4d\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\left(a^2-b^2\right)e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\left(a^2-b^2\right)e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-$$

$$\frac{2\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{7/2}d}-\frac{2\sqrt[4]{a^2-b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{7/2}d}+$$

$$\frac{4e\sqrt{e\sin(c+dx)}b}{a^3d}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*Ar
cTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)
*d) + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/
4)*Sqrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(
3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(a^(7/2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2
*a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2
 - b^2)*e^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[
e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqr
t[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*
d) - (2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin
[c + d*x]])/(a^3*d*(b + a*Cos[c + d*x]))
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.244.4 Maple [A] (warning: unable to verify)

Time = 26.56 (sec) , antiderivative size = 1350, normalized size of antiderivative = 1.53

method	result	size
default	Expression too large to display	1350

input
```
int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(4*e*a*b*((e*sin(d*x+c))^(1/2)/a^4+e^2/a^4*(1/4*(e*sin(d*x+c))^(1/2)*b^2/(
-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/16*(4*a^2-5*b^2)*(e^2*(a^2-b^2)/a^2)^(1/4
)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/
((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^
(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*
(-1/3/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/
2))+2*cos(d*x+c)^2*sin(d*x+c))+3*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/a^4*b^2*(3*a^2-5*b^2)*(-1/2/(a^2-b^2)^
(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))+2*b^4*(a^2-b^2)/a^4*(-1/2*a^2/e/b^2/(
a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2
/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/
2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1...
```

3.244.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.244.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.244.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{3/2}}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x))^2, x)
```

3.245 $\int \frac{\sqrt{e \sin(c+dx)}}{(a+b \sec(c+dx))^2} dx$

3.245.1 Optimal result 1914
3.245.2 Mathematica [C] (warning: unable to verify) 1915
3.245.3 Rubi [A] (verified) 1916
3.245.4 Maple [A] (warning: unable to verify) 1919
3.245.5 Fricas [F(-1)] 1919
3.245.6 Sympy [F] 1920
3.245.7 Maxima [F] 1920
3.245.8 Giac [F] 1920
3.245.9 Mupad [F(-1)] 1921

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 809

$$\begin{aligned}\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx &= \frac{b^3\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} + \frac{2b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} \\ &- \frac{b^3\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} \\ &- \frac{2b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} \\ &- \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} \\ &- \frac{b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} \\ &+ \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}\end{aligned}$$

output
```
b^2*(e*sin(d*x+c))^(3/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))+1/2*b^3*arctan(a
^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-
b^2)^(5/4)/d+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/
2))*e^(1/2)/a^(5/2)/(a^2-b^2)^(1/4)/d-1/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c
))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^2)^(5/4)/d-2*b*ar
ctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2
)/(a^2-b^2)^(1/4)/d+2*b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2
)+1/2*b^4*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^
2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/a^3/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4
*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/(a^2-b^
2)/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+b^2*(sin(1/2*c+1/4*P...
```

3.245.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.87 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$= (b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{(-2a^2+3b^2)\cos^2(c+dx)\left(3\sqrt{2}b\left(-a^2+b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx}}{\sqrt[4]{-a^2+b^2}}\right.\right.\right.\right.$$

$$+\frac{b^2(b+a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((-2*a^2 + 3*
b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
 + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2
, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Si
n[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
qrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^
2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[
c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Si
n[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a
^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1
 - Sin[c + d*x]^2])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[
Sin[c + d*x]]) + (b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[e*Sin[c + d*x
]]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
```

3.245.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{e \cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)\sqrt{e \sin(c+dx)}}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2\sqrt{e\sin(c+dx)}}{a^2(a\cos(c+dx)+b)^2}-\frac{2b\sqrt{e\sin(c+dx)}}{a^2(a\cos(c+dx)+b)}+\frac{\sqrt{e\sin(c+dx)}}{a^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-$$

$$\frac{e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}-$$

$$\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}+\frac{(e\sin(c+dx))^{3/2}b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}-$$

$$\frac{E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}}-$$

$$\frac{2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-$$

$$\frac{2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}d}-$$

$$\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}d}+\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)
*d) - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/
4)*Sqrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt
[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^
2)^(1/4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
+ d*x]]) - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*
Sin[c + d*x]]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*
d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
 + d*x]])/(a^2*d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*
Sin[c + d*x])^(3/2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

3.245.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.245.4 Maple [A] (warning: unable to verify)

Time = 19.71 (sec) , antiderivative size = 1313, normalized size of antiderivative = 1.62

method	result	size
default	Expression too large to display	1313

input
```
int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*e*a*b*(-1/2*b^2/a^2/(a^2-b^2)*(e*sin(d*x+c))^(3/2)/(-a^2*e^2*cos(d*x+c
)^2+b^2*e^2)-1/8*(4*a^2-3*b^2)/a^4/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*
arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^
(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)
^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*(-1/a^2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2)))+2*b^4/a^2*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(
a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi
((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/
a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d...
```

3.245.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.245.6 Sympy [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.245.7 Maxima [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.245.8 Giac [F]

$$\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e \sin(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\sqrt{e\,\sin(c+dx)}}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x))^2, x)
```

3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

3.246.1 Optimal result . . . 1922
3.246.2 Mathematica [C] (warning: unable to verify) . . . 1923
3.246.3 Rubi [A] (verified) . . . 1924
3.246.4 Maple [A] (warning: unable to verify) . . . 1927
3.246.5 Fricas [F(-1)] . . . 1927
3.246.6 Sympy [F] . . . 1928
3.246.7 Maxima [F(-1)] . . . 1928
3.246.8 Giac [F] . . . 1928
3.246.9 Mupad [F(-1)] . . . 1929

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 838

$$
\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx \\
&= -\frac{3b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} - \frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{3b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} \\
&\quad - \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}
\end{aligned}
$$

output

```
-3/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3
/2)/(a^2-b^2)^(7/4)/d/e^(1/2)-2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-3/2*b^3*arctanh(a^(
1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(7/4)
/d/e^(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a
^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2
*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a
^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2+a*(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a...
```

3.246.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.65 (sec) , antiderivative size = 1246, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\left(\frac{2\left(-2a^2+b^2\right)\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c}}{\sqrt[4]{-a^2+}}\right.\right.\right.}{}\right.}{}$$

$$+\frac{b^2(b+a\cos(c+dx))\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}$$

input

```
Integrate[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output

```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]]*((2*(-2*a^2 + b^
2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
 + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^
2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*
x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^
2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin
[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[
c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan
[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1
 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 -
 b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
 d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[
c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*Ap...
```

3.246.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}\,dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}(-a\cos(c+dx)-b)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2}{a^2\sqrt{e\sin(c+dx)}(-a\cos(c+dx)-b)^2}+\frac{2b}{a^2\sqrt{e\sin(c+dx)}(-a\cos(c+dx)-b)}+\frac{1}{a^2\sqrt{e\sin(c+dx)}}\right)dx$$

$$\downarrow 2009$$

$$\frac{3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+$$

$$\frac{3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}-$$

$$\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}+\frac{\sqrt{e\sin(c+dx)}b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}+$$

$$\frac{\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}-$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$

input
```
Int[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
)/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Si
n[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sq
rt[e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*
Sqrt[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]
*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^
(3/4)*d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*Elliptic
Pi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(a^2*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*Ell
ipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin
[c + d*x]]) + (3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d
*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 -
 b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)
*d*e*(b + a*Cos[c + d*x]))
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.246.4 Maple [A] (warning: unable to verify)

Time = 23.71 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.48

method	result	size
default	Expression too large to display	1239

input
```
int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4*a*b*e*(1/4*b^2/a^2/(a^2-b^2)*(e*sin(d*x+c))^(1/2)/(-a^2*e^2*cos(d*x+c)^
2+b^2*e^2)+1/16*(4*a^2-b^2)/a^2/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*
e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(
d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e
^2*(a^2-b^2)/a^2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-1/a^2*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/a^2*b^4*(-
1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a
^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2
-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/
2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1
/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellip
ticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/8/(a^2-
b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1...
```

3.246.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.246.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))**2), x)
```

3.246.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.246.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos(c+dx)^2}{\sqrt{e\,\sin(c+dx)}\,(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))^2), x)
```

3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

3.247.1 Optimal result 1931
3.247.2 Mathematica [C] (warning: unable to verify) 1932
3.247.3 Rubi [A] (verified) 1933
3.247.4 Maple [A] (warning: unable to verify) 1936
3.247.5 Fricas [F(-1)] 1936
3.247.6 Sympy [F(-1)] 1937
3.247.7 Maxima [F(-1)] 1937
3.247.8 Giac [F] 1937
3.247.9 Mupad [F(-1)] 1938

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 1054

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{5b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\,(a^2-b^2)^{9/4}\,de^{3/2}}$$
$$+\frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{5/4}\,de^{3/2}}-\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\,(a^2-b^2)^{9/4}\,de^{3/2}}$$
$$-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{5/4}\,de^{3/2}}-\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}$$
$$+\frac{4b(a-b\cos(c+dx))}{a^2\,(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}+\frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\,(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a\,(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a\,(a^2-b^2)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$-\frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}$$
$$-\frac{4b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\,(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$-\frac{b^2(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\,(a^2-b^2)^2\,de^2\sqrt{\sin(c+dx)}}$$

output

```
5/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-
b^2)^(9/4)/d/e^(3/2)/a^(1/2)+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-5/2*b^3*arctanh(a^(1
/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(3/2
)/a^(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)/(a^2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-2*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(1
/2)+b^2/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+4*b*(a-b*cos
(d*x+c))/a^2/(a^2-b^2)/d/e/(e*sin(d*x+c))^(1/2)+b^2*(5*a*b-(3*a^2+2*b^2)*c
os(d*x+c))/a^2/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2
/d/e/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*
x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a-(a
^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(
a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a+(a^2-b^2
)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2)...
```

3.247.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.23 (sec) , antiderivative size = 772, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = (b+a\cos(c+dx))\left(-\frac{(a^2-b^2)\left(b+a\sqrt{\cos^2(c+dx)}\right)\sec^3(c+dx)\sin^{\frac{3}{2}}(c+dx)}{\qquad}\right.$$

input

```
Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
((b + a*Cos[c + d*x])*(-(((a^2 - b^2)*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c +
 d*x]^3*Sin[c + d*x]^(3/2)*((2*a^3 + 3*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-
a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2
 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 +
b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a
]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*App
ellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
*Sin[c + d*x]^(3/2)) + (1 + I)*a*(6*a^2*b + 4*b^3)*Sqrt[Cos[c + d*x]^2]*(3
*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2
 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]
*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqr
t[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(3/2))))/(a^(3/2)*(a - b)^2*(a + b)^2)) + 24*(-2*(b
 + a*Cos[c + d*x])*(-2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x] + a*b^
2*Sin[c + d*x])*Tan[c + d*x]^2))/(24*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^
2*(e*Sin[c + d*x])^(3/2))
```

3.247.3 Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1054, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2}{a^2(e\sin(c+dx))^{3/2}(-a\cos(c+dx)-b)^2}+\frac{2b}{a^2(e\sin(c+dx))^{3/2}(-a\cos(c+dx)-b)}+\frac{1}{a^2(e\sin(c+dx))^{3/2}}\right.$$

$$\downarrow 2009$$

$$-\frac{5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-$$

$$\frac{5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{5\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}-$$

$$\frac{5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}-\frac{\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}-$$

$$\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}+\frac{\left(5ab-\left(3a^2+2b^2\right)\cos(c+dx)\right)b^2}{a^2\left(a^2-b^2\right)^2de\sqrt{e\sin(c+dx)}}-$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+$$

$$\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}+\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}-$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}+\frac{4(a-b\cos(c+dx))b}{a^2\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}}-$$

$$\left.\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}\right)$$

input
```
Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin
[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(
3/2)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*
Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(
5/4)*d*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*
Cos[c + d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b -
(3*a^2 + 2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]
) - (5*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Si
n[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b
^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b
^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[
a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*Ellipti...
```

3.247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.247.4 Maple [A] (warning: unable to verify)

Time = 22.48 (sec) , antiderivative size = 1417, normalized size of antiderivative = 1.34

method	result	size
default	Expression too large to display	1417

input
```
int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*a*b*(-2/e^2/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2)-2/e^2/(a-b)^2/(a+b)^2*(
1/4*(e*sin(d*x+c))^(3/2)*b^2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/4*(a^2+1/4*
b^2)/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^
2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e
*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/e*((a^2+b^2)/(a^2-b^2)^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)+2*b^4/(a-b)/(a+b)*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2
/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*
x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2)...
```

3.247.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.247.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.247.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)^2}{(e\sin(c+dx))^{3/2}(b+a\cos(c+dx))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))^2), x)
```

3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

3.248.1 Optimal result 1940
3.248.2 Mathematica [C] (warning: unable to verify) 1941
3.248.3 Rubi [A] (verified) 1942
3.248.4 Maple [A] (warning: unable to verify) 1945
3.248.5 Fricas [F(-1)] 1945
3.248.6 Sympy [F(-1)] 1946
3.248.7 Maxima [F(-1)] 1946
3.248.8 Giac [F] 1946
3.248.9 Mupad [F(-1)] 1947

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 1089

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{7\sqrt{a}b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}$$
$$-\frac{2\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}}-\frac{7\sqrt{a}b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}$$
$$-\frac{2\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}}-\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}}$$
$$+\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$
$$+\frac{4b(a-b\cos(c+dx))}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}+\frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}}$$
$$+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

output
```
-2/3*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)+b^2/a/(a^2-b^2)/d/e/(b+a*cos(
d*x+c))/(e*sin(d*x+c))^(3/2)+4/3*b*(a-b*cos(d*x+c))/a^2/(a^2-b^2)/d/e/(e*s
in(d*x+c))^(3/2)+1/3*b^2*(7*a*b-(5*a^2+2*b^2)*cos(d*x+c))/a^2/(a^2-b^2)^2/
d/e/(e*sin(d*x+c))^(3/2)-7/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctan(a^(1/2)*
(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^
(5/2)-7/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)*a^(1/2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1
/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-2/3*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)-
4/3*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/
e^2/(e*sin(d*x+c))^(1/2)-1/3*b^2*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)-7/2*b^4*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/(a^2-b^2)^2/d/e^2/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticP...
```

3.248.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.52 (sec) , antiderivative size = 1320, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output

```
-1/6*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(5/2)*((2*(-2*a^3
 - 5*a*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan
[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1
 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
 + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) -
 (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c +
 d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*S
in[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*
(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2
*(10*a^2*b + 4*b^3)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b
^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)
^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/...
```

3.248.3 Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 1089, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int\left(\frac{b^2}{a^2(e\sin(c+dx))^{5/2}(-a\cos(c+dx)-b)^2}+\frac{2b}{a^2(e\sin(c+dx))^{5/2}(-a\cos(c+dx)-b)}+\frac{1}{a^2(e\sin(c+dx))^{5/2}}\right.$$

$$\downarrow 2009$$

$$\frac{7\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2-\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+$$

$$\frac{7\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2+\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}-\frac{7\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}-$$

$$\frac{7\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}+\frac{\left(5a^2+2b^2\right)\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{3a^2\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}}+$$

$$\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+\frac{\left(7ab-\left(5a^2+2b^2\right)\cos(c+dx)\right)b^2}{3a^2\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}}+$$

$$\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\left(a^2-b^2\right)^{7/4}de^{5/2}}-$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\left(a^2-b^2\right)^{7/4}de^{5/2}}+\frac{4(a-b\cos(c+dx))b}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}+$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}}$$

input
```
Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*
Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e
^(5/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^
2)^(1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan
h[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2
)^(7/4)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) +
 b^2/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*
b*(a - b*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (
b^2*(7*a*b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Si
n[c + d*x])^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(3*a^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/
2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) +
 (b^2*(5*a^2 + 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(3*a^2*(a^2 - b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a
)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^
2 - b^2)^2*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (
2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/...
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.248.4 Maple [A] (warning: unable to verify)

Time = 31.29 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.22

method	result	size
default	Expression too large to display	1331

input
```
int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4*e*a*b*(1/3/e^2/(a^2-b^2)^2/(e*sin(d*x+c))^(3/2)+1/e^2/(a-b)^2/(a+b)^2*(
1/4*(e*sin(d*x+c))^(1/2)*b^2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/16*(4*a^2+3
*b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/
2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1
/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))))+(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/e^2*(1/3*(a^2+b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(
d*x+c)^2*sin(d*x+c))+2*b^4/(a-b)/(a+b)*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b
^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/
2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/...
```

3.248.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.248.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.248.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)^2}{(e\sin(c+dx))^{5/2}(b+a\cos(c+dx))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))^2), x)
```

3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$

3.249.1 Optimal result 1948
3.249.2 Mathematica [A] (verified) 1948
3.249.3 Rubi [A] (verified) 1949
3.249.4 Maple [A] (verified) 1950
3.249.5 Fricas [F] 1950
3.249.6 Sympy [F] 1951
3.249.7 Maxima [F] 1951
3.249.8 Giac [F] 1951
3.249.9 Mupad [F(-1)] 1952

3.249.1 Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}(a+b\sec(e$$
$$\sqrt{a+b}f$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b)^(1/2)/(a+b*sec(f*x+e))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sec(f*x+e))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)
*(b*(1+sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2\right.}{f(b+a\cos(e+fx))}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/(f*(b + a*Cos[e + f*x]))
```

3.249.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4267}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\,dx$$

$$\downarrow 4267$$

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
 + f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f
*x]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*
x]))/(Sqrt[a + b]*f)
```

3.249.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

3.249.4 Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)b-2a\,\text{EllipticPi}\left(\cot(fx+e\right.\right.}{f(b+a\cos(fx+e))}$

input
```
int((a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a
-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot
(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/(b+a*cos(f*x+e))
```

3.249.5 Fricas [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a), x)
```

3.249.6 Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{a+b\sec(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)), x)
```

3.249.7 Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a), x)
```

3.249.8 Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{a+\frac{b}{\cos(e+fx)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2), x)
```

3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$

3.250.1 Optimal result 1953
3.250.2 Mathematica [A] (verified) 1953
3.250.3 Rubi [A] (verified) 1954
3.250.4 Maple [A] (verified) 1956
3.250.5 Fricas [F] 1956
3.250.6 Sympy [F] 1956
3.250.7 Maxima [F] 1957
3.250.8 Giac [F] 1957
3.250.9 Mupad [F(-1)] 1957

3.250.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$$
$$= \frac{\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$
$$-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

output
```
cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2
))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/f-cot(f*x+e)*(a+b*sec(f*x+e))^(1/2)/f
```

3.250.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$$
$$= \frac{-\left((b+a\cos(e+fx))\csc(e+fx)\sqrt{\frac{1}{1+\sec(e+fx)}}\right)+b\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\sqrt{\frac{a+b\sec}{(a+b)(1+s}}}{f\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-((b + a*Cos[e + f*x])*Csc[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + b*El
lipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x
])/((a + b)*(1 + Sec[e + f*x]))])/(f*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[a
+ b*Sec[e + f*x]])
```

3.250.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4363, 25, 3042, 4319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a-b\csc\left(e+fx-\frac{\pi}{2}\right)}}{\cos\left(e+fx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4363$$

$$-\frac{1}{2}b\int -\frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

$$\downarrow 25$$

$$\frac{1}{2}b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

$$\downarrow 3042$$

$$\frac{1}{2}b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

$$\downarrow 4319$$

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

3.250.4 Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.78

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,b\cos(fx+e)+\text{EllipticF}\left(\cot(fx\right.\right.}{f(b+a\cos(fx+e))}$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(EllipticF(cot(f*x+e)-csc(f*x
+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*b*cos(f*x+e)+EllipticF(cot(f*x+e)-csc(f*x
+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*b+a*cos(f*x+e)*cot(f*x+e)+b*cot(f*x+e))
```

3.250.5 Fricas [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

3.250.6 Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{a+b\sec(e+fx)}\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*csc(e + f*x)**2, x)
```

3.250.7 Maxima [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

3.250.8 Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)}}}{\sin(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$

3.251.1 Optimal result . . . 1958
3.251.2 Mathematica [B] (verified) . . . 1959
3.251.3 Rubi [A] (verified) . . . 1960
3.251.4 Maple [B] (verified) . . . 1963
3.251.5 Fricas [F] . . . 1963
3.251.6 Sympy [F] . . . 1964
3.251.7 Maxima [F] . . . 1964
3.251.8 Giac [F] . . . 1964
3.251.9 Mupad [F(-1)] . . . 1965

3.251.1 Optimal result

Integrand size = 14, antiderivative size = 309

$$\int (a+b\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$-\frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

output

```
-2*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/
(a-b))^(1/2)/f+2*(2*a-b)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e
))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

3.251.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(309) = 618$.

Time = 6.73 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.21

$$\int (a + b\sec(e+fx))^{3/2}\,dx = \frac{2b\cos(e+fx)(a+b\sec(e+fx))^{3/2}\sin(e+fx)}{f(b+a\cos(e+fx))}$$
$$+\frac{2(a+b\sec(e+fx))^{3/2}\left(ab\tan\left(\frac{1}{2}(e+fx)\right)+b^2\tan\left(\frac{1}{2}(e+fx)\right)-2ab\tan^3\left(\frac{1}{2}(e+fx)\right)+ab\tan^5\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b*Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(f*(b + a*Cos[e
 + f*x])) + (2*(a + b*Sec[e + f*x])^(3/2)*(a*b*Tan[(e + f*x)/2] + b^2*Tan[
(e + f*x)/2] - 2*a*b*Tan[(e + f*x)/2]^3 + a*b*Tan[(e + f*x)/2]^5 - b^2*Tan
[(e + f*x)/2]^5 - 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 +
b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)
/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqr
t[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + b*(a +
b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(a + b)] + (a^2 - 2*a*b - b^2)*EllipticF[ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e +
f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
 b)]))/(f*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[(1 - Tan[(e +
 f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e +
f*x)/2]^2)])
```

3.251.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4268, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\downarrow 4268$$

$$\int \frac{a^2+(2a-b)b\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx + b^2\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\downarrow 3042$$

$$\int \frac{a^2+(2a-b)b\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + b^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4409$$

$$a^2\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx + b^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + b(2a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\downarrow 3042$$

$$a^2\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + b^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + b(2a-b)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4271$$

$$b^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + b(2a-b)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx - \frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\downarrow 4319$$

$$b^2 \int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx + \frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f} - \frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\downarrow 4492$$

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f} - \frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f} - \frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e
 + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/
(a - b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e
 + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

3.251.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4268
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Simp[b^2   Int[Csc[
c + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.251.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. 2(282) = 564.

Time = 8.89 (sec) , antiderivative size = 1599, normalized size of antiderivative = 5.17

method	result	size
default	Expression too large to display	1599

input
```
int((a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(
f*x+e)^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*
b*cos(f*x+e)^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*b^2*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(
1/2))*a*b*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*b^2*cos(f*x+e)^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,(
(a-b)/(a+b))^(1/2))*a^2*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a
-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(f*x+e)-4*EllipticF(cot(f*x+e)-csc(f*x+e)
,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)-2*EllipticF(cot(f*x+e)-csc(f*
x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x+e
))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(co...
```

3.251.5 Fricas [F]

$$\int (a + b \sec(e + fx))^{3/2}\, dx = \int (b \sec(fx + e) + a)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^(3/2), x)
```

3.251.6 Sympy [F]

$$\int (a + b \sec(e + fx))^{3/2}\, dx = \int (a + b \sec(e + fx))^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2), x)
```

3.251.7 Maxima [F]

$$\int (a + b \sec(e + fx))^{3/2}\, dx = \int (b \sec(fx + e) + a)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2), x)
```

3.251.8 Giac [F]

$$\int (a + b \sec(e + fx))^{3/2}\, dx = \int (b \sec(fx + e) + a)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \sec(e + fx))^{3/2}\, dx = \int \left(a + \frac{b}{\cos(e + f\,x)}\right)^{3/2} dx$$

input
```
int((a + b/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2), x)
```

3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

3.252.1 Optimal result 1966
3.252.2 Mathematica [A] (verified) 1967
3.252.3 Rubi [A] (verified) 1967
3.252.4 Maple [B] (verified) 1970
3.252.5 Fricas [F] 1971
3.252.6 Sympy [F(-1)] 1971
3.252.7 Maxima [F] 1971
3.252.8 Giac [F] 1972
3.252.9 Mupad [F(-1)] 1972

3.252.1 Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{3(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

output

```
-cot(f*x+e)*(a+b*sec(f*x+e))^(3/2)/f-3*(a-b)*cot(f*x+e)*EllipticE((a+b*sec
(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+
e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+3*(a-b)*cot(f*x+e)*Elli
pticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*
(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

3.252.2 Mathematica [A] (verified)

Time = 11.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.21

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \frac{\cos(e+fx)(a+b\sec(e+fx))^{3/2}((-b-a\cos(e+fx))\csc(e+fx)+3b\sin(e+fx))}{f(b+a\cos(e+fx))}$$
$$+\frac{3b(a+b\sec(e+fx))^{3/2}\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\arcsin(\tan(\frac{1}{2}(e+fx)))|\frac{a-b}{a+b}\right)-\text{EllipticF}\left(\arcsin(\tan(\frac{1}{2}(e+fx))),\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}-\right.}{f(b+a\cos(e+fx))^2\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)}\sec}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
(Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((-b - a*Cos[e + f*x])*Csc[e + f*
x] + 3*b*Sin[e + f*x]))/(f*(b + a*Cos[e + f*x])) + (3*b*(a + b*Sec[e + f*x
])^(3/2)*(-(((a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])
)]*(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSi
n[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(f*(b + a*Cos[e + f*x])^2*S
qrt[Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e +
 f*x]])
```

3.252.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4363, 25, 3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-b\csc\left(e+fx-\frac{\pi}{2}\right)\right)^{3/2}}{\cos\left(e+fx-\frac{\pi}{2}\right)^2}dx$$
$$\downarrow 4363$$

$$-\frac{3}{2}b\int -\sec(e+fx)\sqrt{a+b\sec(e+fx)}dx-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 25$$

$$\frac{3}{2}b\int \sec(e+fx)\sqrt{a+b\sec(e+fx)}dx-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 3042$$

$$\frac{3}{2}b\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 4316$$

$$\frac{3}{2}b\left((a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+b\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx\right)-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 3042$$

$$\frac{3}{2}b\left((a-b)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 4319$$

$$\frac{3}{2}b\left(b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+}{a-b}}}{bf}-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

$$\downarrow 4492$$

$$\frac{3}{2}b\left(\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2(a}{}-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

3.252. $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

output
```
-((Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f) + (3*b*((-2*(a - b)*Sqrt[a
+ b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
 + f*x]))/(a - b))])/(b*f) + (2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f))
)/2
```

3.252.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

rule 4492

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.252.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(208) = 416$.

Time = 9.18 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.35

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab\cos(fx+e)+3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^2\cos(fx+e)-3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,b^2\cos(fx+e)-3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab\cos(fx+e)+3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^2\cos(fx+e)-3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,b^2\cos(fx+e)-3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab+3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^2-3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,b^2-3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab+3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^2-3\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,b^2-2b^2\csc(fx+e)\right)$ co

input

```
int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(3*EllipticF(cot(f*x+e)-csc(f
*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)+3*EllipticF(cot(f*x+e)-c
sc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)-3*(1/(a+b)*(b+a*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*cos(f*x+e)-3*(1/(a+b)*(b+a*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)+3*(1/(a+b)*(
b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b+3*(1/(a+b)*(b+a*cos
(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2-3*(1/(a+b)*(b+a*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b-3*(1/(a+b)*(b+a*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2+a^2*cos(f*x+e)*cot(f*x+e)+3*a*b*cos(f*x+
e)*cot(f*x+e)-a*b*cot(f*x+e)+3*b^2*cot(f*x+e)-2*b^2*csc(f*x+e))
```

3.252.5 Fricas [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e)^2*sec(f*x + e) + a*csc(f*x + e)^2)*sqrt(b*sec(f*x
 + e) + a), x)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

3.252.8 Giac [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)}\right)^{3/2}}{\sin(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.253.1 Optimal result 1973
3.253.2 Mathematica [A] (verified) 1973
3.253.3 Rubi [A] (verified) 1974
3.253.4 Maple [A] (verified) 1975
3.253.5 Fricas [F] 1975
3.253.6 Sympy [F] 1976
3.253.7 Maxima [F] 1976
3.253.8 Giac [F] 1976
3.253.9 Mupad [F(-1)] 1977

3.253.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

output
```
-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e
))/(a-b))^(1/2)/a/f
```

3.253.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipt}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*C
os[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticF[ArcSin[Tan[(e + f*x)
/2]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b
)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

3.253.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4271}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4271$$

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

3.253.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

3.253.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1}}}{f(b+a\cos(fx+e))}$

input
```
int(1/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))-2
*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*c
os(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*se
c(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

3.253.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sec(f*x + e) + a), x)
```

3.253.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)), x)
```

3.253.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

3.253.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x))^(1/2), x)
```

3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.254.1 Optimal result . . . 1978
3.254.2 Mathematica [A] (warning: unable to verify) . . . 1979
3.254.3 Rubi [A] (verified) . . . 1979
3.254.4 Maple [B] (verified) . . . 1983
3.254.5 Fricas [F] . . . 1984
3.254.6 Sympy [F] . . . 1984
3.254.7 Maxima [F] . . . 1984
3.254.8 Giac [F] . . . 1985
3.254.9 Mupad [F(-1)] . . . 1985

3.254.1 Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}
&\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx \\
&= \frac{\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f} \\
&\quad - \frac{\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f} \\
&\quad - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}}
\end{aligned}$$

output

```
cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2
))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b)^
(1/2)-cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/
(a+b)^(1/2)-cot(f*x+e)/f/(a+b*sec(f*x+e))^(1/2)+b^2*tan(f*x+e)/(a^2-b^2)/f
/(a+b*sec(f*x+e))^(1/2)
```

3.254.2 Mathematica [A] (warning: unable to verify)

Time = 7.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= \frac{\sqrt{\sec(e+fx)}\left(\frac{(b+a\cos(e+fx))(-a+b\cos(e+fx))\csc(e+fx)}{(a^2-b^2)\sqrt{\sec(e+fx)}} + \frac{b\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\text{Elliptic}\right.\right.}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}}{(-a^2+b^2)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{}}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(Sqrt[Sec[e + f*x]]*(((b + a*Cos[e + f*x])*(-a + b*Cos[e + f*x])*Csc[e + f
*x])/((a^2 - b^2)*Sqrt[Sec[e + f*x]]) + (b*(-(((a + b)*Sqrt[(b + a*Cos[e +
 f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticE[ArcSin[Tan[(e + f*x)/2]],
(a - b)/(a + b)] - EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/
Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x
)/2]))/((-a^2 + b^2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[
e + f*x]])))/(f*Sqrt[a + b*Sec[e + f*x]])
```

3.254.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4363, 25, 3042, 4320, 27, 3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(e+fx-\frac{\pi}{2}\right)^2\sqrt{a-b\csc\left(e+fx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4363$$

$$\frac{1}{2}b\int -\frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 25$$

$$-\frac{1}{2}b\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$-\frac{1}{2}b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4320$$

$$-\frac{1}{2}b\left(-\frac{2\int -\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec(e+fx)}dx}{a^2-b^2} - \frac{2b\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}\right) - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 27$$

$$-\frac{1}{2}b\left(\frac{\int \sec(e+fx)\sqrt{a+b\sec(e+fx)}dx}{a^2-b^2} - \frac{2b\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}\right) - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$-\frac{1}{2}b\left(\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{2b\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}\right) - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4316$$

$$-\frac{1}{2}b\left(\frac{(a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx + b\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a^2-b^2} - \frac{2b\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}\right) - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$-\frac{1}{2}b\left(\frac{(a-b)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}\right)-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4319$$

$$-\frac{1}{2}b\left(\frac{b\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+}}{\sqrt{a+b}}\right.\right.\right.}{bf}}{a^2-b^2}\right)-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4492$$

$$-\frac{1}{2}b\left(\frac{\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1}{}}}{a^2-b^2}\right)-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
-(Cot[e + f*x]/(f*Sqrt[a + b*Sec[e + f*x]])) - (b*(((-2*(a - b)*Sqrt[a + b
]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
f*x]))/(a - b))])/(b*f) + (2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f))/(a
^2 - b^2) - (2*b*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])))/
2
```

3.254.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
 + Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

rule 4492

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.254.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(233) = 466$.

Time = 5.73 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab\cos(fx+e)-\text{EllipticF}\left(\cot$

input

```
int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/f/(a-b)/(a+b)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(-EllipticF(cot(f
*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)-EllipticF(cot
(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)+(1/(a+b)*(b
+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*cos(f*x+e)+(1/(a+b)*
(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)-(1/(a+b
)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b-(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2+(1/(a+b)*(b+a*cos(f*x+e))
/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2)
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x
+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*b^2+a^2*cos(f*x+e)*cot(f*x+e)-a*b*cos(f*x+e)*co
t(f*x+e)+a*b*cot(f*x+e)-b^2*cot(f*x+e))
```

3.254.5 Fricas [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

3.254.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x)
```

3.254.7 Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

3.254.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sin(e+fx)^2\sqrt{a+\frac{b}{\cos(e+fx)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

3.255.1 Optimal result 1986
3.255.2 Mathematica [B] (verified) 1987
3.255.3 Rubi [A] (verified) 1988
3.255.4 Maple [B] (warning: unable to verify) 1991
3.255.5 Fricas [F] 1992
3.255.6 Sympy [F] 1993
3.255.7 Maxima [F] 1993
3.255.8 Giac [F] 1993
3.255.9 Mupad [F(-1)] 1994

3.255.1 Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$
$$-\frac{2\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$
$$-\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^2 f}$$
$$+\frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}}$$

output

```
2*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a/f/(a
+b)^(1/2)-2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/a/f/(a+b)^(1/2)-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(
1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/
2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b^2*tan(f*x+e)/a/(a^2-b^2)/f/(a
+b*sec(f*x+e))^(1/2)
```

3.255.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. $2(347) = 694$.

Time = 6.60 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{(b+a\cos(e+fx))^2\sec^2(e+fx)\left(\frac{2b\sin(e+fx)}{a(-a^2+b^2)}+\frac{2b^2\sin(e+fx)}{a(a^2-b^2)(b+a\cos(e+fx))}\right)}{f(a+b\sec(e+fx))^{3/2}}$$
$$+\frac{2(b+a\cos(e+fx))^{3/2}\sec^{\frac{3}{2}}(e+fx)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}\left(ab\tan\left(\frac{1}{2}(e+fx)\right)+b^2\tan\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(-3/2),x]
```

output
```
((b + a*Cos[e + f*x])^2*Sec[e + f*x]^2*((2*b*Sin[e + f*x])/(a*(-a^2 + b^2)
) + (2*b^2*Sin[e + f*x])/(a*(a^2 - b^2)*(b + a*Cos[e + f*x])))))/(f*(a + b*
Sec[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sq
rt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x
)/2]^2)]*(a*b*Tan[(e + f*x)/2] + b^2*Tan[(e + f*x)/2] - 2*a*b*Tan[(e + f*x
)/2]^3 + a*b*Tan[(e + f*x)/2]^5 - b^2*Tan[(e + f*x)/2]^5 + 2*a^2*EllipticP
i[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]
^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] -
2*b^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]
^2)/(a + b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a +
 b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(
e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcS
in[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e
+ f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a
 + b)] + b*(a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e
 + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - a*(a + b)*EllipticF[ArcSin
[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan
[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2...
```

3.255.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4272, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4272$$

$$\frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}} - \frac{2\int -\frac{a^2-b\sec(e+fx)a-b^2-b^2\sec^2(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^2-b\sec(e+fx)a-b^2-b^2\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2-b\csc\left(e+fx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4546$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx - b^2\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx - b^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4409$$

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx + b^2\left(-\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx\right) - b(a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)} + \frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx + b^2\left(-\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx\right) - b(a-b)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx}{a(a^2-b^2)} + \frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$$\downarrow 4271$$

$$\frac{b^2\left(-\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx\right) - b(a-b)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx - \frac{2\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-}}{a(a^2-b^2)}}{\frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}$$

$$\downarrow 4319$$

$$\frac{b^2\left(-\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx\right) - \frac{2\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}}{af}\right)\right.}{a(a^2-b^2)}}{\frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}$$

$$\downarrow 4492$$

$$\frac{-\frac{2\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af} - \frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{}{}}}{}}{\frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}$$

input
```
Int[(a + b*Sec[e + f*x])^(-3/2),x]
```

3.255. $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

output
```
((2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f - (2*(a - b)*Sqrt[a + b]*Cot[e +
 f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a
 - b))])/f - (2*Sqrt[a + b]*(a^2 - b^2)*Cot[e + f*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f))
/(a*(a^2 - b^2)) + (2*b^2*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e
+ f*x]])
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x],
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.255.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(318) = 636$.

Time = 6.48 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1799

input
```
int(1/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/a/(a+b)/(a-b)*(-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))
^2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*a^2-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^
2*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/
(a+b))^(1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2
*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))
^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(
f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*
csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^
2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a
+b))^(1/2))*b^2+2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2
*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))
^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,(...
```

3.255.5 Fricas [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
 + a^2), x)
```

3.255.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(-3/2), x)
```

3.255.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

3.255.8 Giac [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x))^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x))^(3/2), x)
```

3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

3.256.1 Optimal result 1995
3.256.2 Mathematica [A] (verified) 1996
3.256.3 Rubi [A] (verified) 1996
3.256.4 Maple [B] (verified) 2001
3.256.5 Fricas [F] 2001
3.256.6 Sympy [F] 2002
3.256.7 Maxima [F] 2002
3.256.8 Giac [F] 2002
3.256.9 Mupad [F(-1)] 2003

3.256.1 Optimal result

Integrand size = 23, antiderivative size = 318

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{4a\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx)}{a-b}}}{(a-b)(a+b)^{3/2}f}$$
$$-\frac{(3a-b)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f}$$
$$-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}}$$
$$+\frac{4ab^2\tan(e+fx)}{(a^2-b^2)^2\,f\sqrt{a+b\sec(e+fx)}}$$

output

```
-cot(f*x+e)/f/(a+b*sec(f*x+e))^(3/2)+4*a*cot(f*x+e)*EllipticE((a+b*sec(f*x
+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)
*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f-(3*a-b)*cot(f*x+e)*El
lipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(
f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f+b
^2*tan(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+4*a*b^2*tan(f*x+e)/(a^2-b
^2)^2/f/(a+b*sec(f*x+e))^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.81

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{-((a-b)((3a-b)b+a(a-3b)\cos(e+fx))\csc(e+fx))+8ab(a+b)\cos^2}{}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
(-((a - b)*((3*a - b)*b + a*(a - 3*b)*Cos[e + f*x])*Csc[e + f*x]) + 8*a*b*
(a + b)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e +
 f*x]))]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]
*Sqrt[(1 + Sec[e + f*x])^(-1)] - 2*b*(3*a^2 + 4*a*b + b^2)*Cos[(e + f*x)/2
]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticF[ArcS
in[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x]
)^(-1)])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

3.256.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4363, 25, 3042, 4320, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cos\left(e+fx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(e+fx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4363$$

$$\frac{3}{2}b\int -\frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}dx-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 25$$

$$-\frac{3}{2}b\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}dx-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3}{2}b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 4320$$

$$-\frac{3}{2}b\left(-\frac{2\int -\frac{\sec(e+fx)(3a-b\sec(e+fx))}{2(a+b\sec(e+fx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 27$$

$$-\frac{3}{2}b\left(\frac{\int \frac{\sec(e+fx)(3a-b\sec(e+fx))}{(a+b\sec(e+fx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3}{2}b\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(3a-b\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 4491$$

$$-\frac{3}{2}b\left(\frac{-\frac{2\int -\frac{\sec(e+fx)\left(3a^2+4b\sec(e+fx)a+b^2\right)}{2\sqrt{a+b\sec(e+fx)}}dx}{a^2-b^2}-\frac{8ab\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 27$$

$$-\frac{3}{2}b\left(\frac{\frac{\int \frac{\sec(e+fx)\left(3a^2+4b\sec(e+fx)a+b^2\right)}{\sqrt{a+b\sec(e+fx)}}dx}{a^2-b^2}-\frac{8ab\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3}{2}b\left(\frac{\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(3a^2+4b\csc\left(e+fx+\frac{\pi}{2}\right)a+b^2\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(e+fx)}{f(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}\right)-$$

$$\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 4493$$

$$-\frac{3}{2}b\left(\frac{\frac{(a-b)(3a-b)\int\frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+4ab\int\frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a^2-b^2}-\frac{8ab\tan(e+fx)}{f(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(e+fx)}{3f\left(a^2-b^2\right)(a+b\sec(e+\right.}$$

$$\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3}{2}b\left(\frac{\frac{(a-b)(3a-b)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+4ab\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(e+fx)}{f(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(}{3f\left(a^2-b^2\right)(a+}\right)$$

$$\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 4319$$

$$-\frac{3}{2}b\left(\frac{\frac{4ab\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right.\right.}{bf}}{a^2-b^2}}{3\left(a^2-b^2\right)}\right)$$

$$\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

$$\downarrow 4492$$

$$-\frac{3}{2}b\left(\frac{\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{8a(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticE}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}}{a^2-b^2}}{3\left(a^2-b^2\right)}\right.$$

$$\left.-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}\right)$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
-(Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2))) - (3*b*((-2*b*Tan[e + f*x])
/(3*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (((-8*a*(a - b)*Sqrt[a + b
]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
f*x]))/(a - b))])/(b*f) + (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[e + f*x]*El
lipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])
/(b*f))/(a^2 - b^2) - (8*a*b*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a + b*Sec[e
 + f*x]]))/(3*(a^2 - b^2))))/2
```

3.256.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(294) = 588.

Time = 6.86 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.07

method	result	size
default	Expression too large to display	977

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a-b)^2/(a+b)^2*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(4*EllipticE(
cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)+4*Elli
pticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)-
3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f
*x+e)-4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),
((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2
*cos(f*x+e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x
+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b
^3*cos(f*x+e)+4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc
(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*a^2*b+4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+
e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*
b^2-3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((
a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b-4
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/
(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b...
```

3.256.5 Fricas [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2/(b^2*sec(f*x + e)^2 + 2*a
*b*sec(f*x + e) + a^2), x)
```

3.256.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x))**(3/2), x)
```

3.256.7 Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

3.256.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+fx)^2\left(a+\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)), x)
```

3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

3.257.1 Optimal result 2004
3.257.2 Mathematica [A] (verified) 2005
3.257.3 Rubi [A] (verified) 2005
3.257.4 Maple [F] 2007
3.257.5 Fricas [F] 2008
3.257.6 Sympy [F] 2008
3.257.7 Maxima [F] 2008
3.257.8 Giac [F] 2009
3.257.9 Mupad [F(-1)] 2009

3.257.1 Optimal result

Integrand size = 23, antiderivative size = 249

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$= \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{3a^2b\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+\frac{b^3\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+\frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

output

```
3*a^2*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^
(1+m)/d/e/(1+m)+b^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*
sin(d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2
+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+
3*a*b^2*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e
*sin(d*x+c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

3.257.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.73

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$= \frac{\left(a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)+b\left(3a^2\cos(c+dx)\,\text{Hypergeometric2}\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

output
```
((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2] + b*(3*a^2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, Sin[c + d*x]^2] + b*(3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2
, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1
[2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])))*(e*Sin[c + d*x])^m*Tan[c + d*
x])/(d*(1 + m))
```

3.257.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4360, 25, 25, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$\downarrow 3042$$
$$\int \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$
$$\downarrow 4360$$
$$\int \sec^3(c+dx)\left(-(-a\cos(c+dx)-b)^3\right)(e\sin(c+dx))^m dx$$
$$\downarrow 25$$
$$-\int -(b+a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m dx$$

$$\downarrow 25$$

$$\int \sec^3(c+dx)(a\cos(c+dx)+b)^3(e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 3391$$

$$\int (a^3(e\sin(c+dx))^m + 3a^2b\sec(c+dx)(e\sin(c+dx))^m + 3ab^2\sec^2(c+dx)(e\sin(c+dx))^m + b^3\sec^3(c+dx)(e$$

$$\downarrow 2009$$

$$\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} +$$
$$\frac{3a^2b(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} +$$
$$\frac{3ab^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} +$$
$$\frac{b^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

input
```
Int[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

output
```
(a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2
*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)
/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqr
t[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]  Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.257.4 Maple [F]

$$\int (a + b \sec (dx + c))^3 (e \sin (dx + c))^m \, dx$$

input
```
int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

3.257.5 Fricas [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x +
c) + a^3)*(e*sin(d*x + c))^m, x)
```

3.257.6 Sympy [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (e\sin(c+dx))^m\,(a+b\sec(c+dx))^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

output
```
Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**3, x)
```

3.257.7 Maxima [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.257.8 Giac [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^3 dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3, x)
```

3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

3.258.1 Optimal result 2010
3.258.2 Mathematica [A] (verified) 2011
3.258.3 Rubi [A] (verified) 2011
3.258.4 Maple [F] 2014
3.258.5 Fricas [F] 2014
3.258.6 Sympy [F] 2015
3.258.7 Maxima [F] 2015
3.258.8 Giac [F] 2015
3.258.9 Mupad [F(-1)] 2016

3.258.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+dx))^m}{d(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+\frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}$$

output

```
2*a*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1
+m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*
x+c)^2)*sin(d*x+c)*(e*sin(d*x+c))^m/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hyper
geom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^m*(cos(d*x+
c)^2)^(1/2)*tan(d*x+c)/d/(1+m)
```

3.258.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$= \frac{(e\sin(c+dx))^m\left(2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}(a^2\,\text{Hy}\right.}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

output
```
((e*Sin[c + d*x])^m*(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(a^2*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b^2*Hypergeometric2F1[3/2, (1 +
m)/2, (3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

3.258.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4360, 3042, 3390, 3042, 3044, 27, 278, 4898, 3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$\downarrow 3042$$
$$\int \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$
$$\downarrow 4360$$
$$\int \sec^2(c+dx)(-a\cos(c+dx)-b)^2(e\sin(c+dx))^m dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$
$$\downarrow 3390$$

$$\int \left(b^2+a^2\cos^2(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^m dx + 2ab\int \sec(c+dx)(e\sin(c+dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(b^2+a^2\cos(c+dx)^2\right)(e\sin(c+dx))^m}{\cos(c+dx)^2} dx + 2ab\int \frac{(e\sin(c+dx))^m}{\cos(c+dx)} dx$$

$$\downarrow 3044$$

$$\int \frac{\left(b^2+a^2\cos(c+dx)^2\right)(e\sin(c+dx))^m}{\cos(c+dx)^2} dx + \frac{2ab\int \frac{e^2(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)} d(e\sin(c+dx))}{de}$$

$$\downarrow 27$$

$$\int \frac{\left(b^2+a^2\cos(c+dx)^2\right)(e\sin(c+dx))^m}{\cos(c+dx)^2} dx + \frac{2abe\int \frac{(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)} d(e\sin(c+dx))}{d}$$

$$\downarrow 278$$

$$\int \frac{\left(b^2+a^2\cos(c+dx)^2\right)(e\sin(c+dx))^m}{\cos(c+dx)^2} dx + \frac{2ab(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 4898$$

$$\sin^{-m}(c+dx)(e\sin(c+dx))^m\int \left(b^2+a^2\cos^2(c+dx)\right)\sec^2(c+dx)\sin^m(c+dx)dx + \frac{2ab(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 3042$$

$$\sin^{-m}(c+dx)(e\sin(c+dx))^m\int \frac{\left(b^2+a^2\cos(c+dx)^2\right)\sin(c+dx)^m}{\cos(c+dx)^2} dx + \frac{2ab(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 4901$$

$$\sin^{-m}(c+dx)(e\sin(c+dx))^m\int \left(a^2\sin^m(c+dx)+b^2\sec^2(c+dx)\sin^m(c+dx)\right) dx + \frac{2ab(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 2009$$

$$\sin^{-m}(c+dx)(e\sin(c+dx))^m\left(\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b^2\sqrt{\cos^2(c+dx)}\sec(c+\right.$$

$$\left.\frac{2ab(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}\right)$$

input
```
Int[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

output
```
(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c
 + d*x])^(1 + m))/(d*e*(1 + m)) + ((e*Sin[c + d*x])^m*((a^2*Cos[c + d*x]*H
ypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(
1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b^2*Sqrt[Cos[c + d*x]^2]*Hyper
geometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c
 + d*x]^(1 + m))/(d*(1 + m))))/Sin[c + d*x]^m
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3390
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[2*a*(b/d)
Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*
x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4898
```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Simp[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p])   In
t[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ
[v]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.258.4 Maple [F]

$$\int (a + b \sec (dx + c))^2 (e \sin (dx + c))^m \, dx$$

input
```
int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

3.258.5 Fricas [F]

$$\int (a + b \sec(c + dx))^2 (e \sin(c + dx))^m \, dx = \int (b \sec (dx + c) + a)^2 (e \sin (dx + c))^m \, dx$$

input
```
integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*(e*sin(d*x + c))^
m, x)
```

3.258.6 Sympy [F]

$$\int (a + b\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (e\sin(c + dx))^m (a + b\sec(c + dx))^2 \, dx$$

input
```
integrate((a+b*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

output
```
Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**2, x)
```

3.258.7 Maxima [F]

$$\int (a + b\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (b\sec(dx + c) + a)^2 (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.258.8 Giac [F]

$$\int (a + b\sec(c + dx))^2 (e\sin(c + dx))^m \, dx = \int (b\sec(dx + c) + a)^2 (e\sin(dx + c))^m \, dx$$

input
```
integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^2 dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2, x)
```

3.259 $\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx$

3.259.1 Optimal result 2017
3.259.2 Mathematica [A] (verified) 2017
3.259.3 Rubi [A] (verified) 2018
3.259.4 Maple [F] 2020
3.259.5 Fricas [F] 2021
3.259.6 Sympy [F] 2021
3.259.7 Maxima [F] 2021
3.259.8 Giac [F] 2022
3.259.9 Mupad [F(-1)] 2022

3.259.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx$$
$$= \frac{a\cos(c + dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c + dx)\right)(e\sin(c + dx))^{1+m}}{de(1 + m)\sqrt{\cos^2(c + dx)}}$$
$$+ \frac{b\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c + dx)\right)(e\sin(c + dx))^{1+m}}{de(1 + m)}$$

output
```
b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/
d/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2
)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx$$
$$= \frac{\left(a\sqrt{\cos^2(c + dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c + dx)\right) + b\cos(c + dx)\,\text{Hypergeometric2F1}\left(1\right.\right.}{d(1 + m)}$$

input
```
Integrate[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

output
```
((a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

3.259.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 27, 278, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a - b\csc\left(c + dx - \frac{\pi}{2}\right)\right)\left(e\cos\left(c + dx - \frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4360$$

$$\int -(\sec(c + dx)(-a\cos(c + dx) - b)(e\sin(c + dx))^m)\,dx$$

$$\downarrow 25$$

$$-\int -((b + a\cos(c + dx))\sec(c + dx)(e\sin(c + dx))^m)\,dx$$

$$\downarrow 25$$

$$\int \sec(c + dx)(a\cos(c + dx) + b)(e\sin(c + dx))^m dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\sin\left(c + dx + \frac{\pi}{2}\right) + b\right)\left(-e\cos\left(c + dx + \frac{\pi}{2}\right)\right)^m}{\sin\left(c + dx + \frac{\pi}{2}\right)} dx$$

$$\downarrow 3317$$

$$a\int (e\sin(c + dx))^m dx + b\int \sec(c + dx)(e\sin(c + dx))^m dx$$

$$\downarrow 3042$$

$$a\int (e\sin(c + dx))^m dx + b\int \frac{(e\sin(c + dx))^m}{\cos(c + dx)} dx$$

$$\downarrow 3044$$

$$a\int (e\sin(c+dx))^m dx + \frac{b\int \frac{e^2(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)} d(e\sin(c+dx))}{de}$$

$$\downarrow 27$$

$$a\int (e\sin(c+dx))^m dx + \frac{be\int \frac{(e\sin(c+dx))^m}{e^2-e^2\sin^2(c+dx)} d(e\sin(c+dx))}{d}$$

$$\downarrow 278$$

$$a\int (e\sin(c+dx))^m dx + \frac{b(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 3122$$

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} + \frac{b(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)}$$

input
```
Int[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

output
```
(a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^
2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

3.259.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.259.4 Maple [F]

$$\int (a + b \sec(dx + c)) (e \sin(dx + c))^m \, dx$$

input
```
int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

3.259.5 Fricas [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.259.6 Sympy [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (e\sin(c+dx))^m\,(a+b\sec(c+dx))\,dx$$

input
```
integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

output
```
Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x)), x)
```

3.259.7 Maxima [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.259.8 Giac [F]

$$\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx = \int (b\sec(dx + c) + a)(e\sin(dx + c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))(e\sin(c + dx))^m\,dx = \int (e\,\sin(c + d\,x))^m \left(a + \frac{b}{\cos(c + d\,x)}\right) dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)), x)
```

3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$

3.260.1 Optimal result . . . 2023
3.260.2 Mathematica [B] (warning: unable to verify) . . . 2024
3.260.3 Rubi [A] (verified) . . . 2025
3.260.4 Maple [F] . . . 2027
3.260.5 Fricas [F] . . . 2027
3.260.6 Sympy [F] . . . 2028
3.260.7 Maxima [F] . . . 2028
3.260.8 Giac [F] . . . 2028
3.260.9 Mupad [F(-1)] . . . 2029

3.260.1 Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = -\frac{be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^2d(1-m)} + \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{ade(1+m)\sqrt{\cos^2(c+dx)}}$$

output

```
-b*e*AppellF1(1-m,-1/2*m+1/2,-1/2*m+1/2,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/
(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(a*(1+
cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-
m)+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(
d*x+c))^(1+m)/a/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

3.260.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 687 vs. $2(232) = 464$.

Time = 4.96 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.96

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

$$= \frac{}{d(a+b\sec(c+dx))}\left(\left(-b\,\text{AppellF1}\left(\tfrac{1+m}{2}, m, 1, \tfrac{3+m}{2}, -\tan^2\left(\tfrac{1}{2}(c+dx)\right), \tfrac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right) + (a+b)\,\text{Hy}\right.\right.$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

output
```
(2*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 +
m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(
d*(a + b*Sec[c + d*x])*((-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c
+ d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometr
ic2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^
2 + 2*m*Cot[c + d*x]*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2
F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2] + 2
*m*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 +
m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2 + ((1 + m)*Sec[(c +
 d*x)/2]^2*(-((a + b)^2*(Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -T
an[(c + d*x)/2]^2] - (Sec[(c + d*x)/2]^2)^(-1 - m))) + (2*b*((-a + b)*Appe
llF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^
2)/(3 + m)))/(a + b)))
```

3.260.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4360, 25, 25, 3042, 3346, 3042, 3122, 3182}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos(c+dx)(e\sin(c+dx))^m}{-a\cos(c+dx)-b}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos(c+dx)(e\sin(c+dx))^m}{b+a\cos(c+dx)}dx$$

$$\downarrow 25$$

$$\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\downarrow 3346$$

$$\frac{\int (e\sin(c+dx))^m dx}{a} - \frac{b\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int (e\sin(c+dx))^m dx}{a} - \frac{b\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3122$$

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}} - \frac{b\int\frac{(e\cos(c+dx-\frac{\pi}{2}))^m}{b-a\sin(c+dx-\frac{\pi}{2})}dx}{a}$$

$$\downarrow 3182$$

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}} - \frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx}\right.}{a^2d(1-m)}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

output
```
-((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*
Cos[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^
((1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

3.260.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3182
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b  Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b  Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.260.4 Maple [F]

$$\int \frac{(e \sin (dx+c))^m}{a+b \sec (dx+c)} dx$$

input
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

output
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

3.260.5 Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{a+b \sec(c+dx)} dx = \int \frac{(e \sin (dx+c))^m}{b \sec (dx+c)+a} dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

3.260.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x)), x)
```

3.260.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

3.260.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\,\sin(c+dx))^m}{b+a\,\cos(c+dx)}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x)), x)
```

3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$

3.261.1 Optimal result 2030
3.261.2 Mathematica [B] (warning: unable to verify) 2031
3.261.3 Rubi [A] (verified) 2031
3.261.4 Maple [F] 2033
3.261.5 Fricas [F] 2034
3.261.6 Sympy [F] 2034
3.261.7 Maxima [F] 2034
3.261.8 Giac [F] 2035
3.261.9 Mupad [F(-1)] 2035

3.261.1 Optimal result

Integrand size = 23, antiderivative size = 405

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{2be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right.}{a^3d(1-m)}$$
$$+\frac{b^2e\,\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right.}{a^3d(2-m)(b+a\cos(c+dx))}$$
$$+\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^2de(1+m)\sqrt{\cos^2(c+dx)}}$$

output
```
-2*b*e*AppellF1(1-m,-1/2*m+1/2,-1/2*m+1/2,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b
)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(a*(
1+cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)/a^3/d/(
1-m)+b^2*e*AppellF1(2-m,-1/2*m+1/2,-1/2*m+1/2,3-m,(-a+b)/(b+a*cos(d*x+c)),
(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*
(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)/a^3
/d/(2-m)/(b+a*cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m
],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/a^2/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

3.261.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1433 vs. $2(405) = 810$.

Time = 11.66 (sec) , antiderivative size = 1433, normalized size of antiderivative = 3.54

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-4*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])*Sec[c + d*x]^2*(e*Sin[c +
 d*x])^m*Tan[(c + d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(AppellF1[(1 + m)
/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
 + b)]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c +
 d*x)/2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, (
(a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a
 + b)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (2*b^2*((a + b)*Appel
lF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^2*(e*Sin[c + d*
x])^m*Tan[(c + d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(((a + b)*AppellF1[(
1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^
2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a + b)
*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan...
```

3.261.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4360, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-a\cos(c+dx)-b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(-a\sin\left(c+dx+\frac{\pi}{2}\right)-b\right)^2}dx$$

$$\downarrow 3391$$

$$\int \left(\frac{b^2(e\sin(c+dx))^m}{a^2(a\cos(c+dx)+b)^2}-\frac{2b(e\sin(c+dx))^m}{a^2(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^m}{a^2}\right)dx$$

$$\downarrow 2009$$

$$\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+d}\right.}{a^3d(2-m)(a\cos(c+dx)+b)}$$

$$\frac{2be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+d}\right.}{a^3d(1-m)}$$

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^2de(m+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[
c + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a
*Cos[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))
^((1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a +
b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^
((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*S
in[c + d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x
]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c +
d*x])^(1 + m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

3.261.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.261.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{(a+b\sec(dx+c))^2} dx$$

input
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

output
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

3.261.5 Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2
), x)
```

3.261.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**2, x)
```

3.261.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

3.261.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^m}{(b+a\,\cos(c+dx))^2}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^2, x)
```

3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$

3.262.1 Optimal result 2036
3.262.2 Mathematica [B] (warning: unable to verify) 2037
3.262.3 Rubi [A] (verified) 2038
3.262.4 Maple [F] 2041
3.262.5 Fricas [F] 2041
3.262.6 Sympy [F] 2041
3.262.7 Maxima [F] 2042
3.262.8 Giac [F] 2042
3.262.9 Mupad [F(-1)] 2042

3.262.1 Optimal result

Integrand size = 23, antiderivative size = 580

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx =$$

$$-\frac{3be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right.}{a^4d(1-m)}$$

$$-\frac{b^3e\,\text{AppellF1}\left(3-m,\frac{1-m}{2},\frac{1-m}{2},4-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right.}{a^4d(3-m)(b+a\cos(c+dx))^2}$$

$$+\frac{3b^2e\,\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx)}{b+a\cos(c+dx)}\right.}{a^4d(2-m)(b+a\cos(c+dx))}$$

$$+\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^3de(1+m)\sqrt{\cos^2(c+dx)}}$$

output
```
-3*b*e*AppellF1(1-m,-1/2*m+1/2,-1/2*m+1/2,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b
)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(a*(
1+cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)/a^4/d/(
1-m)-b^3*e*AppellF1(3-m,-1/2*m+1/2,-1/2*m+1/2,4-m,(-a+b)/(b+a*cos(d*x+c)),
(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*
(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*(e*sin(d*x+c))^(-1+m)/a^4
/d/(3-m)/(b+a*cos(d*x+c))^2+3*b^2*e*AppellF1(2-m,-1/2*m+1/2,-1/2*m+1/2,3-m
,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*c
os(d*x+c)))^(-1/2*m+1/2)*(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(-1/2*m+1/2)*
(e*sin(d*x+c))^(-1+m)/a^4/d/(2-m)/(b+a*cos(d*x+c))+cos(d*x+c)*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/a^3/d/e/(1+m
)/(cos(d*x+c)^2)^(1/2)
```

3.262.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. $2(580) = 1160$.

Time = 16.99 (sec) , antiderivative size = 2700, normalized size of antiderivative = 4.66

$$\int \frac{(e \sin(c+dx))^m}{(a+b \sec(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

output

```
(-6*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*(e*Sin[c
 + d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(AppellF1[(1 +
m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c
 + d*x)/2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-
a + b)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5
+ m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (6*b^2*((a + b)*App
ellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d
*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*(b + a*Cos[c + d*x])*Sec[c
 + d*x]^3*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])
^3*((a + b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a
 - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)
/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d
*x)/2]^2 + 2*m*((a + b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + ...
```

3.262.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4360, 25, 25, 3042, 3391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\downarrow 4360$$

$$\int -\frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-a\cos(c+dx)-b)^3}dx$$

$$\downarrow 25$$

$$-\int -\frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(b+a\cos(c+dx))^3}dx$$

$$\downarrow 25$$

$$\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(a\cos(c+dx)+b)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}dx$$

$$\downarrow 3391$$

$$\int \left(-\frac{b^3(e\sin(c+dx))^m}{a^3(a\cos(c+dx)+b)^3}+\frac{3b^2(e\sin(c+dx))^m}{a^3(a\cos(c+dx)+b)^2}-\frac{3b(e\sin(c+dx))^m}{a^3(a\cos(c+dx)+b)}+\frac{(e\sin(c+dx))^m}{a^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^3e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(3-m,\frac{1-m}{2},\frac{1-m}{2},4-m,-\frac{a-b}{b+a\cos(c+}\right.}{a^4d(3-m)(a\cos(c+dx)+b)^2}$$

$$\frac{3b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+}\right.}{a^4d(2-m)(a\cos(c+dx)+b)}$$

$$\frac{3be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+a}\right.}{a^4d(1-m)}$$

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^3de(m+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

output
```
(-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[
c + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a
*Cos[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))
^((1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[
3 - m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a +
b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^
((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*S
in[c + d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*A
ppellF1[2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])
), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c +
d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)
/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos
[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*
Sin[c + d*x])^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3391
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

3.262.4 Maple [F]

$$\int \frac{(e \sin(dx+c))^m}{(a+b\sec(dx+c))^3}\,dx$$

input
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

output
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

3.262.5 Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{(e \sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.262.6 Sympy [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{(e \sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**3, x)
```

3.262.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

3.262.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(c+dx)^3\,(e\,\sin(c+dx))^m}{(b+a\,\cos(c+dx))^3}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^3, x)
```

3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

3.263.1 Optimal result . . . 2043
3.263.2 Mathematica [N/A] . . . 2043
3.263.3 Rubi [N/A] . . . 2044
3.263.4 Maple [N/A] (verified) . . . 2045
3.263.5 Fricas [N/A] . . . 2045
3.263.6 Sympy [F(-1)] . . . 2045
3.263.7 Maxima [N/A] . . . 2046
3.263.8 Giac [N/A] . . . 2046
3.263.9 Mupad [N/A] . . . 2046

3.263.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \text{Int}\left((a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

3.263.2 Mathematica [N/A]

Not integrable

Time = 39.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

3.263.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4365$$

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

output
```
$Aborted
```

3.263.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.263.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+b\sec(dx+c))^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

3.263.5 Fricas [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
```

output
```
Timed out
```

3.263.7 Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.263.8 Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.263.9 Mupad [N/A]

Not integrable

Time = 18.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.264 $\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$

3.264.1 Optimal result 2047
3.264.2 Mathematica [N/A] 2047
3.264.3 Rubi [N/A] 2048
3.264.4 Maple [N/A] (verified) 2049
3.264.5 Fricas [N/A] 2049
3.264.6 Sympy [N/A] 2049
3.264.7 Maxima [N/A] 2050
3.264.8 Giac [N/A] 2050
3.264.9 Mupad [N/A] 2050

3.264.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \text{Int}\left(\sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m, x\right)$$

output
```
Unintegrable((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

3.264.2 Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

3.264.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4365$$

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

output
```
$Aborted
```

3.264.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.264.4 Maple [N/A] (verified)

Not integrable

Time = 0.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (e \sin (dx+c))^m \sqrt{a+b \sec (dx+c)} dx$$

input
```
int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

3.264.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b \sec(c+dx)} (e \sin(c+dx))^m \, dx = \int \sqrt{b \sec (dx+c)+a} (e \sin (dx+c))^m \, dx$$

input
```
integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.264.6 Sympy [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b \sec(c+dx)} (e \sin(c+dx))^m \, dx = \int (e \sin (c+dx))^m \sqrt{a+b \sec (c+dx)} \, dx$$

input
```
integrate((e*sin(d*x+c))**m*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sin(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

3.264.7 Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\sin(dx+c))^m\,dx$$

input
```
integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.264.8 Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\sin(dx+c))^m\,dx$$

input
```
integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.264.9 Mupad [N/A]

Not integrable

Time = 15.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.265.1 Optimal result 2051
3.265.2 Mathematica [N/A] 2051
3.265.3 Rubi [N/A] 2052
3.265.4 Maple [N/A] (verified) 2053
3.265.5 Fricas [N/A] 2053
3.265.6 Sympy [N/A] 2053
3.265.7 Maxima [N/A] 2054
3.265.8 Giac [N/A] 2054
3.265.9 Mupad [N/A] 2054

3.265.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}, x\right)$$

output
```
Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

3.265.2 Mathematica [N/A]

Not integrable

Time = 21.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

3.265.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\sqrt{a-b\csc\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4365$$
$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.265.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.265.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e \sin(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

3.265.5 Fricas [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e \sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e \sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.265.6 Sympy [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e \sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e \sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sin(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

3.265.7 Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.265.8 Giac [N/A]

Not integrable

Time = 78.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.265.9 Mupad [N/A]

Not integrable

Time = 18.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\,\sin(c+d\,x))^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.266.1 Optimal result 2055
3.266.2 Mathematica [N/A] 2055
3.266.3 Rubi [N/A] 2056
3.266.4 Maple [N/A] (verified) 2057
3.266.5 Fricas [N/A] 2057
3.266.6 Sympy [N/A] 2057
3.266.7 Maxima [N/A] 2058
3.266.8 Giac [N/A] 2058
3.266.9 Mupad [N/A] 2058

3.266.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}, x\right)$$

output
```
Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

3.266.2 Mathematica [N/A]

Not integrable

Time = 20.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

3.266.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m}{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4365$$
$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.266.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.266.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

3.266.5 Fricas [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.266.6 Sympy [N/A]

Not integrable

Time = 30.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

3.266.7 Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.266.8 Giac [N/A]

Not integrable

Time = 38.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.266.9 Mupad [N/A]

Not integrable

Time = 25.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\,\sin(c+d\,x))^m}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.267 $\int (a + b\sec(c + dx))^n (e\sin(c + dx))^m \, dx$

3.267.1 Optimal result 2059
3.267.2 Mathematica [N/A] 2059
3.267.3 Rubi [N/A] 2060
3.267.4 Maple [N/A] (verified) 2061
3.267.5 Fricas [N/A] 2061
3.267.6 Sympy [N/A] 2061
3.267.7 Maxima [N/A] 2062
3.267.8 Giac [N/A] 2062
3.267.9 Mupad [N/A] 2062

3.267.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a + b\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \text{Int}((a + b\sec(c + dx))^n (e\sin(c + dx))^m, x)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

3.267.2 Mathematica [N/A]

Not integrable

Time = 7.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b\sec(c + dx))^n (e\sin(c + dx))^m \, dx = \int (a + b\sec(c + dx))^n (e\sin(c + dx))^m \, dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

3.267.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^m (a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^m \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4365$$

$$\int (e\sin(c+dx))^m (a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

output
```
$Aborted
```

3.267.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.267.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b \sec (dx + c))^n (e \sin (dx + c))^m \, dx$$

input
```
int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

3.267.5 Fricas [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b \sec(c + dx))^n (e \sin(c + dx))^m \, dx = \int (b \sec (dx + c) + a)^n (e \sin (dx + c))^m \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.267.6 Sympy [N/A]

Not integrable

Time = 161.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a + b \sec(c + dx))^n (e \sin(c + dx))^m \, dx = \int (e \sin (c + dx))^m (a + b \sec (c + dx))^n \, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
```

output
```
Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

3.267.7 Maxima [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^n(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.267.8 Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^n(e\sin(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.267.9 Mupad [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
```

output
```
int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.268 $\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx$

3.268.1 Optimal result 2063
3.268.2 Mathematica [B] (verified) 2064
3.268.3 Rubi [A] (verified) 2064
3.268.4 Maple [F] 2066
3.268.5 Fricas [F] 2066
3.268.6 Sympy [F(-1)] 2066
3.268.7 Maxima [F] 2067
3.268.8 Giac [F] 2067
3.268.9 Mupad [F(-1)] 2067

3.268.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\begin{aligned}
&\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx \\
&= \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2 d(1+n)} \\
&\quad - \frac{2b^3\,\text{Hypergeometric2F1}\left(4,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^4 d(1+n)} \\
&\quad + \frac{b^5\,\text{Hypergeometric2F1}\left(6,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^6 d(1+n)}
\end{aligned}$$

output

```
b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/
(1+n)-2*b^3*hypergeom([4, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1
+n)/a^4/d/(1+n)+b^5*hypergeom([6, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*
x+c))^(1+n)/a^6/d/(1+n)
```

3.268.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 562 vs. $2(150) = 300$.

Time = 6.79 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.75

$$\int (a + b\sec(c + dx))^n \sin^5(c + dx)\,dx =$$

$$-\frac{\cos^6\left(\frac{1}{2}(c + dx)\right)\cos(c + dx)\left(192a^3(-1 + n)(b + a\cos(c + dx))^2 - 240a^3(-1 + n)(b + a\cos(c + dx))^2\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

output
```
-1/120*(Cos[(c + d*x)/2]^6*Cos[c + d*x]*(192*a^3*(-1 + n)*(b + a*Cos[c + d
*x])^2 - 240*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 - 24*a
^2*(2*a - b*(-4 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 +
 40*a^2*(2*a - b*(-3 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2
]^4 + a*(1 - n)*(96*a^2 + 4*a*b*(6 - 4*n) - 4*b^2*(12 - 7*n + n^2))*(b + a
*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4 - 10*a*((-1 + n)*(-14*a^2 + 2*a*b*(-1
+ n) + b^2*(6 - 5*n + n^2))*(b + a*Cos[c + d*x])^2 + b*(24*a^3 + 12*a^2*b*
(-1 + n) - 4*a*b^2*(2 - 3*n + n^2) - b^3*(-6 + 11*n - 6*n^2 + n^3))*Hyperg
eometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(
c + d*x)/2]^6 + ((-1 + n)*(-84*a^3 + 2*a^2*b*(18 - 7*n) + 4*a*b^2*(9 - 9*n
 + 2*n^2) + b^3*(-24 + 26*n - 9*n^2 + n^3))*(b + a*Cos[c + d*x])^2 + b*(12
0*a^4 + 120*a^3*b*(-1 + n) - 10*a*b^3*(-6 + 11*n - 6*n^2 + n^3) - b^4*(24
- 50*n + 35*n^2 - 10*n^3 + n^4))*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos
[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/2]^6)*(a + b*Sec[c + d*x])
^n)/(a^4*d*(-1 + n)*(b + a*Cos[c + d*x]))
```

3.268.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4362, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c + dx)(a + b\sec(c + dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^5 \left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4362$$

$$-\frac{\int \cos^6(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)+1)^2(a+b\sec(c+dx))^n d(-\sec(c+dx))}{d}$$

$$\downarrow 198$$

$$-\frac{\int \left(\cos^6(c+dx)(a+b\sec(c+dx))^n - 2\cos^4(c+dx)(a+b\sec(c+dx))^n + \cos^2(c+dx)(a+b\sec(c+dx))^n\right) d(-}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^5(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(6,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^6(n+1)} + \frac{2b^3(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(4,n+1,n+2,\frac{b\sec(c+}{a^4(n+1)}}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

output
```
-((-((b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*
Sec[c + d*x])^(1 + n))/(a^2*(1 + n))) + (2*b^3*Hypergeometric2F1[4, 1 + n,
 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*(1 + n)
) - (b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b
*Sec[c + d*x])^(1 + n))/(a^6*(1 + n)))/d)
```

3.268.3.1 Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-f^(-1)   Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e
, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.268.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \sin (dx + c)^5 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

3.268.5 Fricas [F]

$$\int (a + b \sec(c + dx))^n \sin^5(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \sin (dx + c)^5 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n*si
n(d*x + c), x)
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \sin^5(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

output
```
Timed out
```

3.268.7 Maxima [F]

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.268.8 Giac [F]

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \int \sin(c+d\,x)^5 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.269 $\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$

3.269.1 Optimal result 2068
3.269.2 Mathematica [A] (verified) 2068
3.269.3 Rubi [A] (verified) 2069
3.269.4 Maple [F] 2071
3.269.5 Fricas [F] 2071
3.269.6 Sympy [F(-1)] 2071
3.269.7 Maxima [F] 2072
3.269.8 Giac [F(-2)] 2072
3.269.9 Mupad [F(-1)] 2072

3.269.1 Optimal result

Integrand size = 21, antiderivative size = 121

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$$
$$= \frac{b(6a^2-b^2(2-3n+n^2))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{6a^4d(1+n)}$$
$$+\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d}$$

output
```
1/6*b*(6*a^2-b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(
a+b*sec(d*x+c))^(1+n)/a^4/d/(1+n)+1/6*cos(d*x+c)^3*(a+b*sec(d*x+c))^(1+n)*
(2*a-b*(2-n)*sec(d*x+c))/a^2/d
```

3.269.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$$
$$= \frac{\cos(c+dx)\left(-\frac{2(2a-b(-2+n))(b+a\cos(c+dx))^2}{a}+8\cos^2\left(\frac{1}{2}(c+dx)\right)(b+a\cos(c+dx))^2-\frac{2b(-6a^2+b^2(2-3n+n^2))\,\text{H}}{12ad(b+a\cos(c+dx))}\right.$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

output
```
(Cos[c + d*x]*((-2*(2*a - b*(-2 + n))*(b + a*Cos[c + d*x])^2)/a + 8*Cos[(c
 + d*x)/2]^2*(b + a*Cos[c + d*x])^2 - (2*b*(-6*a^2 + b^2*(2 - 3*n + n^2))*
Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])
/(a*(-1 + n)))*(a + b*Sec[c + d*x])^n)/(12*a*d*(b + a*Cos[c + d*x]))
```

3.269.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4362, 25, 162, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4362$$

$$-\frac{\int -\cos^4(c+dx)(1-\sec(c+dx))(\sec(c+dx)+1)(a+b\sec(c+dx))^n d(-\sec(c+dx))}{d}$$

$$\downarrow 25$$

$$\frac{\int \cos^4(c+dx)(1-\sec(c+dx))(\sec(c+dx)+1)(a+b\sec(c+dx))^n d(-\sec(c+dx))}{d}$$

$$\downarrow 162$$

$$-\frac{\frac{1}{6}\left(6-\frac{b^2(1-n)(2-n)}{a^2}\right)\int \cos^2(c+dx)(a+b\sec(c+dx))^n d(-\sec(c+dx)) - \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{n+1}(2a-b(2-n)\sec}{6a^2}}{d}$$

$$\downarrow 75$$

$$-\frac{-\frac{b\left(6-\frac{b^2(1-n)(2-n)}{a^2}\right)(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{6a^2(n+1)} - \frac{\cos^3(c+dx)(2a-b(2-n)\sec(c+dx))(a+b\sec(c}{6a^2}}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

output
```
-((-1/6*(b*(6 - (b^2*(1 - n)*(2 - n))/a^2)*Hypergeometric2F1[2, 1 + n, 2 +
 n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^2*(1 + n)) -
(Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]
))/(6*a^2))/d)
```

3.269.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-f^(-1)   Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e
, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.269.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \sin (dx + c)^3 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

3.269.5 Fricas [F]

$$\int (a + b \sec(c + dx))^n \sin^3(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \sin (dx + c)^3 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \sin^3(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

output
```
Timed out
```

3.269.7 Maxima [F]

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.269.8 Giac [F(-2)]

Exception generated.

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = \int \sin(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.270 $\int (a + b \sec(c + dx))^n \sin(c + dx)\, dx$

3.270.1 Optimal result . . . 2073
3.270.2 Mathematica [A] (verified) . . . 2073
3.270.3 Rubi [A] (verified) . . . 2074
3.270.4 Maple [F] . . . 2075
3.270.5 Fricas [F] . . . 2075
3.270.6 Sympy [F] . . . 2075
3.270.7 Maxima [F] . . . 2076
3.270.8 Giac [F] . . . 2076
3.270.9 Mupad [B] (verification not implemented) . . . 2076

3.270.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a + b \sec(c + dx))^n \sin(c + dx)\, dx = \frac{b \operatorname{Hypergeometric2F1}\left(2, 1 + n, 2 + n, 1 + \frac{b \sec(c+dx)}{a}\right) (a + b \sec(c + dx))^{1+n}}{a^2 d (1 + n)}$$

output
```
b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/
(1+n)
```

3.270.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int (a + b \sec(c + dx))^n \sin(c + dx)\, dx = \frac{b \cos(c + dx) \operatorname{Hypergeometric2F1}\left(2, 1 - n, 2 - n, \frac{a \cos(c+dx)}{b + a \cos(c+dx)}\right) (a + b \sec(c + dx))^n}{d(-1 + n)(b + a \cos(c + dx))}$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

output
```
(b*Cos[c + d*x]*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a
*Cos[c + d*x])]*(a + b*Sec[c + d*x])^n)/(d*(-1 + n)*(b + a*Cos[c + d*x]))
```

3.270.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4362, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4362$$

$$-\frac{\int \cos^2(c+dx)(a+b\sec(c+dx))^n d(-\sec(c+dx))}{d}$$

$$\downarrow 75$$

$$\frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^2 d(n+1)}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

output
```
(b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^2*d*(1 + n))
```

3.270.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-f^(-1)   Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e
, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.270.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \sin (dx + c)\, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

3.270.5 Fricas [F]

$$\int (a + b \sec(c + dx))^n \sin(c + dx)\, dx = \int (b \sec (dx + c) + a)^n \sin (dx + c)\ dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.270.6 Sympy [F]

$$\int (a + b \sec(c + dx))^n \sin(c + dx)\, dx = \int (a + b \sec (c + dx))^n \sin (c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c),x)
```

output
```
Integral((a + b*sec(c + d*x))**n*sin(c + d*x), x)
```

3.270.7 Maxima [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.270.8 Giac [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \frac{\cos(c+dx)\left(a+\frac{b}{\cos(c+dx)}\right)^n {}_2\mathrm{F}_1\left(1-n,-n;\ 2-n;\ -\frac{a\cos(c+dx)}{b}\right)}{d\left(\frac{a\cos(c+dx)}{b}+1\right)^n (n-1)}$$

input
```
int(sin(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

output
```
(cos(c + d*x)*(a + b/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -(a*cos
(c + d*x))/b))/(d*((a*cos(c + d*x))/b + 1)^n*(n - 1))
```

3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$

3.271.1 Optimal result 2077
3.271.2 Mathematica [A] (verified) 2077
3.271.3 Rubi [A] (verified) 2078
3.271.4 Maple [F] 2080
3.271.5 Fricas [F] 2080
3.271.6 Sympy [F] 2080
3.271.7 Maxima [F] 2081
3.271.8 Giac [F] 2081
3.271.9 Mupad [F(-1)] 2081

3.271.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}$$
$$- \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)}$$

output
```
1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*
sec(d*x+c))^(1+n)/(a+b)/d/(1+n)
```

3.271.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \frac{\left(\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right) - 2^n\,\text{Hypergeometric2F1}\left(-n,-n,1-n,\frac{(-a+b)\cos(c+\right.\right.}{2dn}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

output
```
((Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*
x])] - (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a
+ b*Sec[c + d*x])^n)/(2*d*n)
```

3.271.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4362, 25, 82, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4362$$

$$-\frac{\int -\frac{(a+b\sec(c+dx))^n}{(1-\sec(c+dx))(\sec(c+dx)+1)}d(-\sec(c+dx))}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{(a+b\sec(c+dx))^n}{(1-\sec(c+dx))(\sec(c+dx)+1)}d(-\sec(c+dx))}{d}$$

$$\downarrow 82$$

$$\frac{\int \frac{(a+b\sec(c+dx))^n}{1-\sec^2(c+dx)}d(-\sec(c+dx))}{d}$$

$$\downarrow 485$$

$$\frac{\int \left(\frac{(a+b\sec(c+dx))^n}{2(1-\sec(c+dx))}+\frac{(a+b\sec(c+dx))^n}{2(\sec(c+dx)+1)}\right)d(-\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{2(n+1)(a+b)}-\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2(n+1)(a-b)}}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

output
```
-((-1/2*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*
(a + b*Sec[c + d*x])^(1 + n))/((a - b)*(1 + n)) + (Hypergeometric2F1[1, 1
+ n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2
*(a + b)*(1 + n)))/d)
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-f^(-1)   Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e
, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.271.4 Maple [F]

$$\int \csc(dx+c)\,(a+b\sec(dx+c))^n\,dx$$

input
```
int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

3.271.5 Fricas [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.271.6 Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (a+b\sec(c+dx))^n \csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*csc(c + d*x), x)
```

3.271.7 Maxima [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.271.8 Giac [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x), x)
```

3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$

3.272.1 Optimal result . . . 2082
3.272.2 Mathematica [B] (warning: unable to verify) . . . 2083
3.272.3 Rubi [A] (verified) . . . 2083
3.272.4 Maple [F] . . . 2085
3.272.5 Fricas [F] . . . 2085
3.272.6 Sympy [F(-1)] . . . 2085
3.272.7 Maxima [F] . . . 2086
3.272.8 Giac [F] . . . 2086
3.272.9 Mupad [F(-1)] . . . 2086

3.272.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx \\
&= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)d(1+n)} \\
&\quad + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} \\
&\quad + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}
\end{aligned}$$

output

```
1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)-1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*
sec(d*x+c))^(1+n)/(a+b)/d/(1+n)+1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*
x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+
n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

3.272.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1520 vs. $2(231) = 462$.

Time = 16.83 (sec) , antiderivative size = 1520, normalized size of antiderivative = 6.58

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

output
```
((a + b)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((-a + b)*Cos[c + d*x]*Sec[(
c + d*x)/2]^2)/b)^n*(a + b*Sec[c + d*x])^n*((1 - Tan[(c + d*x)/2]^2)^(-1))
^n*(1 - Tan[(c + d*x)/2]^4)^n*(b + (a - a*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
+ d*x)/2]^2))^n*(-((Hypergeometric2F1[n, 1 + n, 2 + n, (a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(2*b)]*(((a - b)*(-1 + Tan[(c + d*x)/2]
^2))/b)^n*(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)^(1 + n))/(
(a - b)*(1 + n)*(2 - 2*Tan[(c + d*x)/2]^2)^n)) + (Hypergeometric2F1[1, -n,
 1 - n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1 + Tan[(c + d*x)/2]^2) -
 b*(1 + Tan[(c + d*x)/2]^2))]*(a + b + (-a + b)*Tan[(c + d*x)/2]^2)^n)/(n*
(1 - Tan[(c + d*x)/2]^2)^n) - (Cot[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2]^2)
^(1 - n)*(a + b + (-a + b)*Tan[(c + d*x)/2]^2)^(1 + n))/(a + b) - (2^n*Hyp
ergeometric2F1[-n, -n, 1 - n, ((a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*(
a + b + (-a + b)*Tan[(c + d*x)/2]^2)^n)/(n*(1 - Tan[(c + d*x)/2]^2)^n*((a
+ b + (-a + b)*Tan[(c + d*x)/2]^2)/b)^n) - ((Hypergeometric2F1[n, 1 + n, 2
 + n, (a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(2*b)]*(((a -
b)*(-1 + Tan[(c + d*x)/2]^2))/b)^n*(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)^(1 + n))/((1 + n)*(2 - 2*Tan[(c + d*x)/2]^2)^n) + ((a - b)*
Hypergeometric2F1[1, -n, 1 - n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1
 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2))]*(a + b + (-a + b)*Ta
n[(c + d*x)/2]^2)^n)/(n*(1 - Tan[(c + d*x)/2]^2)^n) - (2^n*(a - b)*Hype...
```

3.272.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4362, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4362$$

$$-\frac{\int \frac{\sec^2(c+dx)(a+b\sec(c+dx))^n}{(1-\sec(c+dx))^2(\sec(c+dx)+1)^2}d(-\sec(c+dx))}{d}$$

$$\downarrow 198$$

$$-\frac{\int \left(\frac{(a+b\sec(c+dx))^n}{2(\sec^2(c+dx)-1)}+\frac{(a+b\sec(c+dx))^n}{4(-\sec(c+dx)-1)^2}+\frac{(a+b\sec(c+dx))^n}{4(1-\sec(c+dx))^2}\right)d(-\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{4(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right.}{4(n+1)(a+b)}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

output
```
-((-1/4*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*
(a + b*Sec[c + d*x])^(1 + n))/((a - b)*(1 + n)) + (Hypergeometric2F1[1, 1
+ n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4
*(a + b)*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d
*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*(1 + n)) - (b*Hyp
ergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c
 + d*x])^(1 + n))/(4*(a + b)^2*(1 + n)))/d)
```

3.272.3.1 Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4362
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-f^(-1)   Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e
, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.272.4 Maple [F]

$$\int \csc(dx+c)^3 (a+b\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

3.272.5 Fricas [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^3 \, dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n \, dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.272.8 Giac [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+dx)}\right)^n}{\sin(c+dx)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.273 $\int (a + b \sec(c + dx))^n \sin^4(c + dx)\, dx$

3.273.1 Optimal result . . . 2087
3.273.2 Mathematica [N/A] . . . 2087
3.273.3 Rubi [N/A] . . . 2088
3.273.4 Maple [N/A] (verified) . . . 2089
3.273.5 Fricas [N/A] . . . 2089
3.273.6 Sympy [F(-1)] . . . 2089
3.273.7 Maxima [N/A] . . . 2090
3.273.8 Giac [N/A] . . . 2090
3.273.9 Mupad [N/A] . . . 2090

3.273.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a + b \sec(c + dx))^n \sin^4(c + dx)\, dx = \text{Int}\left((a + b \sec(c + dx))^n \sin^4(c + dx), x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

3.273.2 Mathematica [N/A]

Not integrable

Time = 7.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b \sec(c + dx))^n \sin^4(c + dx)\, dx = \int (a + b \sec(c + dx))^n \sin^4(c + dx)\, dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

3.273.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^4\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4365$$

$$\int \sin^4(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

output
```
$Aborted
```

3.273.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.273.4 Maple [N/A] (verified)

Not integrable

Time = 2.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a + b \sec(dx + c))^n \sin(dx + c)^4 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

3.273.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int (a + b \sec(c + dx))^n \sin^4(c + dx) \, dx = \int (b \sec(dx + c) + a)^n \sin(dx + c)^4 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n, x
)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \sin^4(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**4,x)
```

output
```
Timed out
```

3.273.7 Maxima [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b\sec(c + dx))^n \sin^4(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \sin(dx + c)^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.273.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b\sec(c + dx))^n \sin^4(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \sin(dx + c)^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.273.9 Mupad [N/A]

Not integrable

Time = 18.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a + b\sec(c + dx))^n \sin^4(c + dx)\,dx = \int \sin(c + d\,x)^4 \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.274 $\int (a + b \sec(c + dx))^n \sin^2(c + dx)\, dx$

3.274.1 Optimal result 2091
3.274.2 Mathematica [N/A] 2091
3.274.3 Rubi [N/A] 2092
3.274.4 Maple [N/A] (verified) 2093
3.274.5 Fricas [N/A] 2093
3.274.6 Sympy [N/A] 2093
3.274.7 Maxima [N/A] 2094
3.274.8 Giac [N/A] 2094
3.274.9 Mupad [N/A] 2094

3.274.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a + b \sec(c + dx))^n \sin^2(c + dx)\, dx = \text{Int}\big((a + b \sec(c + dx))^n \sin^2(c + dx), x\big)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

3.274.2 Mathematica [N/A]

Not integrable

Time = 6.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b \sec(c + dx))^n \sin^2(c + dx)\, dx = \int (a + b \sec(c + dx))^n \sin^2(c + dx)\, dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

3.274.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4365$$

$$\int \sin^2(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

output
```
$Aborted
```

3.274.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.274.4 Maple [N/A] (verified)

Not integrable

Time = 1.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a + b \sec (dx + c))^n \sin (dx + c)^2 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

3.274.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int (a + b \sec(c + dx))^n \sin^2(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \sin (dx + c)^2 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n, x)
```

3.274.6 Sympy [N/A]

Not integrable

Time = 37.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a + b \sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a + b \sec (c + dx))^n \sin^2 (c + dx) \, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*sin(c + d*x)**2, x)
```

3.274.7 Maxima [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.274.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.274.9 Mupad [N/A]

Not integrable

Time = 15.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int \sin(c+d\,x)^2 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$

3.275.1 Optimal result . . . 2095
3.275.2 Mathematica [B] (warning: unable to verify) . . . 2095
3.275.3 Rubi [A] (verified) . . . 2096
3.275.4 Maple [F] . . . 2099
3.275.5 Fricas [F] . . . 2099
3.275.6 Sympy [F] . . . 2099
3.275.7 Maxima [F] . . . 2100
3.275.8 Giac [F] . . . 2100
3.275.9 Mupad [F(-1)] . . . 2100

3.275.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}bn\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{t}}{(a+b)d\sqrt{1+\sec(c+dx)}}$$

output
```
-cot(d*x+c)*(a+b*sec(d*x+c))^n/d+b*n*AppellF1(1/2,1-n,1/2,3/2,b*(1-sec(d*x
+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/(a+b)
/d/(((a+b*sec(d*x+c))/(a+b))^n)/(1+sec(d*x+c))^(1/2)
```

3.275.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3614 vs. 2(136) = 272.

Time = 17.33 (sec) , antiderivative size = 3614, normalized size of antiderivative = 26.57

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Result too large to show}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

output
```
((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Csc[c + d*x]^2*Sec[c + d*x]^n*(a
+ b*Sec[c + d*x])^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a
 - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/((
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1
[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2
]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n
, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] +
(a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/(2*d*(-1/4*((b + a*Cos[c + d*
x])^n*Csc[(c + d*x)/2]^2*Sec[c + d*x]^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) +
 (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n,
3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a
+ b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2))) - (a*n*(b
 + a*Cos[c + d*x])^(-1 + n)*Cot[(c + d*x)/2]*Sec[c + d*x]^n*Sin[c + d*x...
```

3.275.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4363, 25, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4363$$

$$-bn\int -\sec(c+dx)(a+b\sec(c+dx))^{n-1}dx - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

$$\downarrow 25$$

$$bn\int \sec(c+dx)(a+b\sec(c+dx))^{n-1}dx - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

$$\downarrow 3042$$

$$bn\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

$$\downarrow 4321$$

$$-\frac{bn\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{n-1}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}} - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

$$\downarrow 156$$

$$-\frac{bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{n-1}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d(a+b)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}} - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right.}{d(a+b)\sqrt{\sec(c+dx)+1}} - \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

output
```
-((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/
2, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a +
b*Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*
Sec[c + d*x])/(a + b))^n)
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4363
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Simp[b*m
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
 m}, x]
```

3.275.4 Maple [F]

$$\int \csc(dx+c)^2 (a+b\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

3.275.5 Fricas [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.275.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n \, dx = \int (a+b\sec(c+dx))^n \csc^2(c+dx) \, dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*csc(c + d*x)**2, x)
```

3.275.7 Maxima [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.275.8 Giac [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+dx)}\right)^n}{\sin(c+dx)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$

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3.276.1 Optimal result

Integrand size = 21, antiderivative size = 424

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx =$$

$$-\frac{3\,\text{AppellF1}\left(-\frac{1}{2},\frac{5}{2},-n,\frac{1}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\cot(c+dx)\sqrt{1+\sec(c+dx)}(a+b\sec(c+$$
$$2\sqrt{2}d$$

$$-\frac{\text{AppellF1}\left(-\frac{3}{2},\frac{5}{2},-n,-\frac{1}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\cot^3(c+dx)(1+\sec(c+dx))^{3/2}(a+b\sec($$
$$6\sqrt{2}d$$

$$+\frac{\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c+$$
$$\sqrt{2}d\sqrt{1+\sec(c+dx)}$$

$$+\frac{\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c+$$
$$2\sqrt{2}d\sqrt{1+\sec(c+dx)}$$

output

```
-1/12*AppellF1(-3/2,-n,5/2,-1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*cot(d*x+c)^3*(1+sec(d*x+c))^(3/2)*(a+b*sec(d*x+c))^n/d/(((a+b*sec(d*x+c))
/(a+b))^n)*2^(1/2)-3/4*AppellF1(-1/2,-n,5/2,1/2,b*(1-sec(d*x+c))/(a+b),1/2
-1/2*sec(d*x+c))*cot(d*x+c)*(a+b*sec(d*x+c))^n*(1+sec(d*x+c))^(1/2)/d/(((a
+b*sec(d*x+c))/(a+b))^n)*2^(1/2)+1/2*AppellF1(1/2,-n,3/2,3/2,b*(1-sec(d*x+
c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*sec(d
*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*AppellF1(1/2,-n,5/2,3/2,
b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d
/(((a+b*sec(d*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

3.276.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5928 vs. $2(424) = 848$.

Time = 23.44 (sec) , antiderivative size = 5928, normalized size of antiderivative = 13.98

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Result too large to show}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

output
```
Result too large to show
```

3.276.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4365$$

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

output
```
$Aborted
```

3.276.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.276.4 Maple [F]

$$\int \csc(dx+c)^4 (a+b\sec(dx+c))^n \, dx$$

input
```
int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

3.276.5 Fricas [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^4 \, dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n \, dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

output
```
Timed out
```

3.276.7 Maxima [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.276.8 Giac [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+dx)}\right)^n}{\sin(c+dx)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.277 $\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

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3.277.9 Mupad [N/A] . . . 2108

3.277.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \text{Int}\left((a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx), x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

3.277.2 Mathematica [N/A]

Not integrable

Time = 8.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

3.277.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4365$$

$$\int \sin^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

3.277.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.277.4 Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + b \sec (dx + c))^n \sin (dx + c)^{\frac{3}{2}} \, dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

3.277.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b \sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \sin (dx + c)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.277.7 Maxima [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.277.8 Giac [N/A]

Not integrable

Time = 22.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.277.9 Mupad [N/A]

Not integrable

Time = 16.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int \sin(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.278 $\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$

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3.278.2 Mathematica [N/A] . . . 2109
3.278.3 Rubi [N/A] . . . 2110
3.278.4 Maple [N/A] (verified) . . . 2111
3.278.5 Fricas [N/A] . . . 2111
3.278.6 Sympy [N/A] . . . 2111
3.278.7 Maxima [N/A] . . . 2112
3.278.8 Giac [N/A] . . . 2112
3.278.9 Mupad [N/A] . . . 2112

3.278.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \text{Int}\left((a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

3.278.2 Mathematica [N/A]

Not integrable

Time = 11.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

3.278.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(c+dx)}(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4365$$

$$\int \sqrt{\sin(c+dx)}(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

output
```
$Aborted
```

3.278.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.278.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a+b\sec(dx+c))^n \sqrt{\sin(dx+c)}dx$$

input
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

output
```
int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

3.278.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (b\sec(dx+c)+a)^n \sqrt{\sin(dx+c)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.278.6 Sympy [N/A]

Not integrable

Time = 16.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n*sqrt(sin(c + d*x)), x)
```

3.278.7 Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (b\sec(dx+c)+a)^n \sqrt{\sin(dx+c)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.278.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (b\sec(dx+c)+a)^n \sqrt{\sin(dx+c)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.278.9 Mupad [N/A]

Not integrable

Time = 15.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int \sqrt{\sin(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.279 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

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3.279.3 Rubi [N/A] 2114
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3.279.5 Fricas [N/A] 2115
3.279.6 Sympy [N/A] 2115
3.279.7 Maxima [N/A] 2116
3.279.8 Giac [N/A] 2116
3.279.9 Mupad [N/A] 2116

3.279.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

3.279.2 Mathematica [N/A]

Not integrable

Time = 10.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

3.279.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4365$$
$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

output
```
$Aborted
```

3.279.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.279.4 Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\sin(dx+c)}}dx$$

input
```
int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

output
```
int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

3.279.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.279.6 Sympy [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n/sqrt(sin(c + d*x)), x)
```

3.279.7 Maxima [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.279.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.279.9 Mupad [N/A]

Not integrable

Time = 16.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

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3.280.2 Mathematica [N/A] 2117
3.280.3 Rubi [N/A] 2118
3.280.4 Maple [N/A] (verified) 2119
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3.280.6 Sympy [N/A] 2119
3.280.7 Maxima [N/A] 2120
3.280.8 Giac [N/A] 2120
3.280.9 Mupad [N/A] 2120

3.280.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

3.280.2 Mathematica [N/A]

Not integrable

Time = 10.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

3.280.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4365}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-b\csc\left(c+dx-\frac{\pi}{2}\right)\right)^n}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}dx$$
$$\downarrow 4365$$
$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

3.280.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4365
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(g*Cos[e + f*x])^p*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.280.4 Maple [N/A] (verified)

Not integrable

Time = 0.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sin(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

output
```
int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

3.280.5 Fricas [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1),
x)
```

3.280.6 Sympy [N/A]

Not integrable

Time = 111.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n/sin(c + d*x)**(3/2), x)
```

3.280.7 Maxima [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.280.8 Giac [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.280.9 Mupad [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.281 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

3.281.1 Optimal result 2121
3.281.2 Mathematica [A] (verified) 2122
3.281.3 Rubi [A] (verified) 2122
3.281.4 Maple [C] (warning: unable to verify) 2127
3.281.5 Fricas [C] (verification not implemented) 2128
3.281.6 Sympy [F(-1)] 2129
3.281.7 Maxima [F] 2129
3.281.8 Giac [F] 2129
3.281.9 Mupad [F(-1)] 2130

3.281.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}
&\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} \\
&+\frac{ae^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&+\frac{ae^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&+\frac{2ae^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d}
\end{aligned}$$

output

```
-2/3*a*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2/3*a*e^2*csc(d*x+c)*(e*csc(d
*x+c))^(1/2)/d+a*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)/d+a*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/d
```

3.281.2 Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx =$$

$$-\frac{a(e\csc(c+dx))^{5/2}\left(6\arctan\left(\sqrt{\csc(c+dx)}\right)+4\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}+3\log\left(1-\sqrt{\csc(c+dx)}\right.\right.}{6d\csc^{\frac{5}{2}}(c+$$

input
```
Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
-1/6*(a*(e*Csc[c + d*x])^(5/2)*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 4*Cot[(c +
d*x)/2]*Sqrt[Csc[c + d*x]] + 3*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqr
t[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-2*c + Pi - 2*d*x)/4, 2
]*Sqrt[Sin[c + d*x]]))/(d*Csc[c + d*x]^(5/2))
```

3.281.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 264, 266, 756, 216, 219, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\csc(c+dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\downarrow 4366$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sec(c+dx)a+a}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}}dx$$

$$\downarrow 4360$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{(-\cos(c+dx)a-a)\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{5/2}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(a\int -\frac{\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx-a\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}dx\right)$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}dx-a\int \frac{\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx\right)$$

$$\downarrow 3042$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int \frac{1}{\sin(c+dx)^{5/2}}dx-a\int \frac{1}{\cos(c+dx)\sin(c+dx)^{5/2}}dx\right)$$

$$\downarrow 3044$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)(1-\sin^2(c+dx))}d\sin(c+dx)}{d}-a\int \frac{1}{\sin(c+dx)^{5/2}}dx\right)$$

$$\downarrow 264$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(\int\frac{1}{\sqrt{\sin(c+dx)}(1-\sin^2(c+dx))}d\sin(c+dx)-\frac{2}{3\sin^{\frac{3}{2}}(c+dx)}\right)}{d}-a\int\frac{1}{\sin(c+dx)^{5/}}\right.$$

$$\downarrow 266$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\int\frac{1}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}-\frac{2}{3\sin^{\frac{3}{2}}(c+dx)}\right)}{d}-a\int\frac{1}{\sin(c+dx)^{5/2}}dx\right)$$

$$\downarrow 756$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\int\frac{1}{\sin(c+dx)+1}d\sqrt{\sin(c+dx)}\right)-\frac{2}{3\sin^{\frac{3}{2}}(}\right.\right.$$

$$\downarrow 216$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3\sin^{\frac{3}{2}}(c+dx}\right.\right.$$

$$\downarrow 219$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int\frac{1}{\sin(c+dx)^{5/2}}dx-\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c-}\right.\right.\right.$$

$$\downarrow 3116$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}\right)-\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx}\right.\right.\right.\right.$$

$$\downarrow 3042$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}\right)-\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right.\right.\right.\right.$$

$$\downarrow 3120$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3\sin^{\frac{3}{2}}(c+dx)}\right)}{d}\right.$$

input
```
Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
-(e^2*Sqrt[e*Csc[c + d*x]]*(-((a*(2*(ArcTan[Sqrt[Sin[c + d*x]]]/2 + ArcTan
h[Sqrt[Sin[c + d*x]]]/2) - 2/(3*Sin[c + d*x]^(3/2))))/d) - a*((2*EllipticF
[(c - Pi/2 + d*x)/2, 2])/(3*d) - (2*Cos[c + d*x])/(3*d*Sin[c + d*x]^(3/2))
))*Sqrt[Sin[c + d*x]])
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.281.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.82

method	result
default	$\frac{a\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{2}\sqrt{-i(i+\cot(dx+c)-\csc(dx-}\right.}{\left.6d\sqrt{(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)}\sqrt{(1-co}\right.}$
parts	$\frac{a\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{2}\sqrt{-i(i+\cot(dx+c)-\csc(dx-}\right.}{\left.6d\sqrt{(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)}\sqrt{(1-co}\right.}$

input

```
int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/6*a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c))
)^(5/2)*(1-cos(d*x+c))^2*(2*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)
*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*E
llipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+cs
c(d*x+c))+(1-cos(d*x+c))^4*csc(d*x+c)^4-1)/((1-cos(d*x+c))^3*csc(d*x+c)^3+
csc(d*x+c)-cot(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^
2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2*csc(d*x+c)^2+1/
3*a/d*e^2*(e*csc(d*x+c))^(1/2)*(3*cos(d*x+c)*arctan((sin(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))-3*cos(d*x+c)*arctanh((sin(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))-2*(sin(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)-3*arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*
x+c)))+3*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c
))))/(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*csc(d*x+c)
```

3.281.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.59

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left[-\frac{6\,a\sqrt{-e}e^2\arctan\left(-\frac{\left(\cos(dx+c)^2-6\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\sin(dx+c)-3\,a\sqrt{-e}e^2\,\right.$$

$$\left.-\frac{6\,ae^{\frac{5}{2}}\arctan\left(\frac{\left(\cos(dx+c)^2+6\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\sin(dx+c)-3\,ae^{\frac{5}{2}}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(\right.\right.}{}\right.$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/24*(6*a*sqrt(-e)*e^2*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)
*sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*sin(d*x + c) - 3*a*sq
rt(-e)*e^2*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4
 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sq
rt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos
(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
*sin(d*x + c) + 8*I*a*sqrt(2*I*e)*e^2*sin(d*x + c)*weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*e^2*sin(d*x + c)*we
ierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 16*(a*e^2*cos(d*x
 + c) + a*e^2)*sqrt(e/sin(d*x + c)))/(d*sin(d*x + c)), -1/24*(6*a*e^(5/2)*
arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x +
c))/(e*sin(d*x + c) - e))*sin(d*x + c) - 3*a*e^(5/2)*log((e*cos(d*x + c)^4
 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x
 + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*
x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin(d*x + c) + 8*I*a*sqrt(2*I*e)
*e^2*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))
 - 8*I*a*sqrt(-2*I*e)*e^2*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 16*(a*e^2*cos(d*x + c) + a*e^2)*sqrt(e/sin(d*x + c
)))/(d*sin(d*x + c))]
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int (e \csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.281.7 Maxima [F]

$$\int (e \csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

3.281.8 Giac [F]

$$\int (e \csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2} dx$$

input
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2), x)
```

3.282 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

3.282.1 Optimal result 2131
3.282.2 Mathematica [C] (verified) 2132
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3.282.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\begin{aligned}
&\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} \\
&-\frac{ae\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&+\frac{ae\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&-\frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{d}
\end{aligned}$$

output

```
-2*a*e*(e*csc(d*x+c))^(1/2)/d-2*a*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-a*e*
arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*e*arcta
nh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a*e*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

3.282.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

$$\int (e\csc(c+dx))^{3/2}(a + a\sec(c+dx))\,dx = \frac{a(e\csc(c+dx))^{3/2}\left(2\arctan\left(\sqrt{\csc(c+dx)}\right) - 4(1+\cos(c+dx))\sqrt{\csc(c+dx)} - \log$$

input
```
Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(e*Csc[c + d*x])^(3/2)*(2*ArcTan[Sqrt[Csc[c + d*x]]] - 4*(1 + Cos[c + d
*x])*Sqrt[Csc[c + d*x]] - Log[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c
 + d*x]]] + (2*Csc[c + d*x]^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c
+ d*x]^2]*Sin[2*(c + d*x)])/Sqrt[-Cot[c + d*x]^2]))/(2*d*Csc[c + d*x]^(3/2
))
```

3.282.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 264, 266, 827, 216, 219, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\csc(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2} dx$$

$$\downarrow 4366$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{\sec(c+dx)a+a}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}dx$$

$$\downarrow 4360$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int-\frac{(-\cos(c+dx)a-a)\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int-\frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int-\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{3/2}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(a\int-\frac{\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx-a\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}dx-a\int\frac{\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\downarrow 3042$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int\frac{1}{\sin(c+dx)^{3/2}}dx-a\int\frac{1}{\cos(c+dx)\sin(c+dx)^{3/2}}dx\right)$$

$$\downarrow 3044$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)(1-\sin^2(c+dx))}d\sin(c+dx)}{d}-a\int\frac{1}{\sin(c+dx)^{3/2}}dx\right)$$

$$\downarrow 264$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(\int\frac{\sqrt{\sin(c+dx)}}{1-\sin^2(c+dx)}d\sin(c+dx)-\frac{2}{\sqrt{\sin(c+dx)}}\right)}{d}-a\int\frac{1}{\sin(c+dx)^{3/2}}dx\right)$$

$$\downarrow 266$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\int\frac{\sin(c+dx)}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}-\frac{2}{\sqrt{\sin(c+dx)}}\right)}{d}-a\int\frac{1}{\sin(c+dx)^{3/2}}dx\right)$$

$$\downarrow 827$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int\frac{1}{\sin(c+dx)^{3/2}}dx-\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}-\frac{1}{2}\int\frac{1}{\sin(c+dx)+1}d\right.\right.\right.}{d}$$

$$\downarrow 216$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{\sqrt{\sin(c+dx)}}\right.}{d}\right.$$

$$\downarrow 219$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\int\frac{1}{\sin(c+dx)^{3/2}}dx-\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+\right.\right.\right.}{d}\right.$$

$$\downarrow 3116$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\left(-\int\sqrt{\sin(c+dx)}dx-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}\right)-\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right.\right.\right.}{}$$

$$\downarrow 3042$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-a\left(-\int\sqrt{\sin(c+dx)}dx-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}\right)-\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right.\right.\right.}{}$$

$$\downarrow 3119$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{a\left(2\left(\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{\sqrt{\sin(c+dx)}}\right)}{d}-\right.$$

input
```
Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
-(e*Sqrt[e*Csc[c + d*x]]*(-((a*(2*(-1/2*ArcTan[Sqrt[Sin[c + d*x]]] + ArcTa
nh[Sqrt[Sin[c + d*x]]]/2) - 2/Sqrt[Sin[c + d*x]]))/d) - a*((-2*EllipticE[(
c - Pi/2 + d*x)/2, 2])/d - (2*Cos[c + d*x])/(d*Sqrt[Sin[c + d*x]])))*Sqrt[
Sin[c + d*x]])
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.282.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.78

method	result
default	$\frac{a\sqrt{2}\,e\sqrt{e\csc(dx+c)}\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-}\right.\right.$
parts	$\frac{a\sqrt{2}\,e\sqrt{e\csc(dx+c)}\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-}\right.\right.$

input
```
int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)
*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*E
llipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*
(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2),1/2*2^(1/2))*cos(d*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+
cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE
((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d
*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-2^
(1/2))-a/d*(e*csc(d*x+c))^(1/2)*e/(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(2*(
sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*(cot(d*x+c)+csc(d*x+c)))*cot(d*x+c)+arctanh((sin(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*cot(d*x+c)-arctan((sin(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c)-arctanh((sin(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c))
```

3.282.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.60

$$\int (e\csc(c+dx))^{3/2}(a$$

$$+a\sec(c+dx))\,dx = \left[-\frac{2\,a\sqrt{-e}e\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)-a\sqrt{-e}e\log\left(\frac{e\cos(dx+c)^4-}{}\right.\right.$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/8*(2*a*sqrt(-e)*e*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) - a*sqrt(-e)*e*log((e*co
s(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2
+ (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) +
 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*a*sqrt(2*I*e)*e
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 8*a*sqrt(-2*I*e)*e*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 16*(a*e*cos(d*x + c) + a*e)*sqrt(e/sin
(d*x + c)))/d, 1/8*(2*a*e^(3/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c
) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e)) + a*e^(3/2)*log(
(e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x +
c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c
)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 8*a*sqrt(2*I*
e)*e*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 8*a*sqrt(-2*I*e)*e*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 16*(a*e*cos(d*x + c) + a*e)*sqrt(e
/sin(d*x + c)))/d]
```

3.282.6 Sympy [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = a\left(\int (e\csc(c+dx))^{\frac{3}{2}}\,dx + \int (e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx\right)$$

input
```
integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral((e*csc(c + d*x))**(3/2), x) + Integral((e*csc(c + d*x))**(3/2)
*sec(c + d*x), x))
```

3.282.7 Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Timed out
```

3.282.8 Giac [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2} dx$$

input
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2), x)
```

3.283 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$

3.283.1 Optimal result 2141
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3.283.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}
&\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx \\
&= \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{2a\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}
\end{aligned}$$

output

```
a*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*arcta
nh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-2*a*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

3.283.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx =$$
$$-\frac{a\sqrt{e\csc(c+dx)}\left(2\arctan\left(\sqrt{\csc(c+dx)}\right)+\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(1+\sqrt{\csc(c+dx)}\right)+4\sqrt{}\right.}{2d\sqrt{\csc(c+dx)}}$$

input
```
Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*(a*Sqrt[e*Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] + Log[1 - Sqrt[
Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*Ellipt
icF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]))/(d*Sqrt[Csc[c + d*x]])
```

3.283.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3317, 25, 3042, 3044, 266, 756, 216, 219, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\sqrt{e\csc(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}dx$$
$$\downarrow 4366$$
$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{\sec(c+dx)a+a}{\sqrt{\sin(c+dx)}}dx$$
$$\downarrow 3042$$
$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4360$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{(-\cos(c+dx)a-a)\sec(c+dx)}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\int -\frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{(\cos(c+dx)a+a)\sec(c+dx)}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{a-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\int \frac{a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3317$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(a\int -\frac{\sec(c+dx)}{\sqrt{\sin(c+dx)}}dx-a\int \frac{1}{\sqrt{\sin(c+dx)}}dx\right)$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-a\int \frac{1}{\sqrt{\sin(c+dx)}}dx-a\int \frac{\sec(c+dx)}{\sqrt{\sin(c+dx)}}dx\right)$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-a\int \frac{1}{\sqrt{\sin(c+dx)}}dx-a\int \frac{1}{\cos(c+dx)\sqrt{\sin(c+dx)}}dx\right)$$

$$\downarrow 3044$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-\frac{a\int \frac{1}{\sqrt{\sin(c+dx)}(1-\sin^2(c+dx))}d\sin(c+dx)}{d}-a\int \frac{1}{\sqrt{\sin(c+dx)}}dx\right)$$

$$\downarrow 266$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-\frac{2a\int \frac{1}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}}{d}-a\int \frac{1}{\sqrt{\sin(c+dx)}}dx\right)$$

$$\downarrow 756$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-a\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2a\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\int\frac{1}{\sin(c+dx)+1}d\right.\right.}{d}$$

$$\downarrow 216$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-\frac{2a\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)}{d}-a\int\frac{}{\sqrt{\sin}}\right.$$

$$\downarrow 219$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-a\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2a\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+\right.\right.}{d}$$

$$\downarrow 3120$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(-\frac{2a\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)}{d}-\frac{2a\,\text{EllipticF}\,(}{}\right.$$

input
```
Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
-(Sqrt[e*Csc[c + d*x]]*((-2*a*(ArcTan[Sqrt[Sin[c + d*x]]]/2 + ArcTanh[Sqrt
[Sin[c + d*x]]]/2))/d - (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/d)*Sqrt[Sin
[c + d*x]])
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.283.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

method	result
default	$\frac{(\frac{1}{2}-\frac{i}{2})a\left(i\,\mathrm{EllipticPi}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticPi}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)(\cos(dx+$
parts	$\frac{ia(\cos(dx+c)+1)\sqrt{2}\,\sqrt{e\csc(dx+c)}\,\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\mathrm{El}$

input
```
int((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)*a/d*(I*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,
1/2*2^(1/2)))*(cos(d*x+c)+1)*2^(1/2)*(e*csc(d*x+c))^(1/2)*(-I*(I-cot(d*x+c
)+csc(d*x+c)))^(1/2)*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+
csc(d*x+c)))^(1/2)
```

3.283.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.45

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$$

$$= \left[-\frac{2\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right) - a\sqrt{-e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(dx\right.\right.\right.}{}\right.$$

$$\left. -\frac{2\,a\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right) - a\sqrt{e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(dx+c)^4-9\,\cos(dx\right.\right.\right.}{\cos(dx\right.}\right]$$

input
```
integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt
(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) - a*sqrt(-e)*log((e*cos(d*
x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7
*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*
(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x
+ c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*I*a*sqrt(2*I*e)*wei
erstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)
*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/d, -1/8*(2*a*sq
rt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(
d*x + c))/(e*sin(d*x + c) - e)) - a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*c
os(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 -
 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2
- 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*
x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*I*a*sqrt(2*I*e)*weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weierstrassPInver
se(4, 0, cos(d*x + c) - I*sin(d*x + c)))/d]
```

3.283.6 Sympy [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{e\csc(c+dx)}\,dx + \int \sqrt{e\csc(c+dx)}\sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))**(1/2),x)
```

output
```
a*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(sqrt(e*csc(c + d*x))*sec(c
 + d*x), x))
```

3.283.7 Maxima [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

3.283.8 Giac [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e \csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+dx)}\right)\sqrt{\frac{e}{\sin(c+dx)}}\,dx$$

input
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2), x)
```

3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$

3.284.1 Optimal result 2150
3.284.2 Mathematica [C] (verified) 2150
3.284.3 Rubi [A] (verified) 2151
3.284.4 Maple [C] (warning: unable to verify) 2154
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3.284.9 Mupad [F(-1)] 2157

3.284.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
-a*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arct
anh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2*a*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.284.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.76 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$$
$$= \frac{a\left(-4\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right)+\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\left(2\arctan\right.\right.}{2d\sqrt{-\cot^2(c+dx)}\sqrt{e\csc(c+d}}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

output
```
(a*(-4*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] + Sq
rt[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] - Log
[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))/(2*d*Sqrt[-Cot[c
 + d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

3.284.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 266, 827, 216, 219, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{e\csc(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}\,dx$$

$$\downarrow 4366$$

$$\frac{\int(\sec(c+dx)a+a)\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int-\left((-\cos(c+dx)a-a)\sec(c+dx)\sqrt{\sin(c+dx)}\right)dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int-\left((\cos(c+dx)a+a)\sec(c+dx)\sqrt{\sin(c+dx)}\right)dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int (\cos(c+dx)a+a)\sec(c+dx)\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3317$$

$$\frac{a\int \sqrt{\sin(c+dx)}dx + a\int \sec(c+dx)\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sqrt{\sin(c+dx)}dx + a\int \frac{\sqrt{\sin(c+dx)}}{\cos(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{a\int \frac{\sqrt{\sin(c+dx)}}{1-\sin^2(c+dx)}d\sin(c+dx)}{d} + a\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\frac{2a\int \frac{\sin(c+dx)}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}}{d} + a\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 827$$

$$\frac{a\int \sqrt{\sin(c+dx)}dx + \frac{2a\left(\frac{1}{2}\int \frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)} - \frac{1}{2}\int \frac{1}{\sin(c+dx)+1}d\sqrt{\sin(c+dx)}\right)}{d}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 216$$

$$\frac{\frac{2a\left(\frac{1}{2}\int \frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)} - \frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)}{d} + a\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 219$$

$$\frac{a\int \sqrt{\sin(c+dx)}dx + \frac{2a\left(\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right) - \frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)}{d}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3119$$

$$\frac{\frac{2a\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{d}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

output
```
((2*a*(-1/2*ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]]/2))/d
 + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/d)/(Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]])
```

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.284.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.39 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.46

method	result
parts	$-\frac{a\sqrt{2}\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+\right.\right.$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d*2^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc
(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*
x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(I-cot(d*x+c)+csc(d*x+
c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)
))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d
*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c))
)^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+c
ot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF(
(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/
2))/(e*csc(d*x+c))^(1/2)*csc(d*x+c)+a/d*(arctanh((sin(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(cot(d*x+c)+csc(d*x+c))))/(cos(d*x+c)+1)/(e*csc(d*x+c))^(1/2)/(sin(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

3.284.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.54

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$$

$$= \left[-\frac{2\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right) + a\sqrt{-e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(dx\right.\right.\right.}{\left.\left.\left.\right.\right.\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt
(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + a*sqrt(-e)*log((e*cos(d*
x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7
*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*
(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x
+ c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 8*a*sqrt(2*I*e)*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 8*a*sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c))))/(d*e), 1/8*(2*a*sqrt(e)*arctan(1/4*(cos(d*x + c
)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e
)) + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x +
c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)
*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(
cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) +
8)) + 8*a*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 8*a*sqrt(-2*I*e)*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(d*e)]
```

3.284.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\csc(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\csc(c + dx)}}\,dx + \int \frac{\sec(c + dx)}{\sqrt{e\csc(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

output
```
a*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*csc(
c + d*x)), x))
```

3.284.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\csc(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{e\csc(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

3.284.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\csc(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{e\csc(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\csc(c + dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+dx)}}{\sqrt{\frac{e}{\sin(c+dx)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2), x)
```

3.285 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

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3.285.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
-2*a/d/e/(e*csc(d*x+c))^(1/2)-2/3*a*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+a*
arctan(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arcta
nh(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2/3*a*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 9.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx =$$
$$-\frac{a\left(12 + 4\cos(c+dx) + 6\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\csc(c+dx)} + 3\sqrt{\csc(c+dx)}\log\left(1-\sqrt{\csc(c+dx)}\right)\right.}{6de\sqrt{e\csc(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

output
```
-1/6*(a*(12 + 4*Cos[c + d*x] + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d
*x]] + 3*Sqrt[Csc[c + d*x]]*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Sqrt[Csc[c + d
*x]]*Log[1 + Sqrt[Csc[c + d*x]]] + (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])
/Sqrt[Sin[c + d*x]]))/(d*e*Sqrt[e*Csc[c + d*x]])
```

3.285.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.70, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 262, 266, 756, 216, 219, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\csc(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a - a\csc\left(c+dx-\frac{\pi}{2}\right)}{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4366$$
$$\frac{\int(\sec(c+dx)a+a)\sin^{\frac{3}{2}}(c+dx)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$
$$\downarrow 3042$$

$$\frac{\int \cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\Big((-\cos(c+dx)a-a)\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\Big)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\Big((\cos(c+dx)a+a)\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\Big)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int (\cos(c+dx)a+a)\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3317$$

$$\frac{a\int \sin^{\frac{3}{2}}(c+dx)dx+a\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sin(c+dx)^{3/2}dx+a\int \frac{\sin(c+dx)^{3/2}}{\cos(c+dx)}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{a\int \frac{\sin^{\frac{3}{2}}(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+a\int \sin(c+dx)^{3/2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 262$$

$$\frac{\frac{a\left(\int \frac{1}{\sqrt{\sin(c+dx)}\left(1-\sin^2(c+dx)\right)}d\sin(c+dx)-2\sqrt{\sin(c+dx)}\right)}{d}+a\int \sin(c+dx)^{3/2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\frac{a\left(2\int\frac{1}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}-2\sqrt{\sin(c+dx)}\right)}{d}+a\int\sin(c+dx)^{3/2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 756$$

$$\frac{a\int\sin(c+dx)^{3/2}dx+\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\int\frac{1}{\sin(c+dx)+1}d\sqrt{\sin(c+dx)}\right)-2\sqrt{\sin(c+dx)}\right)}{d}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 216$$

$$\frac{\frac{a\left(2\left(\frac{1}{2}\int\frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}+\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-2\sqrt{\sin(c+dx)}\right)}{d}+a\int\sin(c+dx)^{3/2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 219$$

$$\frac{a\int\sin(c+dx)^{3/2}dx+\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)-2\sqrt{\sin(c+dx)}\right)}{d}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3115$$

$$\frac{a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right)+\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)-2\sqrt{\sin(c+dx)}\right)}{d}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right)+\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)-2\sqrt{\sin(c+dx)}\right)}{d}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3120$$

$$\frac{\frac{a\left(2\left(\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)+\frac{1}{2}\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\right)-2\sqrt{\sin(c+dx)}\right)}{d}+a\left(\frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{3d}-\frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right)}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

output
```
((a*(2*(ArcTan[Sqrt[Sin[c + d*x]]]/2 + ArcTanh[Sqrt[Sin[c + d*x]]]/2) - 2*
Sqrt[Sin[c + d*x]]))/d + a*((2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (
2*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/(3*d)))/(e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]])
```

3.285. $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

3.285.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a  Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d  Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]  Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.285.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{a\sqrt{2}\left(i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{-i(i-\cot(dx+c}\right.\right.$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*a/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)
-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I*(-I*(I-cot(d*x+c)+cs
c(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(
d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))
-cos(d*x+c)*sin(d*x+c)*2^(1/2))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/(cos
(d*x+c)+1)*sin(d*x+c)+a/d*(arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot
(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)-(sin(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*
(cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-(sin(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(c
ot(d*x+c)+csc(d*x+c)))+2*sin(d*x+c))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e
/(cos(d*x+c)+1)*sin(d*x+c)
```

3.285.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.34

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \left[-\frac{6\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)+3\,a\sqrt{-e}\log\left(\frac{e\cos(d}{}\right.\right.$$

$$\left.\left. -\frac{6\,a\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)-3\,a\sqrt{e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(dx+c)^4-9\,\cos(\right.}{\cos(dx+\right.}\right.\right.$$

input

```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(6*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqr
t(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + 3*a*sqrt(-e)*log((e*cos
(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 +
 (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) +
28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*cos(d*x + c)
 + 3*a)*sqrt(e/sin(d*x + c))*sin(d*x + c) + 8*I*a*sqrt(2*I*e)*weierstrassP
Inverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weierstr
assPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(d*e^2), -1/24*(6*a*sqrt
(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*
x + c))/(e*sin(d*x + c) - e)) - 3*a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*c
os(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 -
 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2
- 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*
x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*cos(d*x + c) + 3*a)*sqrt(e/sin(d*
x + c))*sin(d*x + c) + 8*I*a*sqrt(2*I*e)*weierstrassPInverse(4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos
(d*x + c) - I*sin(d*x + c)))/(d*e^2)]
```

3.285.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{(e\csc(c + dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\csc(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c + dx)}{(e\csc(c + dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*csc(c
+ d*x))**(3/2), x))
```

3.285.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{(e\csc(c + dx))^{3/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\csc(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

3.285.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\csc(c + dx))^{3/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\csc(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\csc(c + dx))^{3/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+dx)}}{\left(\frac{e}{\sin(c+dx)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2), x)
```

3.286 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

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3.286.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}$$

output

```
-2/3*a*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*sin(d*x+c)/d
/e^2/(e*csc(d*x+c))^(1/2)-a*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^
(1/2)/sin(d*x+c)^(1/2)+a*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1
/2)/sin(d*x+c)^(1/2)-6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2/(e*csc(d
*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.286.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \frac{a\Big(-72\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right) - 2\sqrt{-\cot^2(c+}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

output
```
(a*(-72*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] - 2
*Sqrt[-Cot[c + d*x]^2]*(-30*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]]
+ 15*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c
+ d*x]]]) + 20*Sin[c + d*x] + 6*Sin[2*(c + d*x)])))/(60*d*e^2*Sqrt[-Cot[c
+ d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

3.286.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.65, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3317, 3042, 3044, 262, 266, 827, 216, 219, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\csc(c+dx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a - a\csc\left(c+dx-\frac{\pi}{2}\right)}{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int(\sec(c+dx)a+a)\sin^{\frac{5}{2}}(c+dx)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\Big((-\cos(c+dx)a-a)\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\Big)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\Big((\cos(c+dx)a+a)\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\Big)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int (\cos(c+dx)a+a)\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3317$$

$$\frac{a\int \sin^{\frac{5}{2}}(c+dx)dx+a\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sin(c+dx)^{5/2}dx+a\int \frac{\sin(c+dx)^{5/2}}{\cos(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{a\int \frac{\sin^{\frac{5}{2}}(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+a\int \sin(c+dx)^{5/2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 262$$

$$\frac{\frac{a\left(\int \frac{\sqrt{\sin(c+dx)}}{1-\sin^2(c+dx)}d\sin(c+dx)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}+a\int \sin(c+dx)^{5/2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\frac{a\left(2\int \frac{\sin(c+dx)}{1-\sin^2(c+dx)}d\sqrt{\sin(c+dx)}-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}+a\int \sin(c+dx)^{5/2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 827$$

$$\frac{\frac{a\left(2\left(\frac{1}{2}\int \frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}-\frac{1}{2}\int \frac{1}{\sin(c+dx)+1}d\sqrt{\sin(c+dx)}\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}+a\int \sin(c+dx)^{5/2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 216$$

$$\frac{\frac{a\left(2\left(\frac{1}{2}\int \frac{1}{1-\sin(c+dx)}d\sqrt{\sin(c+dx)}-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}+a\int \sin(c+dx)^{5/2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 219$$

$$\frac{a\int \sin(c+dx)^{5/2}dx+\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3115$$

$$\frac{a\left(\frac{3}{5}\int \sqrt{\sin(c+dx)}dx-\frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}\right)+\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\left(\frac{3}{5}\int \sqrt{\sin(c+dx)}dx-\frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}\right)+\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3119$$

$$\frac{\frac{a\left(2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\sin(c+dx)}\right)\right)-\frac{2}{3}\sin^{\frac{3}{2}}(c+dx)\right)}{d}+a\left(\frac{6E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{5d}-\frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}\right)}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

3.286. $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

output
```
((a*(2*(-1/2*ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]]/2) -
 (2*Sin[c + d*x]^(3/2))/3))/d + a*((6*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5
*d) - (2*Cos[c + d*x]*Sin[c + d*x]^(3/2))/(5*d)))/(e^2*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]])
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3317
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a   Int[(g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Simp[b/d   Int[(g*Cos[e + f*x])^
p*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.286.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.06

method	result
default	$\frac{a\sqrt{2}\left(-6\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c}\right.\right.$
parts	$\frac{a\sqrt{2}\left(-6\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c}\right.\right.$

input
```
int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*a/d*2^(1/2)*(-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)
-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+3*(-I*(I-cot(d*x+c)+cs
c(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(
d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))
*cos(d*x+c)+cos(d*x+c)^3*2^(1/2)-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-
I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Elli
pticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(-I*(I-cot(d*x+c
)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-
csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1
/2))-4*2^(1/2)*cos(d*x+c)+3*2^(1/2))/(e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)-1
/3*a/d*(3*arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c
)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+3*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))+2
*cos(d*x+c)-2)/(cos(d*x+c)-1)/e^2/(e*csc(d*x+c))^(1/2)*sin(d*x+c)
```

3.286.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.32

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \left[-\frac{30\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)+15\,a\sqrt{-e}\log\left(\frac{e\,\text{co}}{}\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/120*(30*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*s
qrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + 15*a*sqrt(-e)*log((e*
cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^
2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c))
 + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*co
s(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 72*a*sqrt(2*I*e
)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 72*a*sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 16*(3*a*cos(d*x + c)^3 + 5*a*cos(d*x +
 c)^2 - 3*a*cos(d*x + c) - 5*a)*sqrt(e/sin(d*x + c)))/(d*e^3), 1/120*(30*a
*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/s
in(d*x + c))/(e*sin(d*x + c) - e)) + 15*a*sqrt(e)*log((e*cos(d*x + c)^4 -
72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x +
c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x +
 c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(
cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 72*a*sqrt(2*I*e)*weierstrassZeta(
4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 72*a*sqr
t(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 16*(3*a*cos(d*x + c)^3 + 5*a*cos(d*x + c)^2 - 3*a*cos(d*
x + c) - 5*a)*sqrt(e/sin(d*x + c)))/(d*e^3)]
```

3.286.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(5/2),x)
```

output
```
a*(Integral((e*csc(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*csc(c
+ d*x))**(5/2), x))
```

3.286.7 Maxima [F]

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

3.286.8 Giac [F]

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+dx)}}{\left(\frac{e}{\sin(c+dx)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2), x)
```

3.287 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

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3.287.1 Optimal result

Integrand size = 25, antiderivative size = 270

$$\begin{aligned}\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx &= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}\\ &-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d}\\ &+\frac{2a^2e^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &+\frac{2a^2e^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &+\frac{7a^2e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d}\\ &+\frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d}\end{aligned}$$

output

```
-2/3*a^2*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-4/3*a^2*e^2*csc(d*x+c)*(e*c
sc(d*x+c))^(1/2)/d-2/3*a^2*e^2*csc(d*x+c)*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/
d+2*a^2*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
/d+2*a^2*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/d-7/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)/d+5/3*a^2*e^2*(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.287.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2e^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\left(-7+6\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sqrt{\csc(c+dx)}-6\text{arc}\right.}{}$$

input
```
Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(a^2*e^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*(-7 + 6*ArcTan[Sqrt[
Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 6*ArcTanh[Sqrt[Cs
c[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 4*Csc[c + d*x]^2 +
4*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^2 + 7*Sqrt[-Cot[c + d*x]^2]*Hypergeome
tric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[ArcCsc[Csc[c + d*x]]/2]^4*Tan[
c + d*x])/d
```

3.287.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2(e\csc(c+dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\downarrow 4366$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(\sec(c+dx)a+a)^2}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}}dx$$

$$\downarrow 4360$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(-\cos(c+dx)a-a)^2\sec^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\sin\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{\sec^2(c+dx)a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{2\sec(c+dx)a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{a^2}{\sin^{\frac{5}{2}}(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{4a^2}{3d\sin^{\frac{3}{2}}(c+dx)}+\right.$$

input
```
Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
e^2*Sqrt[e*Csc[c + d*x]]*((2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/d + (2*a^2*Ar
cTanh[Sqrt[Sin[c + d*x]]])/d + (7*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3
*d) - (4*a^2)/(3*d*Sin[c + d*x]^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*Sin[c +
 d*x]^(3/2)) - (2*a^2*Sec[c + d*x])/(3*d*Sin[c + d*x]^(3/2)) + (5*a^2*Sec[
c + d*x]*Sqrt[Sin[c + d*x]])/(3*d))*Sqrt[Sin[c + d*x]]
```

3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.287.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.70 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.25

method	result
default	$\frac{a^2e^2\sqrt{2}\left(6i\sin(dx+c)\cos(dx+c)\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\ \text{Ellip}\right.$
parts	Expression too large to display

input
```
int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*e^2/d*2^(1/2)*(6*I*sin(d*x+c)*cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x
+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c
)))^(1/2)*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2+1/2*I,1/2*2^
(1/2))+6*I*sin(d*x+c)*cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*
(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*Ellipt
icPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*sin(d
*x+c)*cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-cs
c(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d
*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)*sin(d*x+c)*(-I*(I-cot(d
*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x
+c)+csc(d*x+c)))^(1/2)*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2
+1/2*I,1/2*2^(1/2))-6*cos(d*x+c)*sin(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))
^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(
1/2)*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-7*2^(1/2)*cos(d*x+c)+3*2^(1/2))*(cos(d*x+c)+1)*(e*csc(d*x+c))^(1/2)*sec(
d*x+c)*csc(d*x+c)
```

3.287.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 806, normalized size of antiderivative = 2.99

$$\int (e \csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/12*(6*a^2*sqrt(-e)*e^2*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) -
2)*sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c)*sin(d*
x + c) - 3*a^2*sqrt(-e)*e^2*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(
d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)
*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 -
2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x
+ c)^2 - 2)*sin(d*x + c) + 8))*sin(d*x + c) + 14*I*a^2*sqrt(2*I*e)*e^2*cos
(d*x + c)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 14*I*a^2*sqrt(-2*I*e)*e^2*cos(d*x + c)*sin(d*x + c)*weierstrassPIn
verse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 4*(7*a^2*e^2*cos(d*x + c)^2 +
 4*a^2*e^2*cos(d*x + c) - 3*a^2*e^2)*sqrt(e/sin(d*x + c)))/(d*cos(d*x + c)
*sin(d*x + c)), -1/12*(6*a^2*e^(5/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*
x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c
)*sin(d*x + c) - 3*a^2*e^(5/2)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*c
os(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 -
 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2
- 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*
x + c)^2 - 2)*sin(d*x + c) + 8))*sin(d*x + c) + 14*I*a^2*sqrt(2*I*e)*e^2*c
os(d*x + c)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 14*I*a^2*sqrt(-2*I*e)*e^2*cos(d*x + c)*sin(d*x + c)*weierstra...
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int (e \csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.287.7 Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.287.8 Giac [F]

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+dx)}\right)^2\left(\frac{e}{\sin(c+dx)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2), x)
```

3.288 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

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3.288.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx &= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}\\ &-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d}\\ &-\frac{2a^2e\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &+\frac{2a^2e\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &-\frac{5a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{d}\\ &+\frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d}\end{aligned}$$

output

```
-4*a^2*e*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-
2*a^2*e*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*arctan(sin(d*x+c)^(1/2))
*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^2*e*arctanh(sin(d*x+c)^(1/2))
*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+5*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+3*a^2*e*sin(d*x+c)*(e*csc(
d*x+c))^(1/2)*tan(d*x+c)/d
```

3.288.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.81

$$\int (e\csc(c+dx))^{3/2}(a$$

$$+a\sec(c+dx))^2\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(3\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}+3\text{ar}\right.}{}$$

input
```
Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*(3*ArcTan[Sqrt[Csc[c + d*
x]]]*Sqrt[Cos[c + d*x]^2] + 3*ArcTanh[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x
]^2] - 6*Sqrt[Csc[c + d*x]] - 6*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] +
5*Sqrt[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*Hypergeometric2F1[3/4, 3/2, 7/4
, Csc[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c + d*x]]/2]^4)/(3*d*Csc[c
+ d*x]^(3/2))
```

3.288.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2(e\csc(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\downarrow 4366$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}dx$$

$$\downarrow 4360$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(-\cos(c+dx)a-a)^2\sec^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\sin\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 3352$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{\sec^2(c+dx)a^2}{\sin^{\frac{3}{2}}(c+dx)}+\frac{2\sec(c+dx)a^2}{\sin^{\frac{3}{2}}(c+dx)}+\frac{a^2}{\sin^{\frac{3}{2}}(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{4a^2}{d\sqrt{\sin(c+dx)}}-\right.$$

input
```
Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]*((-2*a^2*ArcTan[Sqrt[Sin[c + d*x
]]])/d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/d - (5*a^2*EllipticE[(c - Pi/
2 + d*x)/2, 2])/d - (4*a^2)/(d*Sqrt[Sin[c + d*x]]) - (2*a^2*Cos[c + d*x])/
(d*Sqrt[Sin[c + d*x]]) - (2*a^2*Sec[c + d*x])/(d*Sqrt[Sin[c + d*x]]) + (3*
a^2*Sec[c + d*x]*Sin[c + d*x]^(3/2))/d)
```

3.288.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.288.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.20 (sec) , antiderivative size = 1069, normalized size of antiderivative = 4.45

method	result	size
parts	Expression too large to display	1069
default	Expression too large to display	1231

input
```
int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/
2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)
*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-
I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-
I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^
(1/2),1/2*2^(1/2))*cos(d*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(
I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Ellipti
cE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d
*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-
2^(1/2))+1/2*a^2/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(6*(-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d
*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*
cos(d*x+c)-3*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*
x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c
)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+6*(-I*(I-cot(d*x+c)+csc(d*x+c
)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c))
)^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-3*(-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*
(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))...
```

3.288.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.04

$$\int (e\csc(c+dx))^{3/2}(a$$

$$+a\sec(c+dx))^2\,dx = \left[-\frac{2\,a^2\sqrt{-e}e\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos(dx+c)-a^2\sqrt{-e}e\,c}{} \right.$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a^2*sqrt(-e)*e*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*
sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) - a^2*sqr
t(-e)*e*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(
d*x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*s
qrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) +
72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) + 10*a^2*sqrt(2*I*e)*e*cos(d*x + c)*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 10*a^2*sqrt(-2*I*e)
*e*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c))) + 4*(5*a^2*e*cos(d*x + c)^2 + 4*a^2*e*cos(d*x + c) -
 a^2*e)*sqrt(e/sin(d*x + c)))/(d*cos(d*x + c)), 1/4*(2*a^2*e^(3/2)*arctan(
1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*
sin(d*x + c) - e))*cos(d*x + c) + a^2*e^(3/2)*cos(d*x + c)*log((e*cos(d*x
+ c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*c
os(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*
cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c
)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 10*a^2*sqrt(2*I*e)*e*cos
(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I
*sin(d*x + c))) - 10*a^2*sqrt(-2*I*e)*e*cos(d*x + c)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 4*(5*a^2*e...
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int (e \csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.288.7 Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.288.8 Giac [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+dx)}\right)^2\left(\frac{e}{\sin(c+dx)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2), x)
```

3.289 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx$

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3.289.1 Optimal result

Integrand size = 25, antiderivative size = 154

$$\begin{aligned}
&\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx \\
&= \frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{3a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

output

```
2*a^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a
^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-3*a^2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+
a^2*(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.289.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.89 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{2a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\left(-1+2\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sqrt{\csc(c+dx)}-2a\right.}{}$$

input
```
Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*Cos[(c + d*x)/2]^5*Sqrt[e*Csc[c + d*x]]*(-1 + 2*ArcTan[Sqrt[Csc[c
+ d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 2*ArcTanh[Sqrt[Csc[c +
d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 3*Sqrt[-Cot[c + d*x]^2]*H
ypergeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[C
sc[c + d*x]]/2]^4*Sin[(c + d*x)/2])/d
```

3.289.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2\sqrt{e\csc(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4366$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(\sec(c+dx)a+a)^2}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4360$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{(-\cos(c+dx)a-a)^2\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)\sin\left(c+dx-\frac{\pi}{2}\right)^2}}dx$$

$$\downarrow 3352$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{\sec^2(c+dx)a^2}{\sqrt{\sin(c+dx)}}+\frac{2\sec(c+dx)a^2}{\sqrt{\sin(c+dx)}}+\frac{a^2}{\sqrt{\sin(c+dx)}}\right)dx$$

$$\downarrow 2009$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{3a^2\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.$$

input
```
Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[e*Csc[c + d*x]]*((2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/d + (2*a^2*ArcTan
h[Sqrt[Sin[c + d*x]]])/d + (3*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/d + (a
^2*Sec[c + d*x]*Sqrt[Sin[c + d*x]])/d)*Sqrt[Sin[c + d*x]]
```

3.289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.289.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.56 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.09

method	result
parts	$\frac{ia^2(\cos(dx+c)+1)\sqrt{2}\sqrt{e\csc(dx+c)}\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}$ E d
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*a^2/d*(cos(d*x+c)+1)*2^(1/2)*(e*csc(d*x+c))^(1/2)*(-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*
x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+1
/2*a^2/d*2^(1/2)*(e*csc(d*x+c))^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2
)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I
*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)
*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)
))^(1/2),1/2*2^(1/2))+2^(1/2)*tan(d*x+c))+2*a^2/d*sin(d*x+c)*(arctanh((sin
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))-arctan((sin(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c))))*(e*csc(d*x+c))^(1/2)/(
cos(d*x+c)+1)/(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

3.289.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.31

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx$$

$$= \left[-\frac{2\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos(dx+c) - a^2\sqrt{-e}\cos(dx+c)\log\left(\frac{e\cos(}{}\right.\right.$$

$$\left.\left. -\frac{2\,a^2\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\cos(dx+c) - a^2\sqrt{e}\cos(dx+c)\log\left(\frac{e\cos(dx+c)^4-7}{}\right.\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) - a^2*sqrt(
-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x
+ c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(
-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
 + 8)) + 6*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 6*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstrassPI
nverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 4*a^2*sqrt(e/sin(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)), -1/4*(2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c
)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e
))*cos(d*x + c) - a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*co
s(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 -
8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 -
 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)) + 6*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierst
rassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 6*I*a^2*sqrt(-2*I*e)*c
os(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 4*a
^2*sqrt(e/sin(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

3.289.6 Sympy [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{e\csc(c+dx)}\,dx + \int 2\sqrt{e\csc(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\csc(c+dx)}\sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*csc(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(2*sqrt(e*csc(c + d*x))*
sec(c + d*x), x) + Integral(sqrt(e*csc(c + d*x))*sec(c + d*x)**2, x))
```

3.289.7 Maxima [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

3.289.8 Giac [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2 \sqrt{\frac{e}{\sin(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2), x)
```

3.290 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$

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3.290.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}$$

output

```
-2*a^2*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*
a^2*arctanh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-a^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a
^2*tan(d*x+c)/d/(e*csc(d*x+c))^(1/2)
```

3.290.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 18.33 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.88

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx =$$
$$-\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\arctan\left(\sqrt{}\right.\right.}{2d\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}(-\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

output
```
-1/2*((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-ArcTan[Sqrt[Csc[c + d*x]]] - A
rcTanh[Sqrt[Csc[c + d*x]]] - (2*Sqrt[Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]^2
]*Hypergeometric2F1[3/4, 3/2, 7/4, Csc[c + d*x]^2])/(3*Sqrt[1 - Sin[c + d*
x]^2]) + (Sqrt[Csc[c + d*x]]*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x
]^2]*Sqrt[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d*x]^2]))/(d*(1 + Cos[2*(c
/2 + (-c + ArcCsc[Csc[c + d*x]])/2)])^2*Csc[c + d*x]^(3/2)*Sqrt[e*Csc[c +
d*x]]*Sqrt[1 - Sin[c + d*x]^2])
```

3.290.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{e\csc(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4366$$

$$\frac{\int (\sec(c+dx)a+a)^2\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2 dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int (-\cos(c+dx)a-a)^2\sec^2(c+dx)\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\sec^2(c+dx)\sqrt{\sin(c+dx)}a^2+2\sec(c+dx)\sqrt{\sin(c+dx)}a^2+\sqrt{\sin(c+dx)}a^2\right)dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{d}+\frac{a^2\sin^{\frac{3}{2}}(c+dx)\sec(c+dx)}{d}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

output
```
((-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]
])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (a^2*Sec[c + d*x]*Sin[c
+ d*x]^(3/2))/d)/(Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.290.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.83 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.51

method	result	size
parts	Expression too large to display	996
default	Expression too large to display	1290

input
```
int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-a^2/d*2^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-c
sc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(
d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(I-cot(d*x+c)+csc(d*
x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+
c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos
(d*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c
)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+c
sc(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Elliptic
F((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(
1/2))/(e*csc(d*x+c))^(1/2)*csc(d*x+c)-1/2*a^2/d*2^(1/2)/(e*csc(d*x+c))^(1/
2)*(-2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))
^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*x+c)+(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2
)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*
EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*x+c)-2*(
-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(
-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))
^(1/2),1/2*2^(1/2))*csc(d*x+c)+(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Ellip...
```

3.290.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.56

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$$

$$= \left[-\frac{2\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos\left(dx+c\right)+a^2\sqrt{-e}\cos\left(dx+c\right)\log\left(\frac{e\cos(}{}\right.\right.$$

input

```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + a^2*sqrt(
-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x
+ c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(
-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
 + 8)) - 2*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*a^2*sqrt(-2*I*e)*cos(d*x
 + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin
(d*x + c))) + 4*(a^2*cos(d*x + c)^2 - a^2)*sqrt(e/sin(d*x + c)))/(d*e*cos(
d*x + c)), 1/4*(2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c)
- 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c) + a^2
*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos
(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*
sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) +
72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) + 2*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2*a^2*sqrt(-2*I*e)*cos
(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I
*sin(d*x + c))) - 4*(a^2*cos(d*x + c)^2 - a^2)*sqrt(e/sin(d*x + c)))/(d...
```

3.290.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e
*csc(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*csc(c + d*x)), x))
```

3.290.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

3.290.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{\sqrt{\frac{e}{\sin(c+dx)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2), x)
```

3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$

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3.291.2 Mathematica [C] (warning: unable to verify) . . . 2205
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3.291.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
-4*a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2
)+a^2*sec(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+2*a^2*arctan(sin(d*x+c)^(1/2))/d
/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d
/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+1/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.291.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.71 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3-6\sqrt{\cos^2(c}\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

output
```
(2*a^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*Sec[ArcCsc[Csc[c + d*x]]/2]
^4*(3 - 6*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[-1/4, 1, 3/4, Csc[c + d*x
]^2] + 3*Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Csc[c + d*
x]^2] + Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[-3/4, 3/2, 1/4, Csc[c + d*
x]^2]*Sin[c + d*x]^2)*Tan[c + d*x])/(3*d*e^2)
```

3.291.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\csc(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int(\sec(c+dx)a+a)^2\sin^{\frac{3}{2}}(c+dx)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int (-\cos(c+dx)a-a)^2 \sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cos(c+dx+\frac{\pi}{2}))^{3/2}(-\sin(c+dx+\frac{\pi}{2})a-a)^2}{\sin(c+dx+\frac{\pi}{2})^2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)a^2+2\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)a^2+\sin^{\frac{3}{2}}(c+dx)a^2\right)dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{4a^2\sqrt{\sin(c+dx)}}{d}-\frac{a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{3d}-\frac{2a^2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}-}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

output
```
((2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]
)/d - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*a^2*Sqrt[Sin[c + d
*x]])/d - (2*a^2*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/(3*d) + (a^2*Sec[c + d*x
]*Sqrt[Sin[c + d*x]])/d)/(e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.291.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.70 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.65

method	result	size
parts	Expression too large to display	810
default	Expression too large to display	1083

input
```
int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*a^2/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I*(-I*(I-cot(d*x+c)+
csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2
))-cos(d*x+c)*sin(d*x+c)*2^(1/2))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/(c
os(d*x+c)+1)*sin(d*x+c)+1/2*a^2/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c))
)^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^
(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+
c)^2+I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))
^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-2^(1/2)*sin(d*x+c))/(e*csc(d*x+c))^
(1/2)/e/(cos(d*x+c)^2-1)*tan(d*x+c)+2*a^2/d*(arctan((sin(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*cos(d*x+c)-(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)+arctan((sin(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)-(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))+2*sin(d*x+c))/(e*csc(d*x+c))^(1/2
)/(cos(d*x+c)-1)/e/(cos(d*x+c)+1)*sin(d*x+c)
```

3.291.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.26

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \left[-\frac{6\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos(dx+c)+3\,a}{} \right.$$

$$\left. -\frac{6\,a^2\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\cos(dx+c)-3\,a^2\sqrt{e}\cos(dx+c)\log\left(\frac{e\cos(dx+c)^4-72}{}\right.}{} \right]$$

input

```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(6*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*s
qrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + 3*a^2*sq
rt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d
*x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sq
rt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 7
2*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x +
 c) + 8)) - 2*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 2*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstras
sPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 4*(2*a^2*cos(d*x + c)^2 +
 12*a^2*cos(d*x + c) - 3*a^2)*sqrt(e/sin(d*x + c))*sin(d*x + c))/(d*e^2*co
s(d*x + c)), -1/12*(6*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x +
 c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c) -
 3*a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 +
8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c)
 + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
 c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*si
n(d*x + c) + 8)) - 2*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*wei
erstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 4*(2*a^2*cos(d*x +
 c)^2 + 12*a^2*cos(d*x + c) - 3*a^2)*sqrt(e/sin(d*x + c))*sin(d*x + c))...
```

3.291.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{(e\csc(c + dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\csc(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c + dx)}{(e\csc(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c + dx)}{(e\csc(c + dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*c
sc(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*csc(c + d*x))**(3/2)
, x))
```

3.291.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

3.291.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{e}{\sin(c+dx)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2), x)
```

3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$

3.292.1 Optimal result 2211
3.292.2 Mathematica [C] (warning: unable to verify) 2212
3.292.3 Rubi [A] (verified) 2212
3.292.4 Maple [C] (warning: unable to verify) 2214
3.292.5 Fricas [C] (verification not implemented) 2215
3.292.6 Sympy [F(-1)] 2216
3.292.7 Maxima [F] 2217
3.292.8 Giac [F] 2217
3.292.9 Mupad [F(-1)] 2217

3.292.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}$$

output

```
-4/3*a^2*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a^2*cos(d*x+c)*sin(d*x+
c)/d/e^2/(e*csc(d*x+c))^(1/2)-2*a^2*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(
d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*cs
c(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+9/5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/
d/e^2/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*tan(d*x+c)/d/e^2/(e*csc(d*
x+c))^(1/2)
```

3.292.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(-10\sqrt{\cos^2(c+dx)}\,\text{Hypergeo}\right.}{\ldots}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Cos[(c + d*x)/2]^4*Sec[ArcCsc[Csc[c + d*x]]/2]^4*(-10*Sqrt[Cos[c +
d*x]^2]*Hypergeometric2F1[-3/4, 1, 1/4, Csc[c + d*x]^2] + 3*Sqrt[-Cot[c +
d*x]^2]*(-10*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2] + Hypergeom
etric2F1[-5/4, 3/2, -1/4, Csc[c + d*x]^2]*Sin[c + d*x]^2))*Tan[c + d*x])/(
15*d*e^2*Sqrt[e*Csc[c + d*x]])
```

3.292.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4366, 3042, 4360, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\csc(c+dx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int(\sec(c+dx)a+a)^2\sin^{\frac{5}{2}}(c+dx)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int (-\cos(c+dx)a-a)^2 \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cos(c+dx+\frac{\pi}{2}))^{5/2}(-\sin(c+dx+\frac{\pi}{2})a-a)^2}{\sin(c+dx+\frac{\pi}{2})^2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(a^2\sin^{\frac{5}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\right)dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{4a^2\sin^{\frac{3}{2}}(c+dx)}{3d}-\frac{9a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{5d}-\frac{2a^2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}+\frac{a}{}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

output
```
((-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]
])/d - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d) - (4*a^2*Sin[c + d*x
]^(3/2))/(3*d) - (2*a^2*Cos[c + d*x]*Sin[c + d*x]^(3/2))/(5*d) + (a^2*Sec[
c + d*x]*Sin[c + d*x]^(3/2))/d)/(e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x
]])
```

3.292.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.292.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.64 (sec) , antiderivative size = 1069, normalized size of antiderivative = 4.53

method	result	size
parts	Expression too large to display	1069
default	Expression too large to display	1333

input
```
int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2/d*2^(1/2)*(-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+3*(-I*(I-cot(d*x+c)+
csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2
))*cos(d*x+c)+cos(d*x+c)^3*2^(1/2)-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*
(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*El
lipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(-I*(I-cot(d*x
+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c
)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^
(1/2))-4*2^(1/2)*cos(d*x+c)+3*2^(1/2))/(e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)
+1/2*a^2/d*2^(1/2)/(e*csc(d*x+c))^(1/2)/e^2*(6*(-I*(I-cot(d*x+c)+csc(d*x+c
)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c))
)^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*
x+c)-3*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))
^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*x+c)+6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2
)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*csc(d*x+c)-3
*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1...
```

3.292.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.29

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
[-1/60*(30*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*
sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + 15*a^2*
sqrt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos
(d*x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*
sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) +
 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x
 + c) + 8)) + 54*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 54*a^2*sqrt(-2*I*e)*
cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 4*(6*a^2*cos(d*x + c)^4 + 20*a^2*cos(d*x + c)^3 - 21*
a^2*cos(d*x + c)^2 - 20*a^2*cos(d*x + c) + 15*a^2)*sqrt(e/sin(d*x + c)))/(
d*e^3*cos(d*x + c)), 1/60*(30*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*s
in(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*
x + c) + 15*a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x
+ c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin
(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*
sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - 2)*sin(d*x + c) + 8)) - 54*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 54*a^2
*sqrt(-2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,...
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\csc(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.292.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

3.292.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{\left(\frac{e}{\sin(c+dx)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2), x)
```

3.293 $\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

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3.293.1 Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{4e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad}$$

output

```
-4/21*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/7*e^2*cot(d*x+c)*csc(d*x+c
)^2*(e*csc(d*x+c))^(1/2)/a/d-2/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a/d
-4/21*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/a/d
```

3.293.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left((2+\cos(c+dx)-2\cos(2(c+dx))-\cos(3(c+dx))\right.}{168a}$$

input
```
Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-1/168*(Csc[(c + d*x)/2]^2*(e*Csc[c + d*x])^(5/2)*Sec[(c + d*x)/2]^4*((2 +
 Cos[c + d*x] - 2*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*EllipticF[(-2*c + P
i - 2*d*x)/4, 2] + 2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[Sin[c +
d*x]])*Sin[c + d*x]^(5/2))/(a*d)
```

3.293.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\csc(c+dx))^{5/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4366$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\cos(c+dx)}{(-\cos(c+dx)a-a)\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int-\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{5/2}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}dx}{a}+\frac{\int-\frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 25$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}dx}{a}-\frac{\int\frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 3042$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{9/2}}dx}{a}-\frac{\int\frac{\cos(c+dx)}{\sin(c+dx)^{9/2}}dx}{a}\right)$$

$$\downarrow 3044$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{9/2}}dx}{a}-\frac{\int\frac{1}{\sin^{\frac{9}{2}}(c+dx)}d\sin(c+dx)}{ad}\right)$$

$$\downarrow 15$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{9/2}}dx}{a}+\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}\right)$$

$$\downarrow 3047$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{7}\int\frac{1}{\sin^{\frac{5}{2}}(c+dx)}dx-\frac{2\cos(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}}{a}+\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{7}\int\frac{1}{\sin(c+dx)^{5/2}}dx-\frac{2\cos(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}}{a}+\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}\right)$$

$$\downarrow 3116$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}\right)-\frac{2\cos(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}}{a}+\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}\right)-\frac{2\cos(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}}{a}+\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}\right)$$

$$\downarrow 3120$$

$$-e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{2}{7ad\sin^{\frac{7}{2}}(c+dx)}+\frac{-\frac{2\cos(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}-\frac{2}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{3d}-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}\right)}{a}\right.$$

input
```
Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-(e^2*Sqrt[e*Csc[c + d*x]]*(((-2*((2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*
d) - (2*Cos[c + d*x])/(3*d*Sin[c + d*x]^(3/2))))/7 - (2*Cos[c + d*x])/(7*d
*Sin[c + d*x]^(7/2)))/a + 2/(7*a*d*Sin[c + d*x]^(7/2)))*Sqrt[Sin[c + d*x]]
)
```

3.293.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.293.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.32

method	result
default	$\frac{\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(8i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{2}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c}\right.}{84ad\left((1-\cos(dx+c))^2\csc(dx+c)^2+\right.}$

input

```
int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/84/a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c)
))^(5/2)*(1-cos(d*x+c))^2*(8*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2
)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*
EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+c
sc(d*x+c))-3*(1-cos(d*x+c))^6*csc(d*x+c)^6-5*(1-cos(d*x+c))^4*csc(d*x+c)^4
-9*(1-cos(d*x+c))^2*csc(d*x+c)^2-7)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/((
1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(
d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))^(1/2)*csc(d*x+c)^2
```

3.293.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{2\left((i\,e^2\cos(dx+c)+i\,e^2)\sqrt{2i\,e}\sin(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+(-i\right.}{}$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/21*((I*e^2*cos(d*x + c) + I*e^2)*sqrt(2*I*e)*sin(d*x + c)*weierstrassPI
nverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*e^2*cos(d*x + c) - I*e^2
)*sqrt(-2*I*e)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin
(d*x + c)) + (2*e^2*cos(d*x + c)^2 + 2*e^2*cos(d*x + c) + 3*e^2)*sqrt(e/si
n(d*x + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.293.7 Maxima [F]

$$\int \frac{(e \csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e \csc(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.293.8 Giac [F]

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\csc(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\left(\frac{e}{\sin(c+dx)}\right)^{5/2}}{a\left(\cos(c+dx)+1\right)}\,dx$$

input
```
int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e/sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.294 $\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

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3.294.7 Maxima [F(-1)] . . . 2232
3.294.8 Giac [F] . . . 2233
3.294.9 Mupad [F(-1)] . . . 2233

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{5ad}$$

output

```
-4/5*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/5*e*cot(d*x+c)*csc(d*x+c)*(e*
csc(d*x+c))^(1/2)/a/d-2/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a/d+4/5*e*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a/d
```

3.294.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.81 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.59

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(\frac{8\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}\left(1+e^{2ic}\right)\right.\right.}{d(1+e^{2ic}}}{15a(1+}$$

input
```
Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(e*Csc[c + d*x])^(3/2)*((8*Sqrt[2]*E^(I*(c - d*x))*Sqr
t[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2*I)*(c + d*x
)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*(1 + E^((
2*I)*c))*Csc[c + d*x]^(3/2)) - (6*(4*Cos[d*x]*Sec[c] + Sec[(c + d*x)/2]^2)
*Tan[c + d*x])/d))/(15*a*(1 + Sec[c + d*x]))
```

3.294.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\csc(c+dx))^{3/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4366$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\cos(c+dx)}{(-\cos(c+dx)a-a)\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 25$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int-\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^{3/2}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a}+\frac{\int-\frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 25$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a}-\frac{\int\frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 3042$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{7/2}}dx}{a}-\frac{\int\frac{\cos(c+dx)}{\sin(c+dx)^{7/2}}dx}{a}\right)$$

$$\downarrow 3044$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{7/2}}dx}{a}-\frac{\int\frac{1}{\sin^{\frac{7}{2}}(c+dx)}d\sin(c+dx)}{ad}\right)$$

$$\downarrow 15$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{\int\frac{\cos(c+dx)^2}{\sin(c+dx)^{7/2}}dx}{a}+\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}\right)$$

$$\downarrow 3047$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{5}\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}dx-\frac{2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)}}{a}+\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{5}\int\frac{1}{\sin(c+dx)^{3/2}}dx-\frac{2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)}}{a}+\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}\right)$$

$$\downarrow 3116$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{5}\left(-\int\sqrt{\sin(c+dx)}dx-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}\right)-\frac{2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)}}{a}+\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{-\frac{2}{5}\left(-\int\sqrt{\sin(c+dx)}dx-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}\right)-\frac{2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)}}{a}+\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}\right)$$

$$\downarrow 3119$$

$$-e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\frac{2}{5ad\sin^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)}-\frac{2}{5}\left(-\frac{2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{d}-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}\right)}{a}\right)$$

input
```
Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-(e*Sqrt[e*Csc[c + d*x]]*(((-2*((-2*EllipticE[(c - Pi/2 + d*x)/2, 2])/d -
(2*Cos[c + d*x])/(d*Sqrt[Sin[c + d*x]])))/5 - (2*Cos[c + d*x])/(5*d*Sin[c
+ d*x]^(5/2)))/a + 2/(5*a*d*Sin[c + d*x]^(5/2)))*Sqrt[Sin[c + d*x]])
```

3.294.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.294.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.11 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.43

method	result
default	$\frac{\sqrt{2}\left(4\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx$

input
```
int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/5/a/d*2^(1/2)*(4*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2
))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)^2-2*(-I*(I+cot(d*x+c)-c
sc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc
(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2)
)*cos(d*x+c)^2+8*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(
d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))
*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-4*(-I*(I+cot(d*x+c)-csc(d
*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*co
s(d*x+c)+4*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)
))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-
I+cot(d*x+c)-csc(d*x+c)))^(1/2)-2*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I
*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*Ellip
ticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(d*x+c
)-3*2^(1/2))*e*(e*csc(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

3.294.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{2\left((e\cos(dx+c)+e)\sqrt{2i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))\right.}{}$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/5*((e*cos(d*x + c) + e)*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (e*cos(d*x + c) + e)*sqrt(-
2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c))) + (2*e*cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(e/sin(d*x
+ c)))/(a*d*cos(d*x + c) + a*d)
```

3.294.6 Sympy [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.294.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Timed out
```

3.294.8 Giac [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\left(\frac{e}{\sin(c+dx)}\right)^{3/2}}{a\left(\cos(c+dx)+1\right)}\,dx$$

input
```
int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e/sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.295 $\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx$

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3.295.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} - \frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad}$$

output
```
2/3*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d-2/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2
)/a/d-4/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/a/d
```

3.295.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{2(e\csc(c+dx))^{3/2}\left(-1+\cos(c+dx)-2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ade}$$

input
```
Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(2*(e*Csc[c + d*x])^(3/2)*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*
d*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*e)
```

3.295.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 3044, 15, 3047, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\csc(c+dx)}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 4366$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4360$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\cos(c+dx)}{(-\cos(c+dx)a-a)\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\int -\frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{\cos(c+dx)}{(\cos(c+dx)a+a)\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3318$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx}{a}+\frac{\int -\frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 25$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx}{a}-\frac{\int \frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx}{a}\right)$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{5/2}}dx}{a}-\frac{\int \frac{\cos(c+dx)}{\sin(c+dx)^{5/2}}dx}{a}\right)$$

$$\downarrow 3044$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{5/2}}dx}{a}-\frac{\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}d\sin(c+dx)}{ad}\right)$$

$$\downarrow 15$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{5/2}}dx}{a}+\frac{2}{3ad\sin^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3047$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{-\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}}{a}+\frac{2}{3ad\sin^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{-\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}}{a}+\frac{2}{3ad\sin^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3120$$

$$\sqrt{\sin(c+dx)}\left(-\sqrt{e\csc(c+dx)}\right)\left(\frac{2}{3ad\sin^{\frac{3}{2}}(c+dx)}+\frac{-\frac{4\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{3d}-\frac{2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}}{a}\right)$$

input
```
Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
-(Sqrt[e*Csc[c + d*x]]*(((-4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (2*
Cos[c + d*x])/(3*d*Sin[c + d*x]^(3/2)))/a + 2/(3*a*d*Sin[c + d*x]^(3/2)))*
Sqrt[Sin[c + d*x]])
```

3.295.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.295.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

method	result
default	$\frac{\sqrt{2}\sqrt{\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}}(1-\cos(dx+c))\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{2}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\right.}{3ad\sqrt{(1-\cos(dx+c))\left((1-\cos(dx+c))^2\csc(dx+c)^2+1\right)}}$

input
```
int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c))
)^(1/2)*(1-cos(d*x+c))*(2*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*(
-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*Ell
ipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(1-cos(d*x+c))^3*
csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-co
t(d*x+c))^(1/2)*csc(d*x+c)
```

3.295.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{2i\,e}(i\cos(dx+c)+i)\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{-2i\,e}(-i\cos(d\right.}{3\,(ad\cos(dx+c)}$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2*I*e)*(I*cos(d*x + c) + I)*weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(-2*I*e)*(-I*cos(d*x + c) - I)*weierstrassPInv
erse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(e/sin(d*x + c))*sin(d*x +
 c))/(a*d*cos(d*x + c) + a*d)
```

3.295.6 Sympy [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.295.7 Maxima [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.295.8 Giac [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\sqrt{\frac{e}{\sin(c+dx)}}}{a\left(\cos(c+dx)+1\right)}\,dx$$

input
```
int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e/sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.296 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx$

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3.296.2 Mathematica [C] (verified) 2241
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3.296.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} - \frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
2*cot(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)-2*csc(d*x+c)/a/d/(e*csc(d*x+c))^(1/2
)-4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
E(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(
1/2)
```

3.296.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{6(2i+\cot(c+dx)-\csc(c+dx)) - 4\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx}\right.}{3ad\sqrt{e\csc(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(6*(2*I + Cot[c + d*x] - Csc[c + d*x]) - 4*Sqrt[1 - E^((2*I)*(c + d*x))]*(
I + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/(
3*a*d*Sqrt[e*Csc[c + d*x]])
```

3.296.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3047, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)\sqrt{e\csc(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sqrt{\sin(c+dx)}}{\sec(c+dx)a+a}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{-\cos(c+dx)a-a}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{\cos(c+dx)a+a}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{\cos(c+dx)a+a}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3318$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a} - \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{\sin(c+dx)^{3/2}}dx}{a} - \frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{3/2}}dx}{a}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}d\sin(c+dx)}{ad} - \frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{3/2}}dx}{a}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 15$$

$$\frac{-\frac{\int \frac{\cos(c+dx)^2}{\sin(c+dx)^{3/2}}dx}{a} - \frac{2}{ad\sqrt{\sin(c+dx)}}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3047$$

$$\frac{-\frac{-2\int \sqrt{\sin(c+dx)}dx - \frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}}{a} - \frac{2}{ad\sqrt{\sin(c+dx)}}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{-\frac{-2\int \sqrt{\sin(c+dx)}dx - \frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}}{a} - \frac{2}{ad\sqrt{\sin(c+dx)}}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3119$$

$$\frac{-\frac{2}{ad\sqrt{\sin(c+dx)}}-\frac{-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}}{a}}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(-(((-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (2*Cos[c + d*x])/(d*Sqrt[Sin
[c + d*x]]))/a) - 2/(a*d*Sqrt[Sin[c + d*x]]))/(Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]])
```

3.296.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.296.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c}\right.\right.$

input
```
int(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/d*2^(1/2)*(4*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-c
sc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(
d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*(-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*
x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*c
os(d*x+c)+4*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x
+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)
+csc(d*x+c)))^(1/2),1/2*2^(1/2))-2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-
I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Elli
pticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)
-2^(1/2))/(e*csc(d*x+c))^(1/2)*csc(d*x+c)
```

3.296.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(\cos(dx+c)-1)+\sqrt{2i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin\right.}{ade}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(e/sin(d*x + c))*(cos(d*x + c) - 1) + sqrt(2*I*e)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(-2*I
*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))))/(a*d*e)
```

3.296.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/
a
```

3.296.7 Maxima [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.296.8 Giac [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\sqrt{\frac{e}{\sin(c+dx)}}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2)),x)
```

output
```
int(cos(c + d*x)/(a*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

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3.297.5 Fricas [C] (verification not implemented) . . . 2252
3.297.6 Sympy [F] . . . 2253
3.297.7 Maxima [F] . . . 2253
3.297.8 Giac [F] . . . 2253
3.297.9 Mupad [F(-1)] . . . 2254

3.297.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
2/a/d/e/(e*csc(d*x+c))^(1/2)-2/3*cos(d*x+c)/a/d/e/(e*csc(d*x+c))^(1/2)+4/3
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1
/2)
```

3.297.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right) - 2(-3+\cos(c+dx))\sqrt{\sin}}{3ad(e\csc(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}$$

input

```
Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

output

```
(4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 2*(-3 + Cos[c + d*x])*Sqrt[Sin[c
+ d*x]])/(3*a*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

3.297.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3049, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\csc(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{-\cos(c+dx)a-a}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3318$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}dx}{a} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}dx}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}dx}{a} - \frac{\int \frac{\cos(c+dx)^2}{\sqrt{\sin(c+dx)}}dx}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{\int \frac{1}{\sqrt{\sin(c+dx)}}d\sin(c+dx)}{ad} - \frac{\int \frac{\cos(c+dx)^2}{\sqrt{\sin(c+dx)}}dx}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 15$$

$$\frac{\frac{2\sqrt{\sin(c+dx)}}{ad} - \frac{\int \frac{\cos(c+dx)^2}{\sqrt{\sin(c+dx)}}dx}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3049$$

$$\frac{\frac{2\sqrt{\sin(c+dx)}}{ad} - \frac{\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx + \frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{2\sqrt{\sin(c+dx)}}{ad} - \frac{\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx + \frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3120$$

$$\frac{\frac{2\sqrt{\sin(c+dx)}}{ad} - \frac{\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d} + \frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}}{a}}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(-(((4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d) + (2*Cos[c + d*x]*Sqrt[Sin[
c + d*x]])/(3*d))/a) + (2*Sqrt[Sin[c + d*x]])/(a*d))/(e*Sqrt[e*Csc[c + d*x
]]*Sqrt[Sin[c + d*x]])
```

3.297.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.297.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.59

method	result
default	$\frac{\sqrt{2}\left(2i\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx$

input
```
int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/d*2^(1/2)*(2*I*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-
csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1
/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)+2*I*(-I*(I+cot(d*x+c)
-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+co
t(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1
/2)+cos(d*x+c)*sin(d*x+c)*2^(1/2)-3*2^(1/2)*sin(d*x+c))/(e*csc(d*x+c))^(1/
2)/e/(cos(d*x+c)-1)/(cos(d*x+c)+1)*sin(d*x+c)
```

3.297.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(\cos(dx+c)-3)\sin(dx+c)-i\sqrt{2ie}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c$$
$$3ade^2$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(e/sin(d*x + c))*(cos(d*x + c) - 3)*sin(d*x + c) - I*sqrt(2*I*e)
*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(-2*I*e)
*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a*d*e^2)
```

3.297.6 Sympy [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2)
), x)/a
```

3.297.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.297.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\left(\frac{e}{\sin(c+dx)}\right)^{3/2}(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2)),x)
```

output
```
int(cos(c + d*x)/(a*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

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3.298.3 Rubi [A] (verified) . . . 2256
3.298.4 Maple [C] (verified) . . . 2259
3.298.5 Fricas [C] (verification not implemented) . . . 2260
3.298.6 Sympy [F(-1)] . . . 2260
3.298.7 Maxima [F] . . . 2260
3.298.8 Giac [F] . . . 2261
3.298.9 Mupad [F(-1)] . . . 2261

3.298.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}$$

output
```
2/3*sin(d*x+c)/a/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)/a/d/
e^2/(e*csc(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^2/(e*c
sc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{8\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e$$
$$30ade^2\sqrt{e\csc(c$$

input
```
Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(8*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2,
 3/4, 7/4, E^((2*I)*(c + d*x))] + 20*Sin[c + d*x] - 6*(4*I + Sin[2*(c + d*
x)]))/(30*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

3.298.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3049, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\csc(c+dx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{-\cos(c+dx)a-a}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3318$$

$$\frac{\frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}dx}{a} - \frac{\int \cos^2(c+dx)\sqrt{\sin(c+dx)}dx}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}dx}{a} - \frac{\int \cos(c+dx)^2\sqrt{\sin(c+dx)}dx}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{\int \sqrt{\sin(c+dx)}d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sqrt{\sin(c+dx)}dx}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 15$$

$$\frac{\frac{2\sin^{\frac{3}{2}}(c+dx)}{3ad} - \frac{\int \cos(c+dx)^2\sqrt{\sin(c+dx)}dx}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3049$$

$$\frac{\frac{2\sin^{\frac{3}{2}}(c+dx)}{3ad} - \frac{\frac{2}{5}\int \sqrt{\sin(c+dx)}dx + \frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{2\sin^{\frac{3}{2}}(c+dx)}{3ad} - \frac{\frac{2}{5}\int \sqrt{\sin(c+dx)}dx + \frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3119$$

$$\frac{\frac{2\sin^{\frac{3}{2}}(c+dx)}{3ad} - \frac{\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5d} + \frac{2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d}}{a}}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

3.298. $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}dx$

output
```
((2*Sin[c + d*x]^(3/2))/(3*a*d) - ((4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5
*d) + (2*Cos[c + d*x]*Sin[c + d*x]^(3/2))/(5*d))/a)/(e^2*Sqrt[e*Csc[c + d*
x]]*Sqrt[Sin[c + d*x]])
```

3.298.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.298.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.59 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.85

method	result
default	$\frac{\sqrt{2}\left(12\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx$

input
```
int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/15/a/d*2^(1/2)*(12*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-
csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1
/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-6*(-I*(I+cot(d*x+c)-c
sc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc
(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2)
)*cos(d*x+c)+12*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d
*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*
(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)-6*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2
)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*
EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^3
*2^(1/2)-5*2^(1/2)*cos(d*x+c)^2+3*2^(1/2)*cos(d*x+c)-2^(1/2))/(e*csc(d*x+c
))^(1/2)/e^2*csc(d*x+c)
```

3.298.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{2\left(\left(3\cos(dx+c)^3-5\cos(dx+c)^2-3\cos(dx+c)+5\right)\sqrt{\frac{}{}}\right.}{}$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
2/15*((3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 3*cos(d*x + c) + 5)*sqrt(e/si
n(d*x + c)) - 3*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(-2*I*e)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*e^3)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.298.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.298.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\left(\frac{e}{\sin(c+dx)}\right)^{5/2}(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2)),x)
```

output
```
int(cos(c + d*x)/(a*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

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3.299.1 Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21ade^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}$$

output
```
-2/21*cos(d*x+c)/a/d/e^3/(e*csc(d*x+c))^(1/2)+2/7*cos(d*x+c)^3/a/d/e^3/(e*
csc(d*x+c))^(1/2)+2/5*sin(d*x+c)^2/a/d/e^3/(e*csc(d*x+c))^(1/2)+4/21*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^3/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\sqrt{e\csc(c+dx)}\left(80\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+}\right.$$

input
```
Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Sqrt[e*Csc[c + d*x]]*(80*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c +
 d*x]] + 126*Sin[c + d*x] + 10*Sin[2*(c + d*x)] - 42*Sin[3*(c + d*x)] + 15
*Sin[4*(c + d*x)]))/(420*a*d*e^4)
```

3.299.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4366, 3042, 4360, 25, 25, 3042, 3318, 3042, 3044, 15, 3048, 3042, 3049, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\csc(c+dx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{7/2}}{a-a\csc\left(c+dx-\frac{\pi}{2}\right)}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int -\frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{-\cos(c+dx)a-a}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{\cos(c+dx)a+a}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3318$$

$$\frac{\frac{\int \cos(c+dx)\sin^{\frac{3}{2}}(c+dx)dx}{a} - \frac{\int \cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)dx}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \cos(c+dx)\sin(c+dx)^{3/2}dx}{a} - \frac{\int \cos(c+dx)^2\sin(c+dx)^{3/2}dx}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3044$$

$$\frac{\frac{\int \sin^{\frac{3}{2}}(c+dx)d\sin(c+dx)}{ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^{3/2}dx}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 15$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\int \cos(c+dx)^2\sin(c+dx)^{3/2}dx}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3048$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\frac{1}{7}\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}dx - \frac{2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\frac{1}{7}\int \frac{\cos(c+dx)^2}{\sqrt{\sin(c+dx)}}dx - \frac{2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3049$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\frac{1}{7}\left(\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx + \frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right) - \frac{2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\frac{1}{7}\left(\frac{2}{3}\int \frac{1}{\sqrt{\sin(c+dx)}}dx + \frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right) - \frac{2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3120$$

$$\frac{\frac{2\sin^{\frac{5}{2}}(c+dx)}{5ad} - \frac{\frac{1}{7}\left(\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d}+\frac{2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}\right)-\frac{2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}}{a}}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(-((((4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d) + (2*Cos[c + d*x]*Sqrt[Sin
[c + d*x]])/(3*d))/7 - (2*Cos[c + d*x]^3*Sqrt[Sin[c + d*x]])/(7*d))/a) + (
2*Sin[c + d*x]^(5/2))/(5*a*d))/(e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])
```

3.299.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.299.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.01 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.12

method	result
default	$\frac{\sqrt{2}\left(-10i\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot$

input
```
int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/105/a/d*2^(1/2)*(-10*I*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(I+cot(d
*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(
-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+15*2^(1/2)*cos(d*
x+c)^3*sin(d*x+c)-10*I*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x
+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I
+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))-21*cos(d*x+c)^2*2^(1/2)*sin(d*
x+c)-5*cos(d*x+c)*sin(d*x+c)*2^(1/2)+21*2^(1/2)*sin(d*x+c))/e^3/(e*csc(d*x
+c))^(1/2)/(cos(d*x+c)-1)^2/(cos(d*x+c)+1)^2*sin(d*x+c)^3
```

3.299.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{2\left(\left(15\cos(dx+c)^3 - 21\cos(dx+c)^2 - 5\cos(dx+c) + 21\right)\right.}{}$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
2/105*((15*cos(d*x + c)^3 - 21*cos(d*x + c)^2 - 5*cos(d*x + c) + 21)*sqrt(
e/sin(d*x + c))*sin(d*x + c) + 5*I*sqrt(2*I*e)*weierstrassPInverse(4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(-2*I*e)*weierstrassPInverse(4, 0,
 cos(d*x + c) - I*sin(d*x + c)))/(a*d*e^4)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.299.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.299.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\left(\frac{e}{\sin(c+dx)}\right)^{7/2}(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(7/2)),x)
```

output
```
int(cos(c + d*x)/(a*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

3.300 $\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

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3.300.1 Optimal result

Integrand size = 25, antiderivative size = 268

$$\begin{aligned}\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d}\\ &+\frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d}\\ &-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}\\ &-\frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}+\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}\\ &+\frac{4e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231a^2d}\end{aligned}$$

output

```
-4/231*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/77*e^2*cot(d*x+c)*csc(
d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*e^2*cot(d*x+c)^3*csc(d*x+c)^2*(e*
csc(d*x+c))^(1/2)/a^2/d-4/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d-2/
11*e^2*cot(d*x+c)*csc(d*x+c)^4*(e*csc(d*x+c))^(1/2)/a^2/d+4/11*e^2*csc(d*x
+c)^5*(e*csc(d*x+c))^(1/2)/a^2/d-4/231*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a^2/d
```

3.300.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.43

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{e^3\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c\right.\right.}{3696a^2d\sqrt{e\csc(c+dx)}}$$

input
```
Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3696*(e^3*Csc[(c + d*x)/2]^2*Sec[(c + d*x)/2]^6*(52 + 97*Cos[c + d*x] +
 4*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*
c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*Sqrt[e*Csc[c + d*x]])
```

3.300.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4366$$
$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)^2\sin^{\frac{5}{2}}(c+dx)}dx$$
$$\downarrow 3042$$
$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4360$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)}{(-\cos(c+dx)a-a)^2\sin^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{13}{2}}(c+dx)}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{13/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}+\frac{a^2\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{4a^2}{7d\sin^{\frac{7}{2}}(c+dx)}+\frac{4a^2}{11d\sin^{\frac{11}{2}}(c+dx)}+\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{231d}-\frac{2a^2\cos^3(c+dx)}{11d\sin^{\frac{11}{2}}(c+dx)}-\frac{4a^2}{231d}\right.}{a^4}$$

input
```
Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^2*Sqrt[e*Csc[c + d*x]]*((4*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(231*d
) + (4*a^2)/(11*d*Sin[c + d*x]^(11/2)) - (2*a^2*Cos[c + d*x])/(11*d*Sin[c
+ d*x]^(11/2)) - (2*a^2*Cos[c + d*x]^3)/(11*d*Sin[c + d*x]^(11/2)) - (4*a^
2)/(7*d*Sin[c + d*x]^(7/2)) + (16*a^2*Cos[c + d*x])/(77*d*Sin[c + d*x]^(7/
2)) - (4*a^2*Cos[c + d*x])/(231*d*Sin[c + d*x]^(3/2)))*Sqrt[Sin[c + d*x]])
/a^4
```

3.300.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.300.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(21(1-\cos(dx+c))^8\csc(dx+c)^8+16i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\right.}{1848a^2d\left((1-\cos($

input
```
int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/1848/a^2/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*
x+c)))^(5/2)*(1-cos(d*x+c))^2*(21*(1-cos(d*x+c))^8*csc(d*x+c)^8+16*I*(-I*(
I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/
2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+
c)))^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-6*(1-cos(d*x+c))^6*csc(d*
x+c)^6-136*(1-cos(d*x+c))^4*csc(d*x+c)^4-186*(1-cos(d*x+c))^2*csc(d*x+c)^2
-77)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/((1-cos(d*x+c))*((1-cos(d*x+c))^2
*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+
c)-cot(d*x+c))^(1/2)*csc(d*x+c)^2
```

3.300.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(\left(i\,e^2\cos(dx+c)^2+2i\,e^2\cos(dx+c)+i\,e^2\right)\sqrt{2i\,e}\sin(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+\right.}{}$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/231*((I*e^2*cos(d*x + c)^2 + 2*I*e^2*cos(d*x + c) + I*e^2)*sqrt(2*I*e)*
sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-
I*e^2*cos(d*x + c)^2 - 2*I*e^2*cos(d*x + c) - I*e^2)*sqrt(-2*I*e)*sin(d*x
+ c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (2*e^2*cos
(d*x + c)^3 + 4*e^2*cos(d*x + c)^2 + 47*e^2*cos(d*x + c) + 24*e^2)*sqrt(e/
sin(d*x + c)))/((a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(
d*x + c))
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.300.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.300.8 Giac [F]

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\csc(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\left(\frac{e}{\sin(c+dx)}\right)^{5/2}}{a^2(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e/sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

3.301.1 Optimal result 2276
3.301.2 Mathematica [C] (verified) 2277
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3.301.7 Maxima [F(-1)] 2281
3.301.8 Giac [F] 2282
3.301.9 Mupad [F(-1)] 2282

3.301.1 Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}\\ &+\frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}\\ &-\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}\\ &-\frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}+\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}\\ &-\frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{15a^2d}\end{aligned}$$

output

```
-4/15*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/45*e*cot(d*x+c)*csc(d*x+c
)*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)^3*csc(d*x+c)*(e*csc(d*x+c))^
(1/2)/a^2/d-4/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)
*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d+4/9*e*csc(d*x+c)^4*(e*csc(d*x+c))
^(1/2)/a^2/d+4/15*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/a^2/d
```

3.301.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \cos^4\left(\tfrac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\sec(c+dx)\left(\frac{16\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c-\right.\right.$$

input
```
Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*Sec[c + d*x]*((16*Sqrt[2]*E^(I*
(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^(
(2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x]
)/(d*(1 + E^((2*I)*c))*Csc[c + d*x]^(3/2)) - (2*(24*Cos[d*x]*Sec[c] + (8 +
 13*Cos[c + d*x])*Sec[(c + d*x)/2]^4)*Tan[c + d*x])/d))/(45*a^2*(1 + Sec[c
 + d*x])^2)
```

3.301.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow \text{4366}$$
$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)^2\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)}{(-\cos(c+dx)a-a)^2\sin^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{11}{2}}(c+dx)}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^{11/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}+\frac{a^2\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{4a^2}{5d\sin^{\frac{5}{2}}(c+dx)}+\frac{4a^2}{9d\sin^{\frac{9}{2}}(c+dx)}-\frac{4a^2E\left(\frac{1}{2}(c+dx-\frac{\pi}{2})|2\right)}{15d}-\frac{2a^2\cos^3(c+dx)}{9d\sin^{\frac{9}{2}}(c+dx)}+\frac{16a^2\cos(c+dx)}{45d\sin^{\frac{5}{2}}(c+dx)}\right.}{a^4}$$

input
```
Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e*Sqrt[e*Csc[c + d*x]]*((-4*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(15*d)
+ (4*a^2)/(9*d*Sin[c + d*x]^(9/2)) - (2*a^2*Cos[c + d*x])/(9*d*Sin[c + d*x
]^(9/2)) - (2*a^2*Cos[c + d*x]^3)/(9*d*Sin[c + d*x]^(9/2)) - (4*a^2)/(5*d*
Sin[c + d*x]^(5/2)) + (16*a^2*Cos[c + d*x])/(45*d*Sin[c + d*x]^(5/2)) - (4
*a^2*Cos[c + d*x])/(15*d*Sqrt[Sin[c + d*x]]))*Sqrt[Sin[c + d*x]])/a^4
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.301.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 859, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	859

input
```
int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/45/a^2/d*2^(1/2)*(12*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x
+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I
-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-6*(-I*(I-cot(d*x+
c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)
-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(
1/2))*cos(d*x+c)^3+36*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-18*(-I*(I-cot(d*x+
c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)
-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(
1/2))*cos(d*x+c)^2+36*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-18*(-I*(I-cot(d*x+c)
+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-c
sc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/
2))*cos(d*x+c)+12*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-c
sc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(
d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2
)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-6*2^(1/2)...
```

3.301.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.73

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(3\left(e\cos(dx+c)^2+2\,e\cos(dx+c)+e\right)\sqrt{2ie}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+\right.}{}$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/45*(3*(e*cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(2*I*e)*weierstrass
Zeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(
e*cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(-2*I*e)*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (6*e*cos(d*x
 + c)^3 + 12*e*cos(d*x + c)^2 + 19*e*cos(d*x + c) + 8*e)*sqrt(e/sin(d*x +
c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.301.6 Sympy [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x
)/a**2
```

3.301.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.301.8 Giac [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\left(\frac{e}{\sin(c+dx)}\right)^{3/2}}{a^2(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e/sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.302 $\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

3.302.1 Optimal result . . . 2283
3.302.2 Mathematica [A] (verified) . . . 2284
3.302.3 Rubi [A] (verified) . . . 2284
3.302.4 Maple [C] (verified) . . . 2286
3.302.5 Fricas [C] (verification not implemented) . . . 2287
3.302.6 Sympy [F] . . . 2287
3.302.7 Maxima [F(-1)] . . . 2288
3.302.8 Giac [F] . . . 2288
3.302.9 Mupad [F(-1)] . . . 2288

3.302.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad - \frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d} \\ &\quad - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad + \frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad + \frac{20\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d}\end{aligned}$$

output

```
16/21*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)^3*(e*csc(d*x+c)
)^(1/2)/a^2/d-4/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)*csc
(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d+4/7*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)
/a^2/d-20/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)/a^2/d
```

3.302.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}\left(2(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)+5\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),\right.\right.}{21a^2d}$$

input
```
Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-4*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]]*(2*(8 + 11*Cos[c + d*x])*Sin[(c +
d*x)/2]^4 + 5*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21
*a^2*d)
```

3.302.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 4366$$
$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)^2\sqrt{\sin(c+dx)}}dx$$
$$\downarrow 3042$$
$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int \frac{1}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4360$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)}{(-\cos(c+dx)a-a)^2\sqrt{\sin(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a\sin\left(c+dx-\frac{\pi}{2}\right)-a\right)^2}dx$$

$$\downarrow 3354$$

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{9}{2}}(c+dx)}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\frac{\sin(c+dx-\frac{\pi}{2})^2\left(\sin(c+dx-\frac{\pi}{2})a+a\right)^2}{\cos(c+dx-\frac{\pi}{2})^{9/2}}dx}{a^4}$$

$$\downarrow 3352$$

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\int\left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}+\frac{a^2\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(-\frac{4a^2}{3d\sin^{\frac{3}{2}}(c+dx)}+\frac{4a^2}{7d\sin^{\frac{7}{2}}(c+dx)}+\frac{20a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{21d}-\frac{2a^2\cos^3(c+dx)}{7d\sin^{\frac{7}{2}}(c+dx)}+\frac{16a^2\cos(c}{21d\sin^{\frac{3}{2}}(c}\right.}{a^4}$$

input
```
Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[e*Csc[c + d*x]]*((20*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*d) +
(4*a^2)/(7*d*Sin[c + d*x]^(7/2)) - (2*a^2*Cos[c + d*x])/(7*d*Sin[c + d*x]^
(7/2)) - (2*a^2*Cos[c + d*x]^3)/(7*d*Sin[c + d*x]^(7/2)) - (4*a^2)/(3*d*Si
n[c + d*x]^(3/2)) + (16*a^2*Cos[c + d*x])/(21*d*Sin[c + d*x]^(3/2)))*Sqrt[
Sin[c + d*x]])/a^4
```

3.302.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.302.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.06 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{2}\sqrt{\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}}(1-\cos(dx+c))\left(20i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{2}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\right.}{42a^2d\sqrt{(1-\cos(dx+c))\left((1-\cos(dx+c)\right.}}$

input
```
int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/42/a^2/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+
c)))^(1/2)*(1-cos(d*x+c))*(20*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/
2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)
*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(1-cos(d*x+
c))^5*csc(d*x+c)^5-16*(1-cos(d*x+c))^3*csc(d*x+c)^3-19*csc(d*x+c)+19*cot(d
*x+c))/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)
/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))^(1/2)*csc(d*x+c)
```

3.302.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(11\cos(dx+c)+8)\sin(dx+c)+5\left(i\cos(dx+c)^2+2i\cos(dx+c)+i\right)\sqrt{2i\,e}\text{weierstr}\right.}{}$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/21*(sqrt(e/sin(d*x + c))*(11*cos(d*x + c) + 8)*sin(d*x + c) + 5*(I*cos(
d*x + c)^2 + 2*I*cos(d*x + c) + I)*sqrt(2*I*e)*weierstrassPInverse(4, 0, c
os(d*x + c) + I*sin(d*x + c)) + 5*(-I*cos(d*x + c)^2 - 2*I*cos(d*x + c) -
I)*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.302.6 Sympy [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a
**2
```

3.302.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.302.8 Giac [F]

$$\int \frac{\sqrt{e \csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e \csc(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\sqrt{\frac{e}{\sin(c+dx)}}}{a^2(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e/sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.303 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$

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3.303.1 Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}$$
$$- \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
16/5*cot(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)^3/a^2/d/(e*csc(d
*x+c))^(1/2)-4*csc(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)*csc(d*
x+c)^2/a^2/d/(e*csc(d*x+c))^(1/2)+4/5*csc(d*x+c)^3/a^2/d/(e*csc(d*x+c))^(1
/2)-28/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/(e*csc(d*x+c))^(1/2)/sin(d
*x+c)^(1/2)
```

3.303.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}\sec^2(c+dx)\left(-\frac{28\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}\left(1+e^{2ic}\right)\sqrt{1-e^{2i(c+dx)}}\right.\right.}{1+e^{2ic}}}{15a^2d\sqrt{}}$$

input
```
Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
(4*Cos[(c + d*x)/2]^4*Sqrt[Csc[c + d*x]]*Sec[c + d*x]^2*((-28*Sqrt[2]*E^(I
*(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^
((2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/(1 + E^((2
*I)*c)) - 3*Sqrt[Csc[c + d*x]]*((-23 + 5*Cos[2*c])*Cos[d*x]*Sec[c] - 2*(-1
0 + Sec[(c + d*x)/2]^2 + 5*Sin[c]*Sin[d*x]))))/(15*a^2*d*Sqrt[e*Csc[c + d*
x]]*(1 + Sec[c + d*x])^2)
```

3.303.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2\sqrt{e\csc(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\sqrt{e\sec\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sqrt{\sin(c+dx)}}{(\sec(c+dx)a+a)^2}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{\cos(c+dx-\frac{\pi}{2})}}{(a-a\csc(c+dx-\frac{\pi}{2}))^2}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int \frac{\cos^2(c+dx)\sqrt{\sin(c+dx)}}{(-\cos(c+dx)a-a)^2}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cos(c+dx+\frac{\pi}{2})}\sin(c+dx+\frac{\pi}{2})^2}{(-\sin(c+dx+\frac{\pi}{2})a-a)^2}dx}{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3354$$

$$\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{7}{2}}(c+dx)}dx}{a^4\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin(c+dx-\frac{\pi}{2})^2(\sin(c+dx-\frac{\pi}{2})a+a)^2}{\cos(c+dx-\frac{\pi}{2})^{7/2}}dx}{a^4\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} + \frac{a^2\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{\frac{4a^2}{5d\sin^{\frac{5}{2}}(c+dx)} - \frac{4a^2}{d\sqrt{\sin(c+dx)}} + \frac{28a^2E(\frac{1}{2}(c+dx-\frac{\pi}{2})|2)}{5d} - \frac{2a^2\cos^3(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)} - \frac{2a^2\cos(c+dx)}{5d\sin^{\frac{5}{2}}(c+dx)} + \frac{16a^2\cos(c+dx)}{5d\sqrt{\sin(c+dx)}}}{a^4\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

3.303. $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}dx$

output
```
((28*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d) + (4*a^2)/(5*d*Sin[c + d*
x]^(5/2)) - (2*a^2*Cos[c + d*x])/(5*d*Sin[c + d*x]^(5/2)) - (2*a^2*Cos[c +
 d*x]^3)/(5*d*Sin[c + d*x]^(5/2)) - (4*a^2)/(d*Sqrt[Sin[c + d*x]]) + (16*a
^2*Cos[c + d*x])/(5*d*Sqrt[Sin[c + d*x]]))/(a^4*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]])
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.303.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.70 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.31

method	result
default	$-\frac{\sqrt{2}\left(28\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot$

input
```
int(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^2/d*2^(1/2)*(28*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c
)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^
(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)^2-14*(-I*(I+cot(d*x
+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c
)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^
(1/2))*cos(d*x+c)^2+56*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c
)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^
(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-28*(-I*(I+cot(d*x+c
)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-
csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1
/2))*cos(d*x+c)+28*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2
))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)-14*(-I*(I+cot(d*x+c)-csc(d*x+c)))^
(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1
/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))+5*2^(1/2)*
cos(d*x+c)^2+2^(1/2)*cos(d*x+c)-6*2^(1/2))/(cos(d*x+c)+1)/(e*csc(d*x+c))^(
1/2)*csc(d*x+c)
```

3.303.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{2\left(7\sqrt{2i\,e}(\cos(dx+c)+1)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(7*sqrt(2*I*e)*(cos(d*x + c) + 1)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7*sqrt(-2*I*e)*(cos(d*x + c)
 + 1)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin
(d*x + c))) + (9*cos(d*x + c)^2 - cos(d*x + c) - 8)*sqrt(e/sin(d*x + c)))/
(a^2*d*e*cos(d*x + c) + a^2*d*e)
```

3.303.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec^2(c+dx)+2\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*csc(c + d*x))*
sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a**2
```

3.303.7 Maxima [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

3.303.8 Giac [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\sqrt{\frac{e}{\sin(c+dx)}}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2)),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.304 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

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3.304.1 Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} + \frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

output

```
4/a^2/d/e/(e*csc(d*x+c))^(1/2)-4/3*cos(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)
-2/3*cos(d*x+c)*cot(d*x+c)^2/a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*cot(d*x+c)*c
sc(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)+4/3*csc(d*x+c)^2/a^2/d/e/(e*csc(d*x
+c))^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e/(e*csc(d*x+c))^(1/2)
/sin(d*x+c)^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.47

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12(1+\cos(c+dx))\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi -\right.\right.}{6a^2d(e\csc(c+}$$

input
```
Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Sec[(c + d*x)/2]^2*(12*(1 + Cos[c + d*x])*EllipticF[(-2*c + Pi - 2*d*x)/4
, 2] + (15 + 10*Cos[c + d*x] - Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]]))/(6*a
^2*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

3.304.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\csc(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4366$$

$$\frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{3/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int \frac{\cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)}{(-\cos(c+dx)a-a)^2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cos(c+dx+\frac{\pi}{2}))^{3/2}\sin(c+dx+\frac{\pi}{2})^2}{(-\sin(c+dx+\frac{\pi}{2})a-a)^2}dx}{e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3354$$

$$\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}dx}{a^4e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin(c+dx-\frac{\pi}{2})^2(\sin(c+dx-\frac{\pi}{2})a+a)^2}{\cos(c+dx-\frac{\pi}{2})^{5/2}}dx}{a^4e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}+\frac{a^2\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx}{a^4e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{\frac{4a^2}{3d\sin^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sqrt{\sin(c+dx)}}{d}-\frac{4a^2\,\text{EllipticF}(\frac{1}{2}(c+dx-\frac{\pi}{2}),2)}{d}-\frac{2a^2\cos^3(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}-\frac{2a^2\cos(c+dx)}{3d\sin^{\frac{3}{2}}(c+dx)}-\frac{4a^2\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}}{a^4e\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((-4*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/d + (4*a^2)/(3*d*Sin[c + d*x]^(
3/2)) - (2*a^2*Cos[c + d*x])/(3*d*Sin[c + d*x]^(3/2)) - (2*a^2*Cos[c + d*x
]^3)/(3*d*Sin[c + d*x]^(3/2)) + (4*a^2*Sqrt[Sin[c + d*x]])/d - (4*a^2*Cos[
c + d*x]*Sqrt[Sin[c + d*x]])/(3*d))/(a^4*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
 + d*x]])
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.304.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.99 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sqrt{2}\left(6i\sqrt{i(-i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(dx$

input
```
int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2/d*2^(1/2)*(6*I*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c
)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^
(1/2))*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*cos(d*x+c)^2+12*I*(I*(-I-cot(d
*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(d*x+c)+csc(d*x+c)
))^(1/2)*cos(d*x+c)+6*I*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+
c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2
^(1/2))*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)+cos(d*x+c)^2*2^(1/2)*sin(d*x+
c)-5*cos(d*x+c)*sin(d*x+c)*2^(1/2)-8*2^(1/2)*sin(d*x+c))/(cos(d*x+c)-1)/(c
os(d*x+c)+1)^2/e/(e*csc(d*x+c))^(1/2)*sin(d*x+c)
```

3.304.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.63

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(\left(\cos(dx+c)^2-5\cos(dx+c)-8\right)\sqrt{\frac{e}{\sin(dx+c)}}\sin(dx+c)+3\sqrt{2i\,e}(-i\cos(dx+c)-i)\text{weierstrassP}\right.}{3\,(a}$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*((cos(d*x + c)^2 - 5*cos(d*x + c) - 8)*sqrt(e/sin(d*x + c))*sin(d*x +
 c) + 3*sqrt(2*I*e)*(-I*cos(d*x + c) - I)*weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 3*sqrt(-2*I*e)*(I*cos(d*x + c) + I)*weierstrass
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a^2*d*e^2*cos(d*x + c) + a
^2*d*e^2)
```

3.304.6 Sympy [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec^2(c+dx)+2(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*csc(c + d*x))**
(3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2)), x)/a**2
```

3.304.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

3.304.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\left(\frac{e}{\sin(c+dx)}\right)^{3/2}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2)),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

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3.305.7 Maxima [F] . . . 2308
3.305.8 Giac [F] . . . 2308
3.305.9 Mupad [F(-1)] . . . 2309

3.305.1 Optimal result

Integrand size = 25, antiderivative size = 215

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}$$
$$+\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{44E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}} - \frac{12\cos(c+dx)\sin(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}}$$

output

```
-2*cot(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-2*cos(d*x+c)^2*cot(d*x+c)/a^2
/d/e^2/(e*csc(d*x+c))^(1/2)+4*csc(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)+4/
3*sin(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-12/5*cos(d*x+c)*sin(d*x+c)/a^2
/d/e^2/(e*csc(d*x+c))^(1/2)+44/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^
2/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.82 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{-123\cot(c+dx)+88\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\text{ Hyperg}}{}$$

input
```
Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-123*Cot[c + d*x] + 88*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*H
ypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))] + Csc[c + d*x]*(140 -
 20*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)] - (264*I)*Sin[c + d*x]))/(30*a^2
*d*e^2*Sqrt[e*Csc[c + d*x]])
```

3.305.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\csc(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4366$$
$$\frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{5/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 4360$$

$$\frac{\int \frac{\cos^2(c+dx)\sin^{\frac{5}{2}}(c+dx)}{(-\cos(c+dx)a-a)^2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cos(c+dx+\frac{\pi}{2}))^{5/2}\sin(c+dx+\frac{\pi}{2})^2}{(-\sin(c+dx+\frac{\pi}{2})a-a)^2}dx}{e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3354$$

$$\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^4e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin(c+dx-\frac{\pi}{2})^2(\sin(c+dx-\frac{\pi}{2})a+a)^2}{\cos(c+dx-\frac{\pi}{2})^{3/2}}dx}{a^4e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\frac{a^2\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} + \frac{a^2\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx}{a^4e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{\frac{4a^2\sin^{\frac{3}{2}}(c+dx)}{3d} + \frac{4a^2}{d\sqrt{\sin(c+dx)}} - \frac{44a^2E(\frac{1}{2}(c+dx-\frac{\pi}{2})|2)}{5d} - \frac{2a^2\cos^3(c+dx)}{d\sqrt{\sin(c+dx)}} - \frac{12a^2\sin^{\frac{3}{2}}(c+dx)\cos(c+dx)}{5d} - \frac{2a^2\cos(c+dx)}{d\sqrt{\sin(c+dx)}}}{a^4e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((-44*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d) + (4*a^2)/(d*Sqrt[Sin[c
+ d*x]]) - (2*a^2*Cos[c + d*x])/(d*Sqrt[Sin[c + d*x]]) - (2*a^2*Cos[c + d*
x]^3)/(d*Sqrt[Sin[c + d*x]]) + (4*a^2*Sin[c + d*x]^(3/2))/(3*d) - (12*a^2*
Cos[c + d*x]*Sin[c + d*x]^(3/2))/(5*d))/(a^4*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.305.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.86 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.15

method	result
default	$\frac{\sqrt{2}\left(132\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(d}$

input
```
int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/15/a^2/d*2^(1/2)*(132*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+
c)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2
^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-66*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)
-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(
1/2))*cos(d*x+c)+3*cos(d*x+c)^3*2^(1/2)+132*(-I*(I+cot(d*x+c)-csc(d*x+c)))
^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(
d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)-66*(-I*(I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+c
ot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2
),1/2*2^(1/2))-10*2^(1/2)*cos(d*x+c)^2+33*2^(1/2)*cos(d*x+c)-26*2^(1/2))/(
e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)
```

3.305.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\left(3\cos(dx+c)^3 - 10\cos(dx+c)^2 - 33\cos(dx+c) + 40\right.\right.}{}$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/15*((3*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 33*cos(d*x + c) + 40)*sqrt(e
/sin(d*x + c)) - 33*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 33*sqrt(-2*I*e)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*e^3)
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.305.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.305.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\left(\frac{e}{\sin(c+dx)}\right)^{5/2}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2)),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

3.306.1 Optimal result . . . 2310
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3.306.8 Giac [F] . . . 2315
3.306.9 Mupad [F(-1)] . . . 2315

3.306.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{52\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}$$

output

```
-4/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+26/21*cos(d*x+c)/a^2/d/e^3/(e*csc(d*x+c)
)^(1/2)+2/7*cos(d*x+c)^3/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+4/5*sin(d*x+c)^2/a
^2/d/e^3/(e*csc(d*x+c))^(1/2)-52/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d
/e^3/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{e\csc(c+dx)}\left(-520\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(-7\right.$$

input
```
Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Sqrt[e*Csc[c + d*x]]*(-520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-756 +
305*Cos[c + d*x] - 84*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt[Sin[c +
 d*x]])*Sqrt[Sin[c + d*x]])/(210*a^2*d*e^4)
```

3.306.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4366, 3042, 4360, 3042, 3354, 3042, 3352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\csc(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2\left(e\sec\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$
$$\downarrow 4366$$
$$\frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)^{7/2}}{\left(a-a\csc\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$
$$\downarrow 4360$$
$$\frac{\int \frac{\cos^2(c+dx)\sin^{\frac{7}{2}}(c+dx)}{(-\cos(c+dx)a-a)^2}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{\left(-\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(-\sin\left(c+dx+\frac{\pi}{2}\right)a-a\right)^2}dx}{e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3354$$

$$\frac{\int \frac{\cos^2(c+dx)(a-a\cos(c+dx))^2}{\sqrt{\sin(c+dx)}}dx}{a^4e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2\left(\sin\left(c+dx-\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}}dx}{a^4e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 3352$$

$$\frac{\int \left(\frac{a^2\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{\sin(c+dx)}}+\frac{a^2\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx}{a^4e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{\frac{4a^2\sin^{\frac{5}{2}}(c+dx)}{5d}-\frac{4a^2\sqrt{\sin(c+dx)}}{d}+\frac{52a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\frac{\pi}{2}),2\right)}{21d}+\frac{2a^2\sqrt{\sin(c+dx)}\cos^3(c+dx)}{7d}+\frac{26a^2\sqrt{\sin(c+dx)}\cos(c+dx)}{21d}}{a^4e^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

input
```
Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((52*a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*d) - (4*a^2*Sqrt[Sin[c + d*
x]])/d + (26*a^2*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/(21*d) + (2*a^2*Cos[c +
d*x]^3*Sqrt[Sin[c + d*x]])/(7*d) + (4*a^2*Sin[c + d*x]^(5/2))/(5*d))/(a^4*
e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3352
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3354
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*
m)   Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x]
)^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] &&
ILtQ[m, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

rule 4366
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Simp[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p]   Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /
; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

3.306.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.86 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{2}\left(130i\sqrt{i(-i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\right.\right.$

input
```
int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/105/a^2/d*2^(1/2)*(130*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-I-cot
(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-
I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+130*I*(-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(
d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1
/2*2^(1/2))+15*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-42*cos(d*x+c)^2*2^(1/2)*sin
(d*x+c)+65*cos(d*x+c)*sin(d*x+c)*2^(1/2)-168*2^(1/2)*sin(d*x+c))/e^3/(e*cs
c(d*x+c))^(1/2)/(cos(d*x+c)-1)^2/(cos(d*x+c)+1)^2*sin(d*x+c)^3
```

3.306.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.65

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\left(15\cos(dx+c)^3 - 42\cos(dx+c)^2 + 65\cos(dx+c) - 1\right.\right.}{}$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/105*((15*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 65*cos(d*x + c) - 168)*sqr
t(e/sin(d*x + c))*sin(d*x + c) - 65*I*sqrt(2*I*e)*weierstrassPInverse(4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 65*I*sqrt(-2*I*e)*weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c)))/(a^2*d*e^4)
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.306.7 Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

3.306.8 Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\left(\frac{e}{\sin(c+dx)}\right)^{7/2}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(7/2)),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 2316

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,`^`) then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,`+`) or type(expn,`*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```